AIRPLANE

STRESS

ANALYSIS

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AIRPLANE STRESS ANALYSIS

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PART ONE

ASSIGNMENT 1

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AIRPLANE STRESS ANALYSIS

Introduction

A knowledge of structural design methods is essential to advancement in nearly all phases of aircraft engineering. Aside from the fact that approximately ten percent of the engineering hours spent on an airplane design are chargeable directly to the obtaining of design loads in members and the strength checking of drawings of the finished parts and assemblies, no aircraft engineer can expect to go far in advanced drafting, layout work, and design without a working knowledge of stress analysis. Thus, whether a man expects to go into the actual analysis work or to engage in some other type of aircraft engineering he will find a good working knowledge of stress analysis to be of great value to him.

The type of analysis which is used in aircraft work is very different in many respects from that used in other types of engineering. The reason for this condition lies in the fact that weight saving must be paramount if an efficient airplane, that is one which carries a high percentage of payload, is to be obtained.

It is necessary therefore that structural analysis and design in aircraft be very precise, and very accurate. It is usual to design aircraft structures so that they are just ready to yield at their maximum loads and to allow a 50 per cent extra margin of strength before they break. Such close design means that the analysis must not only be very thorough and very accurate but that all of the possible effects of deflection of the structure under load must be taken into account.

The volume of analysis necessary to produce an aircraft structure is far greater in proportion to the total engineering work than that which is required in other types of engineering. It can not be reduced to completely standard form, such as is possible in many types of bridge engineering for example, because in the constant seeking after more perfect designs the types of construction and even the materials used change rapidly. For this reason airplane stress analysis must be based on fundamental principles and can never become rule of thumb. The purpose of the Basic Analysis Course is to furnish the student with those fundamental principles so that no matter what type of new problem may confront him, he will be able to develop a method for its solution.

It is common knowledge that very few colleges graduate men, even in the specialized aviation courses, who are ready for aircraft analysis work without a training period. This condition arises from the fact that the professors and the writers of text books find it impossible to keep up with new trends in the industry. The only way a man can train himself to step directly into the industry is to complete "stream-lined" courses such as those given in "Basic and Advanced Stress Analysis", courses which are written and revised each year by men in key positions in the active aviation industry.

After the general features of an airplane design are decided upon the first step is to lay out a structure which will accommodate those features and form a skeleton on which to hang the necessary installations. Then the loading conditions which will cause the highest loads in the structure are determined and a preliminary analysis is made to find the effect of these loads. This preliminary analysis is necessary in order to determine approximate sizes, since aircraft installations are as compact as possible and clearance allowances are low. After the final design features are decided upon and all installations, such as those for power plant, electrical system, control systems and furnishings are placed it is possible to estimate the final dead weight and its distribution and to proceed with the final analysis.

The Basic Course does not carry through a complete analysis of an airplane. It completes the design of representative components and furnishes the tools for detail design after the loads are determined. The method of determining the loads from aerodynamic data and of carrying through an analysis for the loads in the members is given as an appendix to the Basic Course, and is covered completely for monocoque structures in the Advanced Analysis Course.

The student completing the Basic course will be thoroughly familiar with the requirements for routine strength checking and will have the knowledge necessary to design welded tubular trusses, continuous beams, fuselage floor beams, control surface beams continuous over supports, struts, drag trusses and welded and riveted fittings. He will also be able to calculate the deflections of different components. Limiting allowable deflections are often the controlling feature of a structural design. The Basic Course also covers the introduction to indeterminate structures by giving the student the basic theory of Least Work in direct stress, bending and torsion as applied to aircraft problems.

A knowledge of high school algebra and geometry is essential to success in the Basic Course. Trigonometry, analytic geometry and the calculus are also very useful. The course contains enough of the calculus to clarify its application wherever it is used. The student whose mathematics is not well grounded will find it advisable to secure texts written especially for home study, some of which are referenced in the course. He will find it advantageous to study his mathematics in conjunction with the Basic Course which will give him practical applications which will long be remembered.

Throughout the course every effort has been made to put concrete examples and solutions before the student before he is requested to solve similar problems. All steps in the solutions have been given, so that the student may follow the examples through item by item. It is advised strongly that the student follow the form of the examples in outlining his own work, since the forms given have been found more conducive to accurate work than any others. At every hand the student is given alternate methods for solving the same problem so that one method may be used as a check of the result obtained by another.

The serious student who masters all of the work given in the Basic stress Analysis Course and who conscientiously works all of the problems will find himself well on the way to a thorough preparation for success in the field of Aviation Engineering.

BASIC STRESS ANALYSIS

Nomenclature.

The following terms, which are used throughout the course, are grouped for ready reference. The student will find more extensive explanations when the terms appear in the assignments. The symbols and the sign conventions used are those which are standard for the aviation industry and in some cases do not agree with usage in other types of engineering. The student should, however, adhere strictly to the terms and sign conventions given, in order to form good habits which will smooth his path to advancement in the aviation field.

In addition to correct usage of signs and symbols the student should form the habit of using explanatory notes in his computations. This not only clarifies his work for usage by a designer or a checker, but makes review of the subject matter much easier for him after the work has become "cold".

Definitions of Symbols.

L = a load.

R = a reaction, the effect at a support.

w = distributed load per inch run.

W = total load or resultant load.

P = End Load Compression in an axially loaded member.

T = End Load tension in an axially loaded member.

S = shear at any point in a beam

or S = side load on a structure.

V = a vertical load.

H = a fore and aft load.

M = a bending moment at a section in a beam.

M_1 = a bending moment at joint (1) in a continuous beam.

M_2 = a bending moment at joint (2) in a continuous beam, etc.

A = area of a cross section of a member of a structure.

A_n = net area of a cross-section of a structure = total area less area of bolt or rivet holes.

Q = Statical Moment of a beam cross-section
= first moment of the areas of the cross section about the neutral axis

I = Moment of Inertia of a cross-section
= second moment of the areas of the cross-section about the neutral axis.

I_p = Polar Moment of Inertia.
= radius of gyration = $(I/A)^{\frac{1}{2}}$

****

Stress = internal load in a member which lengthens or shortens its fibers.

f_t = unit stress in tension = $\dfrac{\text{Axial tension load}}{\text{net area}}$

F_t = allowable unit stress in tension.

f_c = unit stress in compression = $\dfrac{\text{axial compression}}{\text{Gross area}}$

F_c = allowable unit stress in compression.

f_b = unit stress in bending = $\frac{My}{I}$ where y = distance from neutral axis to fiber being considered.

F_b = allowable unit stress in bending.

f_s = unit stress in shear.

F_s = allowable unit stress in shear.

E = Modulus of Elasticity = $\dfrac{\text{unit stress}}{\text{unit deformation}} = \dfrac{\text{lbs. per. sq. in.}}{\text{inches per inch}}$

G = Modulus of Rigidity = Shear Modulus = $\dfrac{\text{Unit shear stress}}{\text{unit shear deformation}}$

ε = unit deformation in inches per inch.

δ = total deflection in inches.

θ = an angle of the inclination between the deflected and undeflected axis of a beam.

Σ = summation of = a total of.

Δx = a small finite (measurable) segment of length along the x-x axis.

dx = an infinitesimally small segment of length along the x-x axis.

Δy = a small finite segment of length along the y-y axis.

dy = an infinitesimally small segment of length along the y-y axis.

Δs = a small finite distance along a line $= \sqrt{\Delta x^2 + \Delta y^2}$

ds = an infinitesimally small distance along a line $= \sqrt{dx^2 + dy^2}$

ΔA = a small finite portion of area of a cross-section $= (\Delta x)\ (\Delta y)$

dA = an infinitesimally small portion of area of a cross-section = $(dx)\ (dy)$

$d\Theta$ = an infinitesimally small increment of angle.

Definition of Signs.

Loads on an Airplane:

Upward acting loads are plus (+) loads.
Downward acting loads are minus (-) loads.
Backward acting loads are plus (+) loads.
Forward acting loads are minus (-) loads.
Outward acting loads are plus (+) loads.
Inward acting loads are minus (-) loads.

Shears and Moments:

For a free body of a structure

Plus (+) shear means part to the left of cut section tends to go up.

Plus (+) moment means tension on bottom fibers of the beam.

Axial Loads:

Tension in a member is plus (+) axial load.
Compression in a member is minus (-) axial load.
Note: Tension lengthens a member.
Compression shortens a member.

Definitions of Terms Used.

Structure--a device for transferring load.

Truss--a structure made up of a series of members forming triangles in which loads are carried axially along the members.

Beam--a structure which transfers loads at right angles to its longitudinal axis, by bending.

Reaction--a force, acting on a free-body of a structure, which replaces the effect of a support point.

External force--an imposed load on a structure or a member of a structure.

Internal force (or stress)--load stored in a structure due to deformation of its fibers as it deflects in supporting external forces.

Axial Load--a load acting along the longitudinal axis of a member.

Axial Tension Load--a load along the axis of a member tending to increase the length of the member.

Axial Compression Load--a load along the axis of a member tending to decrease its length.

Static Load--a dead weight load.

Dynamic Load--a load involving motion or impact.

Static Equilibrium--a state of balance of all forces acting on a structure of rest.

Column or Strut--a member designed to carry axial compression loads.

Coefficient of End Fixity--a term defining the condition at the end supports of a strut or beam.

Modulus of Rupture--the unit stress, from the beam formula, at which actual rupture of the outer fibers of a beam will occur.

Form Factor--a coefficient of reduction for local crippling by which the allowable theoretical unit stress in a column or on the compression flange of a beam is multiplied to obtain true allowable values.

Note: Both the Modulus of Rupture and the Form Factor are based on tests.

Determinate Structure--one which may be solved by the equations of equilibrium without recourse to calculations of deformation.

Indeterminate Structure--one which has extra or redundant members, and which must be solved by applying the principle of "consistent deformations".

Unstable Structure--one which does not contain enough members to keep it from collapsing under load.

Redundant member--an extra member not necessary to make a complete structure.

Force diagram--a plot of forces to correct direction and to scale.

Scale diagram--a plot of members to correct directions and to scale.

Phantom member--an imaginary member used to rplace several others in effect, or used to replace auxiliary members which are not actually a part of the structure, such as an engine.

Beam Restraint--the resistance the supports offer to rotation of a beam at its ends as it deflects under loads.

Simple Beam--one having only two supports and free to rotate at the support points.

Continuous Beam--one continuous over three or more supports.

Beam Load Curve--a diagram of the loads on a beam, showing their magnitudes and locations.

Beam Shear Curve--a diagram of the shear force acting at a cut face of a beam at any station along its length.

Beam Moment Curve--a diagram of the bending moment acting on any cut face of a beam at any station along its length.

Beam Inclination Curve--a diagram of the angle in radians between the unbent axis of a beam and the axis of the beam after bending, at any station along its length.

Note: 2π radians $=$ 360 degrees or
1 radian $=$ 57.3 degrees

Beam Deflection Curve--a diagram of the deflection of the axis of a beam , at any station along its length.

Neutral Axis--the location of the longitudinal strata of fibers in a bent beam at which neither shortening or lengthening takes place.

Primary Moment--the bending moment due to loads acting at right angles to the longitudinal axis of a beam.

Secondary Moment--the bending moment introduced when a beam deflects, when axial loads are present.

Combined Unit Stress--the combined effects of axial and bending loads reduced to lbs. per square inch on a section of beam fibers.

Inflection Point--a point of zero bending moment in a beam.

GENERAL REFERENCE

Review of the Basic Principles of Mathematics.

The purpose of the following chapter is to furnish a review of those basic principles of mathematics which are of every day use in an engineering office. No attempt is made to cover complete theory of the subject, but a common sense approach will be used. This review will therefore be equally useful to the student to whom parts of the subject matter are entirely new, and to the student who has covered the ground but is "rusty" and feels the need of a brief review.

Algebra.

The use of algebra for the solutions of expressions of equality which are not solvable by simple arithmetic is commonplace in any engineering work. In mathematics a statement of equality is termed an equation. Certain facts about an equation which form the basis for algebraic solutions are stated below.

An equation may be expressed in the general form $A+B = C+D$ which could mean in actual numbers say

$$100 + 50 = 80 + 70$$

Suppose we knew the actual numbers which represented all the symbols except A. We could solve the equation for A as follows:

$$\text{(a)}\quad A + 50 = 80 + 70$$
$$\text{(b)}\quad A = 80 + 70 - 50 = 100$$

We have illustrated the fact that we can subtract the same number from each side of an equation without changing the equality, for we could consider that we have subtracted 50 from each side of the original equation, (a), to obtain equation (b).

It is only common sense to say that we may perform the following operations on an equation without disturbing the balance between the two sides.

(1) We may add the same number (or expression) to each side of an equation without disturbing the balance.

$$\text{If } A + B = C + D \quad \text{then also}$$
$$A + B + N = C + D + N.$$

(2) We may subtract the same number from both sides of an equation without changing the equality

$$A + B - N = C + D - N$$

(3) We may multiply both sides of an equation by the same number without changing the equality. If $A + B = C + D$ then also

$5(A + B) = 5(C + D)$ or using our original values for A, B, C, and D

$$5(100 + 50) = 5(80 + 70) \text{ which reduces to}$$

$$750 = 750 \text{ which is of course true.}$$

(4) We may divide both sides of an equation by the same number whithout changing the equality.

$\frac{A + B}{N} = \frac{C + D}{N}$ and if we make N equal to 3 and the other values are the same as in previous equations

$$\frac{100 + 50}{3} = \frac{80 + 70}{3} \text{ or } 50 = 50$$

The equation above could have been expressed also as

$\frac{A}{N} + \frac{B}{N} = \frac{C}{N} + \frac{D}{N}$ and the final result would have been the same. This introduces the axion: "To divide (or multiply) any expression made up of the sum or difference of a number of terms by a given number (or expression), divide (or multiply) each term of the terms by that expression."

This is not true of the case of an expression made up of the product of several terms. In such a case only one member of the product would be divided (or multiplied) to affect the whole product.

For example if the value A in our illustrative equations were actually made up of the product of three separate parts such that $A = xyz$ (read A equals x times y times z) to divide or multiply A by any given number would involve that operation on only one of the expressions x, y or z. Say that A remains 100, and that x is 2, y is 5, and z is 10. Then in numbers the above equation could be written

$$100 = (2)(5)(10)$$

To multiply both sides of this equation by any number, say 4

$$(4)(100) = (4)(2) \times (5)(10)$$
$$\text{or } (4)(100) = 2 \times (4)(5) \times (10)$$
$$\text{or } (4)(100) = (2)(5) \times (4)(10)$$

Any of the three equations gives the same result, namely that

$$400 = 400.$$

The foregoing statements are based on sound reasoning. The student can convince himself of the validity of any of the statements merely by substituting actual numbers in any equation of his own choosing.

Signs:

We must bear in mind that to express a number in algebra we must indicate its sign. Not 3, but +3 or -3. (Read plus 3 or minus 3). Also that the product of numbers with like signs is plus and with unlike signs is minus thus:

$$(+2)(-2) = -4, \quad (-2)(-3) = +6, \text{ or } (-2)\ (+3)\ (-4) = +24$$

Powers and Exponents:

To raise a number (or a letter representing a number or a group of numbers) to a power means to use that number as a multiplier the number of times indicated by an exponent written alongside the number. Thus 2^2 means $(2)(2)$ or x^2 means $(x)(x)$. In the latter expression 2 is the exponent of x. In expressing physical facts in engineering formulas it oftens happens that expressions appear with exponents of 2, 3, 4 or more. These expressions are referred to as second, third, or fourth power expressions. For purposes of illustration we will take a general expression of the form x + 1 and raise it to various powers.

$$(x+1)^1 = x+1$$

$$(x+1)^2 = (x+1)(x+1) = x^2+2x+1 \text{ obtained thus}$$

$$\begin{array}{rl} & x+1 \\ \text{times} & \underline{x+1} \\ & x^2+x \\ & \underline{\quad +x+1} \\ & x^2+2x+1 \end{array}$$

which is the general form of the quadratic equation.

$$(x+1)^3 = (x+1)(x+1)(x+1) = x^3+3x^2+3x+1$$

Obtained thus, using expression already obtained for $(x+1)^2$

$$\begin{array}{rl} & x^2+2x+1 \\ \text{times} & \underline{\qquad\quad x+1} \\ & x^3+2x^2+x \\ & \underline{\quad +x^2+2x+1} \\ & x^3+3x^2+3x+1 \end{array}$$

which is the general form of the cubic equation.

The cases which involve fractional powers are sometimes confusing. For example $x^{\frac{1}{2}}$, (read x to the one-half power), means the square root of x. To perform the operation signified by a fractional exponent raise the number to the power shown by the numerator of the fraction and from this value extract the root shown by the denominator of the fraction.

In the above example $x^1 = x$, and extracting the root indicated by the denominator gives $\sqrt[2]{x}$.

$$x^{3/2} \text{ means } \sqrt[2]{x^3}$$

$$x^{4/3} \text{ means } \sqrt[3]{x^4}, \text{ etc.}$$

In engineering formulas exponents do not always appear as even fractions. Powers and roots of uneven fractions are handled by the use of Logarithms.

Logarithms.

The Logarithm of a number is the power to which a given base or index number must be raised to give that number. In our engineering system Logarithms are to the base 10. Therefore the logarithm of any number is the power to which 10 must be raised to give that number. Thus the logarithm of 10 is 1.0, the logarithm of 100 is 2.0, that of 1000 is 3.0, etc. Numbers which do not come as even multiples of 10 will have decimal logarithms. These values have been calculated and grouped for reference in Logarithm tables.

Logarithms are useful in performing multiplication, division, in raising numbers to powers, and in extracting roots of numbers. They are not used in addition or subtraction. The principles of the applications of logarithms are as follows:

1. The logarithm of a product is equal to the sum of the logarithms of the factors. That is to multiply, add logarithms.

2. The logarithm of a fraction is equal to the difference obtained by subtracting the logarithm of the denominator from the logarithm of the numerator. That is to divide, subtract logarithms.

3. The logarithm of a power of a number is equal to the logarithm of the number multiplied by the exponent of the power. Thus the logarithm of $10^2 = (2)$ (logarithm of 10). $(2)(1) = 2.0$

4. The logarithm of a root of a number is equal to the logarithm of the number divided by the index of the root. Thus the logarithm of $\sqrt[2]{100} = (\text{Logarithm of } 100)/2 = 2/2 = 1.0$.

We have seen that the logarithm of 10 is 1.0 of 100 is 2.0, etc. Then any number between 10 and 100 will have a logarithm of 1 + a decimal, every number between 100 and 1000 will have a logarithm of 2 + a decimal, etc. It is plain that every number not an even power of 10 will have a logarithm consisting of a whole number and a fraction. The whole number is called the characteristic, and the decimal is the mantissa.

The characteristic determines the position of the decimal point in the number for which the logarithm stands. The mantissa determines the digits in the number itself. Thus 5, 50, 500 and 5000 all have the same mantissa but the characteristics are 0, 1, 2, and 3 respectively. The mantissa in this case is .69897. The following tables will help to keep the value of the characteristic clear.

Numbers above 1

Number	1	10.	100.	1,000	10,000	100,000	1,000,000
Characteristic	0	1	2	3	4	5	6

Decimals

Number	1	.1	.01	.001	.0001	.00001	.000001
Characteristic	0	-1	-2	-3	-4	-5	-6

The rules are as follows:

1. The characteristic of any number greater than 1 is 1 less than the number of digits before the decimal point.
2. The characteristic of any number less than 1 is found by subtracting from 9 the number of ciphers between the decimal point and the first significant digit, and writing -10 after the result.

The use of logarithms will be illustrated in an example involving an uneven power of a decimal.

To take (.25) to the 3/2 power

$$\text{Log } .25 = 9.39794 - 10 = -.60206$$

$$\text{Log } (.25)^{3/2} = (-.60206)(3/2) = -.90309 = 9.09691 - 10$$

$$= \text{Log. } \boxed{.125}$$ = ANSWER IS DECIMAL

We have seen that equations are made up of variables which are expressed by letters and which may have different values, and of constants which are expressed as numbers. In engineering they express a statement of fact which may cover one individual case but in general may represent a whole series of possible cases, which may be solved for by varying the expressions for the variable. For example suppose we drop a body from rest and let it fall freely under the acceleration of gravity. We know from actual measurement that this acceleration is about 32 ft. per second, per second, and that it acts steadily; that is for every second of fall the actual velocity will increase by 32 ft. per second. Then if the body fell for one second its average velocity during that second would have been 16 feet per second (the average of 0 at the start and 32 at the end of the second) and the distance it fell would equal the average velocity times the time elapsed, or 16 feet. Mathematically if we call

a = acceleration
v = velocity
t = time

we can say that the following relations hold. The velocity at any point equals the acceleration times the time elapsed from the start of the fall or $v = a\,t$

The distance fallen for any elapsed time is equal to the average velocity in feet per second for the time fallen multiplied by the time fallen expressed in seconds. If acceleration (increase in velocity) is constant then the average velocity is one-half the final velocity, from which the space, "s", traversed can be expressed as

$$\frac{v}{2} \times t, \text{ but since } v = a\,t \text{ then}$$

$$s = \frac{a\,t}{2} \times t = \tfrac{1}{2}\,a\,t^2.$$

The advantages of such an expression are that we can find the space traversed at any particular time without summing up from the beginning. For example if $t = 1$

$$s = (\tfrac{1}{2})(32)(1)^2 = 16 \text{ ft.}$$

if $t = 2$

$$s = (\tfrac{1}{2})(32)(2)^2 = 64 \text{ ft.} \qquad \text{etc.}$$

In the above expression t is the variable and all other terms are constants.

Another direct application of the simple equation for space travelled is that if we know the space, (height of drop), we can calculate from our formula the velocity at impact or vice-versa. This application is common in aircraft since it forms a basis for landing gear shock absorber design. Suppose for example that one wished to test in free fall a landing gear for an airplane that was calculated to have a maximum sinking speed of 12 feet per second. This would represent the velocity at impact and the height required for the free drop, "s", would be found as follows:

v = impact velocity = 12 ft. per second
a = acceleration of gravity = 32 ft. per sec. per sec. approximately.

The height of free drop, s, is required.

We know that $s = \frac{1}{2}\,a\,t^2$, or $t^2 = \frac{2s}{a}$ and that $v = a\,t$

Then $v^2 = a^2t^2 = a^2\left(\frac{2s}{a}\right) = 2as$

from which $s = v^2/2a = (12)^2/(2)(32) = 2.25$ ft. or 27 inches.

A free drop of 27 inches would therefore produce an impact velocity of 12 ft. per second. The time of fall could be calculated from the relation

$$t^2 = \frac{2s}{a} \quad \text{from which } t^2 = \frac{(2)\ (2.25)}{32} = 0.141 \text{ seconds.}$$

The foregoing example illustrates a basic point in setting up engineering equations; namely, Units must be consistent if equality is to hold. All units of space, weight, or time must be used in the same units. Seconds do not equate with minutes, pounds with tons, or inches with feet.

Trigonometry.

That portion of trigonometry which covers 90 per cent of the uses in an engineering office may be stated very simply by referring for basic formulas to a circle whose radius is unity.

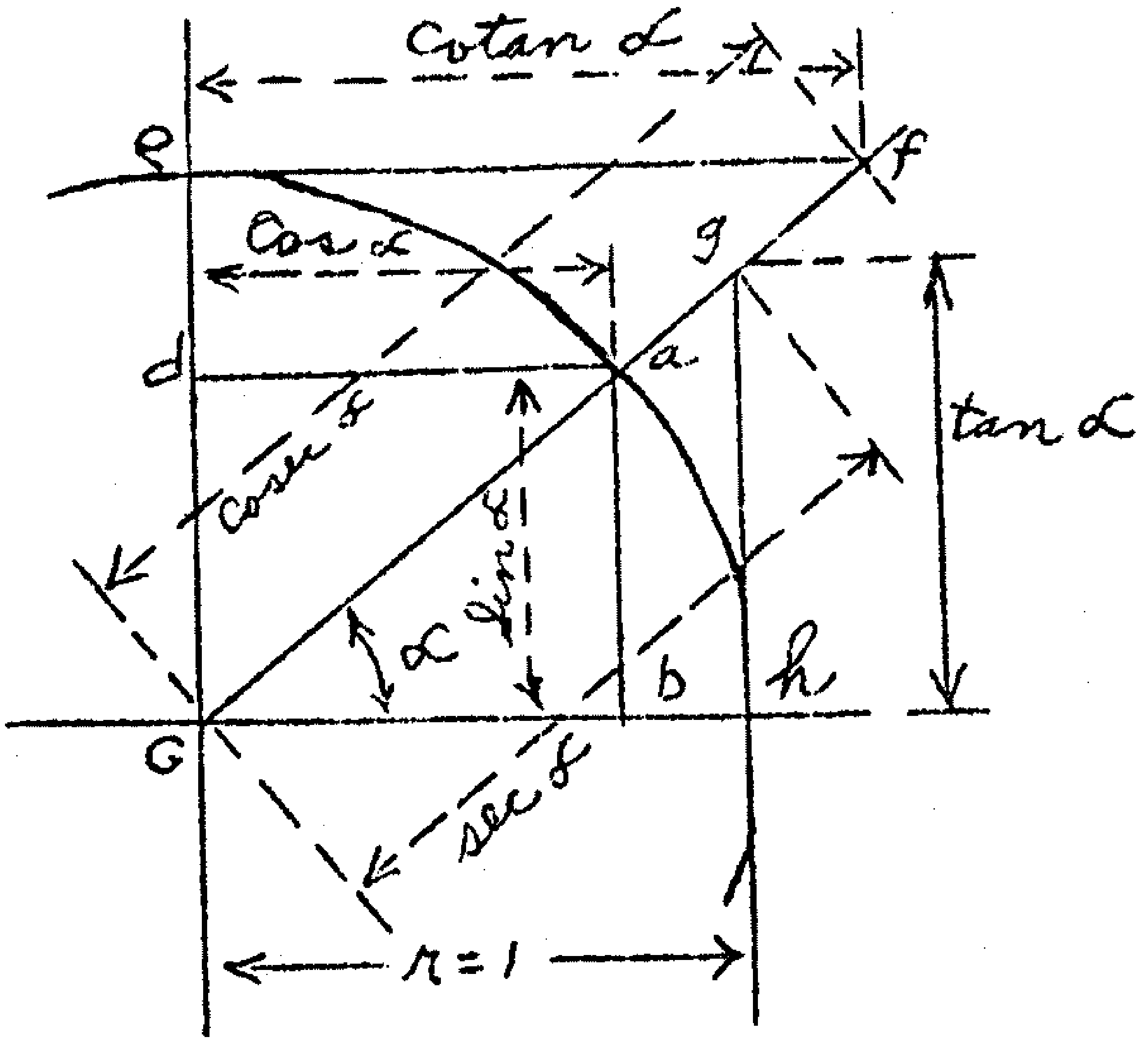


Fig. 1

The functions are diagrammed at the left. Fig. 1.

α = any angle between 0° and 90°

radius $= 1 = \sin^2 + \cos^2$

sine $\alpha = \frac{ab}{r} = \frac{ab}{1}$

cosine $\alpha = \frac{ad}{r} = \frac{ad}{1}$

tangent $\alpha = \frac{gh}{r} = \frac{gh}{1}$

The above are the basic formulas. From them, the following additional functions may be derived:

$$\text{Cotangent } \alpha = \frac{1}{\text{tangent } \alpha}$$

$$\text{Secant } \alpha = \frac{1}{\text{cosine } \alpha}$$

$$\text{Cosecant } \alpha = \frac{1}{\text{sine } \alpha}$$

These functions are also plotted on the diagram. From the basic functions different combinations may be evolved as follows:

$$\text{sine}\,\alpha = \frac{\cos\alpha}{\cot\alpha} = \frac{1}{\text{cosec}\,\alpha} = \cos\alpha\ \tan\ \alpha = (1 - \cos^2\alpha)^{\frac{1}{2}}$$

$$\text{cosine}\,\alpha = \frac{\sin\alpha}{\tan\alpha} = \frac{1}{\sec\alpha} = \sin\alpha\ \ \cot\alpha = (1 - \cos^2\alpha)^{\frac{1}{2}}$$

$$\text{tangent } \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{1}{\cot \alpha} = \sin \alpha \ \sec \alpha$$

$$\text{cotangent } \alpha = \frac{\cos \alpha}{\sin \alpha} = \cos \alpha \ \operatorname{cosec} \alpha$$

$$\text{secant } \alpha = \frac{\tan \alpha}{\sin \alpha}$$

$$\text{cosecant } \alpha = \frac{\cot \alpha}{\cos \alpha}$$

Many other forms of the combinations may be developed. In general they are of academic interest only. The signs to be used with the functions may be coordinated with our standard system of x and y coordinates. Fig. 2

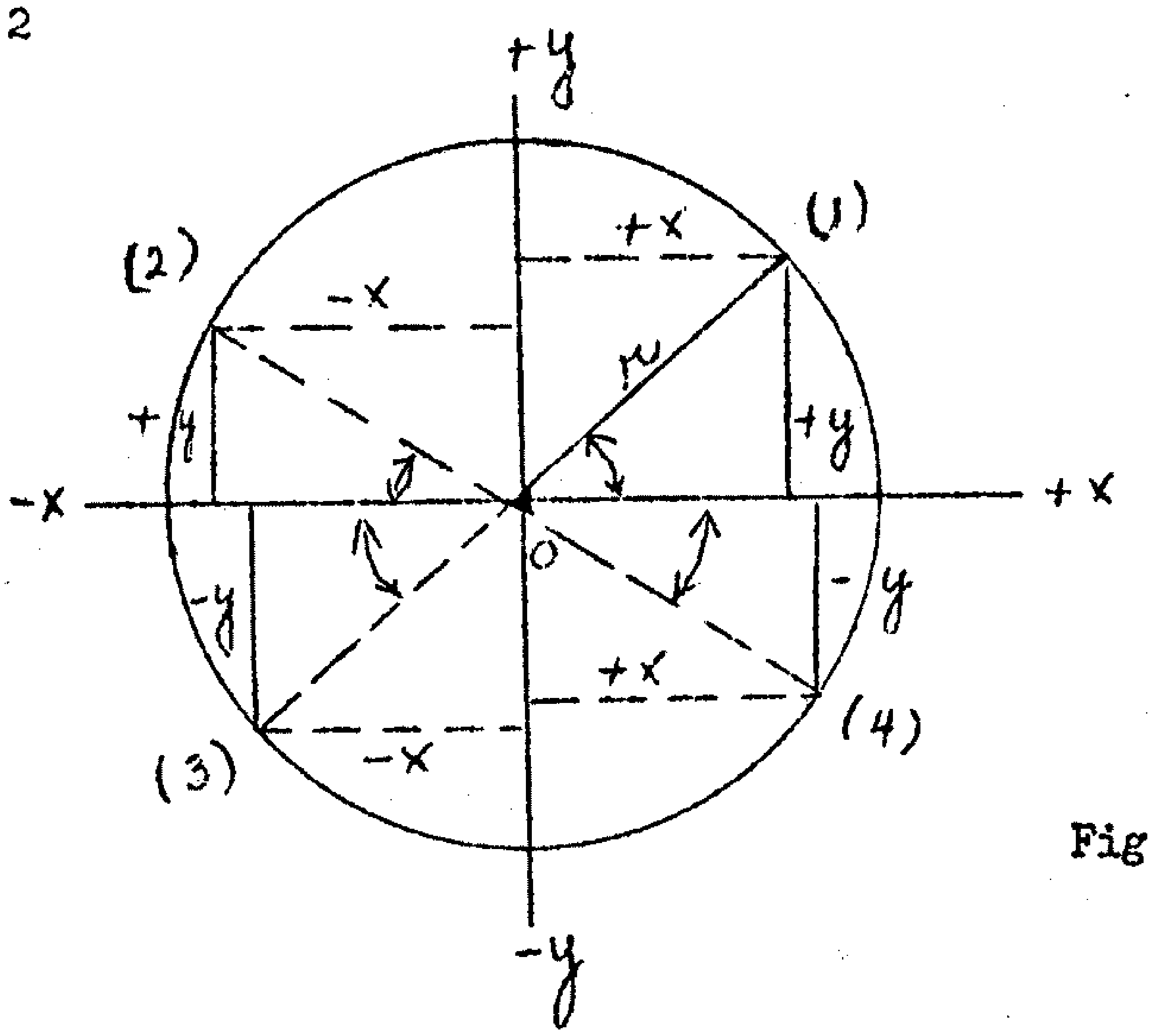


Fig. 2.

From an origin of coordinates we say that distances measured out to the right are plus distances along the x-axis, and distances measured out to the left are minus distances. Along the y-axis distances above the origin are plus and below the origin are minus. The radius (or the hypotenuse) we will always consider to be plus, since it is always the square root of $x^2 + y^2$ which sum is plus in any quadrant, since the square of a number is plus whether the number is minus or plus in sign.

Then in the 1st quadrant (upper right hand) the sine, cosine and tangent are all plus in sign. Thus

$$\text{sine} = \frac{+y}{+r} = (+)$$

$$\text{cosine} = \frac{+x}{+r} = (+)$$

$$\text{tangent} = \frac{+\text{ sine}}{+\text{ cosine}} = (+)$$

The inverse functions, cotangent, secant and cosecant are also plus, since they will always have the same sign as the function from which they are derived.

In the upper left hand (2nd) quadrant signs are as follows:

$\text{sine} = \frac{+y}{r} = (+)$; $\text{cosecant} = (+)$

$\text{Cosine} = \frac{-x}{r} = (-)$; $\text{secant} = (-)$

$\text{tangent} = \frac{+\ \text{sine}}{-\ \text{cosine}} = (-)$; $\text{cotangent} = (-)$

In the lower left hand (3rd) quadrant

$\text{sine} = \frac{-y}{r} = (-)$; $\text{cosecant} = (-)$

$\text{cosine} = \frac{-x}{r} = (-)$; $\text{secant} = (-)$

$\text{tangent} = \frac{-\ \text{sine}}{-\ \text{cosine}} = (+)$; $\text{cotangent} = (+)$

In the lower right hand (4th) quadrant

$\text{sine} = \frac{-y}{r} = (-)$; $\text{cosecant} = (-)$

$\text{cosine} = \frac{+x}{r} = (+)$; $\text{secant} = (+)$

$\text{tangent} = \frac{-\ \text{sine}}{+\ \text{cosine}} = (-)$; $\text{cotangent} = (-)$

Summarizing on Figure 3.

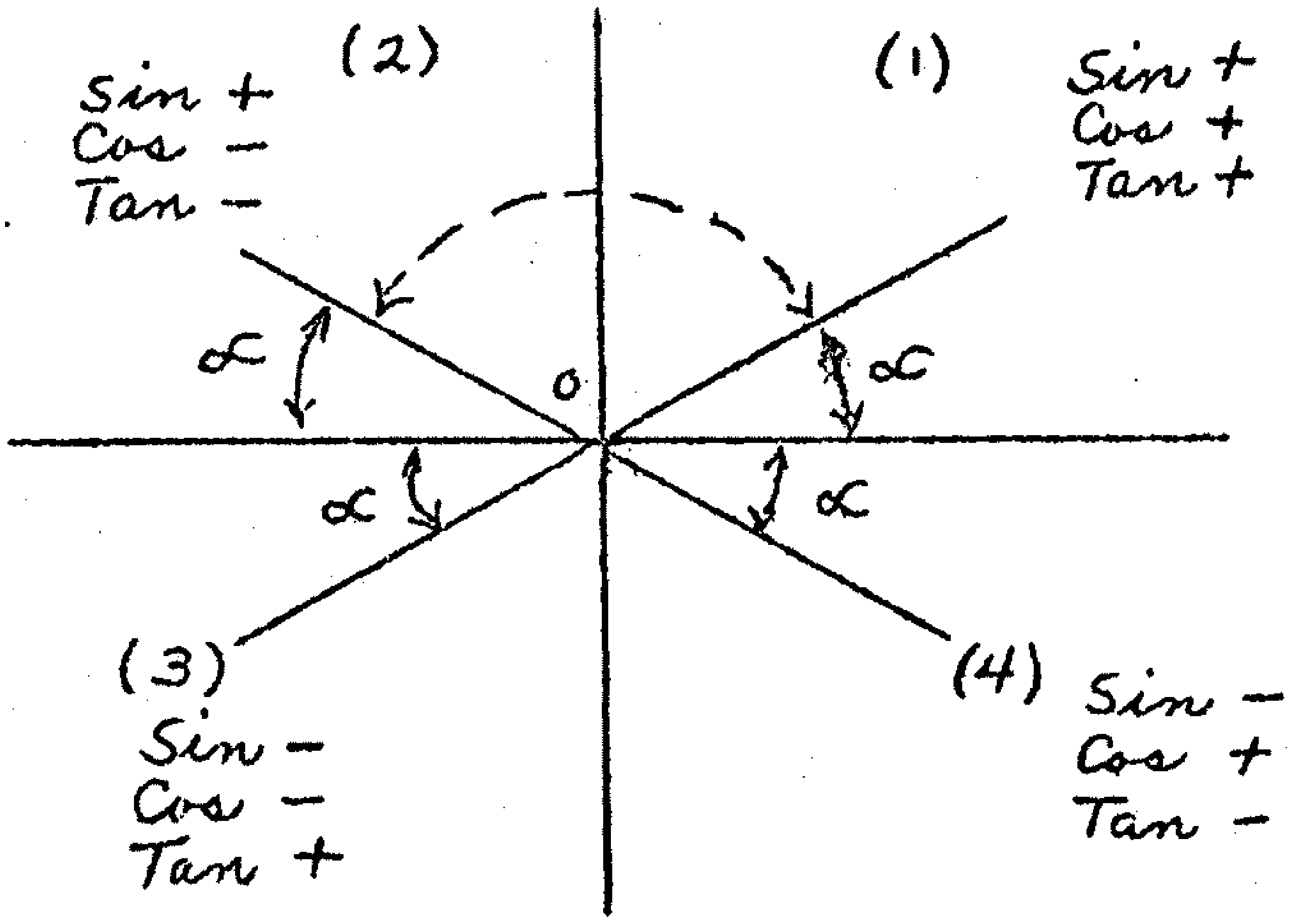


Fig. 3.

From the above figure certain relations between the functions of angles in the different quadrants are readily apparent. Consider angles turned counter-clockwise.

$$\sin\alpha = \sin(180^\circ - \alpha) = -\sin(180^\circ + \alpha) = -\sin(360^\circ - \alpha)$$

$$\cos\alpha = -\cos(180^\circ - \alpha) = -\cos(180^\circ + \alpha) = \cos(360^\circ - \alpha)$$

$$\tan\alpha = -\tan(180^\circ - \alpha) = \tan(180^\circ + \alpha) = -\tan(360^\circ - \alpha)$$

*****

The foregoing discussion covers the basic principles of trigonometry. Other relations between the functions can be obtained from engineering handbooks or from tirgonometric tables. Special usages which arrive in the subject matter of this course are further clarified as they arise.

Points in Space.

For location of points in space with relation to each other we need only two coordinate axes if the points can be located in the same plane. Thus if "a" and "b" both lie in the plane of the paper and we have their "coordinates" with respect to a reference point which we term the origin, o, we can locate them with respect to each other.

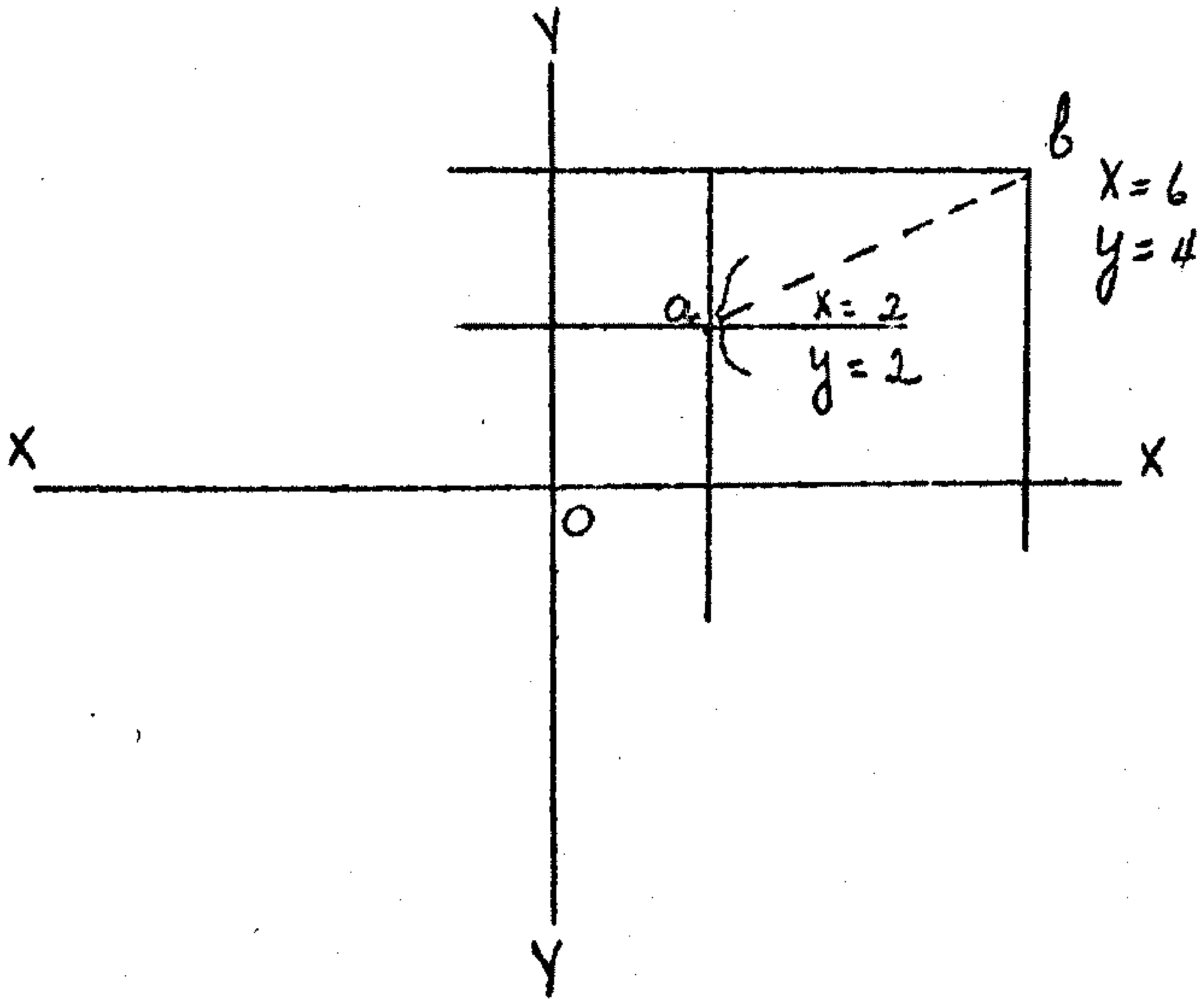


Fig. 4

If the coordinates are shown as in Fig. 4 points a and b are 4 units apart along the x-axis and 2 units apart along the y axis. The actual distance between them is

$$\sqrt{4^2 + 2^2} = \sqrt{20}$$
$$= 4.47 \text{ units.}$$

Now suppose the points do not line in the same plane; that is that one point is also a certain distance above or below the paper while the other lies in the plane of the paper. We must introduce a third axis to completely locate the points relative to each other in space.

Figure 5 illustrates the coordinate axes for three planes.

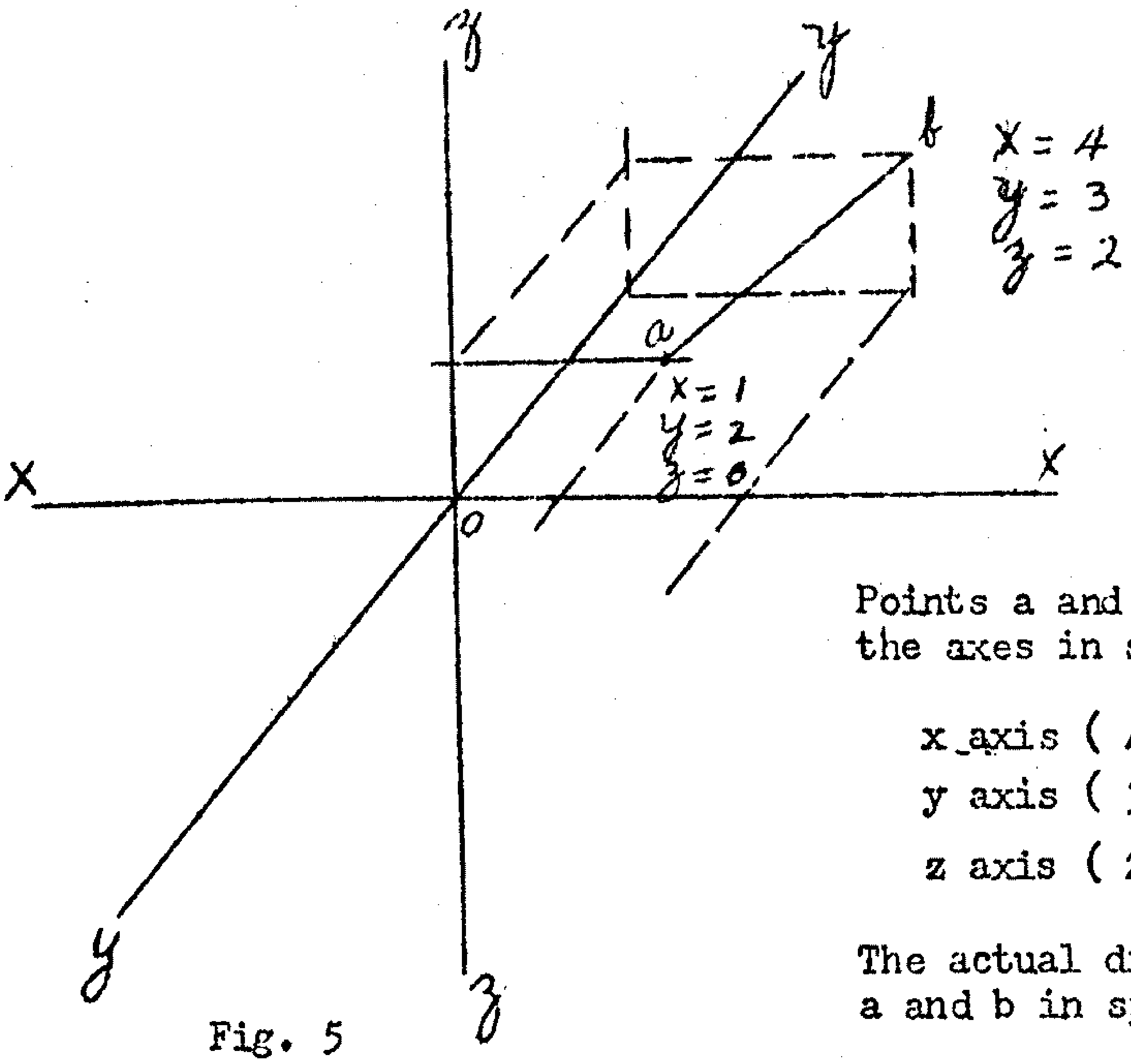


Fig. 5

Points a and b are separated along the axes in space as follows:

x axis $(4 - 1) = 3$ units
y axis $(3 - 2) = 1$ unit
z axis $(2 - 0) = 2$ units

The actual distance between points a and b in space is

$$\sqrt{3^2 + 1^2 + 2^2} = \sqrt{14} = 3.74 \text{ units.}$$

The relation between the length of a line in space and its projections on the axes is sometimes referred to as its direction cosines. The direction cosines of the line a-b are as follows:

Along the x-axis $3/3.74 = .802$

Along the y-axis $1/3.74 = .267$

Along the z-axis $2/3.74 = .534$

The sum of the squares of the direction cosines will equal 1.0 as a check. This principle is much used in structures.

BASIC STRESS ANALYSIS

ASSIGNMENT ONE

Assignment 1

BASIC AIRPLANE STRESS ANALYSIS

Foreword.

The purpose of this course is to furnish the student with the necessary tools to solve the detail problems which arise in the analysis and strength checking of airplane designs. The methods used to analyze airplane structures are built upon the fundamental principles of mechanics. It is therefore imperative that the student be thoroughly grounded in mechanics, and the first portion of this course is designed to obtain this result. For the student who has had college training in engineering this portion of the course will constitute a review of basic principles, presented possibly in a more practical and usable form than is usually given in the college course. For the student who lacks college training this portion of the course will supply the foundation on which the later work will be based. In no case should the first part of the work be covered lightly, for even the most experienced analysts find it advantageous to make a thorough review of fundamental principles at frequent intervals. The most advanced methods of analysis, even as applied to all metal monocoque airplanes are based upon variations of the principles given in this basic course.

The student should supply himself with Army-Navy-Commerce Bulletin-5 for use as a reference. This Bulletin may be obtained by sending 25 cents to Superintendent of Documents, Washington, D. C.

Definitions.

A structure is a part or an assembly of parts subjected to a load or a system of loads, which are known, and which the structure is required to sustain or hold in equilibrium (balance). The function of a structure is to porvide support for an external load or loads by transferring the effect to a point or points where resistance is furnished by other forces, which we term reactions. The types of structures which fulfill this function may be termed trusses, beams, columns, tension ties, torsional tubes or boxes. Trusses, columns and tension ties may be treated together since a truss is essentially an assembly of columns and tension ties. Only axially loaded members and simple beams will be considered in this assignment.

The forces which act on a structure are divided into two types, external and internal. The external forces consist of external or outer loads, and the reactions set up at the support points to resist them. The internal forces are the loads in the members of a structure proper. The internal forces are represented by a strained or deformed condition of the structure. If the external loads are removed, and the structure has not been strained beyond its limit of elasticity, that is it has not taken a "permanent set," the internal loads act to return all parts to the position they were in before any external loads were applied.

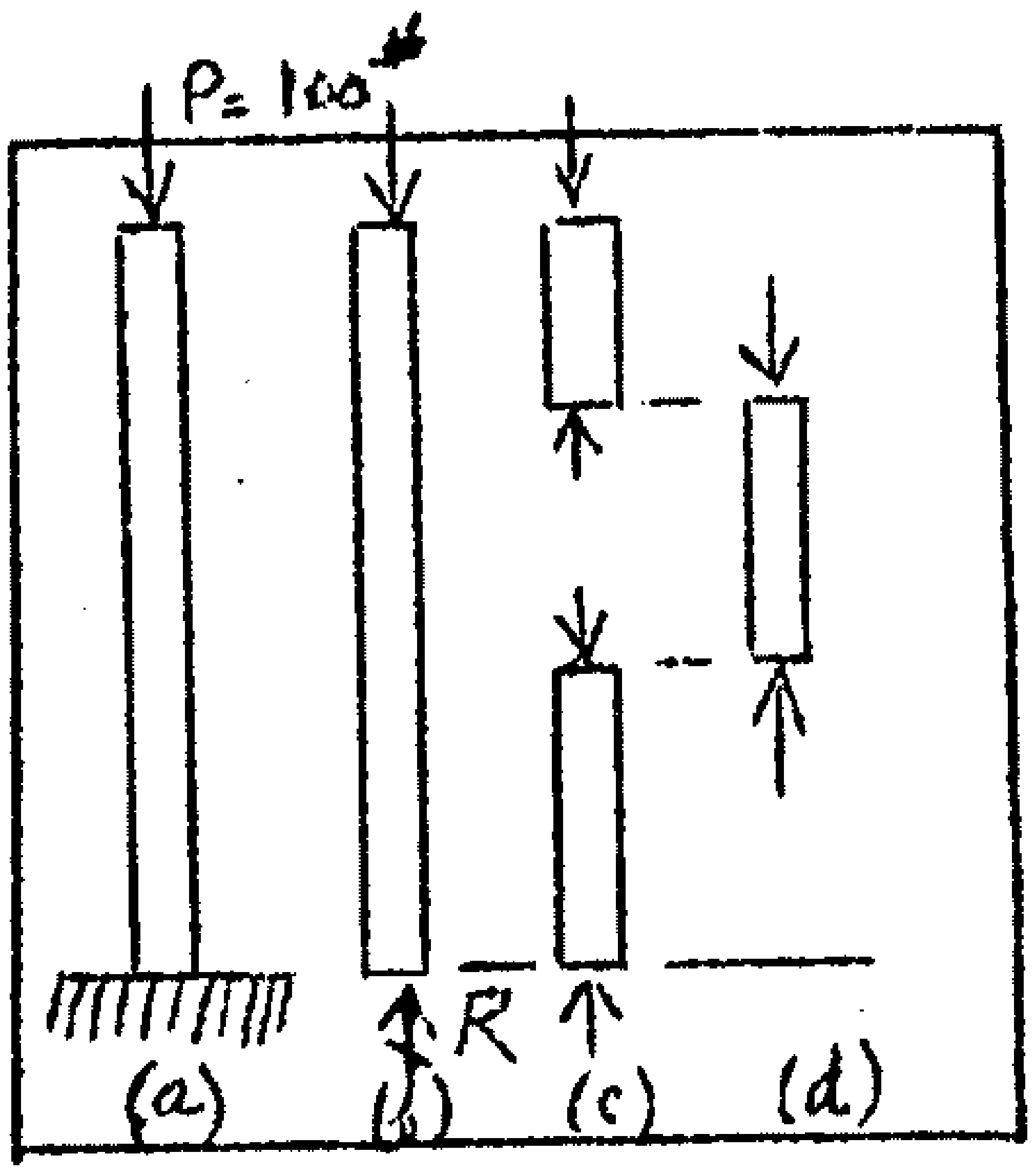
Dead weight loads represent the measure of the pull of gravity upon the parts considered. Weight therefore represents a force.

A force is represented by its amount in pounds and the direction in which it acts. A force therefore has magnitude and sense.

A truss is in general represented by a series of simple members joined together in such a way that they form a series of triangles.

For a trussed structure the internal loads are termed axial, that is they act along the axes (lengths) of the members of the truss. They may be divided into two types, (1) those which tend to lengthen the truss member acted upon, producing tension and (2) those which tend to shorten the member acted upon producing compression. The axial loads in members are uniformly distributed over the cross-sections of the members. Members which are designed primarily to carry compression loads we term struts or columns.

We may think of the simplest form of a truss as consisting of a single member. For the strut of Fig. 1



P is the external load, R is the reaction.

Reactions are indicated by the symbol ↑.

The strut is in static equilibrium under the action of the load and its reaction at the ground, that is there is no motion of the strut in any direction. If the force in one direction were greater than that in the other the strut would move in the direction of the greater force.

Fig. 1

In actual practice in aircraft work the forces which act on the structure are dynamic and the structure actually is in motion, but it may be brought into a condition of static equilibrium by replacing dynamic forces with equivalent static forces, i.e. forces which give the same effect. A simple illustration of how this may be done is given below.

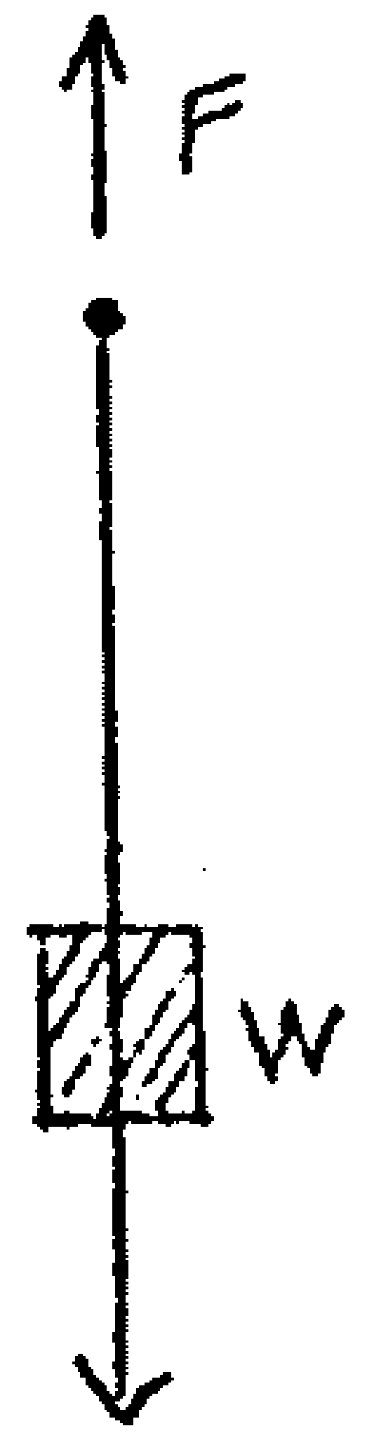


Fig. 2

Consider a weight, W, suspended at rest on a string. The force at the upper end of the string, F, must be equal to the weight, W, in order for the system of forces to remain in balance. Now if an upward acceleration is given to the system the force at F would have to increase. The force required to cause acceleration would, from the principles of physics, be represented by the expression $F = Ma$, and the total force would equal $W + Ma$. But W is actually a measure of the acceleration of gravity and from physics Mass, $M = \frac{W}{g}$. For the purpose of reducing forces to pounds for engineering use we would write the expression of total force acting on the accelerated body as $W + \frac{Wa}{g}$. The second term $\frac{Wa}{g}$ would represent the inertia force due to the acceleration upward and is seen to be given in proportion to the acceleration, g, of gravity. If gravity is trying to accelerate the weight 32.2 ft. per second downward and a force were applied at the upper end to accelerate it 32.2 ft. per second upward, the combined effect would be to double the effective value of the weight, W. A portion of the forces acting would be dynamic but for analysis we could represent the entire system of forces by 2W static load acting downward resisted by an equal load at the upper end. The tension in the string would then equal 2W. The analysis of airplane structures is accomplished by bringing all forces into static equilibrium by elaborations of the method given in this simple illustration.

The Laws of Static Equilibrium.

For a structure to remain in balance in space the forces acting on it in any direction must balance each other. For convenience these forces are generally referred to the planes of the standard coordinate vertical, horizontal and fore and aft axes.

The First Law of Static Equilibrium states that for a body at rest (even momentarily) the summation of vertical forces acting upon it must equal zero (otherwise the body would move in space). This is expressed in equation form

$$\Sigma V = 0 \qquad \text{Eq. 1:1}$$

where Σ is the sign for "the summation of".

In aircraft upward acting loads are considered plus, and for Fig. 1 we would write the above equation

$$\Sigma V = 0, \ +R - P = 0, \ \ R = P$$

The laws of static equilibrium apply not only to an entire member, but also to any part, no matter how small, taken from that member. This is the basis on which all analyses are developed. It is possible to cut out any portion of a structure and still maintain it in static equilibrium if all the forces which act on the cut faces are applied. Any part thus removed from a structure for investigation is termed a "free body". This "free body" method of attack is the basic tool in analysis, since it can be used to solve for unknown loads in members by cutting the members and finding the loads required to replace their effect in the structure. Thus in Fig. 1 if we start with (a) we have a column with a load of 100 lbs. applied at the top. If we wish to find the reaction applied upward at the lower end by the ground we take the whole strut as a "free body" and supply the necessary reaction to make $\Sigma V = 0$, Fig. 1 (b). Similarly in Figs. 1 (c) and 1 (d) we could cut the strut into three segments, each a free body, and each would continue to be held in equilibrium by the end loads, the reaction, and the forces on the cut faces of the column which represent the internal loads in the column. If the column had a cross-section of one sq. inch the unit compression across a cut face would be 100 lbs. per sq. inch. This figure would then represent the unit stress in the strut.

We have said that the simplest structure consists of a single member. The next complication is introduced in the two member truss attached to a rigid support, Fig. 3 (a).

A force in space is determined by its magnitude and line of action. For the two member truss shown the magnitude and line of action of the external

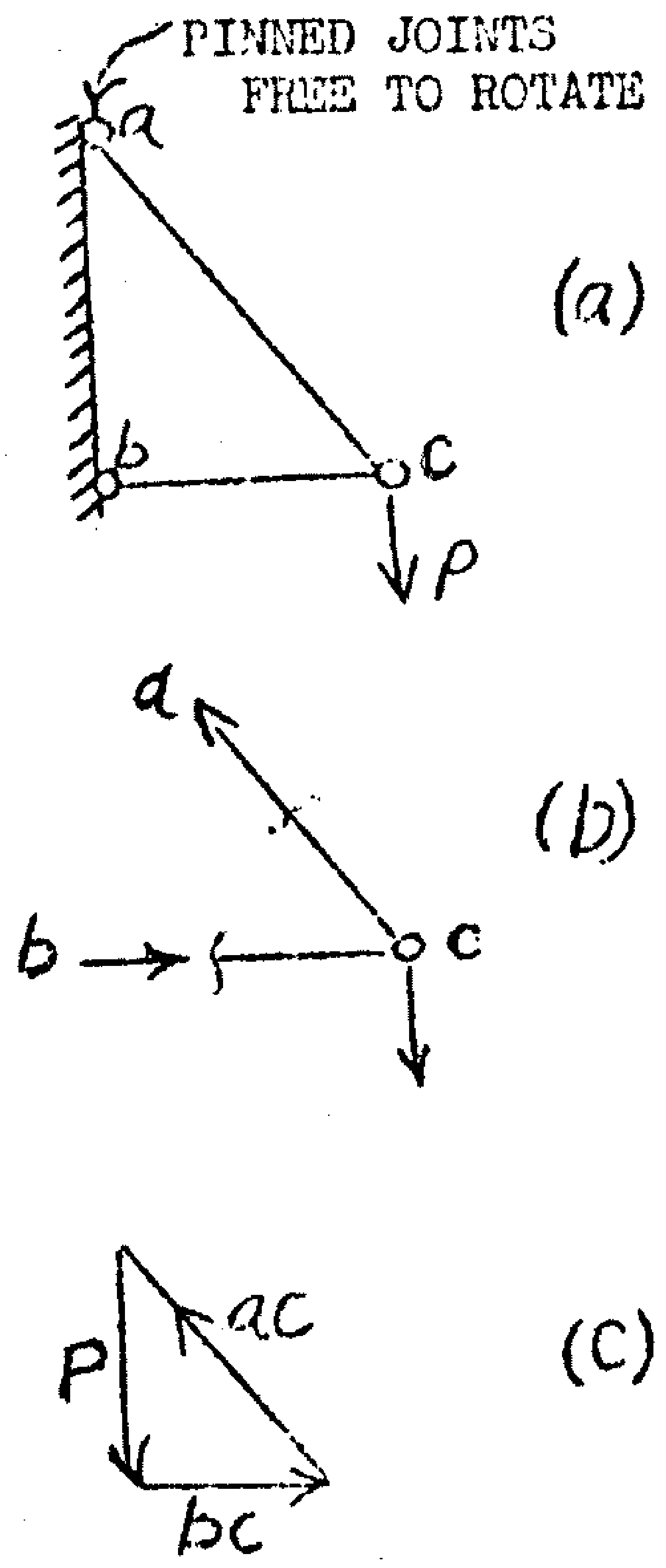


Fig. 3

load P are known, and the lines of action of the forces in truss members ac and bc are known since the axial loads must act along the axes of the members.

It is possible therefore to take joint "c" out of the structure, as a free body and find the forces necessary to hold it stationary (Fig.3,b).

The law of statics from equation (1:1) may be extended to read

"For a body at rest the summation of forces acting upon it in any direction must equal zero." Taking the standard three coordinate axes of reference we may say

The summation of vertical forces equals zero

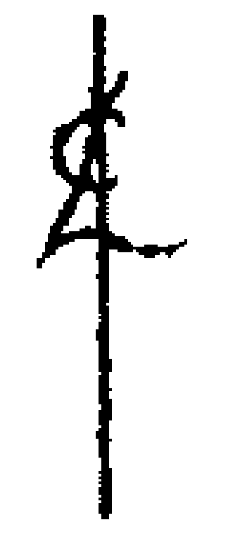
$$\Sigma V = 0 \qquad + \uparrow = \text{up} \qquad (1:1)$$

The summation of horizontal (fore and aft) forces equals zero

$$\Sigma H = 0 \qquad \overset{+}{\rightarrow} = \text{aft} \qquad (1:2)$$

The summation of side forces equals zero

$$\Sigma S = 0 \qquad \overset{+}{\rightarrow} = \text{out} \qquad (1:3)$$



Only two of the equations of equilibrium are required to solve the joint shown in Fig. 3 (b). To satisfy $\Sigma V = 0$ the vertical component of member ac must equal the external load P. (Since bc is horizontal it can have no vertical component.) To satisfy $\Sigma S = 0$ the load in member bc must equal the side component of member ac since the external load P has no side component. The graphical solution is shown in Fig. 3 (c), the vector diagram of the forces on joint c. A force vector is a representation of a force to a given scale and is plotted in the direction of the force. For the example shown the direction and magnitude of P was known. For the other forces on the joint, the direction but not the magnitude was known. The force vector diagram was drawn by plotting force P to scale and drawing lines through the ends of this vector paralleling ac and bc. The intersection of the parallel lines located the ends of the vectors for forces ac and bc and thus fixed their magnitude. The force vectors are plotted in a regular manner, clockwise around the joint and automatically show the type of load in the member. Force P pulls down from the joint, bc acts toward the joint (denoting compression) and ac acts away from the joint (denoting tension).

Moment is defined as force times distance. The moment of a force about any point in a plane is equal to the magnitude of the force multiplied by the perpendicular distance from the point about which moments are taken to the line of action of the force. For a body at rest the summation of moments about any point in the body must equal zero. This is expressed by $\Sigma M = 0$.

If the above condition did not exist the body would rotate in space. Moments may be taken about any point in a body. If a force acts through the point about which moments are taken its lever arm is zero and therefore its moment is zero. Selection of the correct point about which to take moments enables one to solve the truss of Fig. 3 by use of moments alone.

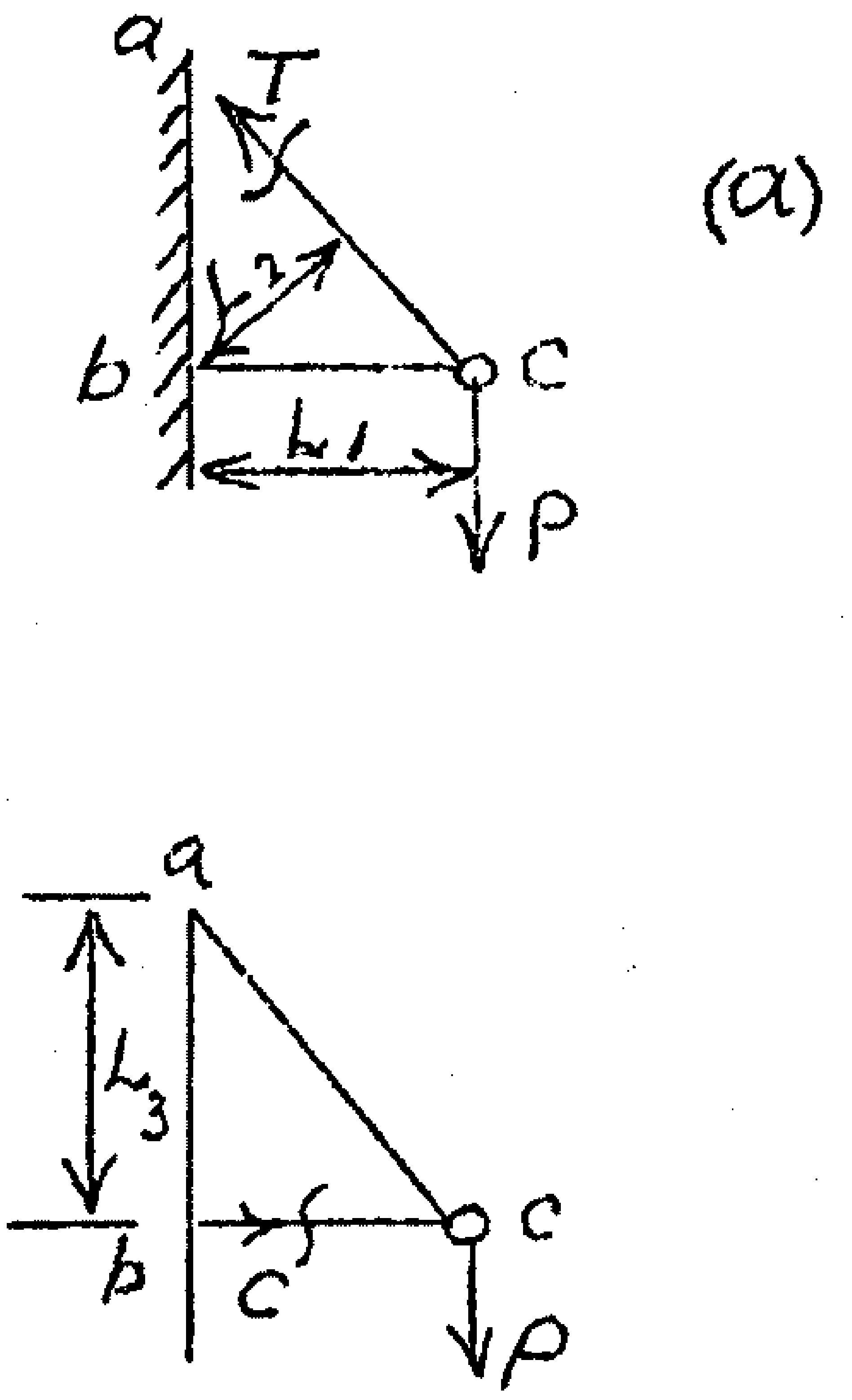


Fig. 4

If member ac is cut and replaced by the force in the member, load P will tend to rotate point c about b. Its moment about b is $P \times L_1$ (Force P x distance L_1).

To satisfy $\Sigma M = 0$ the force in AC, "T", when multiplied by its lever arm L_2 must equal PL_1. Therefore we may write $T \times L_2 = P \times L_1$ and dividing both sides of the equation by L_2

we arrive at the solution

$$T = \frac{P \times L_1}{L_2}$$

Similarly we may solve for the load in member bc by taking moments about "a" with member bc cut. (Fig. 4, b)

$$C \times L_3 = P \times L_1$$

$$C = \frac{P \times L_1}{L_3}$$

A beam is a member which transfers load to support points by bending instead of axial load. A simple beam is considered free to rotate at its points of support. The moment at any section in a beam is equal to the sum of the moments of all forces acting on one side of the section. Moment tends to rotate any section of a beam. (See Fig. 5, a) Shear is defined as the summation of all the forces acting on one side of a section. It is a measure of the force which tends to slide one section of a beam past another as shown in Fig. 5 (b).

Fig. 5

The solution for load shear and moment on a typical pin ended beam is illustrated below. Given the loading and dimensions to the support points, first solve for the reactions R_1 and R_2. Use the equation $\Sigma M = 0$ taking moments about R_1 to solve for R_2.

Taking moments $\curvearrowright +, \Sigma M = 0$

$$R_2 \times 100 - 100 \times 20 - 300 \times 50 = 0$$

$$100\ R_2 = 2{,}000 + 15{,}000 = 17{,}000$$

$$R_2 = 170 \text{ lbs.}$$

Taking moments about R_2, $\overset{+}{\curvearrowright}$

$$R_1 \times 100 - 300 \times 50 - 100 \times 80 = 0$$

$$100\ R_1 = 15{,}000 + 8{,}000 = 23{,}000$$

$$R_1 = 230 \text{ lbs.}$$

Check by $V = 0$

$$170 + 230 = 100 + 300$$

$$400 = 400 \quad \text{check}$$

The shear and moment across the beam are plotted on the diagram.

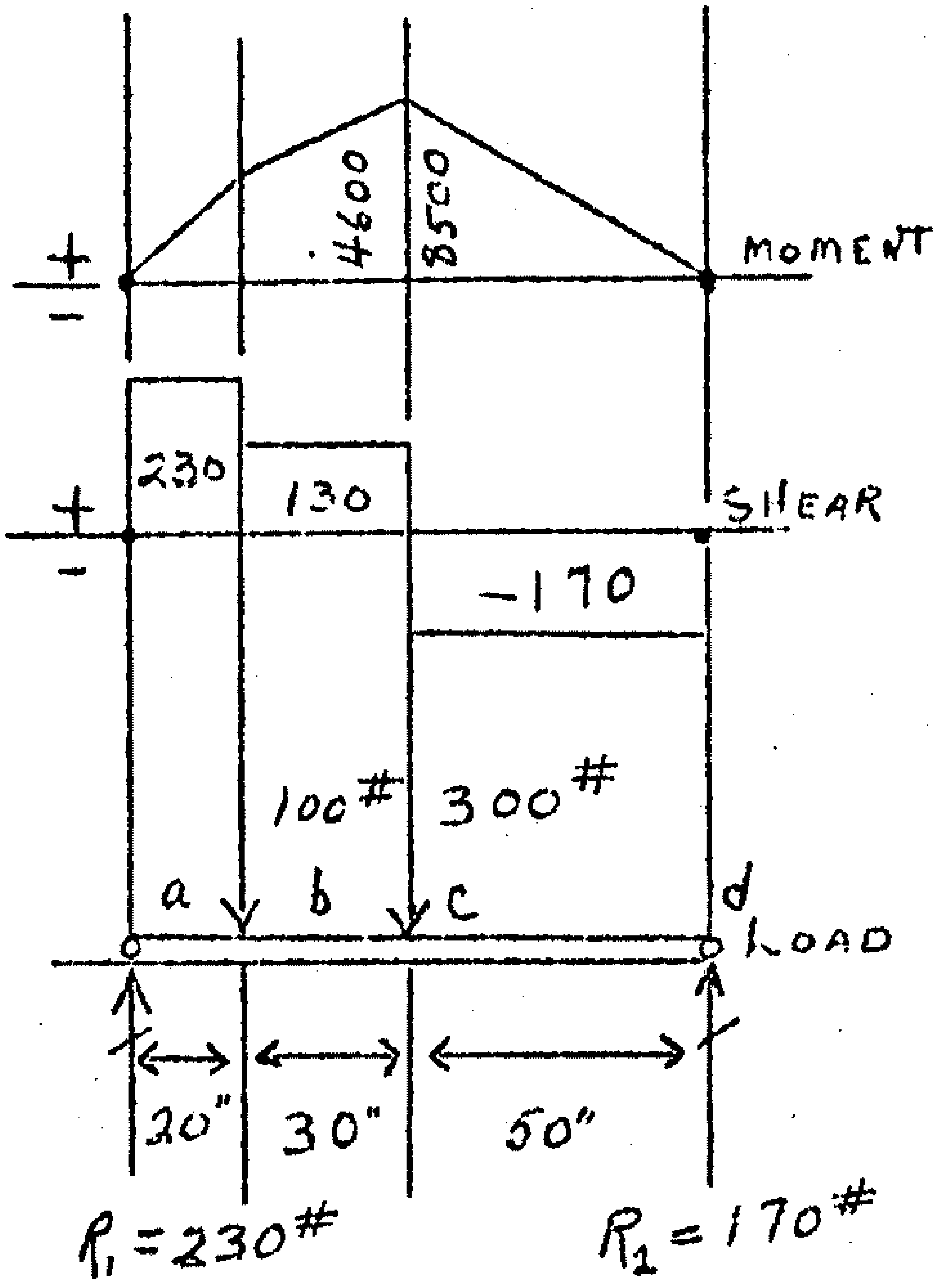


Fig. 6

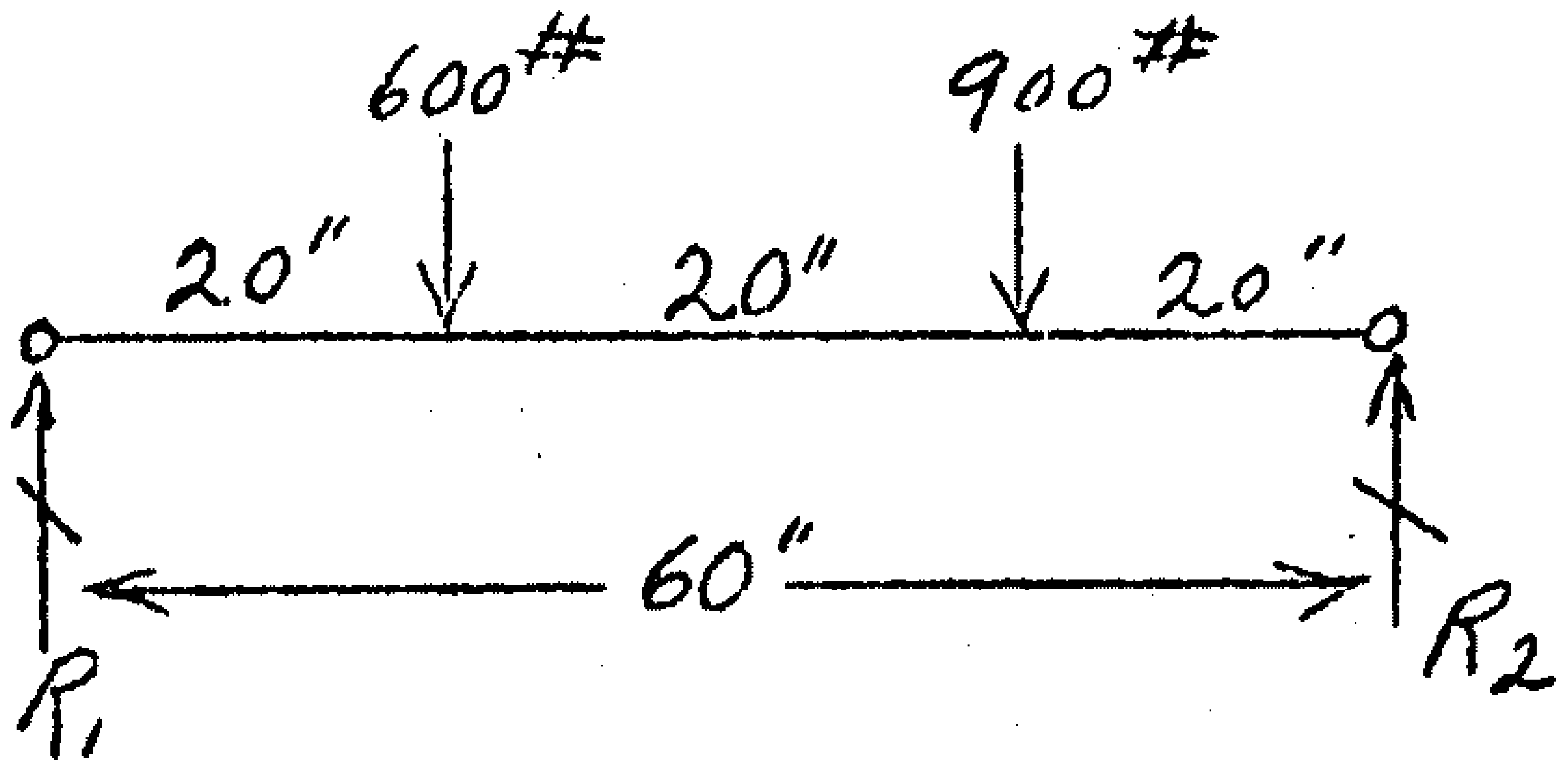
Sign conventions are as follows:
Plus Shear--the part of the beam to the left tends to move up.
Plus Moment--tension on the bottom fibers of the beam.
Plus Loads--upward.

Note: When moments are taken about one point to find the reaction at another, that the sign convention for moments may be chosen optionally. It is generally standard practice to set up the equation so that the sign of the unknown force is plus. Then if the force (or reaction) when solved for comes out minus it may be concluded that the force acts in the opposite direction to that which has been assumed for it.

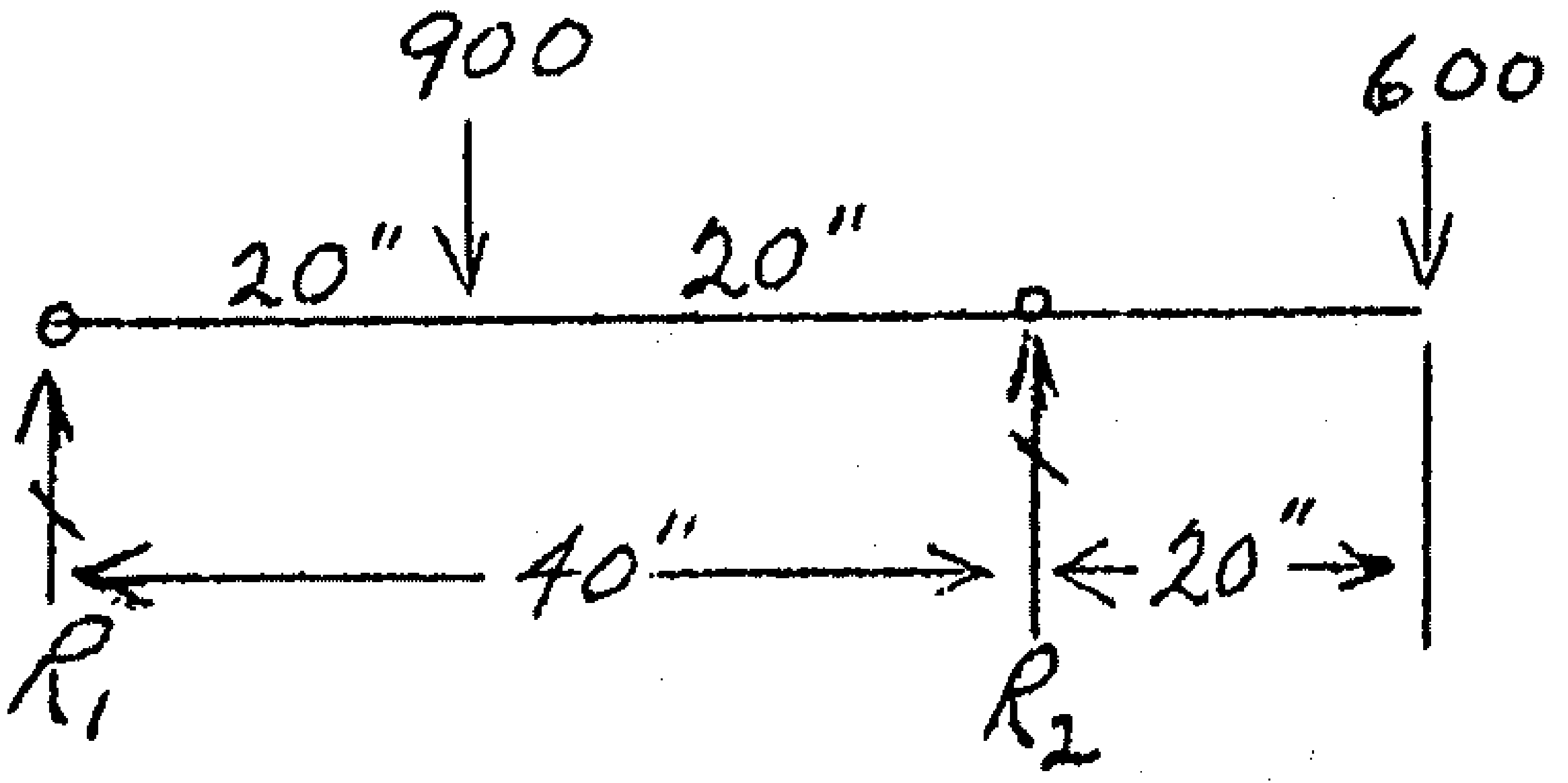
Problem Assignment.

1:1 Calculate reactions, shears and moments for the simple beams shown below and draw load shear and moment curves similar to those of Fig. 6.

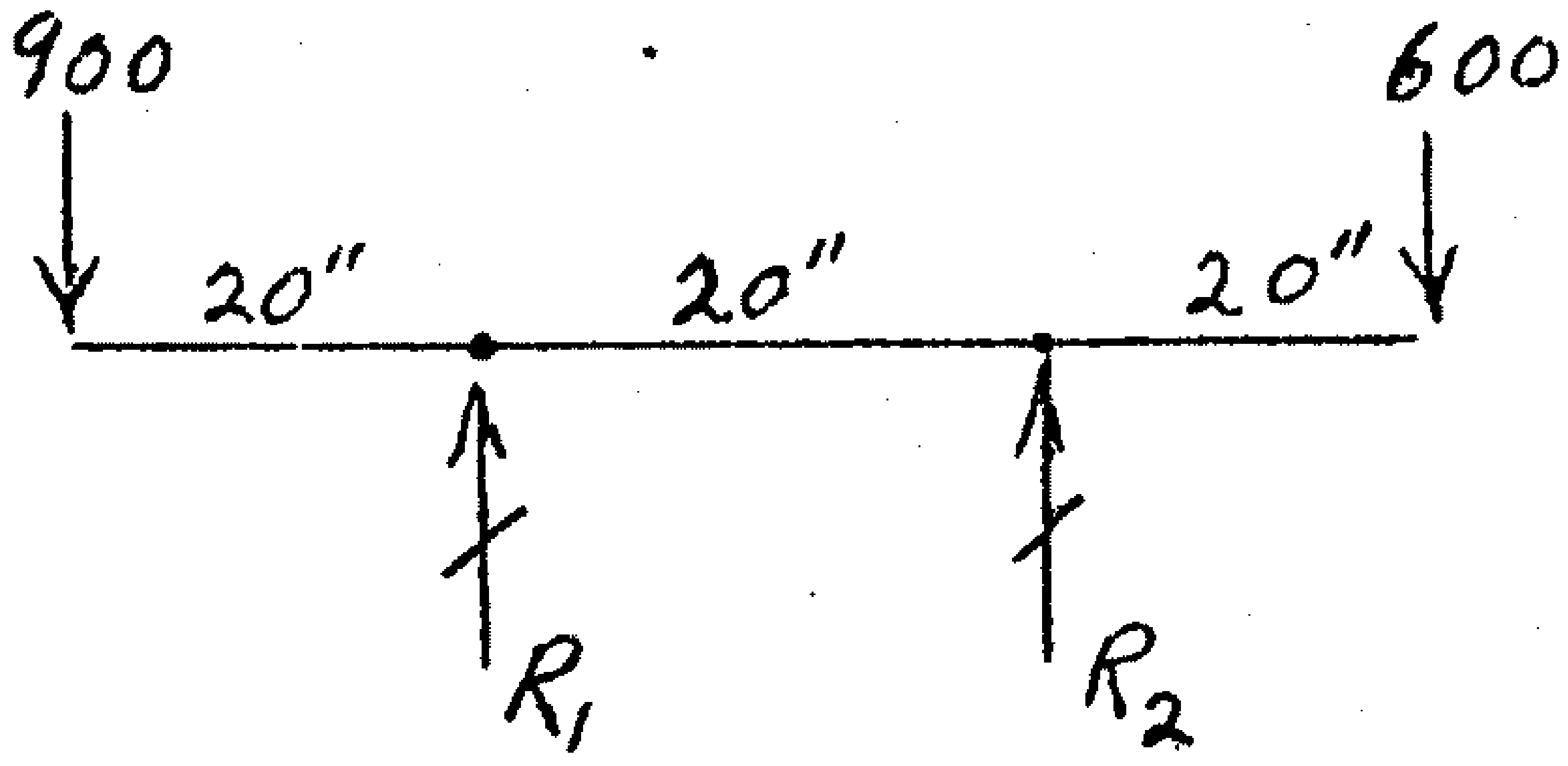
(a)



(b)



(c)



BASIC STRESS ANALYSIS

ASSIGNMENT TWO

Assignment 2

TRUSSES

We have noted that the essential difference between a beam and a truss is that a beam transfers loads in shear and bending, while a truss transfers loads as direct axial loads in the members of the truss.

In aircraft, trusses are used in making up welded steel tubular fuselages or engine mounts and in the "drag" trusses of wings (the trusses whose primary function is to carry loads in the "drag" (chord) direction). The structural members of the wing which carry the vertical lift load are usually beams. A typical wing drag truss will be used to illustrate the methods of truss analysis.

Analysis of Drag Truss.

An airplane drag truss consists of chord members formed by the wing beams, horizontal columns or drag struts, and diagonal tubes or pairs of diagonal wires which transfer shear. See Fig. 1. Tie rods must come in pairs since in aircraft the chord loads on the wing may act either forward or backward depending on the attitude of the airplane, and tie rods can carry tension only, bowing out of action when subjected to compressive loads.

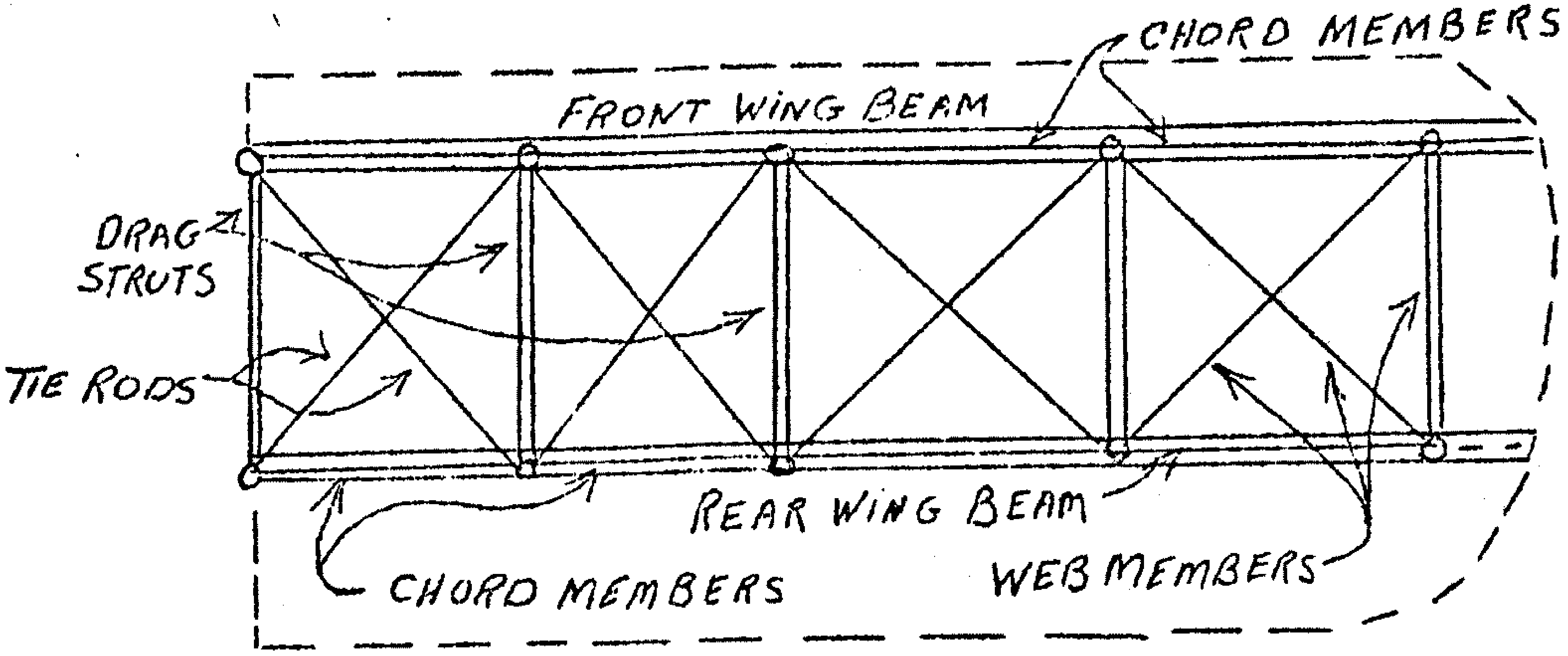


Fig. 1

Plan form of wing showing drag truss

Fig. 2 represents the drag truss taken out as a free body and held in static equilibrium by the forces and reactions. The truss represented would be typical for a biplane lower wing in level flight at high speed. The drag forces from the air stream are transmitted to the nose ribs which transfer them to the panel points 1, 2, 3, 4, and 5. Note that the division of load is made on the assumption that each panel point carries the load between the midpoints of the adjacent spans. The outer panel point, 1, carries the tip load plus half the load in span 1, 2. The load of 900 lbs. shown in parenthesis at panel point 12 represents a forward drag component of load in a strut acting between the upper and lower wings of the biplane.

The reactions to the drag loading are taken at attachments to the fuselage at panel points 5 and 15, and are found by taking moments about each of these points in turn. It is assumed that all of the aft shear is reacted at point 15 since this assumption gives the highest design loads in drag strut 5-15. Members are designated by the panel point numbers at their ends.

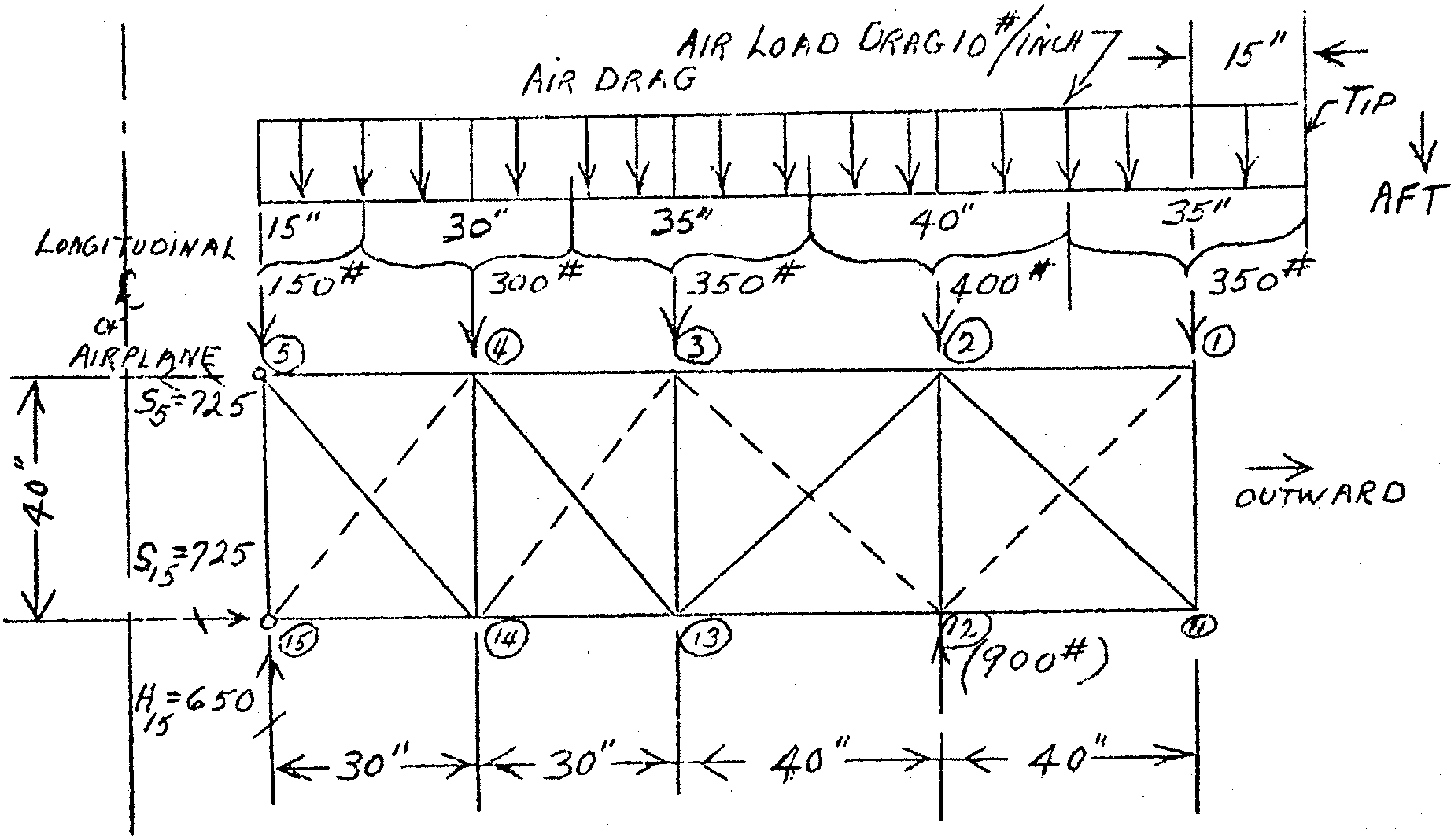


Fig. 2

The full diagonal lines indicate the tie rods which are in action (take tension) under this system of loads. The dashed diagonal lines show those which drop out of action because the loads in them would be compression. The experienced analyst can tell by inspection which members will go out of action. The rule is as follows:

Evaluate the shear loads (sums of the loads to one side) for each panel. In the case in point all the reactions are at the inner end so it is merely a case of summing the loads beginning at the outer end. If when a section is cut through the middle of the panel, isolating the outer portion as a free body, the part to the right tends to go aft, then the tie rod which is in action must have a balancing forward component. If the part cut off tends to go forward the tie rod in action must have an aft component.

For example if we cut through panel 1-2 our free body section is as shown in Fig. 3. Obviously tie rod 2-11 must go into action to furnish a component force to resist the 350 lbs. aft load at 1.

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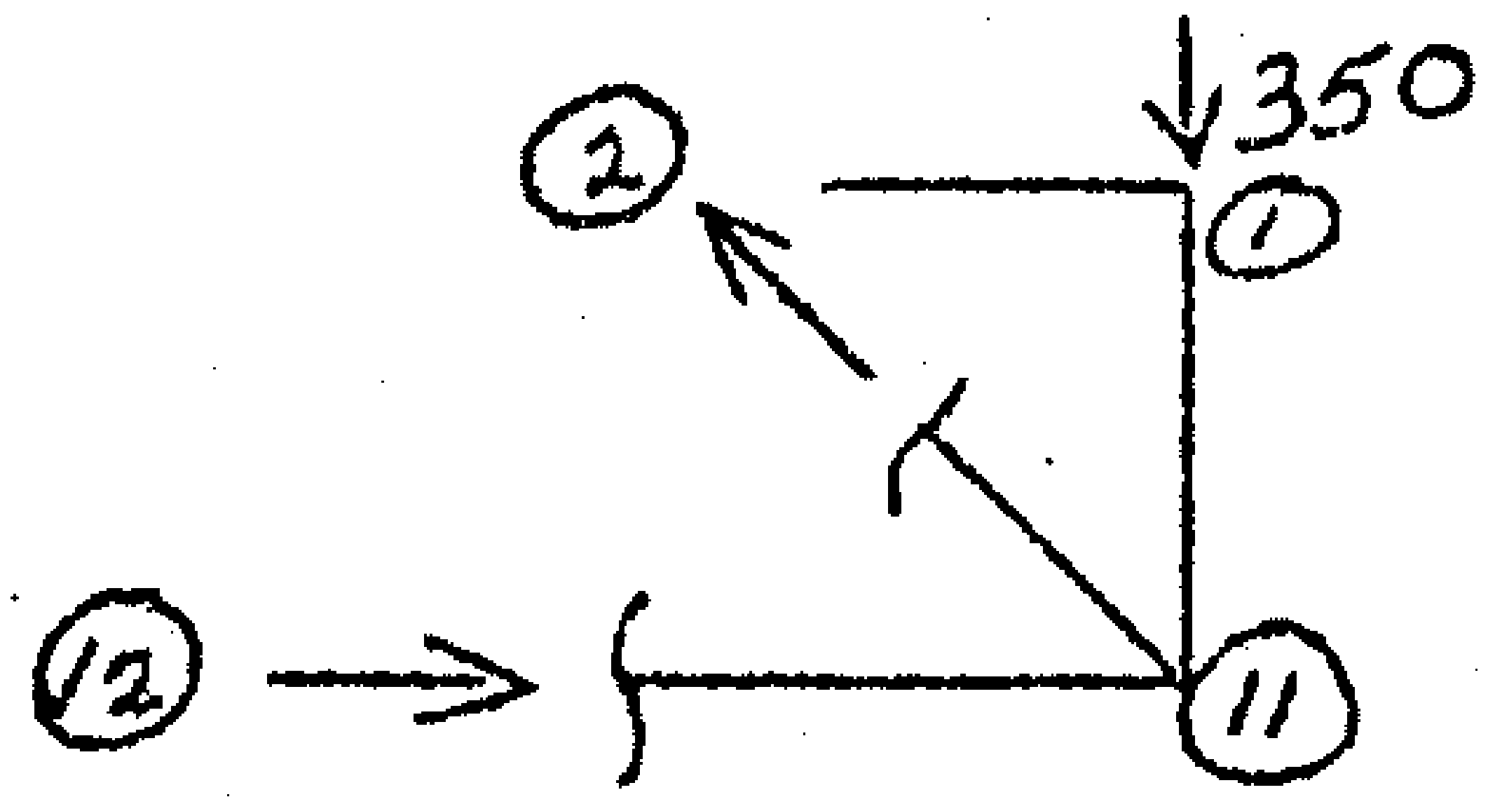


Fig. 3

If we cut through panel 2-3 our free body is as shown in Fig. 4. In this case the 900 lb. load acting forward at 12 has overcome the tendency of the loads at 1 and 2 to move the truss section aft so that an aft component is required in tie rod 2-13 to maintain a balance of fore and aft forces.

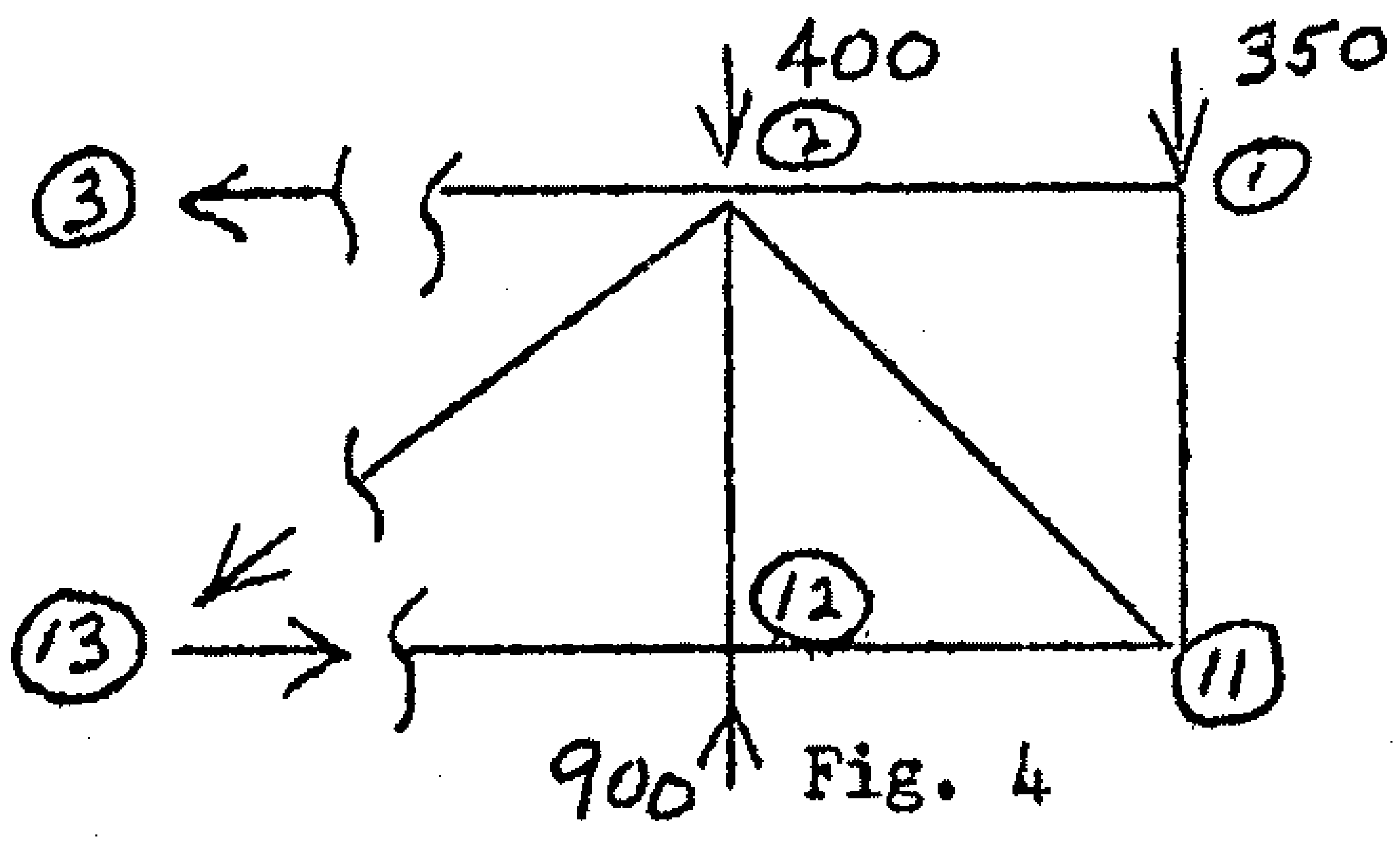


Fig. 4

The order of solution of the truss is as follows:

1. Compute the air loads to the panel points 1 to 5 inclusive.
2. Determine the tie rods which remain in action (in tension).
3. Compute the reations at panel points 5 and 15.

   (a) Drag forces are termed Horizontal-Symbol H

   $\Sigma H = 0 \qquad \therefore H_{15} = \Sigma$ of H forces on truss

   (b) Side forces are termed S

   $\Sigma S = 0$
   S_{15} is found by taking moments about point 5. $\Sigma M_5 = 0$

   (c) S_5 is found by taking moments about point 15. $\Sigma M_{15} = 0$

   (d) S_5 and S_{15} are the only side forces on the truss. They can be checked by applying the equation $\Sigma S = 0$

Let us apply these rules to the solution of the truss of Fig. 2.

1. $\Sigma H = 0 \qquad \uparrow$ taken +
$H_{15} + 900 - (150 + 300 + 350 + 400 + 350) = 0$
$H_{15} = 1550 - 900 = 650$ lbs.

2. $\Sigma M_5 = 0$ taken +

$$(S_{15})(40) + (900)(30 + 30 + 40) - (300)(30) - (350)(60) - (400)(100) - (350)(140) = 0$$

$$40S_{15} + 90{,}000 - 9000 - 21{,}000 - 40{,}000 - 49{,}000 = 0$$

$$40S_{15} = 119{,}000 - 90{,}000 = 29{,}000$$

$$S_{15} = \frac{29{,}000}{40} = 725 \text{ lbs.}$$ acting outward as assumed

3. $\Sigma S = 0$ $S_5 = 725$ lbs. acting inward

The structure is shown in Fig. 2 as a free body in space with the loads and reactions holding it in equilibrium. The analytic (computed) solution is fairly simple because the chord members are parallel. If a section is cut through any member or set of members of the truss the forces acting internally in the cut members must hold the cut sections in equilibrium. That is, the equations $\Sigma S = 0$, $\Sigma H = 0$, and $\Sigma M = 0$ must still be satisfied. Now a force in space may be resolved into compoments as shown in Fig. 5 (a). Conversely, if one of the components of a force or load is known, the actual load may be obtained from geometric ratio only. The ratios of partial loads to total loads are the same as the ratios of partial lengths to total lengths. The method of solving for the loads in the diagonal tie-rods, drag struts and chord members is illustrated in Figs. 5 (b) and 5 (c).

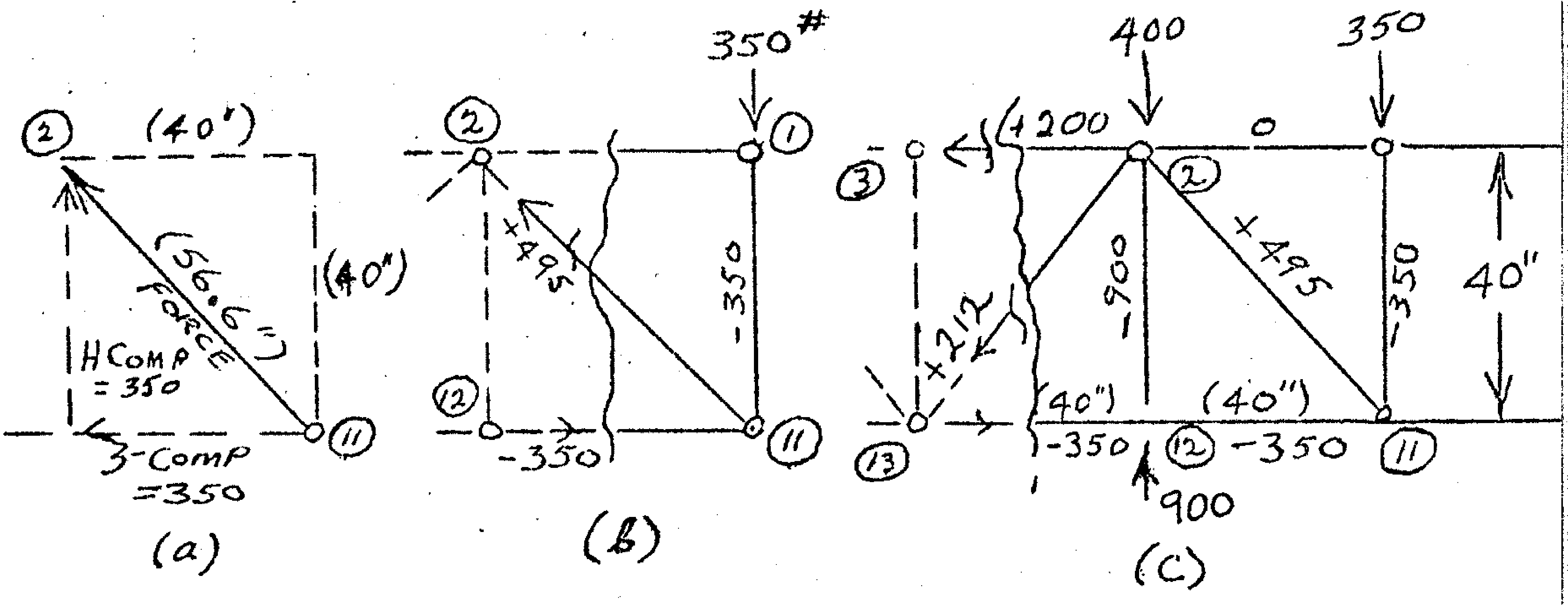


Fig. 5

Considering Fig. 5 (b). There can be no force in member 1-2 since there is nothing to resist side force at 1. The 350 lb. load goes directly into drag strut 1-11 putting compression of 350 lbs. in the member. The horizontal component of tie rod 2-11 must then equal 350 lbs. at 11. The true load in the tie rod is

$$\left(\frac{56.6}{40}\right)(350) = 495 \text{ lbs. tension}$$

The side component is 350 lbs. This must be resisted at 11 by a compression load in 11-12 of this amount. The force in member 11-12 could also have been obtained by taking moments about point 2.

$$\Sigma M_2 = 0 \circlearrowleft +$$

$$(\text{Load } 11\text{-}12)(40) - (350)(40) = 0$$

$$\text{Load } 11\text{-}12 = \frac{(350)(40)}{40} = 350 \text{ lbs.}$$

Note that the plus sign obtained for the load in this equation merely indicates that the load acts in the direction assumed. This load was assumed to act toward joint 11 which indicates compression in the member. In putting member loads on a truss the correct convention is to mark tension loads plus and compression loads minus. Similarly the load in 2-11 could have been obtained by taking moments about point 12.

Consider next Fig. 5 (c). Tie-rod 2-13 must balance the shear load to the right. Its horizontal component is therefore 900 - (400 + 350) = 150 Lbs. acting aft. Its true tension load is $\frac{56.6}{40} \times 150 = 212$ lbs. The load in member 3-2 may be obtained by taking moments about point 13.

$$\Sigma M_{13} = 0 \circlearrowleft +$$

$$(\text{Load } 3\text{-}2)(40) - (400)(40) - (350)(80) + (900)(40) = 0$$

$$(\text{Load } 3\text{-}2)(40) = 16{,}000 + 28{,}000 - 36{,}000 = 8000$$

$$\text{Load } 3\text{-}2 = \frac{8000}{40} = 200 \text{ lbs. Tension as shown.}$$

The load in 13-12 may be obtained by taking moments about point 2.

$$\Sigma M_2 = 0 \circlearrowleft +$$

All forces pass through point 2 except 13-12 and the load at 1.

$$(\text{Load } 13\text{-}12)(40) - (350)(40) = 0$$

$$\text{Load } 13\text{-}12 = 350 \text{ lbs. Compression as shown.}$$

The truss could also be solved graphically by taking each joint of the truss out as a free body and finding the forces on the cut faces of the members necessary to hold the joint in static equilibrium. The free body sketches of typical joints of the truss are shown in Fig.6. The method of joints can be applied only at a joint where the forces in all but two of the members are known since we are working with only two of the equations of equilibrium-- $\Sigma H = 0$ and $\Sigma S = 0$.

Joints 1, 11, 12, 2, 3, 13, 4, and 14 must be solved in the order named in order to use the method of joints.

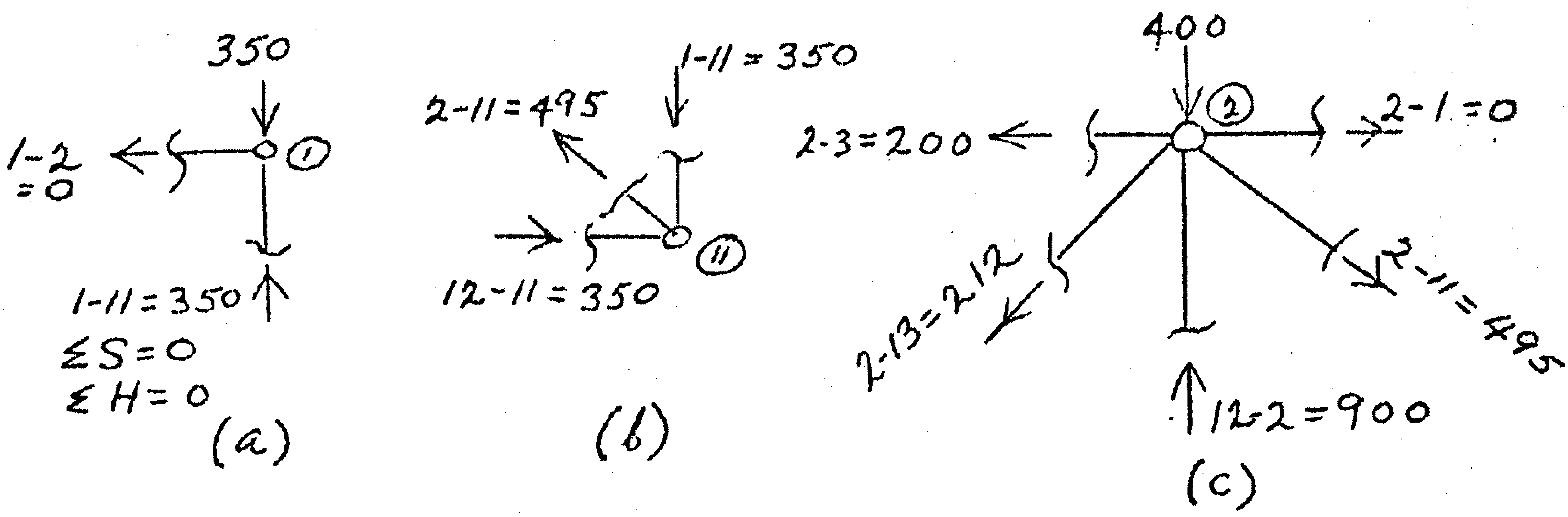


Fig. 6

Problem Assignment.

2:1 Complete the calculation of loads in members by the analytic method.

2:2 Check all joints for static equilibrium by taking them out as free bodies as shown in Fig. 6.

BASIC STRESS ANALYSIS

ASSIGNMENT THREE

Assignment 3

DESIGN OF TENSION AND COMPRESSION MEMBERS

The web members for which loads were found in Assignment 2 are brought together in a design table to show the sizes required and the strength margins beyond those just necessary to sustain the loads.

Loads in the table are for a load factor of 6, whereas our original loads were for a load factor of 1.

TABLE 3-1

Design Table

	Symbol	Matl.	Size	Area	Length	Design LD.	Allew. LD.	M.S.
Round Tie Rods	5-14	Steel				3750		
	4-13	Steel				1500		
	2-13	Steel				1270		
	2-11	Steel				2970		
Drag Struts	5-15	17St	$1\frac{1}{2}$ x		40"	- 3900		
	4-14	17ST	$1\frac{1}{2}$ x		40"	- 3000		
	3-13	17ST	1 1/4 x		40"	- 2100		
	2-12	17ST	1 3/4 x		40"	- 5400		
	1-11	17ST	1 1/4 x		40"	- 2100		

Outside diameters of the drag struts are given as a starting point. Choice of a strut diameter is optional unless clearance is important, but these sizes are given you so that results may be uniform. Your problem is to choose a guage to give ample strength with a small margin of safety. All drag struts are round aluminum alloy 17ST tubes and are assumed pinned at the ends.

$$\text{M.S.} = \text{the margin of safety} = \frac{\text{allowable load}}{\text{actual load}} - 1$$

After studying this assignment and the portions of Army-Navy-Commerce Bulletin 5 which refer to design of columns, the student should fill in the missing columns of Table 3-1. The methods used to design tension and compression members will now be explained.

A tension member or tie is subjected to axial loads tending to stretch it; that is, to increase its length. If the end connections are in line with the axis of the members, a condition which is termed concentric, the distribution of load over a cross-section of a tie is considered uniform and the maximum load which it will carry is equal to the product of the allowable unit tensile stress in the material and the net cross sectional area. The members used generally for ties in airplane construction are stream-lined steel wires, round steel wires, or steel cables. Wires are larger at the threaded ends than in the center portion to allow for a reduction of area due to threads. Standard wires are so designed that the ends are as strong as the center portions. The strengths of the sizes used are tabulated, so that their design consists merely of choosing a correct size from a table. Cable strengths are also tabulated, but the actual strength of a cable is governed by the type of terminal used on the ends. Ordinary spliced ends develop about 85 per cent of the cable strength. Swaged-on terminals will develop 95 per cent or more of the cable strength. The strengths of ties used commonly in aircraft are listed for reference. (See Table 3-2.)

TABLE 3-2

Strength of Ties

				Cables			
Round Tie Rods		Streamline Wire		19-strand non-flex.		7 x 19 extra flex.	
Size and thds/inch	Strength Lbs.	Size and thds/inch	Strength Lbs.	Diameter	Strength Lbs.	Diameter	Strength Lbs.
6-40	1000	6-40	1000	1/32	185		
10-32	2100	10-32	2100	1/16	500		
1/4-28	3400	1/4-28	3400	3/32	1100		
		5/16-24	4600	1/8	2100	1/8	2000
5/16-24	5700	5/16-24	6100	5/32	3200	5/32	2800
3/8-24	8500	3/8-24	8500	3/16	4600	3/16	4200
7/16-20	11500	7/16-20	11500	7/32	6100	7/32	5600
1/2-20	15500	1/2-20	15500	1/4	8000	1/4	7000
		9/16-18	20200	5/16	12500	5/16	9800
		5/8-18	24700				

A column or strut is a member acted upon by two equal and opposite compressive forces at the ends. A pinned end column is free to rotate at its ends but is fixed against motion in any direction. A fixed end column is not only restrained against motion, but the axis of the column is fixed in direction at the support. Fig. 1 (a) and (b) illustrates the two extremes of restraint.

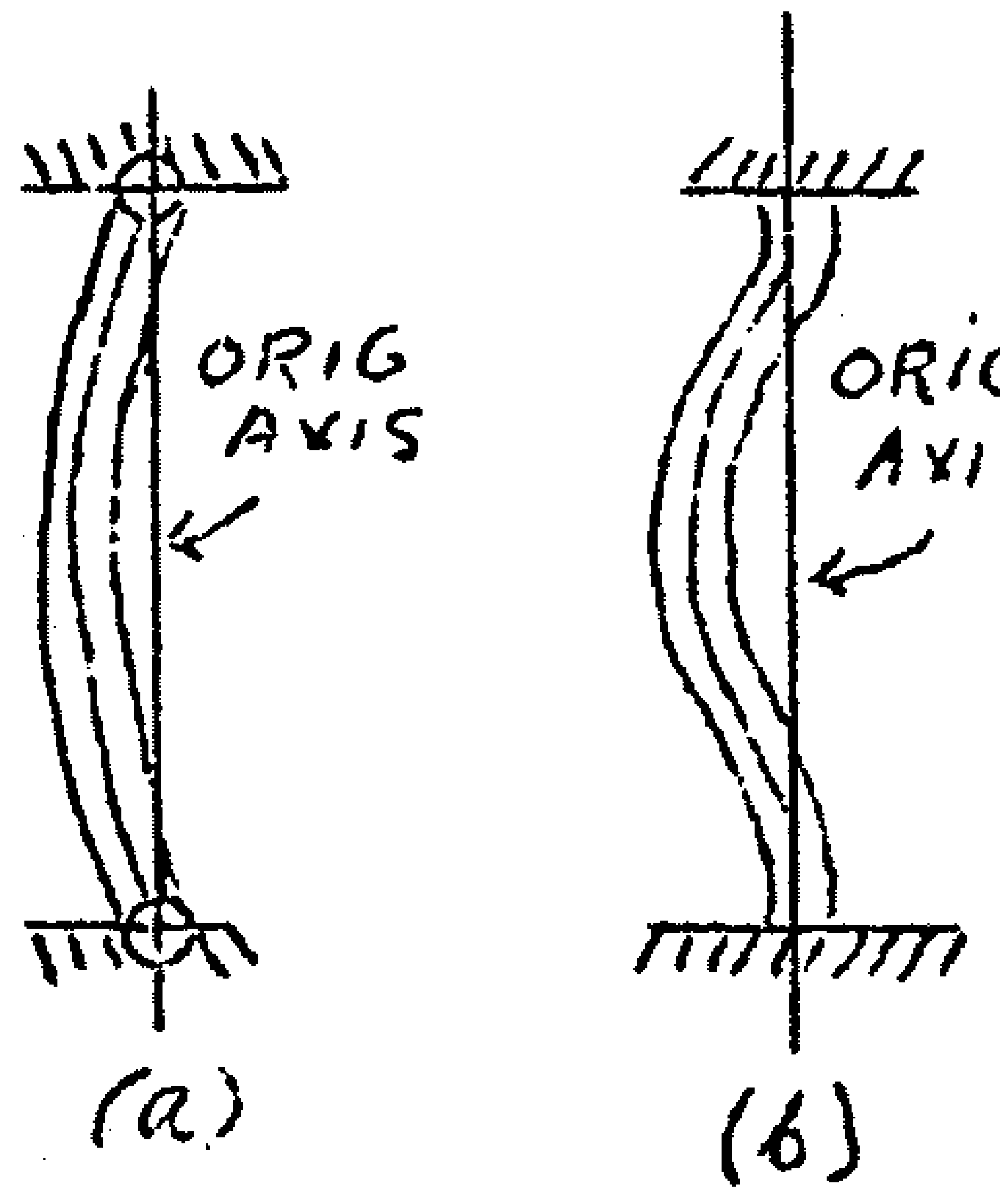


Fig. 1

The column of 1 (a) rotates freely about its ends. The column of 1 (b) is embedded in the supports so that the axis at the edge of the support must align itself with the axis in the support. The end fixity of columns is denoted by the "restraint coefficient", "c".

For the column of Fig. 1 (a), $c=1$; for 1 (b), $c=4$. All columns met with in airplane design lie in the range between pin-ended and partially restrained. The maximum coefficient of fixity allowed is $c=2$. This is used for welded up tubular frames such as certain types of fuselages. The choice of the maximum value is based on destruction tests of assemblies. The use of the restraint coefficient appears later in the column formulas.

Columns fail in one of two ways, depending upon their lengths and stiffness. A short rigid column will undergo a plastic failure, by local flow or fracture of material before it shows any tendency to bend sideways as a spring. A longer more elestic column will bend sideways in the direction of least resistance until failure occurs by combined bending and compression. In most cases failure will occur at or below the yield point of the material in the column. The yield point is defined as the unit stress (lbs. per sq. inch) at which the material fails to return to its original shape after load is removed.

A compression member which has a high ratio of length to width or thickness will fail under a unit stress much below the yield point. A member of this type behaves as an "intermediate" or "long column". Up to a certain load the member will bend sideways as a spring, and when the load is removed will return to its initial position without injury. If the critical load is exceeded the member will fail by bending at the center. Since the bending at the center depends upon the sideward deflection, and the sideward deflection depends on the Modulus of Elasticity of the Material, this type of failure is termed elastic. It must be noted that elastic failure must start from a slight eccentricity in the column; but eccentricities are always present, due to manufacturing tolerances.

Note: The modulus of Elasticity, "E", is defined as unit stress over unit deformation.

$$E = \frac{\text{lbs. per sq. inch}}{\text{inches per inch elongation}}$$

The values of E used in aircraft metals are:

Mild steel--S.A.E. 1025 or lower--28,000,000
Alloy steels--C.M., nickel, etc.--29,000,000
Aluminum alloy--heat treated --10,000,000

Refer to ANC-5 for values for special alloys and for further discussion of the Modulus of Elasticity.

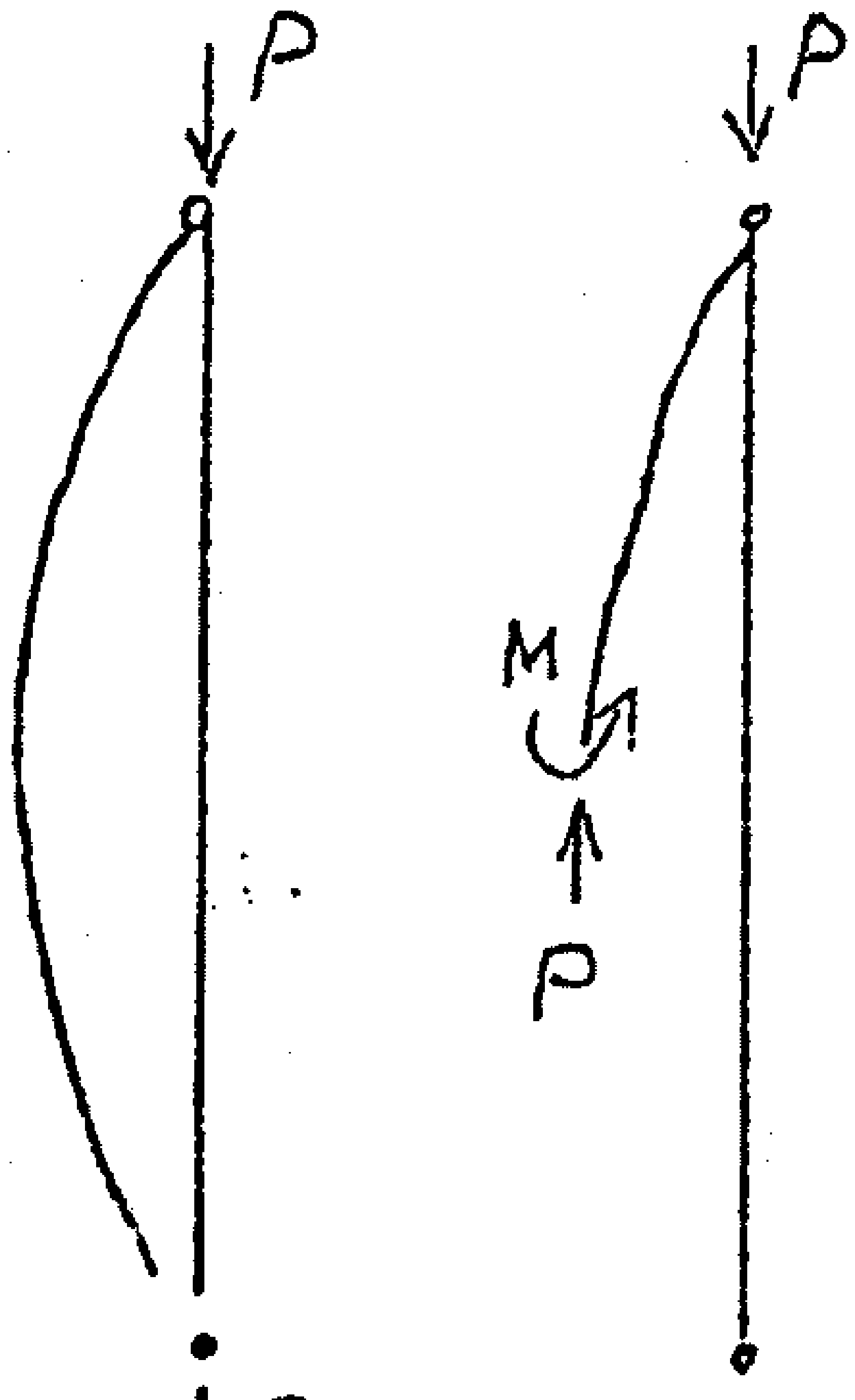


Fig. 2

For a column bent as in Fig. 2, but not beyond its elastic limit the loads on a cross section are a direct unit stress, P/A, where P is the total load and A the area of the cross section; and a bending stress due to the fact that the line of action of the end load is no longer along the axis of the strut.

If the column shown is cut at the center, the forces necessary to hold it against motion or rotation are as shown in Fig. 2. The value of the moment, M, at the center is equal to the load, P, times the deflection, d.

$$M = P \times d$$

At any section along the length of the column this relation holds. Due to the fact that the sideward deflection curve of a pin ended column is a parabola the moment induced in the strut is also a parabola with its maximum at the center.

For the same cross section of column the failing load decreases as the column length increases.

For the same length of column the failing load decreases as the Radius of Gyration decreases. The Radius of Gyration may be defined as the index of resistance to lateral bending. It depends upon the area of the cross section and the location of the material composing it. Mathematically the Radius of Gyration,

$\rho = \sqrt{\frac{I}{A}}$ where I = Moment of Inertia of the cross-section
A = Area of the cross-section

The Moments of Inertia of cross sections will be discussed in a later assignment.

Euler's Formula.

For columns which fail elastically, the formula developed by the Swiss mathematician, Euler, is used. In terms of the critical average unit stress it may be written

$$\frac{P}{A} = \frac{C\pi^2 E}{(L/\rho)^2}$$

where P = critical (failing load in lbs.
A = area of cross section
C = coefficient of end restraint
E = Modulus of Elasticity of Material--lbs. per sq. inch.
π = 3.1416
L = column length-inches
$\rho = \sqrt{I/A}$ and I = moment of inertia of column cross section in the direction in which it fails.

In terms of the critical load, P, this formula may also be written $P = \frac{C\pi^2 EI}{L^2}$, from which it is apparent that if L becomes very small the load which can ve carried becomes very large. Actually there is a practical limitation based on tests which shows that when the indicated allowable P/A of an Euler column becomes greater than one-half of the yield point of the material, the Euler or long column formula does not apply. From this point on short column formulas based largely on tests are used.

The ratio of length to radius of gyration, L/ρ = Slenderness Ratio of a column. The value of L/ρ at which the short column formula curves meet the Euler curve is known as the Critical L/ρ.

In the short column range for steel struts Johnson's Parabolic formula is used:

$$P/A = F - \frac{F^2 (L/\rho)^2}{4C\pi^2 E}$$

where F is the yield point of the material and the other symbols are the same as shown for the Euler column.

For the design of aluminum alloy tubes in compression the Straight-line Formula is used in the short column range. For 17ST material this formula is written

$$P/A = 34{,}500 - 245\ L'/P,\ \text{for yield point of } 30{,}000 \text{ psi}$$

$$P/A = 42{,}500 - 334.5\ L'/P,\ \text{for yield point} = 40{,}000 \text{ psi}$$

A practical limit of 40,000 lbs. per sq. inch is used with the latter material.

The column formulas for aluminum alloys, and steels of various treatments are tabulated in ANC-5, Tables 4-1 and 5-1.

The ANC-5 system of showing allowable unit stresses takes care of the end coefficient of fixity, "C", by using instead of the actual length an equivalent length

$$L' = \frac{L}{C^{\frac{1}{2}}}$$

If a column has an end fixity of four its equivalent length, L', is equal to half its actual length, L.

$$(L' = \frac{L}{2})$$

Note: The most efficient column would from theory be that with the thinnest wall for any given diameter. Practical limitations as to crippling strength and minimum thickness for welding must be taken into account. For this reason .035 wall tubes are used as a minimum in welded construction. If the D/t ratio $\left(\frac{\text{Diameter}}{\text{thickness}}\right)$ is less than 50 it is not necessary to check a tube for crippling strength. A formula for allowable crippling stress is given below:

$$F_{cr} = \left(0.5 - \left(\frac{D/t}{8000}\right)\right) \frac{E}{D/t}$$

24ST Tubing.

For 24ST Aluminum Alloy round tubes which have a tensile yield point of 42,000 psi, the short column formula has been revised to read

$$\text{Allow. } P/A = 50000 - 427\ L/\rho \qquad \text{Critical } L/\rho = 78$$

Note again that the use of the Euler curve above the critical L/ρ i.e. in the short column range will lead to erroneous results. Always check to see if the column is in the short or long column range.

The following points should be noted carefully:

1. The modulus of elasticity of steel does not change to any marked degree with change in heat treatment. Therefore if a column is in the long column class its strength can be increased only by increasing the size and not by raising its heat treatment. For members of the short column class, an increase in heat treatment raises the allowable load by raising the yield point of the material.

2. Any fixed end column may be treated as an equivalent pin ended column of a shorter length by applying the rule that the length of the equivalent pin ended column is equal to the actual length divided by the square root of C, the restraint coefficient.

3. Column formulas are based on round tube values, or equivalent closed sections. Where sections have open flanges they may fail locally, because of lack of support. The ratio of the load such a column will carry to the theoretical load for round tubes is called the Form Factor. The Form Factor is always a value less than one (1) and is determined by test.

4. The column formulas given apply only to members whose cross-sections are practically constant throughout their lengths. When the cross-section varies along the column length special methods of analysis must be used.

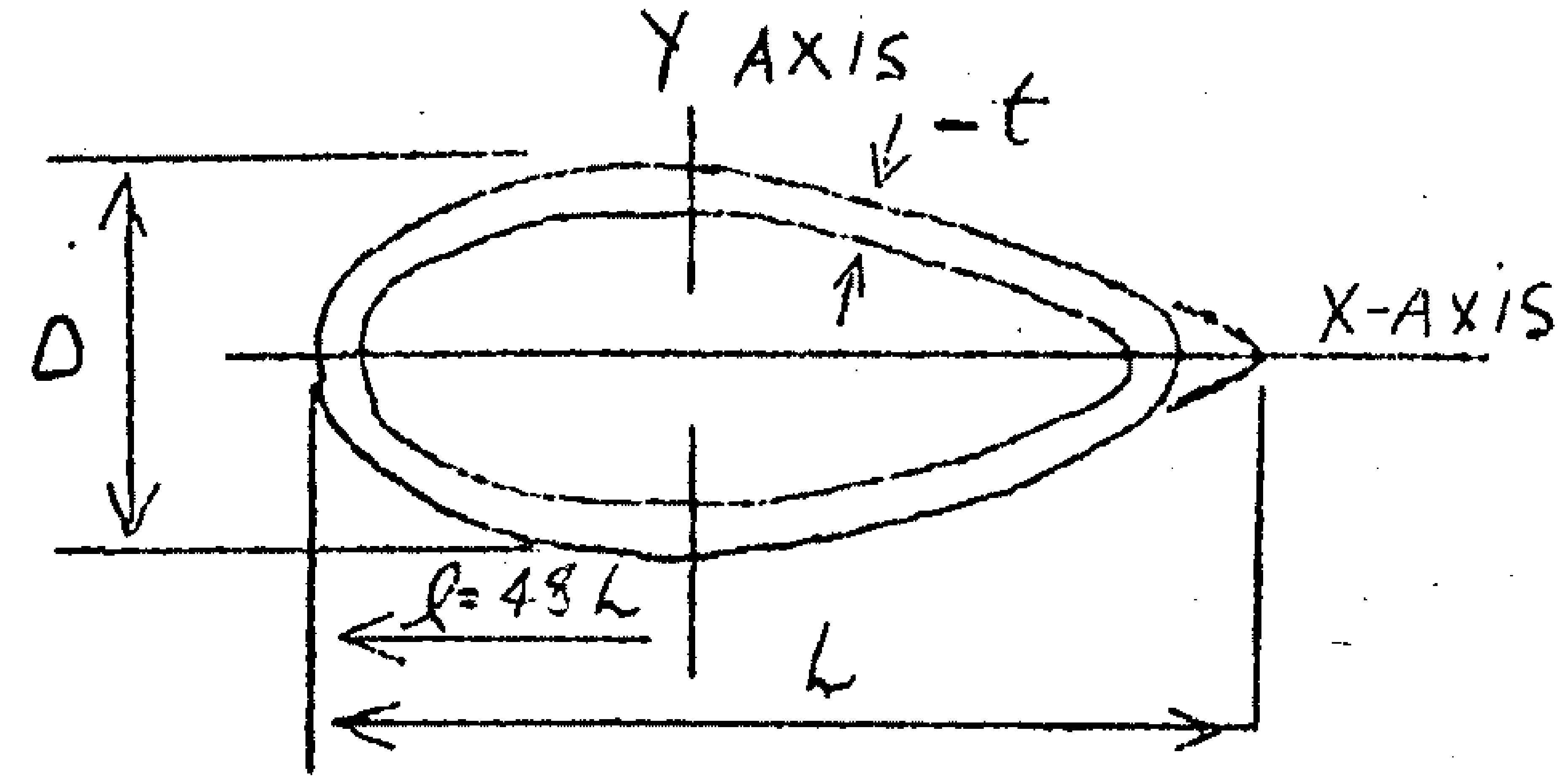
Values of ρ for round tubes are tabulated below.

TABLE 3-3

ρ = Radii of Gyration--Round Tubes

	Wall Thicknessess						
O.D.	.035	.049	1/16	3/32	1/8	5/32	3/16
1/2	.165	.160	.156	.147	.140		
5/8	.209	.204	.200	.191	.182		
3/4	.253	.248	.244	.234	.225	.217	.210
7/8	.297	.293	.288	.278	.269	.260	.252
1	.341	.337	.332	.322	.313	.303	.295
1-1/8	.386	.381	.376	.366	.356	.347	.338
1-1/4	.430	.425	.420	.410	.400	.391	.382
1-3/8	.474	.469	.465	.454	.444	.434	.425
1-1/2	.518	.513	.509	.498	.488	.478	.469
1-3/4			.597	.587	.576	.566	.556
2			.685	.675	.664	.654	.644
2-1/4			.774	.763	.753	.742	.732
2-1/2			.862	.851	.841	.831	.821
2-3/4			.950	.940	.929	.919	.908
3				1.028	1.017	1.007	.997

Streamline Tubes Ratios of L/D from 2 to 4.5.



I = moment of inertia
P = radius of gyration
D = outside max. width
$D' = D - t$
L = overall length to theoretical end of streamline
$R = L/D$

About the x-x axis:

$K_A = 1.875R + .992$ $\qquad A = K_A \times t \times D'$

$K_{I_{x-x}} = .290R + .054$ $\qquad I_{x-x} = K_I \times t \times D'^3$

$K_P = .394 - .060/R$ $\qquad P = K_P \times D'$

It is sometimes required to have the properties about the y-y axis for use in calculation of bending strength. For a solid-stream lined section

$$I_{y-y} = .0439\ L^3D$$

For a tube the moment of inertia is found by calculating two solid sections one of which is bounded by the outside dimensions and the other by the inside dimensions, and subtracting one from the other. For a tube the formula may therefore be written

$$I_{y-y} = .0439\ \left[L^3D - (L - 2t)^3\ (D - 2t)\right]$$

It is customary to reduce the calculated column strength of a streamline tube by 15 per cent to allow for the fact that the material is not as efficiently placed as in a round tube. The form factor for a streamline tube is therefore .85

Problem Assignment.

3:1 is given at the beginning of these notes.

3:2 Design the drag struts of Table 3-1 as round steel tubes heat treated to a yield point of 100,000 lbs. per sq. inch, using no tube with a thinner wall than .035, or smaller O.D. than 1 inch.

AIRPLANE
STRESS
ANALYSIS

PART TWO

ASSIGNMENT 4

ASSIGNMENT 5

ASSIGNMENT 6

NAME________________

DATE________________

RYAN AERONAUTICAL INSTITUTE

STUDENT WORK-GRADE SHEET

INSTRUCTIONS: BEFORE BEGINNING YOUR STUDY, READ THIS CAREFULLY.

As you complete each portion of this assignment, you are to answer the problems listed for you. Each set of problems is to be completed in order.

Upon completion of all portions of this assignment, you are to mail them to us for correction and comment, and then proceed with the work involved in the next assignment. Send each assignment in for correction as completed.

Before sending your work to us, check the completed problems against the list below. Indicate by check mark in the margin that you have done so. DO NOT SEND IN INCOMPLETED ASSIGNMENTS. Title each problem correctly.

This sheet will be returned to you for a permanent work-grade record. Problem solutions to all assignments in part two will be included in part four of this course.

AIRPLANE STRESS ANALYSIS

PART TWO

ASSIGNMENT 4, BASIC STRESS, (1 Problem) __________

ASSIGNMENT 5, BASIC STRESS, (2 Problems) __________

ASSIGNMENT 6, BASIC STRESS, (1 Problem) __________

COMPLETED GRADE: __________

Time spent on assignment, hours:__________

REMOVE THIS SHEET AND ATTACH TO YOUR WORK. This sheet must accompany your work so that you will have a grade record.

BASIC STRESS ANALYSIS

ASSIGNMENT FOUR

Assignment

GRAPHICAL ANALYSIS OF TRUSSES

Before proceeding further with the analysis of trusses let us consider the requirements for a complete truss structure. A completely stable truss, that is one which is restrained against collapse under any load applied in its plane, must be composed of triangles. Removal of any member, leaving a parallelagram or trapezoid of a section, will allow that section to collapse under certain loads. A truss which contains enough members and just enough members to make complete sets of triangles without any members to spare is termed statically determinate. This is the type of truss with which we are concerned.

If members are added to a truss which is just statically determinate, it becomes redundant, or statically indeterminate. The added member is a redundant (or extra) member. To determine if a truss, or any part of a truss, is a statically determinate or redundant apply the following rules:

Let m = number of members
j = number of joints
$m = 2j - 3$ for a truss just statically complete

If m is greater than $2j - 3$ the truss is statically indeterminate (contains one or more redundant members). If m is less than $2j - 3$ the truss is unstable (contains too few members).

At this point it is advisable that the student review the fundamentals of graphical analysis (balance of forces around a joint) as given in Assignment 2.

The truss analyzed algebraically in Assignment 2 was constructed with parallel chord members which simplified the computations. In actual practice, aside from drag trusses of wings, there are very few airplane trusses of such convenient geometry. In general on steel tube engine mounts and fuselages the angles between the chord members are variable from panel to panel and the direction of the posts often varies from the vertical. It is much quicker in such cases to analyze the trusses graphically. This could be done as indicated in Assignment 2 by taking each joint of the structure in turn out as a free body and constructing the force system necessary to hold the joint in static equilibrium. For an entire structure it is simpler and self checking to solve for the entire system of member loads on a single diagram.

Note: The basic assumption on which the graphic analysis of trusses is built is that the truss members carry only axial loads, and that when the truss deflects under load its center-line diagram deflected can, for all practical purposes, be assumed to coincide with the center-line diagram undeflected. The directions of loads at a joint are known since loads act along the axes of the members.

It is necessary to standardize the method and the sign convention for graphic analysis in order to avoid errors.

Procedure:

1. Determine the direction and magnitude of the external loads.

2. Take the truss out as a free body in space and compute the reactions required to hold it in equilibrium.

3. Draw an accurate diagram of the truss to any scale convenient for the paper size. This is the scale diagram.

4. Show all loads and reactions and the directions in which they act on the scale drawing.

5. Beginning at any point on the outside of the truss place capital letters between all external loads. The external loads then become identified as A-B, the load between A and B, B-C, etc. Reactions are treated the same as any other loads.

6. Place a small letter in each triangle formed by the truss. This identifies the loads in the members as forces a-b, b-c etc. for the web members and A-a, etc. for the chord members.

7. Plot the external loads and reactions to scale on the Force Diagram, the directions of the forces being transferred from the space diagram and the values of the loads being measured off to scale. With this diagram if the summation of forces equals zero ($\Sigma V = 0$, $\Sigma S = 0$, etc.) the diagram will return to the starting point. Forces are plotted in order, going around the truss clockwise. The location of forces on the Force Diagram is kept accurately by marking the ends of Force Vectors with corresponding letters from the scale diagram. Thus external force A-B will begin at point A and end at point B, etc.

8. The summation of forces around each joint is plotted in order to obtain the loads in the members. It is necessary to begin at a joint where only two unknowns exist since it is possible to solve graphically for not more than two unknowns. Where more than two unknown forces are present at a joint it is necessary to determine one of them by solving an adjacent joint where only two unknowns are present. (There are some cases in which special methods are required. These will be covered in the next assignment

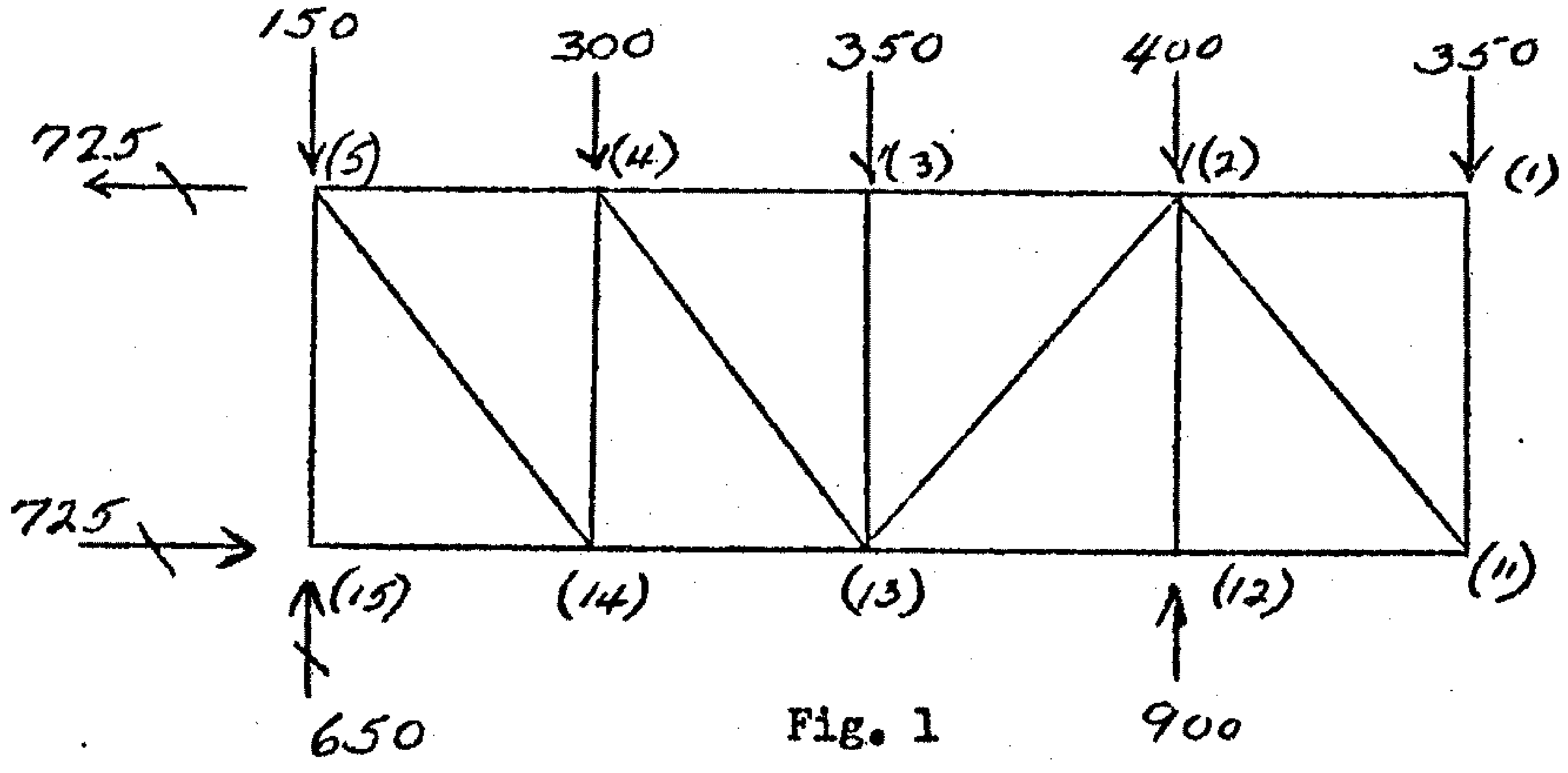


Fig. 1

The simplest method of familiarizing one's self with the graphic method is to follow through an example. Suppose we take the truss we solved analytically in Assignment 2. See Fig. 1. We will leave the joints numbered as in Assignment 2 to give an additional cross reference.

Fig. 2 shows the scale diagram of the truss with all external loads and reactions indicated. The space scale, 1"=30", is noted to the right of the Space Diagram. It is good practice to always indicate the scale in this fashion.

Beginning at the inner end, capital letters are placed between adjacent loads and reactions in order going around the truss to the right. Note that where more than one load or reaction act on a panel, as at the inner end points 5 and 15, a letter is placed between each of these loads, Also if no external loads act at panel points for several spans one letter applies to the entire distance. See G and H on Fig. 2.

Lower case letters a to h inclusive are placed in the triangles formed by the truss. With this notation all external loads and internal loads in members are defined by the adjacent letters through which they pass. Cross referencing between Figs. 1 and 2.

The load at panel point 1 becomes FG, that at panel point 12 becomes GH, etc.

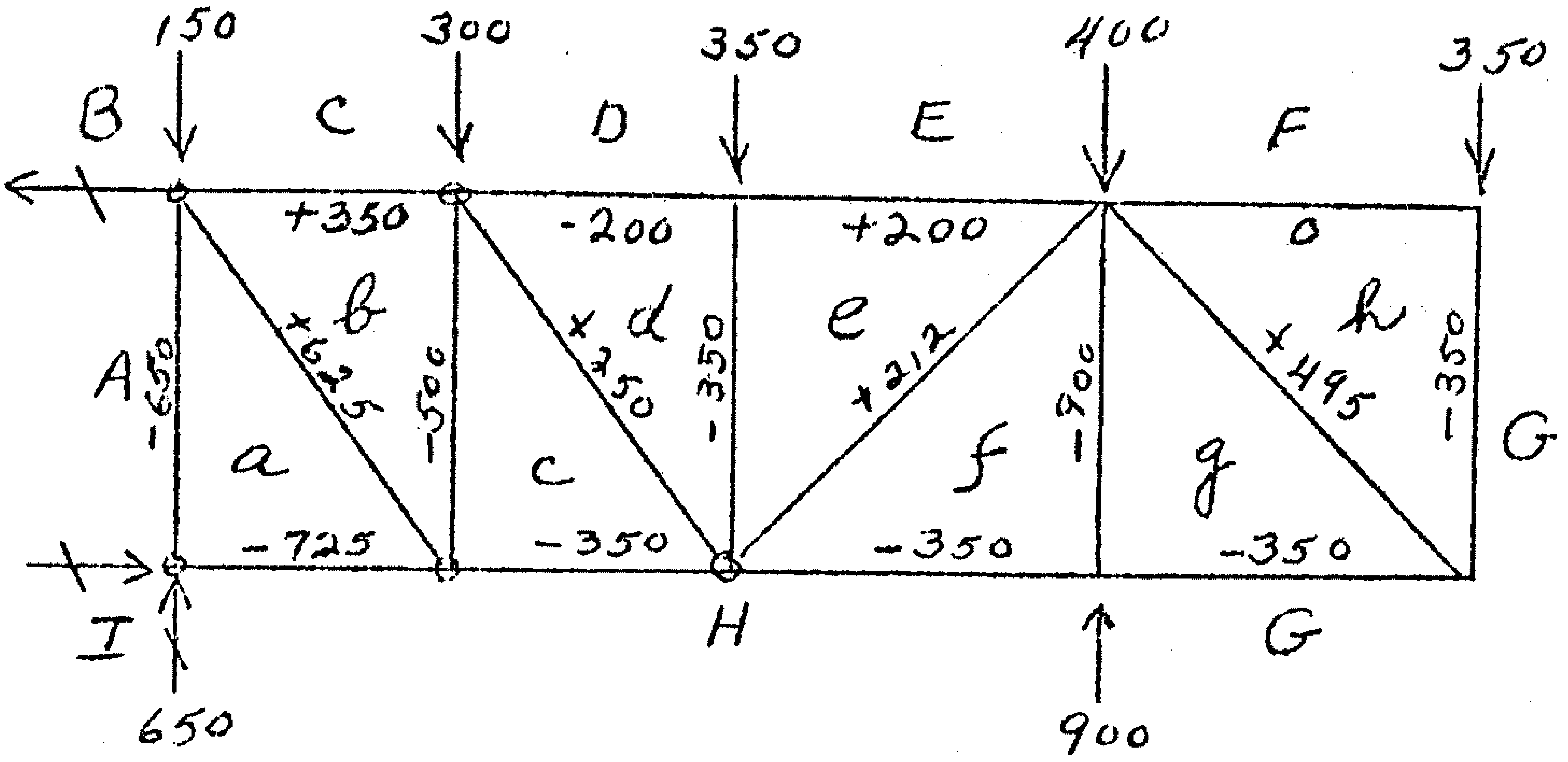
The load in member 1-11 becomes Gh, that in member 2-11 becomes gh, that in member 2-12 becomes fg, etc.

The importance of exact adherence to the same definite order in going around the entire truss and around each joint is apparent as we study the force diagram of Fig. 2. It will be shown that use of a simple rule will enable the student to read the type of load (tension or compression) directly from the Force Diagram. We will follow the procedure through on the Force Diagram of Fig. 2.

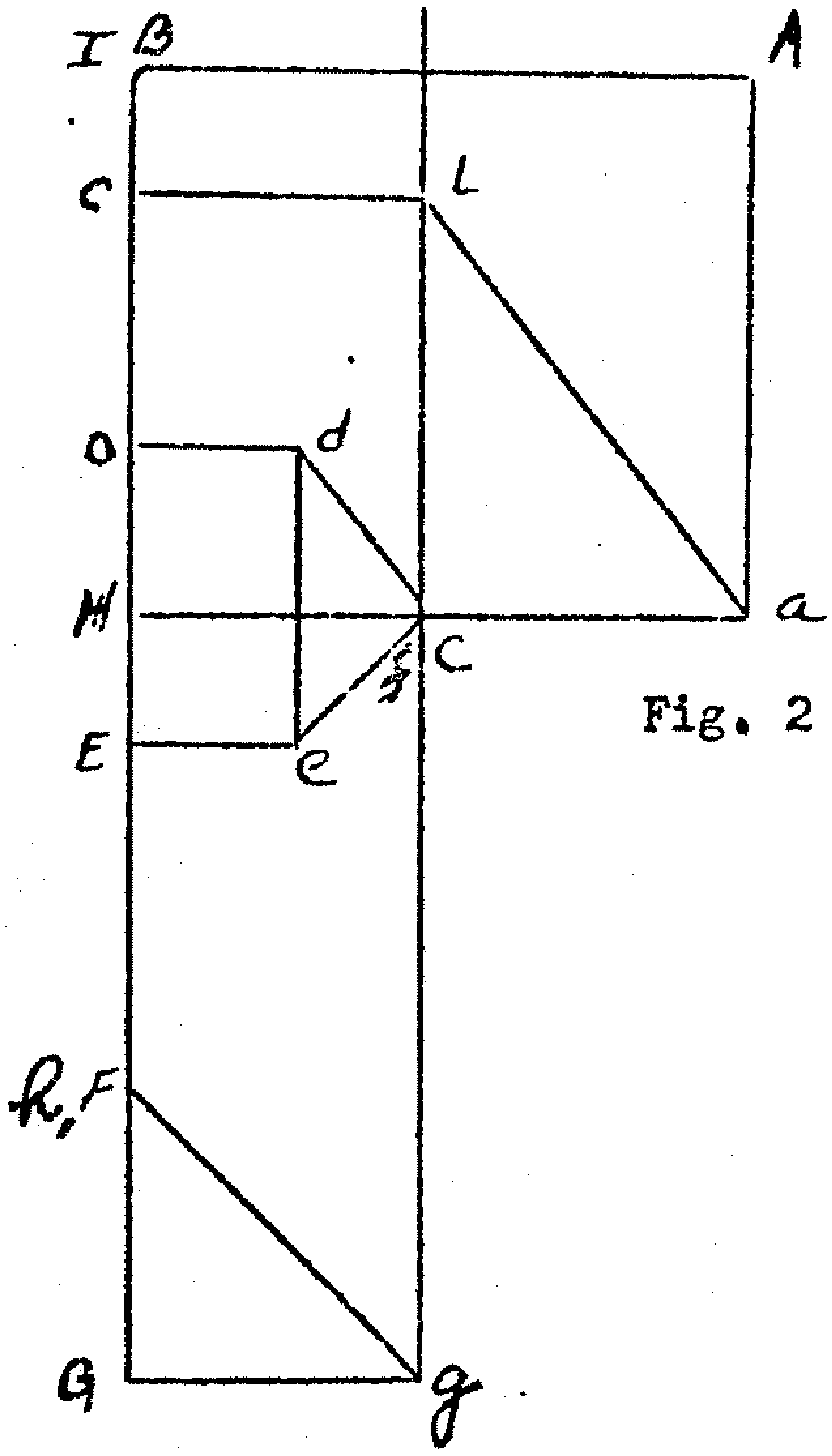
We begin our plot of the diagram of external loads and reactions at A and go round the truss clockwise. Our initial point then is A. A 725 lb. load acts between A and B to the left. This load is laid off to scale from A to B paralleling the direction in which it acts on the truss and measuring off 725 lb to the force scale (1"=400# in this case". Point B is thus established by measurement from A to B.

The next load we encounter is BC, acting from B to C (downward). This load is laid off along a vertical line downward from B and point C is established by measurement. Similarly loads CD, DE, EF and FG are laid off in a downward direction. The termination of each force is shown by the pertinent letter.

When we pass to the bottom chord of the truss load GH acts upward from G to H. This force is measured upward from G on the scale diagram and point H is established. HI, the horizontal reaction at panel point 15, also acts upward from H to I. Point I falls on the same location as point B, which checks the summation of horizontal external forces on the diagram. Force IA acts to the right from I to A, bringing us back to the starting point and checking the summation of side forces. We have therefore plotted the proof that $\Sigma H = 0$ and $\Sigma S = 0$.



SPACE DIAGRAM
1"=30"



LOAD FACTOR=1

FORCE DIAGRAM
1"=400#

Fig. 2

Now we begin our plot of forces in the members of the truss. Either joint 1 or joint 15 could have been chosen as a starting point, since at each of these joints only two unknown forces are acting. We will start with joint 1. We have the locations of points F and G on the diagram which give us the ends of vectors Gh and hF. We know the directions of these forces parallel the same members in the space diagram. We start with our known force FG and go around the joint in a clockwise direction. FG acts downward from F to G. Gh acts upward from G to h. Upward toward joint 1 for this member would indicate a push toward the joint and therefore compression load in Gh. h and F coincide on the diagram. Therefore there is no load in member hF. This is also self evident at joint 1 since hF is the only horizontal member at this joint and there is no horizontal external load.

Having determined the compression in Gh, the only unknown members at joint 11 are Gg and gh. We can therefore go around joint 11. Beginning with our known member and going clockwise, hG is compression and therefore acts downward from h to G at joint 11.

hG and Gg intersect at g out of the right of G, going around the joint following the forces in sequence hG acts toward the joint (compression) so we go from h to G. Gg acts toward the joint, i.e. to the right and is therefore compression. gh acts away from the joint and is tension.

Our next joint is 12 at which now that Gg is established only members Hf and fg remain unknown. We go around this joint clockwise beginning with the known compression gG which acts toward the joint, therefore to the left on the Force Diagram. GH acts upward toward the joint. Hf acts to the right and fg acts downward toward the joint and is therefore compression. The four forces acting around this joint are thus balancing compressions, which fact could have been known from inspection.

We now know the forces in members Fh (= 0), hg, gf at point 2 and of course the external load of 400 lbs. EF. Only fe and eE are unknown, so this joint may now be solved.

Continuing from joint to joint we eventually finish with member Aa. In this case this member closes the diagram by intersecting starting point A. This fact indicates that the moments on the truss balance.

In all cases the closure will not be exact since it depends upon accuracy in transferring parallel lines from the space diagram and in measuring the vectors for the external forces off correctly.

The values of the loads should be transferred to the space diagram giving them the conventional signs of + for tension and - for compression. Inspection of the forces will indicate if an error exists. By laying a triangle across any section of the truss an thus mentally cutting it loose as a free body, the student can readily detect and error in sign or magnitude of a load. Such self-checking is of primary importance in turning out usable accurate work.

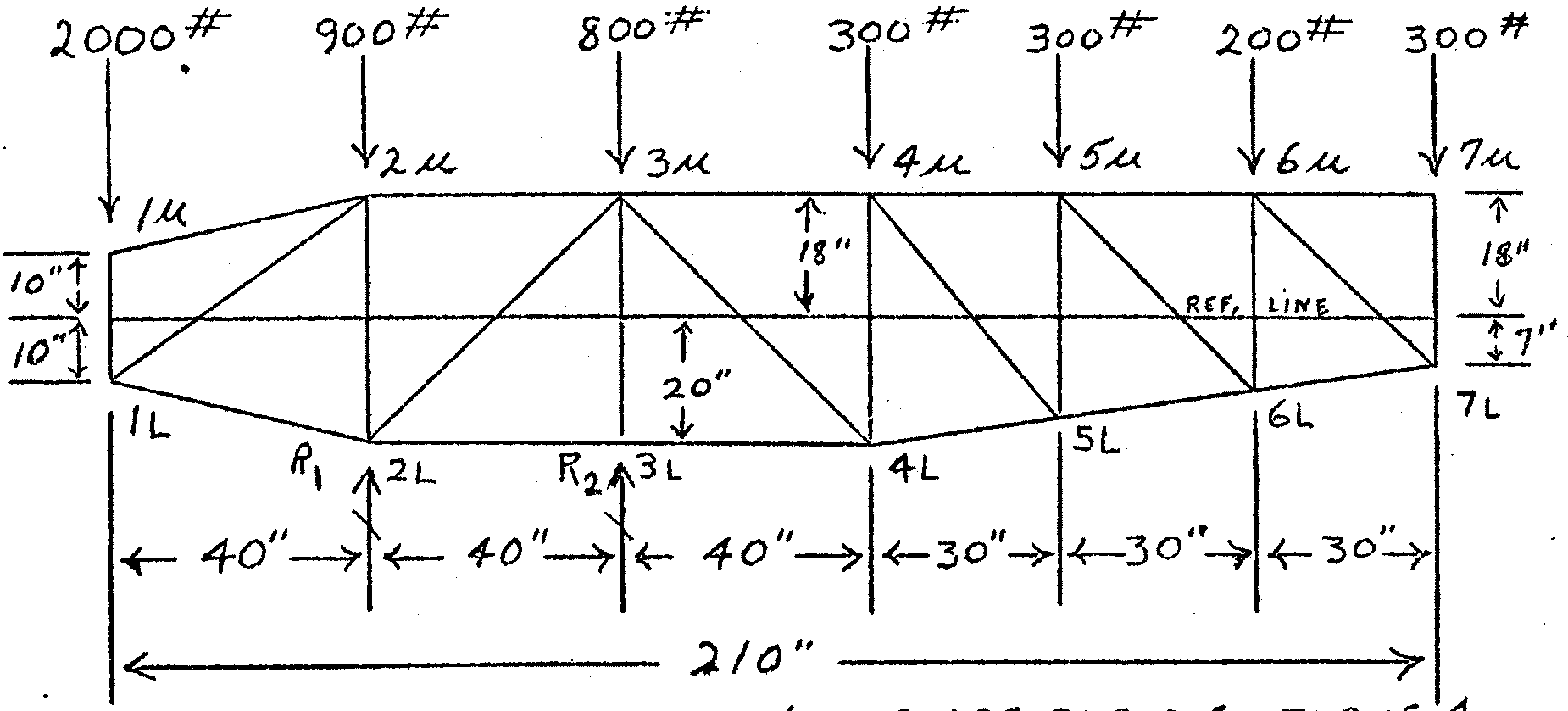
Another advantage in placing the loads on the truss is that it makes them readily available for use by the designer or checker. Note that the loads for which this truss has been analyzed correspond to a load factor of 1.0. This fact is noted on Fig. 2. The load factor, whether for 1.0 or for a design load factor, should always be noted on the diagram. This prevents misunderstandings in using the results.

Always

Note the space scale.
Note the force scale.
Note the load factor.
Transfer the loads to the truss, after they are measured from the force diagram, giving them the proper signs for tension (+) or compression (-).
Check your final results for reasonableness, both as to amount and sign of the values.

Problem Assignment.

4:1 Make a graphical solution for the loads in the members of the fuselage truss shown in the drawing. All members are steel tubes and can take either tension or compression loads.



LOADS ARE FOR A FACTOR OF 4

Procedure:

1. Solve for reactions at R_1 and R_2. Take ΣM about R_1 to obtain R_2 and ΣM about R_2 to obtain R_1. Check ΣV Forces = 0.
2. Draw scale diagram. Suggested scale 1"=30".
3. Follow notation given in preceding notes.
4. Draw Force Diagram. Suggested scale 1"=1000#.
5. Transfer loads to scale diagram.

Note: Either 1u or 7u may be taken as the first joint for plotting the Force Diagram.

BASIC STRESS ANALYSIS

ASSIGNMENT FIVE

Assignment 5

(a) Special Problems in Graphical Solution
(b) Algebraical Solution of the Tripod

Part (a)
THE USE OF PHANTOM MEMBERS

Sometimes cases occur in graphical analysis which are most readily solved by the use of phantom members. The term phantom member is applied to an imaginary member of a truss which is assumed for convenience to replace a group of members whose loads connot be readily determined because more than two unknown members meet at a panel point. Generally a truss requiring a phantom member can be solved by computing loads in some of the members, but the complete graphical solution is not difficult and will be given here for a typical case.

a_1 Phantom members to replace engine section.

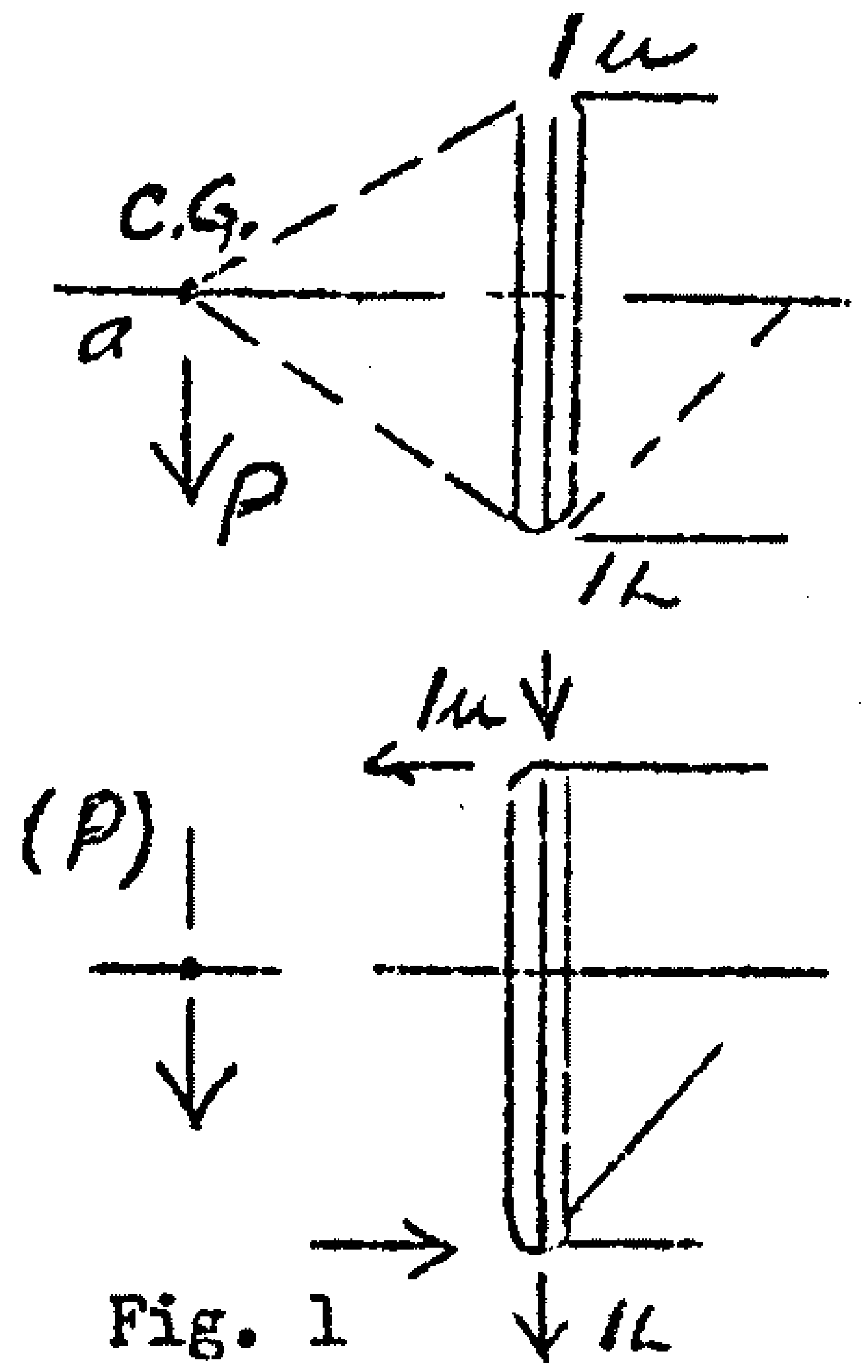


Fig. 1
Side View of Engine Section
O of Fuselage

Suppose member 1_u-1_L in Fig. 1 represents the mounting ring for an engine whose center of gravity is at "a". The effect of a load at "a" on the truss can be calculated by assuming the shear distribution at 1_u and 1_L to balance the moment of P about the ℄ of the ring. Exactly the same result is obtained if we replace the effect of the engine structure in transferring loads to the ring by a **pair** of Phantom members connecting the center of gravity of the engine section with panel points 1_u and 1_L. It is advisable that the student set up a type problem for himself and solve it both ways to convince himself of the truth of this statement.

a_2 K-brace in the side of a fuselage.

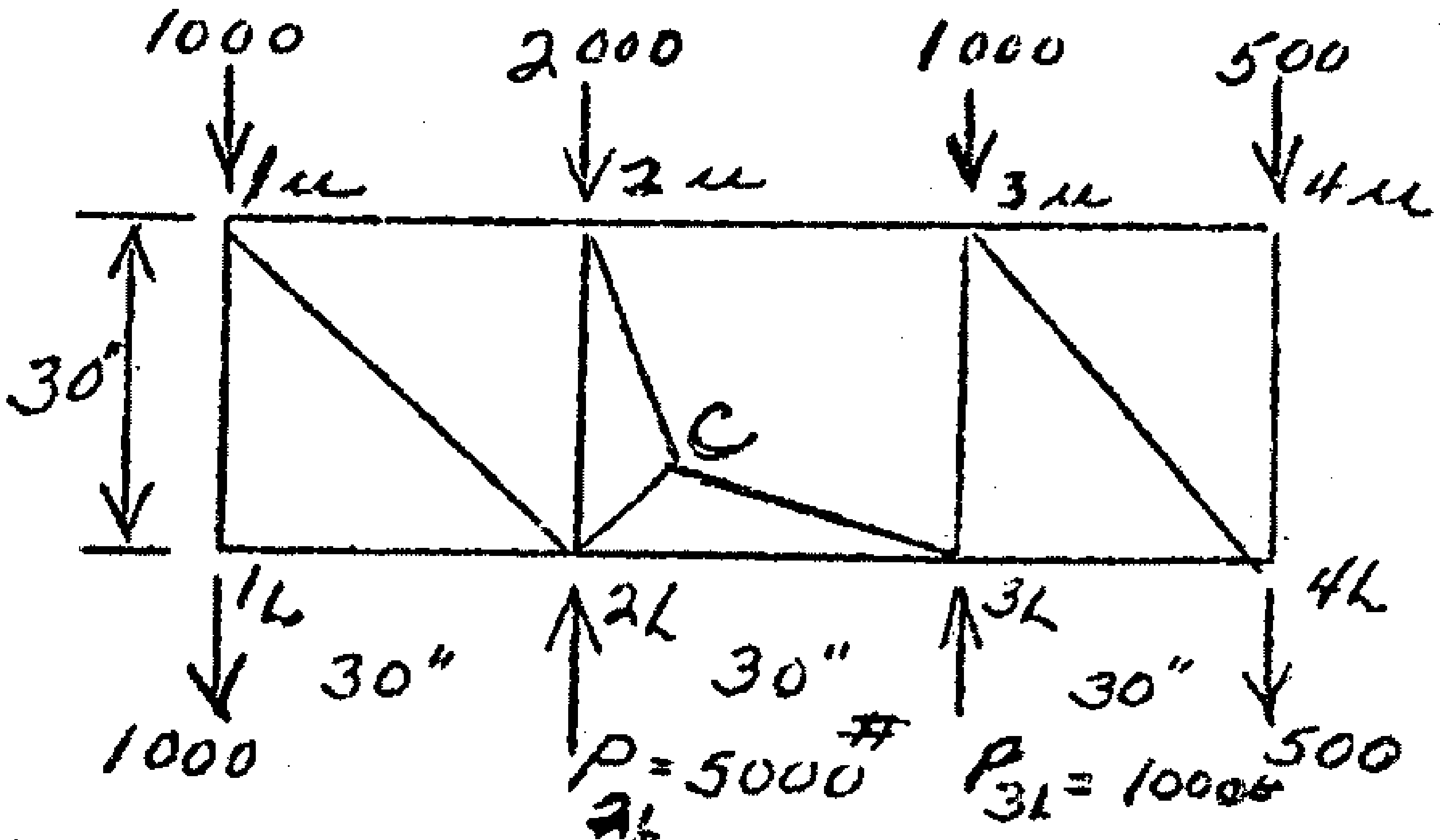


Fig. 2

To provide sufficient opening for a door in the side of a trussed fuselage it is sometimes necessary to resort to a tube pattern as shown in Fig. 2. This figure represents a section of side truss of a fuselage with down loads at the panel points 1_u, 2_u, 3_u, 4_u, 1_L and 4_L resisted by reactions at 2_L and 3_L. The shear bracing in panel 2 to 3 is commonly known as a "k-brace". If one begins the Force Diagram at the front, 1_L, he will proceed without trouble until he reaches panel point 2_u or 2_L at which he will find himself with three unknown forces. Since we can solve graphically for only two unknown forces at a joint one of three methods must be resorted to at this point.

1. The Force Diagram may be plotted from both ends toward the joint 2_u and 2_L This solution is indicated on Fig. 3 (a).

From the rear, joints 4_u, 4_L, and 3_u are solved in order, the solution of joint 3_u giving the load in member 2_u-3_u. From the front joints 1_L and 1_u are solved in the order given the solution of joint 1_u giving the load in member 1_u-2_u. Having the loads in members 1_u-2_u and 2_u-3_u it is possible to complete the solution around joint 2_u since the only unknown values are the loads in members 2_u-2_L and 2_u-C. All other joints are then left open to solution since after solution of joint 2_u joints 2_L and C are left with only two unknown members at each. (This truss could also have been solved from the rear only by taking the joints in the order 4_u, 4_L, 3_u, 3_L, C, 2_u, 2_L, 1_u, 1_L. The student should verify this solution by following through the force diagram.)

The solution of the truss of Fig. 2 follows:

Solving first for the reactions

$$\Sigma M_{2_L} = 0 \qquad \overset{+}{\circlearrowleft}$$

$$(R_{3_L})(30) + (1000 + 1000)(30) - (1000)(30) - (500 + 500)(60) = 0$$

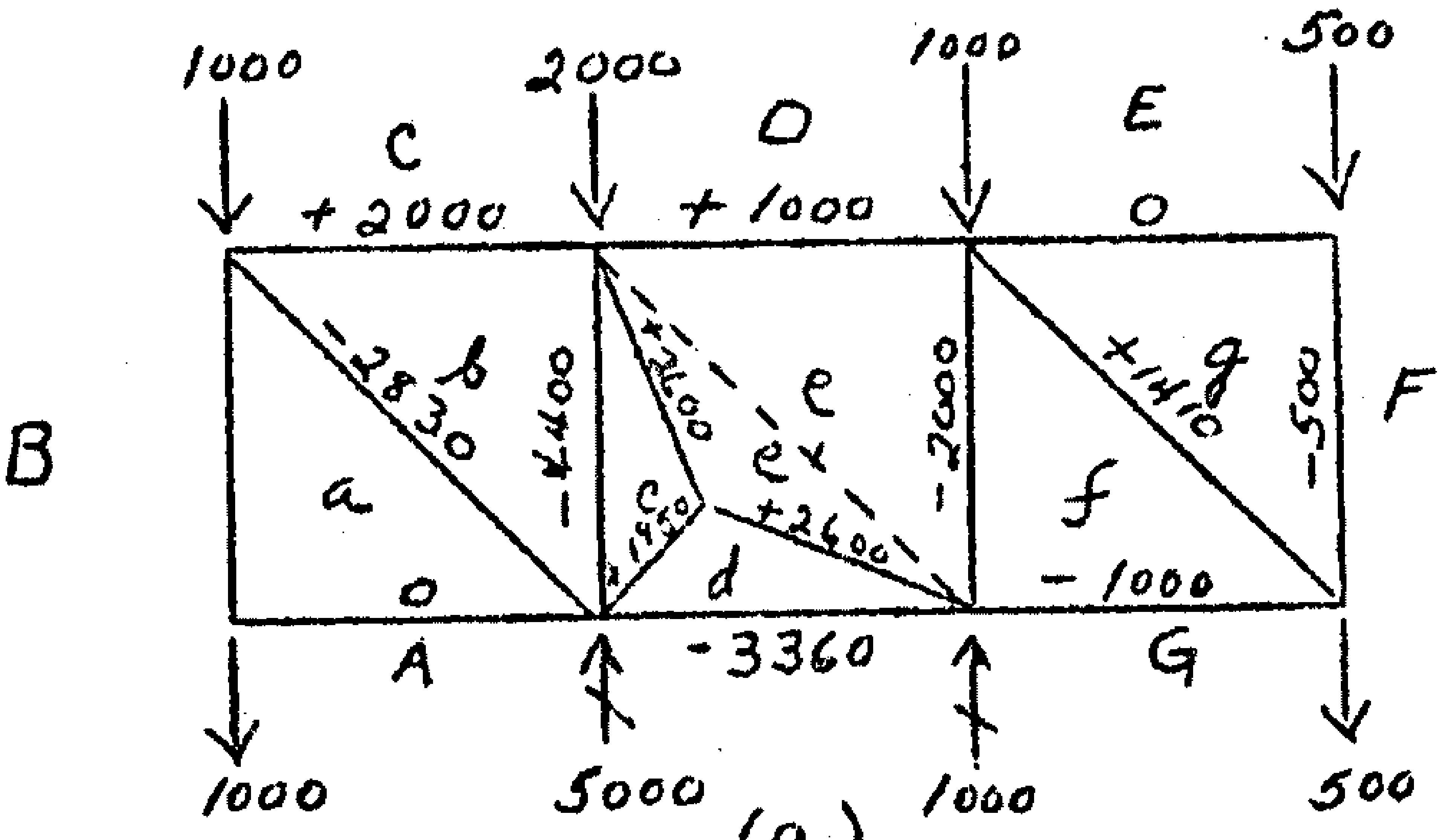
$$(R_{3_L})(30) = 60{,}000 - 30{,}000 - 60{,}000 = 0$$

$$30R_{3_L} = 30{,}000 \qquad R_{3_L} = 1000 \text{ lbs.} \qquad R_{2_L} = 5000 \text{ lbs.}$$

Then plotting the diagram Fig. 3, the dashed lines shown that portion which is solved from rear to front and the full lines that which is solved from front to rear. Force Diagram Fig. 3 (b).

2. A "phantom" member may be substituted for the cluster of members cd, ce, and de, and used to establish point "e" by establishing the force in member De. The phantom member e-e' is shown in broken line on the scale diagram, Fig. 3 (a). The graphic solution is shown on Fig. 3 (c). Use of this member is made to locate point e only. After this point is established, the only unknown members at joint 2_u are ec and bc so that the joint may be solved. Then joint c is solved, the only unknowns being cd and ed, and the remaining joints may be solved in order proceeding toward the rear.

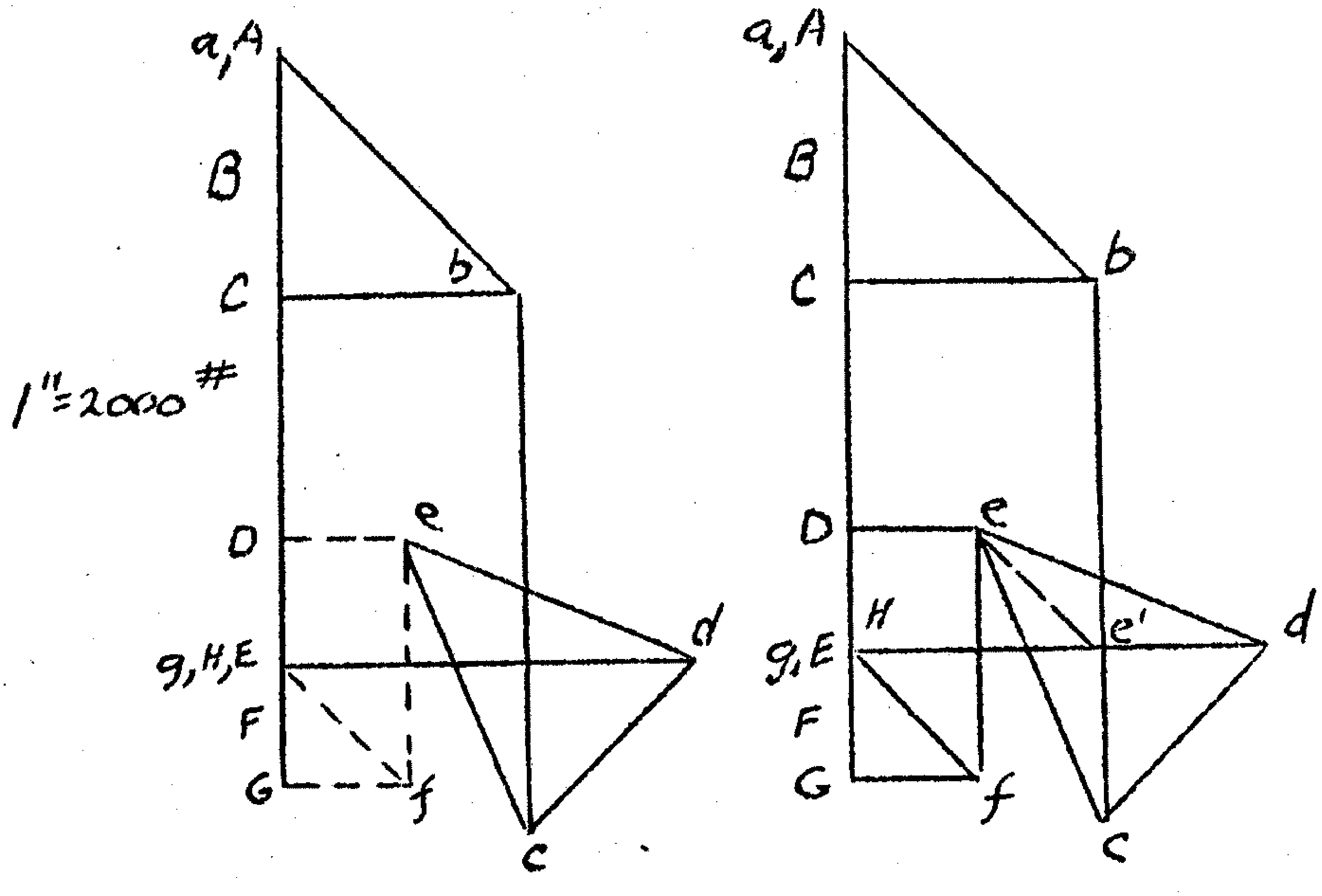
3. The use of the phantom member suggests the third method of solution, which is semigraphical. It consists of taking moments around panel point 3_L with the rear portion of the truss taken out as a free body and computing the load in member 2_u-3_u (De). When this load is known point e may be established on the force diagram by laying off the direction of member De and measuring the correct force distance out from D to e.



(a)

Fig. 3

1" = 30"



(b)

PLOTTING INWARD FROM BOTH ENDS

(c)

USING "PHANTOM" MEMBER e-e'

Part (b)
ALGEBRAICAL SOLUTION OF THE TRIPOD

A problem which appears frequently in aircraft analysis requires the solution of a tripod. This problem takes us into 3-planes and while it may be solved graphically is also susceptible to a straightforward algebraical treatment. The method is based upon the fundamental equations of equilibrium $\Sigma V = 0$, $\Sigma H = 0$, and $\Sigma S = 0$.

A solution will be given for the landing gear truss shown in space diagram on Fig. 4. The solution is based upon the principle that the loads in members are proportional to their lengths and that the components of loads in members are proportional to the components of their lengths. If the relation

$$\frac{\text{Vertical Component of Length}}{\text{Total Length}} = K \quad \text{then the relation}$$

$$\frac{\text{Vertical Component of Load}}{\text{Total Load}} = K \qquad \text{or}$$

$$\frac{\text{Vertical Component of Length}}{\text{Total Length}} = \frac{\text{Vertical Component of Load}}{\text{Total Load}}$$

Similar equations may be written for the horizontal and side components.

The first step in a solution then is to determine the components of lengths of the members involved. For the axle and brace struts of Fig. 4, the computations are shown on the table below the figure. A natural check of results lies in the fact that the square root of the sum of the squares of the components should equal 1.0.

$$(V^2 + H^2 + S^2)^{\frac{1}{2}} = 1$$

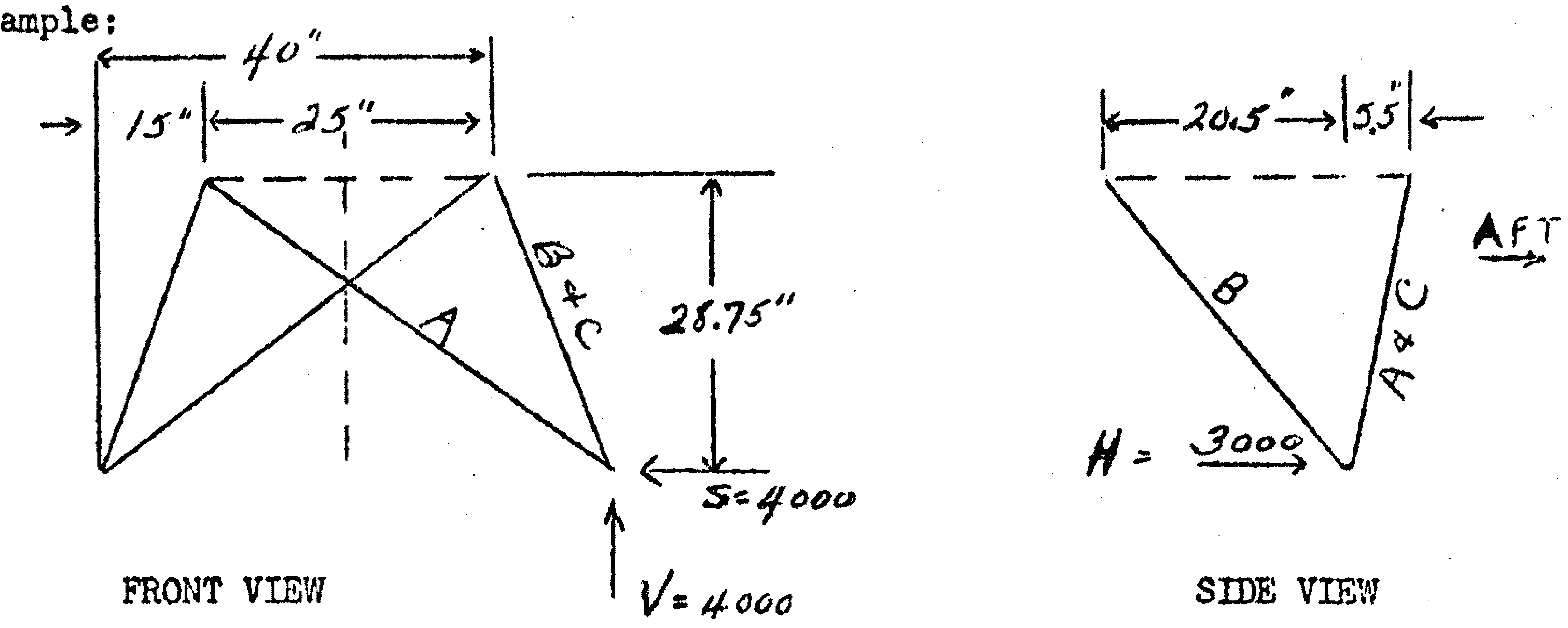
Solution for the tripod in our example consists of balancing the forces at point "a" in the three reference planes. The external applied loads are

V = 4000 lbs. acting upward
H = 3000 lbs. acting aft
S = 4000 lbs. acting inward

The equations of equilibrium are written assuming tension in all members meeting at the joint. Then if a load in a member comes out plus its sign was correctly chosen and it is in tension. If it comes out minus the member is in compression Since these signs coincide with out standard sign conventions for tension and compression we may use the results as they come from the solutions of the equations.

In setting up the equations of equilibrium we must look at the joint to see which direction the particular component of the member would be acting if the member were in tension. We will follow through the solution on the subject example.

Example:



FRONT VIEW

SIDE VIEW

Fig. 4

Member	Length	Length^2	Component	
A Axle	28.75 5.5 40.0 49.57	826.5625 30.25 1,600.00 2,456.8125	= .5800 = .1110 = .8069	V H S
B Strut	28.75 20.5 15.0 38.36	826.5625 420.25 225.0 1,471.8125	= .7495 = .5344 = .3910	V H S
C Strut	28.75 5.5 15.0 32.89	826.5625 30.25 225.0 1,081.8125	= .8741 = .1672 = .4560	V H S

We have three unknowns at joint a, namely the loads in members A, B, and C. We have three known loads acting on the joint as listed on page 4. If external loads had been acting in only two of the reference planes or even only one our solution would still have been valid. It would merely mean that the internal forces in the members would be in complete balance within themselves in the direction in which no external load was applied.

Having three equations ≤V, ≤H, ≤S = 0 and only three unknowns we may solve for the loads in the members by solution of three simultaneous equations.

Setting up our equations and using the symbols A, B, and C for the loads in these respective members:

$\Sigma V = 0$ $\quad \uparrow +$ up $\quad$ A, B, and C would all have upward components at "a" if they were in tension.

Then

$$.5800A + .7495B + .8741C + 4000 = 0$$

$$.5800A + .7495B + .8741C = -4000 \qquad (1)$$

$\Sigma H = 0$ $\quad \rightarrow +$ aft

$$.1110A - .5344B + .1672C + 3000 = 0$$

$$.1110A - .5344B + .1672C = -3000 \qquad (2)$$

$\Sigma S = 0$ $\quad \leftarrow + =$ in

$$.8069A + .3910B + .4560C + 4000 = 0$$

$$.8069A + .3910B + .4560C = -4000 \qquad (3)$$

Dividing equation (1) through by .58

$$1.000A + 1.29B + 1.505C = -6900 \qquad (1a)$$

Dividing equation (2) through by .1110

$$1.000A - 4.81B + 1.505C = -27000 \qquad (2a)$$

Dividing equation (3) through by .8069

$$1.000A + .485B + .565C = -4960 \qquad (3a)$$

Subtracting equation (3a) from equation (1a)

$$\begin{array}{rrrrl} & \cancel{1.00A} + & 1.29B + & 1.505C = & -6900 \\ - & \cancel{1.00A} + & .485B + & .565C = & -4960 \\ \hline & & .805B + & .940C = & -1940 \qquad (4) \end{array}$$

Subtracting equation (3a) from equation (2a)

$$\begin{array}{rrrrl} & \cancel{1.00A} - & 4.81B + & 1.505C = & -27000 \\ - & \cancel{1.00A} + & .485B + & .565C = & -4960 \\ \hline & & -5.295B + & .940C = & -22040 \qquad (5) \end{array}$$

It happens that in this case the coefficients of C are equal in equations (4) and (5). We can therefore subtract equation (5) from equation (4) and eliminate C. This will leave us with an equation in which B is the only unknown. If the coefficients of C had not been equal we would have divided through each equation by the respective coefficient to give C a coefficient of unity in each case and then completed by eliminating C from the final equation.

Subtracting equation (5) from equation (4)

$$\begin{array}{rrrl} & .805B + & \cancel{.940C} = & -1940 \quad (4) \\ - & -5.295B + & \cancel{.940C} = & -22040 \quad (5) \\ \hline & 6.100B & & 20100 \end{array}$$

$$B = +3300 \text{ lbs.}$$

$$C = -4880 \text{ lbs.}$$

$$A = -3800 \text{ lbs.}$$

Problem Assignment.

5:1 Check the Σ V, Σ H, and Σ S about point "a" by taking the external loads and the components of the loads in members found on page 6.

5:2 Find the loads in members if the external side load of 4000 lbs. is removed, leaving the external loads at "a" V = 4000 lbs. up, H = 3000 lbs. aft.

BASIC STRESS ANALYSIS

ASSIGNMENT SIX

Assignment 6

USE OF THE CALCULUS

Before we enter the more involved phases of the beam theory it is well to review our fundamental algebra and calculus in regard to equations of curves and the areas under curves. For those already conversant with this phase of the work this assignment will represent a brief review of fundamentals; for those who have had no calculus this assignment will furnish the necessary tools for the solution of beam curves. For further home study of calculus the student is referred to Palmer "Calculus for Home Study".

From algebra we have the fundamental idea of referring a line in space to the coordinate x-x and y-y axes as shown in Fig. 1. Consider a segment of a line whose equation with respect to the coordinate axes is $y = x^2 + 2x + 3$.

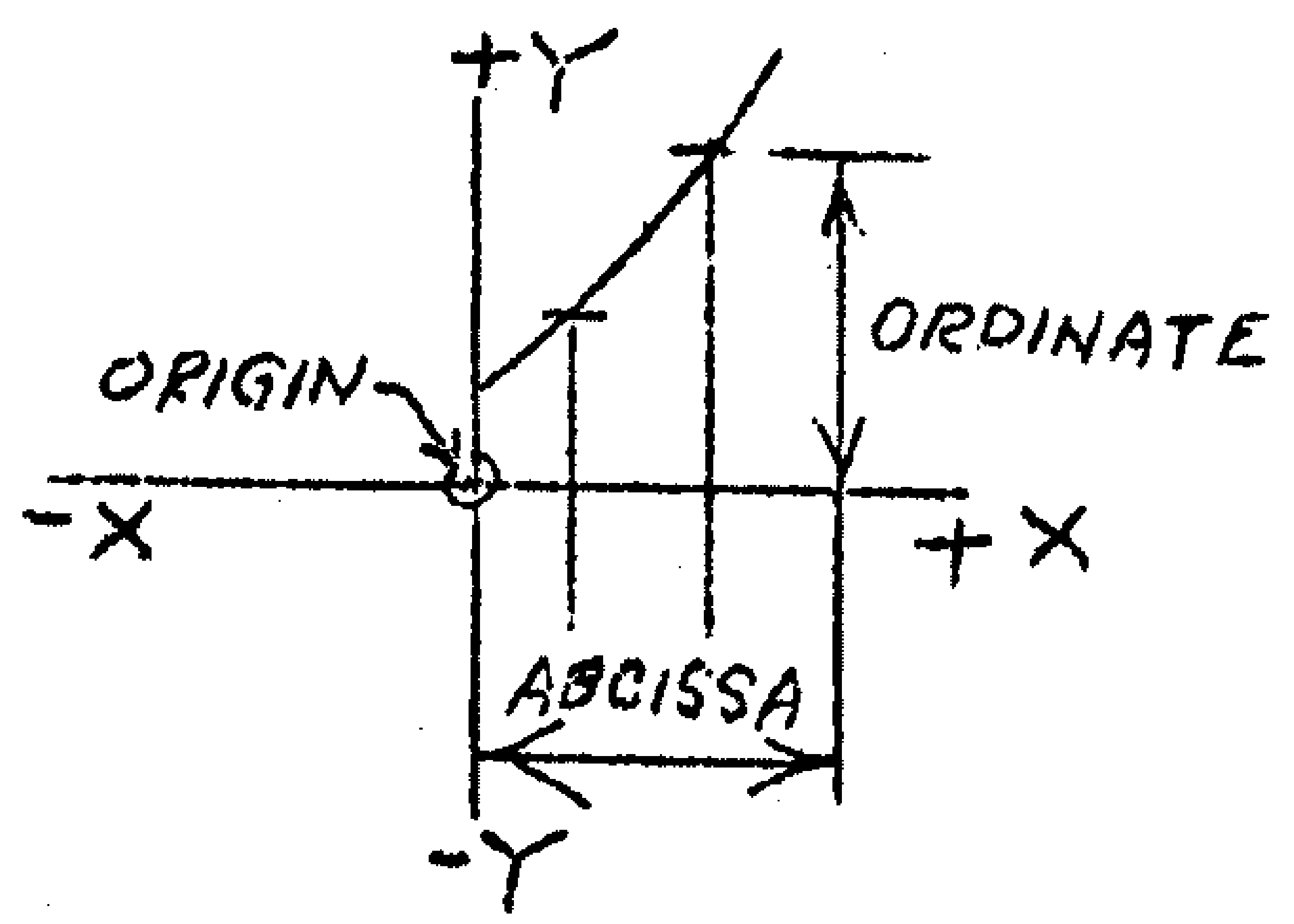


Fig. 1

+y represents the distance above the x-x axis, the ordinate to the curve, at any value of the abcissa, x.

To obtain values for the ordinate, y distance, at any point on the curve we substitute the corresponding abscissa, x value, and solve the equation for y. For example when $x = 0$

$$y = (0)^2 + (2)(0) + 3 = 3$$

when $x = 1$, $y = (1)^2 + (2)(1) + 3 = 6$

when $x = 2$, $y = (2)^2 + (2)(2) + 3 = 11$, etc.

Simple algebra then gives us the equation of the line. Suppose that we wish the area enclosed between the line and the reference axes, shown cross-hatched in Fig. 2 (b). An approximation could be obtained by cutting the area into a series of strips and taking the area of each strip as the product of its width and the height of the ordinate at the center of the strip. The only error introduced would be in the fact that the slope of the line at the top of each segment was not constant, but was changing, and this error could be made very small by taking the strips in narrow widths. If we sum up all the areas in the narrow strips Δx, we could write the equation of the area

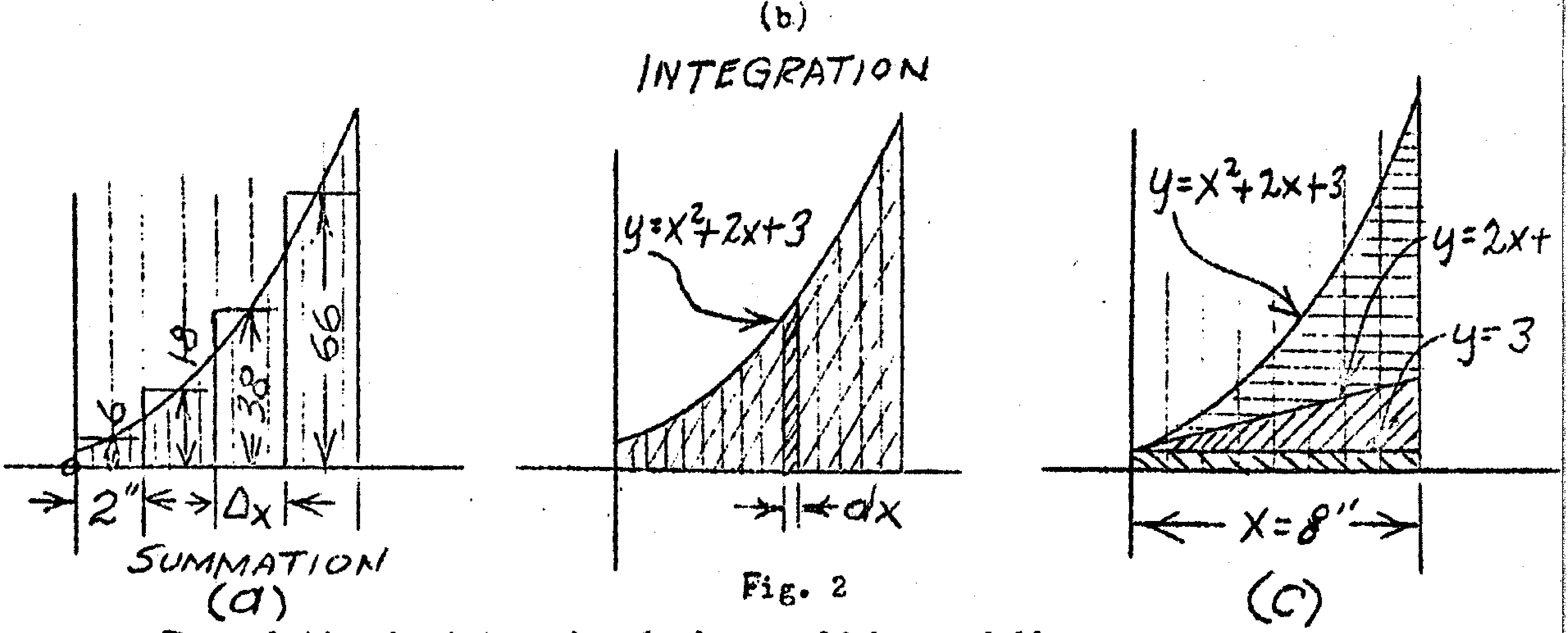
$$A = \sum (\Delta x)(y), \text{ where } \sum \text{ is the summation sign}$$

Now if we take the widths of the strips infinitesimally small we term the widths, dx. If we can visualize the widths so small that the ordinate to the curve is the true height over that width the summation of all the products dx times y would give the exact area under the curve. Finding such a summation is the calculus function termed integration. Suppose for example that we wished to determine the area under the curve $y = x^2 + 2x + 3$, between $x = 0$ and $x = 8$. We will say that for taking the summation $\sum(\Delta X)(y)$ we will divide the area into 4 strips of width $\Delta_x = 2$" (Fig. 2, a). We must next determine the middle ordinate for each strip, that is the values of "y" at $x = 1, 3, 5$, and 7.

Where $x = 1$, $y = 6$; where $x = 3$, $y = (3)^2 + (2)(3) + 3 = 18$
Where $x = 5$, $y = (5)^2 + (2)(5) + 3 = 38$
Where $x = 7$, $y = (7)^2 + (2)(7) + 3 = 66$

Then the summation of area under the curve between $x = 0$ and $x = 8$ based on a value $\Delta_x = 2$ would be $\sum \Delta_{xy}$ .....

$$\sum = (2)(6) + (2)(18) + (2)(38) + (2)(66) = 256\,\square''$$



SUMMATION
(a)

Fig. 2

(C)

The solution by integral calculus would be as follows:

Ordinate $y = x^2 + 2x + 3$

Width $w = dx$ (infinitesimally small)

$$\sum ydx = \int_{x=0}^{x=8} ydx = \int_{x=0}^{x=8} (x^2 + 2x + 3)dx = \Big[\frac{x^3}{3} + \frac{2x^2}{2} + 3x\Big]_{x=0}^{x=8} =$$

$$= \Big[0.333x^3 + x^2 + 3x\Big]_{x=0}^{x=8} = \Big[(0.333)(8)^3 + (8)^2 + (3)(8)\Big] -$$

$$\Big[(0.333)(0)^3 + (0)^2 + (3)(0)\Big] = \Big[170.67 + 64 + 24\Big] - \Big[0\Big] = 258.67\,\square''$$ which is the actual area. The student will note that the approximate solution, $(\sum \Delta xy)$, above was low by approximately one per cent.

Now let us discuss the steps in the integration. The integral sign, $\int$ indicates that we have taken the width of strips into which we have divided the area as infinitely small. The limits, values of x between which we have taken the area, are noted at the top at bottom of the integral sign thus $\int_{x=0}^{x=8}$

They indicate the boundaries of the inegration, along the x-x axis. In our example $x = 8$ is the upper limit and $x = 0$ is the lower limit.

Our solution consists of substituting for the ordinate at any point along the curve its value in terms of x and performing on this term. $(x^2 + 2x + 3)(dx)$, what we call integration. Let us first perform the indicated multiplication. The expression then becomes $(x^2dx + 2x\,dx + 3dx)$. To integrate we raise the exponent of the variable, x, by one in each term and divide the affected term by the new exponent. The express dx is dropped when integration is made, since the performance of the integration results in a summing up of area in terms of x. Performing the indicated integration

$$x^2dx \text{ becomes } \frac{x^{2+1}}{2+1} = \frac{x^3}{3}$$

$$2x^1\,dx \text{ becomes } \frac{2x^{1+1}}{2} = \frac{2x^2}{2} = x^2$$

$3dx$ could be written $3x^0dx$ since any quantity to the zero exponent is equal to 1.

$$\text{Then } 3x^0dx \text{ becomes } \frac{3x^{0+1}}{0+1} = \frac{3x}{1} = 3x.$$

Now on Fig. 2 (c) let us determine just what each of these terms meant in terms of actual area. The ordinate at each point $y = x^2 + 2x + 3$ could be divided into 3 parts, each being that portion represented by one term of the equation. This division is shown by drawing 3 separate curves, superimposed one on the other as shown by the different cross hatchings on Fig. 2 (c). The area of the rectangle at the bottom is the product of its height $y = 3$ and its width x or 3x. Where $x = 8$ its area is 24 sq. in.

The area of the triangular portion next above is equal to $\frac{1}{2}$ (partial ordinate $= 2x$) $(x = \text{base}) = \frac{2x^2}{2} = x^2$. Where $x = 8$, the triangular area $= 64$ sq. in.

The area of the segment bounded at the bottom by the line $y = 2x + 3$ and at the top by $y = x^2 + 2x + 3$ is the remaining term of the area equation or $\frac{x^3}{3}$.

Where $x = 8$ this area is 170.67 sq. in.

In the case of the area bounded by $x = 0$ and $x = 8$ the second portion of the solution for area was zero because $x = 0$. The lower limit, however, might be any value of x depending on the area desired. For example, suppose we wished the area between $x = 2$ and $x = 8$. We would then write our integral equation

$$A = \lessgtr y\,dx = \int_{x=2}^{x=8} (x^2 + 2x + 3) = \Big[0.333x^3 + x^2 + 3x \Big]_{x=2}^{x=8}$$

$$= \Big[(0.333)(8)^3 + (8)^2 + (3)(8) \Big] - \Big[(0.333)(2)^3 + (2)^2 + (3)(2) \Big]$$

$$= \Big[170.67 + 64 + 24 \quad - \Big[2.67 + 4 + 6 \Big] = 246.$$

What we have done is to sum up all the areas out to $x = 8$, sum up the areas out to $x = 2$, and by subtracting the latter from the form, determined the area between $x = 8$ and $x = 2$.

Had the same summation been made by the approximate method, see Fig. 2 (a), we would have summed up the areas of the three right hand rectangles or $(2)(18) + (2)(38) + (2)(66) = 244$ sq. in.; 2 sq. in. lower than the actual area obtained by integration.

Differentiation.

The reverse of the integral process is termed Differentiation. If we differentiate an equation our result is termed the Derivative. If we have an equation of the form $y = 0.333x^3 + x^2 + 3x$ and wish to obtain its Derivative we would write

$$\frac{dy}{dx} = d(0.333x^3 + x^2 + 3x)$$

$$\text{or} \quad \frac{dy}{dx} = (3)(0.333)x^{3-1} + 2x^{2-1} + 3x^{1-1}$$

$$\text{or} \quad \frac{dy}{dx} = x^2 + 2x + 3 \qquad \longleftarrow \text{The Derivative}$$

This is read, the derivative of "y" with respect to "x" is $x^2 + 2x + 3$. What we have done is to reduce each exponent of the variable, x, by one and multiply the term being worked with by its original exponent. If our original equation had read $y = 0.333x^3 + x^2 + 3x + 10$, the Derivative, $\frac{dy}{dx}$ would still have been $x^2 + 2x + 3$ for the Derivative of the last term, 10, a constant with no variable factor, is zero. Conversely if there is no constant term in an equation, a constant term might appear in the integral of the equation, in order to make it fit the physical facts. Therefore the complete solution for an integral equation should contain an additional Constant of Integration, C. This value turns out frequently to be zero, but should be included as a reminder every time an integral equation is written. Knowing the physical facts the Constant of Integration, C, is solved for by setting the integral equation equal to a known value of y at a given value of x, and solving the equation for the constant. The use of the Constant of Integration will be illustrated by solving for the load, shear and moment curves on the beam of Fig. 3.

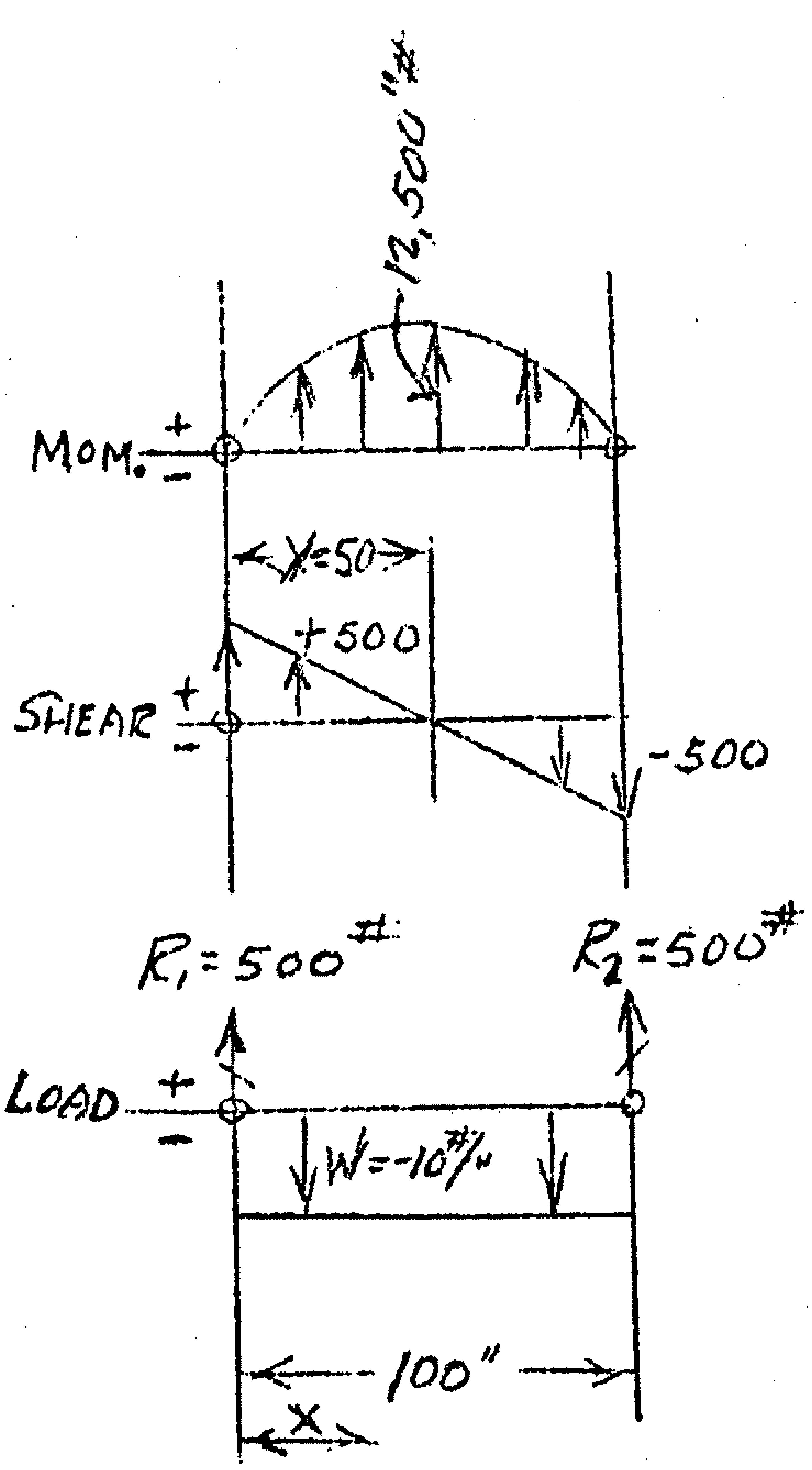


Fig. 3

Moment = $\int_0^x$ Shear = $\int_0^x 500 - 10x$

= $500x - 5x^2 + C_2$, but where x = 0 the moment is zero (because pin end beam). Therefore $C_2 = 0$ and moment,

$M = 500x - 5x^2$.

At x = 100, $M = (500)(100) - (5)(100)^2 = 0$ Check.

****

Shear = $\int_0^x$ Load = $\int_0^x -10 = -10x + C_1$

Where x = 0, Shear = 500 = C_1

Therefore Shear = $-10x + 500$

Where x = 100, Shear = $(-10)(100) + 500 = -500$ Checks.

****

Load equation $w = -10$

****

The Constant of Integration does appear and is used in the Shear Equation but drops out in the moment equation, in each case because of physical facts.

Problem Assignment.

6;1 (a) Integrate the equation $y = 3x^5 + 25x^4 + 16x^3 + 12x^2 + 10x + 8$ and find the area between x = 2 and x = 1.

(b) Differentiate the equation $y = 8x^3 + 9x^2 + 12x + 24$

(c) Find the area under the curve bounded by $y = x^2 - 3$, between x = 10 and x = 0. (Note: The total area desired is the algebraic sum of the plus and minus areas. Draw a sketch to clarify your thinking.) As a check on your final results determine the area under the x axis (- area) and the area above the x -axis (+ area) separately and add the results. To plot the curve to clarify your work give x values and solve for the corresponding values of y. For example to find where the curve crosses the x-x axis (y = 0), $y = x^2 - 3 = 0$ then $x^2 - 3 = 0$, $x^2 = 3$ and $x = \sqrt{3}$. Then the area below this value of x is negative and above it plus. The total area may be obtained at one summation or by two separate summations.

AIRPLANE
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PART THREE

ASSIGNMENT 7

ASSIGNMENT 8

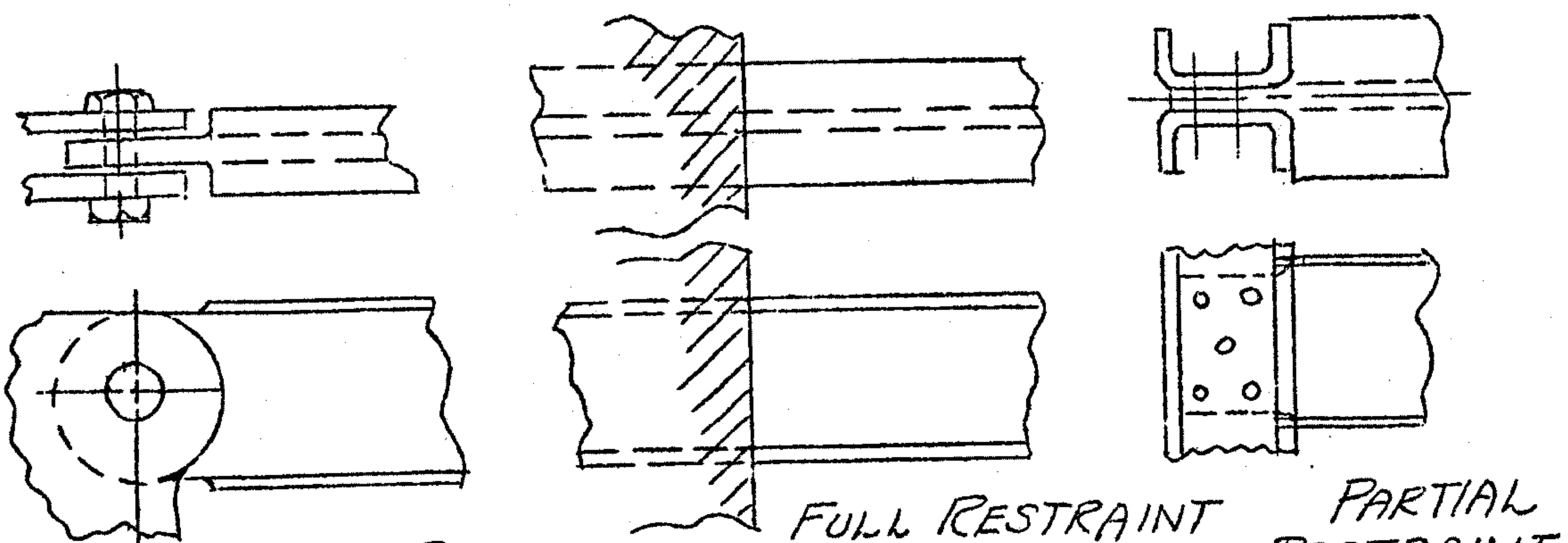
ASSIGNMENT 9

BASIC STRESS ANALYSIS

ASSIGNMENT SEVEN

BEAM FIXITY--LOADS, SHEARS AND MOMENTS WITH DISTRIBUTED LOADS

When we say that a beam is pinned at the ends we mean that the attachment to the supports is of such a type that the end of the beam may rotate freely when the beam deflects under loads. Such a joint would imply attachment with a frictionless pin. Actually such a condition is rarely realized. A certain amount of judgment is necessary to estimate what the end fixity or restraint of a beam actually is. In practical problems it is generally assumed when designing a beam of doubtful fixity at the ends, that the end fixity is one, i.e. the beam is pin ended and free to rotate. Quite often this effect is almost gained when the ends of the beam are rigidly riveted or bolted to a support, which in itself rotates freely. As an example, a heavy cross floor beam of a transport fuselage might be attached to light frames at the sides. These frames would generally be much more flexible than the floor beams themselves. They would therefore offer very little more resistance to bending of the floor beams than as if they were pinned to the beams, and the design of the floor beams would be made on the assumption that they were pin ended. (Fig. 1,c)


PINNED END BEAMS (a)

FULL RESTRAINT (b)

PARTIAL RESTRAINT (c)

Fig. 1

Full restraint is illustrated in Fig. 1, b. Full restraint at the supports implies that the support is so rigid that the bending axis of the beam if horizontal at the support before load is applied remains horizontal under the load. The actual amount of restraint will nearly always be more than pin end but never as much as full fixity, since no support will be perfectly rigid. If the rotation of the support can be calculated when the load is applied the actual fixity and hence the actual moment at the end of the beam can be computed. Methods for determining these values are given later. For the present we will deal only with simple beams pin ended on the supports.

The loads which act on beams may be represented by uniform (equally distributed) loads, distributed loads which vary by some mathematical law, such as triangular, or by a series of concentrated loads. For finding reactions, which is the first step in the solution of a beam, a distributed load may be represented by a single concentrated load, equal in magnitude to the total of the concentrated load and acting at its center of gravity. For example take:

(a) A simple beam loaded uniformly from end to end. The total load on the beam may be represented for calculation of reactions as 10 x 50 = 500 lbs. concentrated at the center. Shown ↓ (500) to indicate that it is an equivalent load.

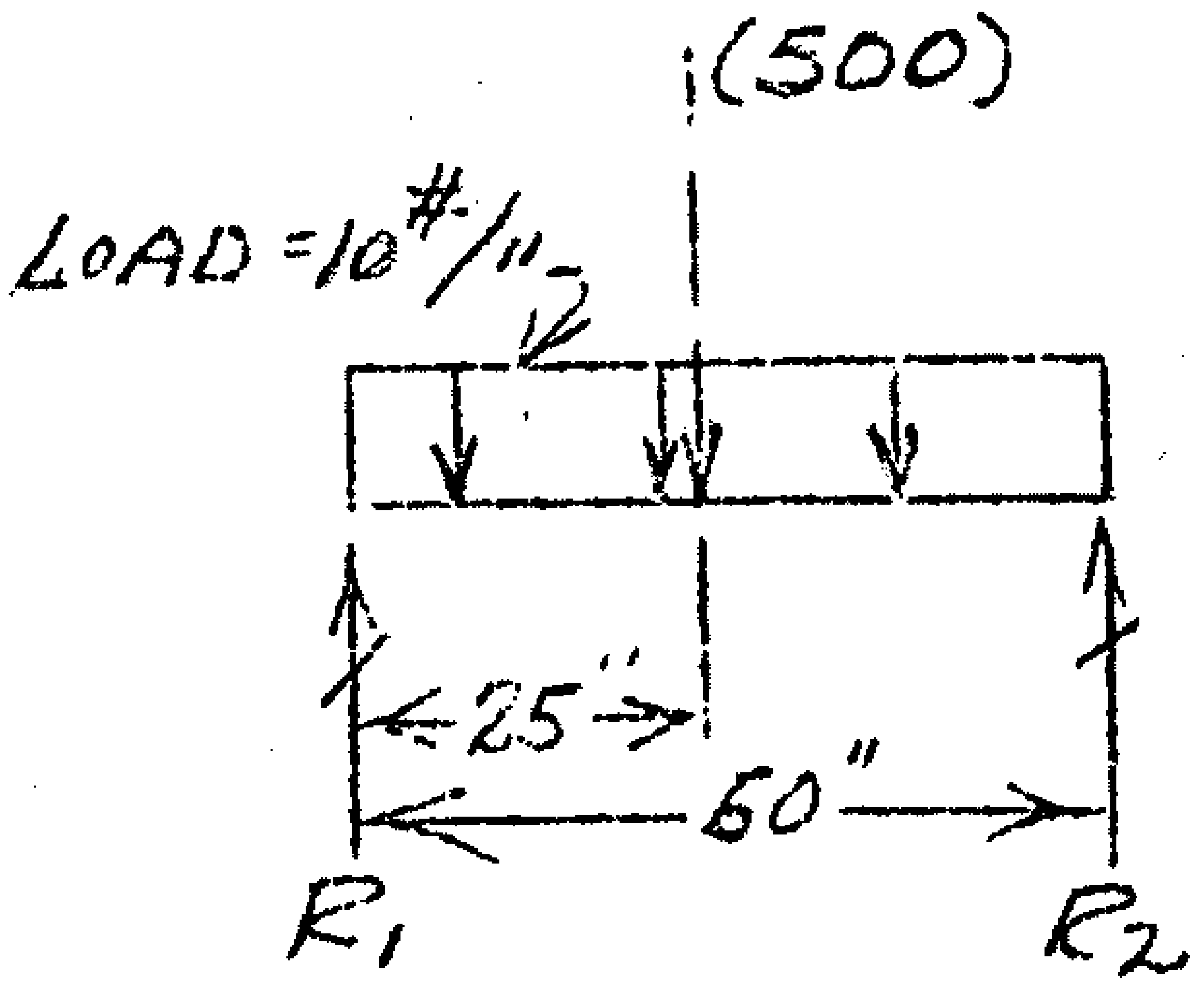


Fig. 2

Then for calculation of reactions

ΣM about $R_1 = 0$ ↶+)

$(R_2)(50) - (500)(25) = 0$

$50R_2 - 12{,}500 = 0, \quad 50R_2 = 12{,}500$

$R_2 = \frac{12{,}500}{50} = 250$ lbs.

Similarly $R_1 = 250$ lbs.

We could have anticipated from the symmetry of this beam that R_1 and R_2 would each have been half the total load, but it is well to develop fixed habits in setting down the moment equations, as this practice will avoid mistakes when more involved computations are made later on. Note that the equivalent concentrated load is useful only until the reactions are found. The true load distribution must be used in computing the shears and moments.

(b) A simple beam with a triangular loading. The load varies from zero at R_1 to 20 lbs./inch at R_2. The total load on the beam is $\frac{20 \cancel{+0}}{2}$ (50) = 500 lbs.

OK AS IT WAS.

Since the center of gravity of a triangle is one-third the distance from its base the point of application of this equivalent load is 16.67 inches from R_2.

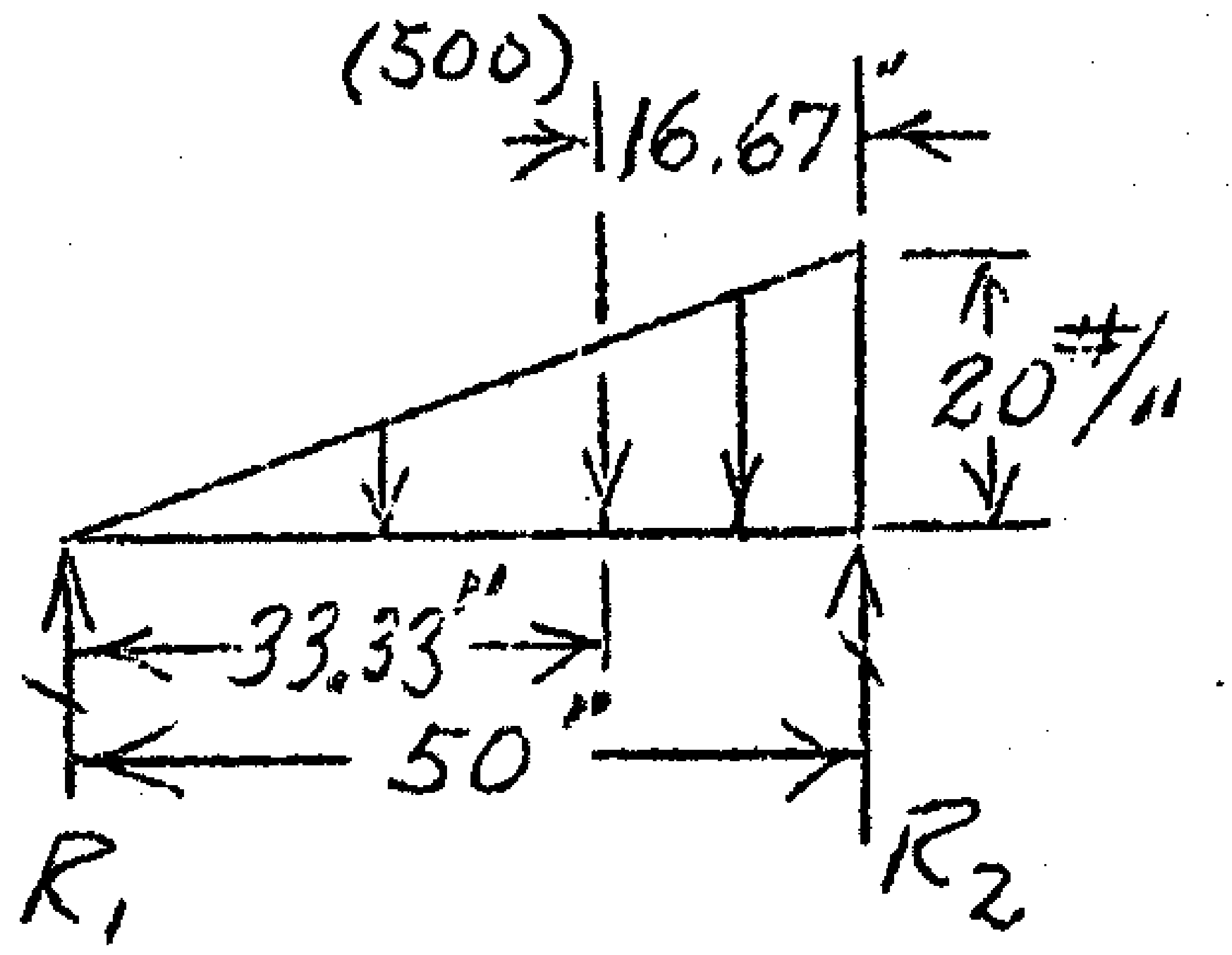


Fig. 3

ΣM about $R_1 = 0$ ↶+)

$(R_2)(50) - (500)(33.33) = 0$

$50R_2 - 16{,}667 = 0$

$R_2 = \frac{16{,}667}{50} = 333$ lbs., $R_1 = 167$ lbs.

by taking moments about R_2.

$R_1 + R_2 = 333 + 167 = 500$ lbs. Check. ΣV = zero.

(c) A simple beam with a uniform load in part of the span.

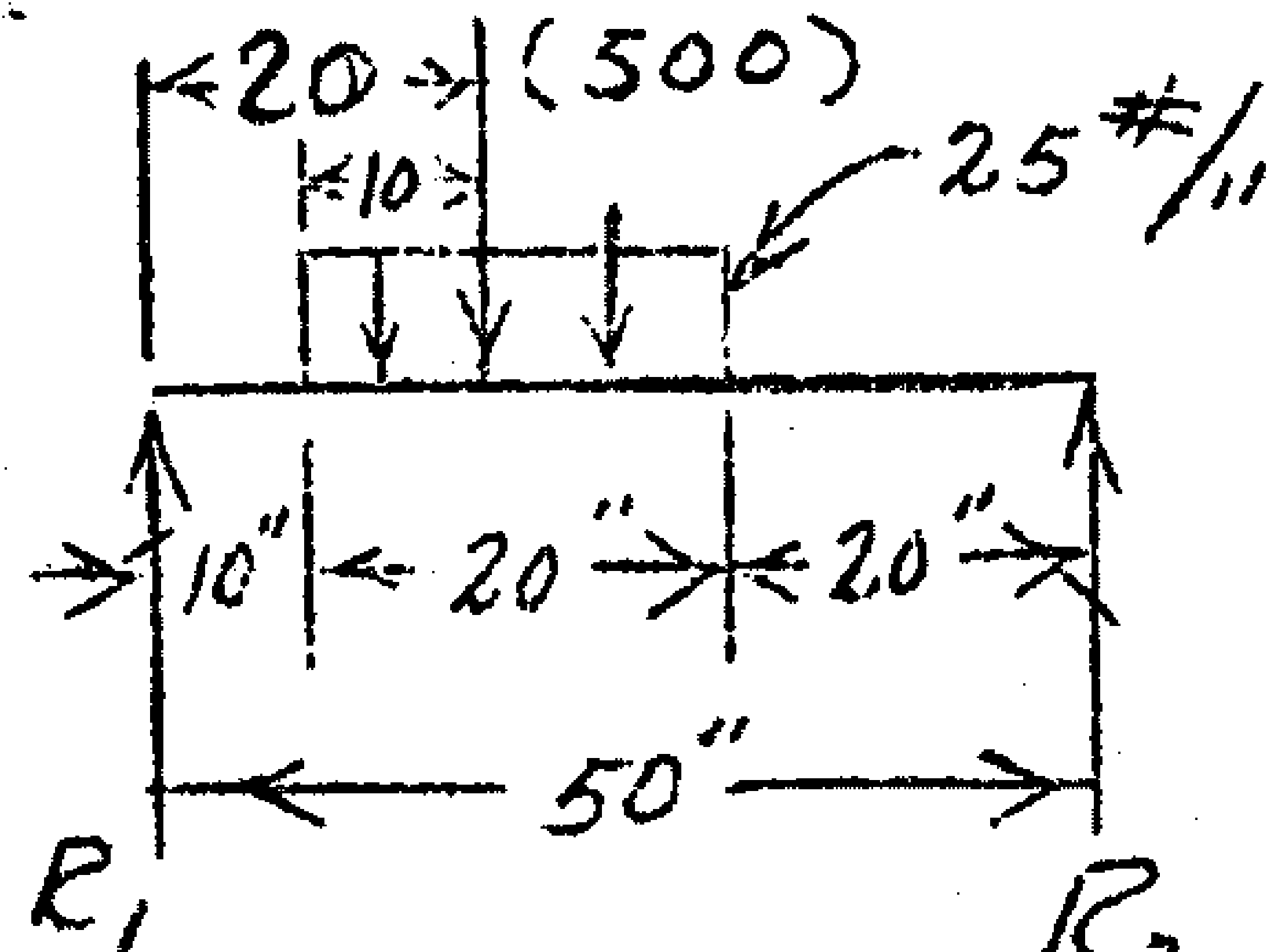


Fig. 4

Total load = (20)(25) = 500 lbs.

ΣM about $R_1 = 0$ ↶+

$(R_2)(50) - (500)(20) = 0$

$50R_2 - 10{,}000 = 0$

$50R_2 = 10{,}000$

$R_2 = 200$ lbs.

By taking moments about R_2, ΣM about $R_2 = 0$ $(\curvearrowright +)$

$(R_1)(50) - (500)(30) = 0$; $50R_1 - 15,000 = 0$

$50R_1 = 15,000$, $R_1 = 300$ lbs. $\Sigma V = 0$ $R_1 + R_2 = \text{Load}$

$300 + 200 = 500$ (Check)

(d) A simple beam with a trapezoidal load. This load may be treated by finding the center of gravity of the trapezoid but it is more direct to break it into a uniform load and a triangular load and treat them separately.

$\Sigma M_{R_1} = 0$, $(\curvearrowleft +)$

$(R_2)(50) - (250)(33.33) - (250)(25) = 0$

$50R_2 - 8333 - 6250 = 0$

$50R_2 = 8333 + 6250 = 14,583$

$R_2 = 291$ lbs., $R_1 = 209$ lbs.

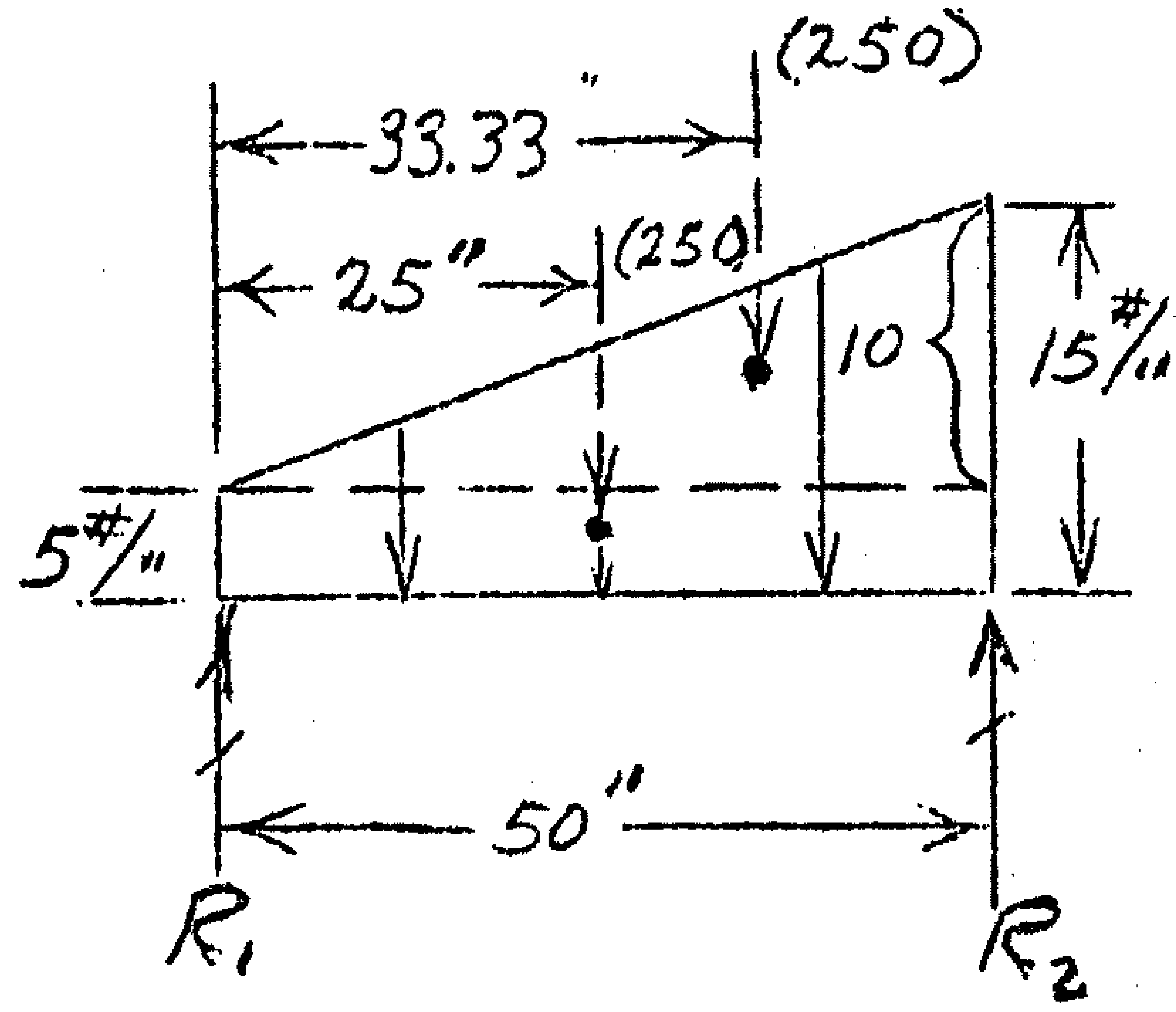


Fig. 5

Any combination of distributed and concentrated loads may be treated in a similar manner.

Development of Shear and Moment Curves.

Typical shear and moment curves for a beam loaded by concentrated loads were shown on Fig. 6, Assignment 1. These curves may be developed from the definitions of shears and moments given on page 1:5. We have stated that the shear in a beam is equal to the summation of all the forces acting on one side of a section. These forces must include also all reactions, since the reactions are a part of the force system holding the beam in equilibrium when it is taken as a free body. For the beam of Fig. 6, Assignment 1, we have an upward reaction, $R_1 = 230$ lbs., at the left end. Therefore if we isolate all of the section to the left of b the section will tend to move upward with a force of 230 lbs. The shear in this section is therefore a constant value of plus 230 lbs. If we isolate the section to the left of point c, for the section to the left of b the shear would still be plus 230 lbs., but at the instant we go to the right of point b, a downward force of 100 lbs. comes into action. Therefore for any point between point b and point c the value of the shear is $+230 - 100 = +130$ lbs. When we pass to the right of point c the shear changes sign $+230 - 100 - 300 = -170$ lbs. When we reach point d the upward reaction, R_2, of 170 lbs. brings the value of the shear back to zero. This is obviously correct since from the physical facts there is no load and therefore no shear to the right of point d.

The calculation of the moments may be made by taking the summation of force times distance, cutting the beam at any section and using either side as a free body. For example:

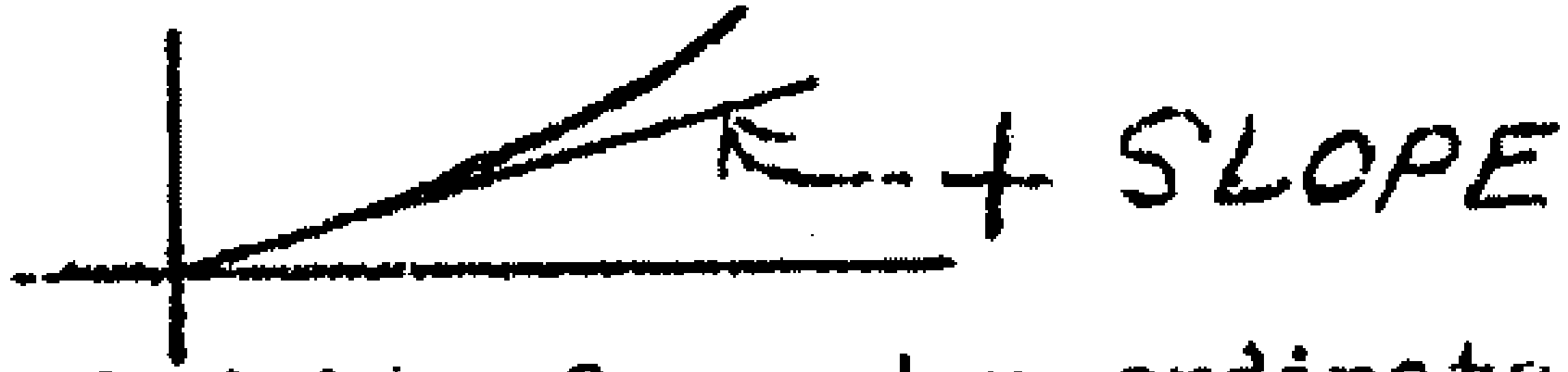
The moments at a and d are zero because the beam has pinned ends. The moment at b may be obtained simplest by isolating the part to the left of b. Then the only force acting is R_1 the moment of which around point b is (230)(20) = 4600"#. The same result could have been obtained by taking the section to the right of b. The moment would be (170)(50 + 30) - (300)(30) = 13,600 - 9000 = 4600"#. Similarly the moment at c, taking the section to the right is (170)(50) = 8500"# and taking the section to the left is (230)(20 + 30) - (100)(30) = 8500"#.

Note carefully that the moment in the span is a maximum where the shear curve passes through zero.

* * * * *

It is obvious that with complicated loadings a better organized method of computing moment and shear curves is necessary. A tool is at hand in the laws of integral curves which are the standard for solution of beams. They are based upon the following mathematical laws:

1. The slope of any curve is equal to the ordinate in the next lower curve.



2. The difference in the height of any two ordinates in any curve is equal to the area between the corresponding ordinates in the next lower curve.

3. The point where a curve passes through zero corresponds to a maximum or a minimum in the next higher curve. For example with a complicated system of loads it is always possible to find the maximum bending moment by finding the point where the shear passes through zero and checking that point for bending.

* * * * *

The curves may be constructed (piece-meal) by taking actual measured areas, or integration may be carried forward by using calculus formulas. Almost identical results will be obtained with either method, as was shown in Assignment 6. Referring again to the curves of Fig. 6, Assignment 1, the shear curve would be constructed by integral curves as follows:

a to b, there is no ordinate in the load curve and therefore the slope of the shear curve in this section is zero, i.e. horizontal. Hence a horizontal line in the shear curve with ordinate 230. Between b and c, a reduction in ordinate to 230 - 100 = 130 lbs. and again a horizontal line because the ordinate in the load curve is zero. Between c and d a further reduction of 300 lbs. (to -170) and again a horizontal line in the shear curve because there is no load curve ordinate. The check for the shear curve is that the reaction at the right must return the shear curve to zero.

Taking the shear curve, the moment curve is developed as follows:

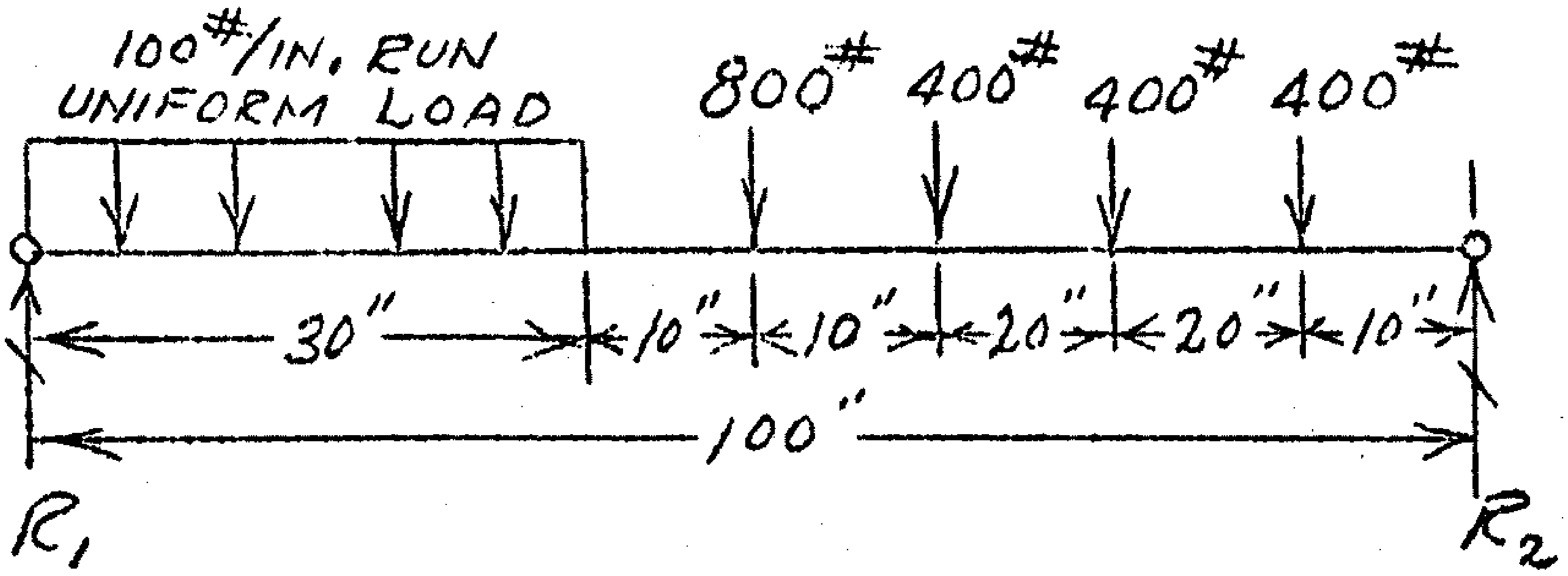
The moment ordinate is zero at "a" by definition. The difference in ordinates between b and a is equal to the area between these ordinates in the next lower (shear) curve, or (230)(20) = 4600"#. The ordinate at b is therefore 0 + 4600 = 4600"# (check). The difference between the ordinates at b and c is equal to the area between b and c in the shear curve or (130)(30) = 3900"#. The ordinate at c is therefore 4600 + 3900 = 8500"# which checks our previous result.

Continuing to point d the difference in ordinates between c and d would be (-170)(50) = -8500"#. The moment at d is therefore 8500 - 8500 = 0 which coincides with the physical facts. This also serves as a natural check for the moment computations for a pin ended beam.

Problem Assignment.

7:1 Develop the load, shear, and moment curves for the simple beams illustrated in Figs. 2, 3, 4, and 5.

7:2 Develop the load, shear, and moment curves for a fuselage floor beam loaded as shown below.



BASIC STRESS ANALYSIS

ASSIGNMENT EIGHT

Assignment 8

THE BEAM THEORY

The behaviour of a beam depends upon its material and the way the material is distributed in the beam cross-section. It therefore depends upon the physical properties of the material and the section properties (distribution of area) of the beam cross-section. The entire theory is based upon the following fundamental assumptions:

1. Any cross section of the beam which is a plane before bending remains a plane after bending.
2. The stretch or compression of any longitudinal fiber of a bent beam is proportional to the load on it, up to the point of failure.

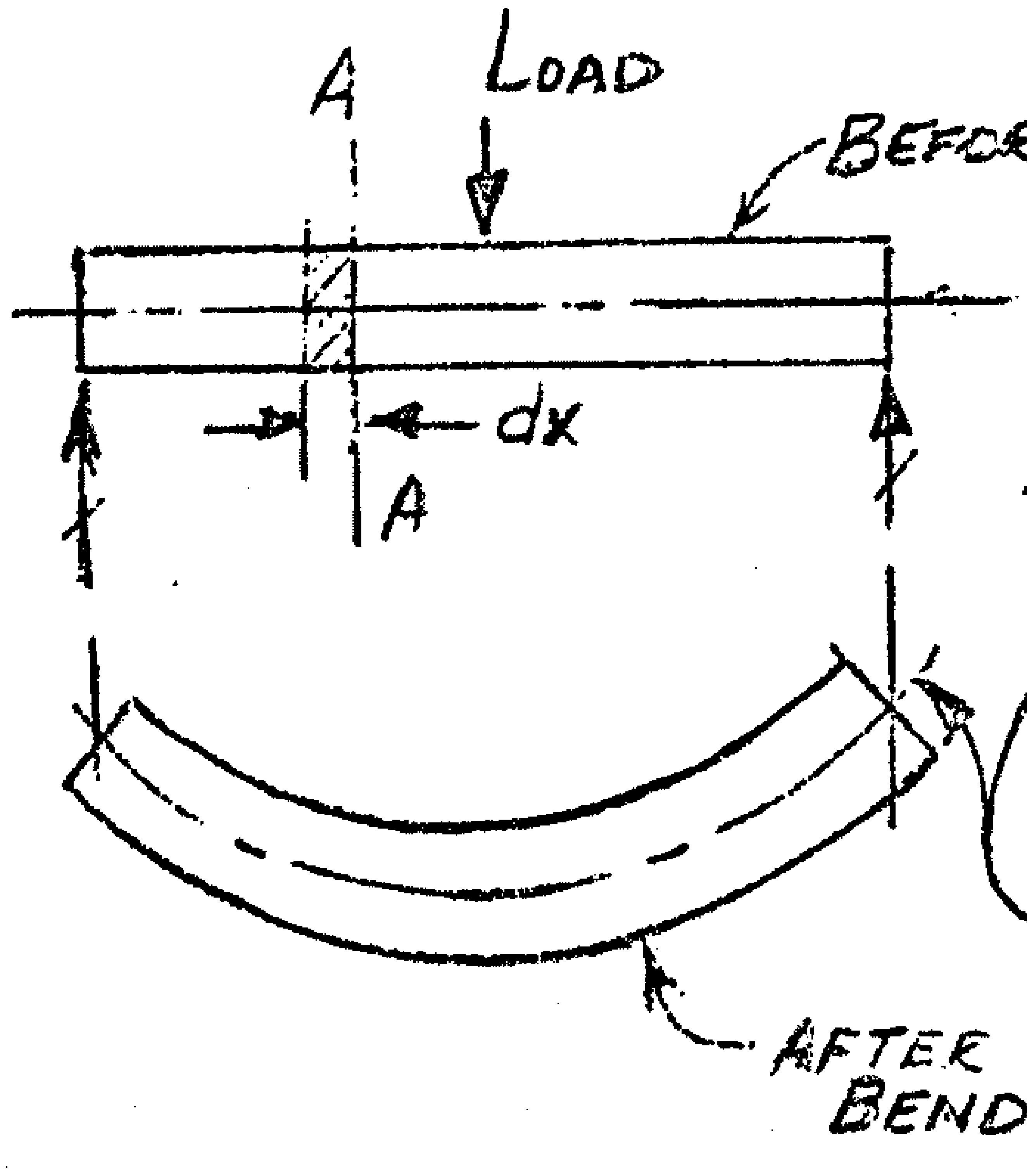


Fig. 1

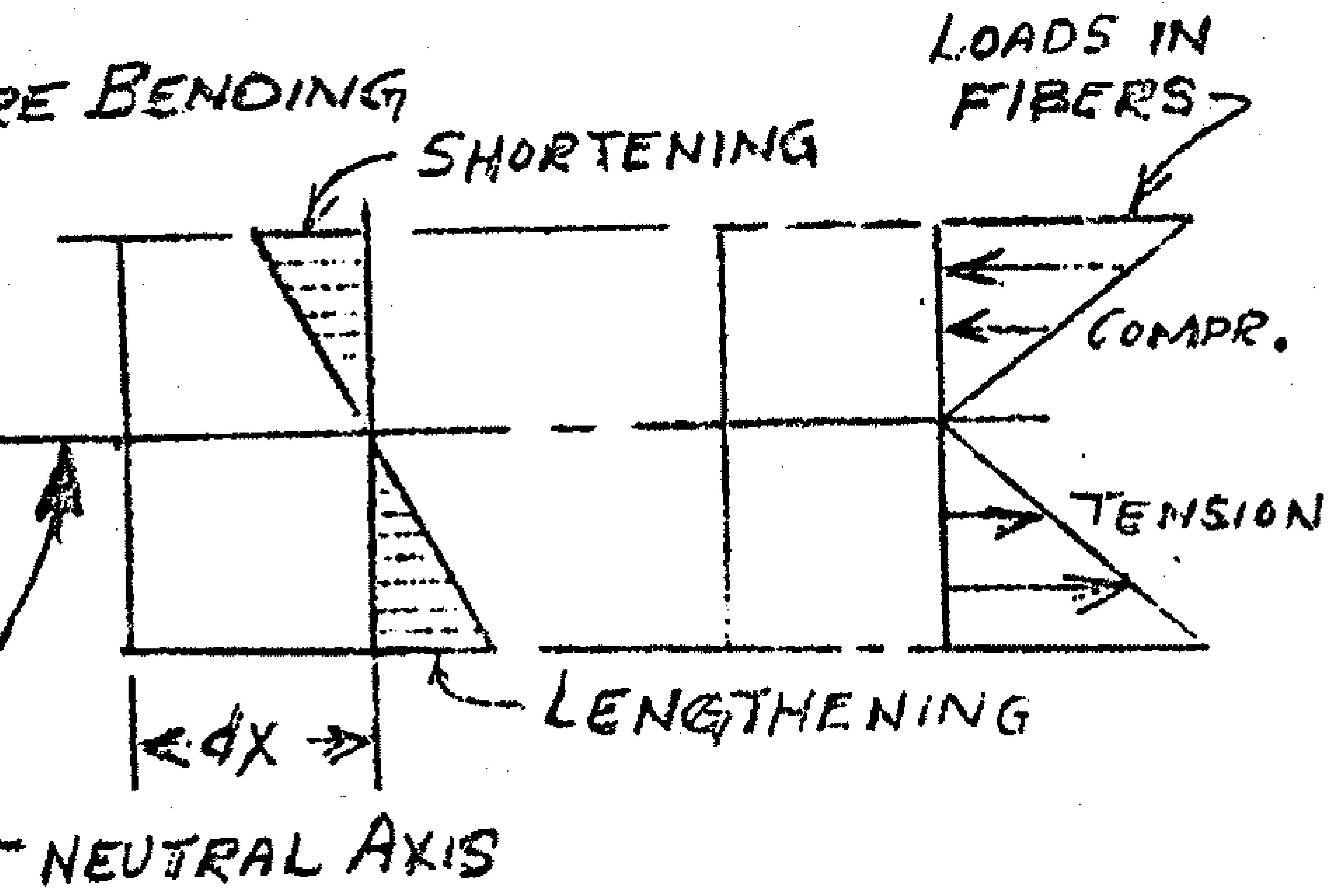


Fig. 2

From the fundamental assumptions the maximum unit stresses from bending in a beam will appear at the top or bottom fibers, the points where the most stretching or compressing takes place.

When a beam bends as shown in Fig. 1 the bottom fibers are stretched and the top fibers are compressed in directions lengthwise of the beam. At some horizontal plane in any beam section there must be a layer of fibers which are neither stretched or compressed. The location of this layer of fibers defines the neutral axis of the beam. (Fibers with neutral stress).

Fig. 2 shows a plot of unit stress intensities along the cross-section of a rectangular beam. Since we assume that a plane before bending remains a plane after bending, the deformation of any fiber and hence its unit stress is directly proportional to its distance from the neutral axis.

To find the reactions on a beam we took it as a free body in space acte upon by external loads and calculated the reactions at the points of support. To find the internal forces in a beam, and from them the unit stresses, we cut the beam at the section where the forces are desired and calculated the internal forces required on the cut face to hold the system of external loads and reactions on one side (either side) of the cut face in equilibrium. Any section of a beam taken out as a free body must satisfy the fundamental equations of equilibrium: $\Sigma V = 0$, $\Sigma H = 0$, and $\Sigma M = 0$. A plot of the forces acting at section A-A on the beam of Fig. 1, would appear as shown on Fig. 3.

The external moment on the beam at section A-A = (R)(x). This moment must be resisted by the moment of the internal forces acting across the cut face of the beam. These internal forces, represented by a tension-compression couple, set up the internal moment, or resisting moment in the beam.

To satisfy $\Sigma V = 0$ there must also be a vertical shear force acting on the cut face of the beam. In the case illustrated the shear equals the reaction R.

To satisfy $\Sigma H = 0$, T must equal C or the summation of all the compression forces acting longitudinally along the beam must equal the summation of the tension forces.

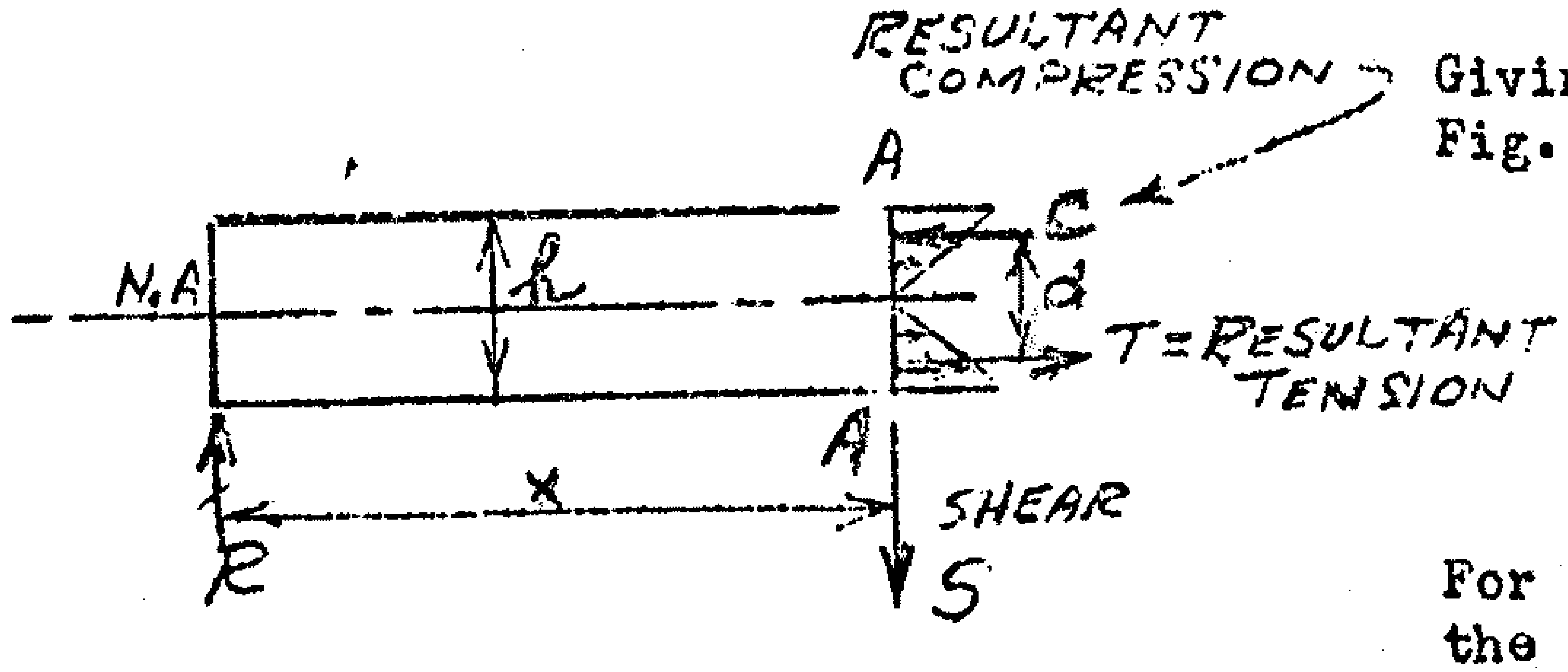


Fig. 3

Giving values to the beam of Fig. 3,

R = 1000#
x = 10"
h = beam height = 6"
b = beam breadth = 1"

For a homogeneous rectangular beam the neutral axis is at the center.

Moment on section A-A =

R times x = 10,000"#

The resultant of a triangular load lies at a point one-third the distance from the base. Therefore the resisting arm, "d", of the tension compression couple equals 4" and the total value of T or C must be 10,000/4 = 2500#. At section A-A the area in tension is one-half the cross section or 3 square inches. The average unit stress is 2500/3 = 833#/□", but the maximum at the top or bottom is twice the average or 1666#/□" since the distribution of load is triangular.

The same result is obtained from the beam formulas as follows:

I of rectangular section = $\frac{bh^3}{12}$ (see handbook formulas)

y = distance from neutral axis to extreme fiber = h/2

I/y = Section Modulus = $Z = \frac{bh^3}{12} \div \frac{h}{2} = \frac{bh^2}{6}$

Maximum fiber stress in bending $f_b = M/Z$

For this beam $Z = \frac{1 \times 6^2}{6} = 6^{in^3}$ and $f_b = \frac{10,000}{6} = 1666\#/\square''$

Section properties of beams are discussed in Assignment 9. If we are to consider the inclination or deflections of the neutral axes it becomes necessary to know what the section properties, particularly the moment of Inertia are. The importance of the section properties can best be illustrated by extending the 3 beam curves of previous assignments to 5. Those then are Load, Shear Moment, Inclination and Deflection Curves.

The 5-Beam Curves.

In Assignment 1, page 4, we stated the Laws in Integrated curves as follows:

1. The Slope in any curve is equal to the ordinate in the next lower curve.
2. The difference in height of the ordinates in any curve is equal to the area between the corresponding ordinates in the next lower curve.
3. The point where a curve passes through zero (cuts the x-axis) corresponds to a maximum or a minimum in the next higher curve.

****

If we apply these laws to the construction of Beam Curves, keeping always in mind the physical facts with which we are dealing, it is readily apparent that:

1. The Equation of the Shear Curve is the Integral of the Equation of the Load Curve.

$$\text{Shear} = \int \text{Load} \qquad (S = \int L\,dx)$$

2. The Equation of the Moment Curve is the Integral of the Equation of the Shear Curve.

$$\text{Moment} = \int \text{Shear} \qquad (M = \int S\,dx)$$

We now extend our Integral Curves further by stating:

3. The Equation of the Inclination of the Neutral Axis of the Beam is equal to 1/EI times the Integral of the Equation of the Moment Curve.

$$\theta = \frac{1}{EI}\int M\,dx \qquad \text{or} \qquad EI\,\theta = \int M\,dx$$

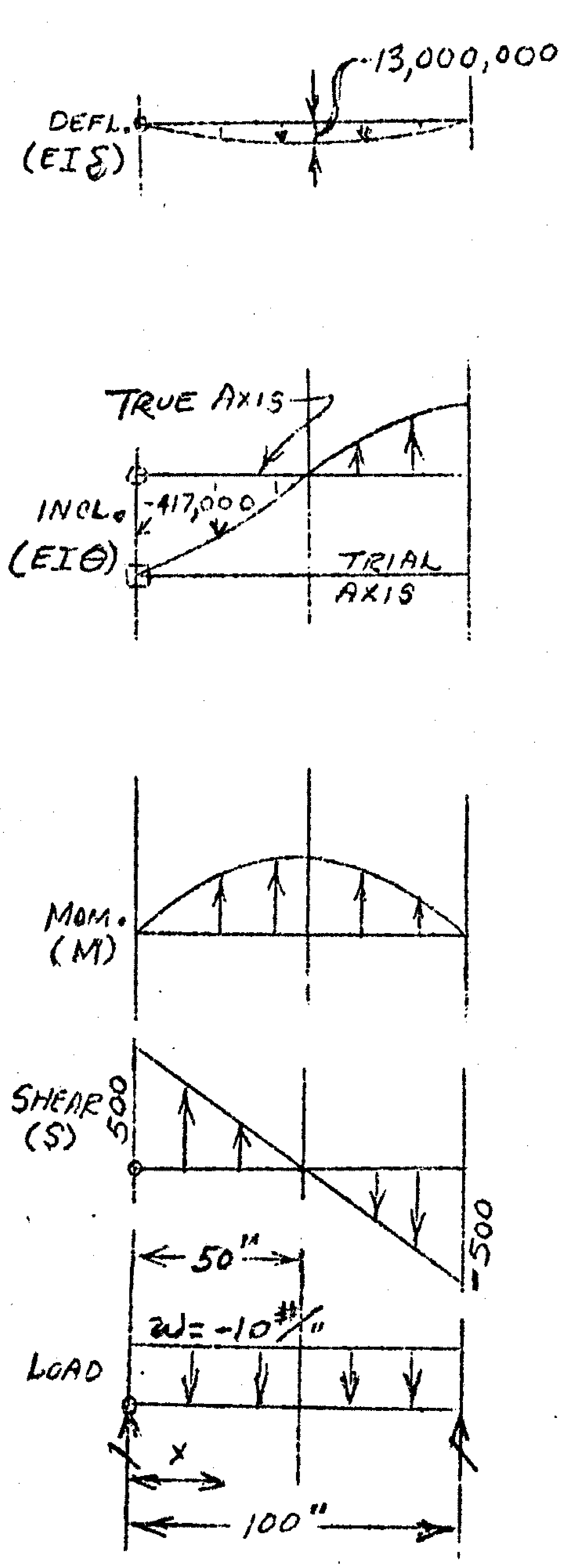
Where E is the Modulus of Elasticity of the Beam and I is the Moment of Inertia of the Beam. θ is the angle of inclination of the Beam expressed in radians. (1 radian = 57.3°)

4. The Equation of the Deflection Curve of the Neutral Axis of a Beam is equal to the Integral of the Equation of the Inclination curve of the Beam

$$EI\delta = \int \theta\,dx$$

Then to calculate the deflection of a beam under any given load it is necessary only to Integrate the Load, Shear, Moment and Inclination curves, taking into account Constants of Integration, and to divide the final ordinates in the deflection curve by a constant EI to obtain actual values of deflection. To illustrate the method let us extend the work on the uniformly loaded beam, Fig. 3 of Assignment 6, to include the Inclination and deflection curves.

Five Integrated Curves.



$$EI\delta = \int_0^x (-417{,}000 + 250x^2 - 1.667x^3)dx$$

$$= -417{,}000x + 83.3x^3 - .416x^4 + C_4$$

when $x = 0$, deflection $= 0$. Therefore $C_4 = 0$.

****

$$EI\theta = -417{,}000 + 250x^2 - 1.667x^3 \text{ (true eq.)}$$

$$EI\theta = \int_0^x (500x - 5x^2)dx = C_3 + 250x^2 - 1.667x^3 \text{ (trial eq.)}$$

$C_3 = -250x^2 + 1.667x^3$ when $EI\theta = 0$

$EI\theta = 0$ at $x = 50$ from physical properties and loading.

Subst.
$C_3 = (-250)(50)^2 + (1.667)(50^3) = -417{,}000$

****

$$\text{Mom.} = \int \text{Shr.} = \int_0^x (500 - 10x)dx$$

$$= C_2 + 500x - 5x^2 \qquad C_2 = 0$$

$\text{Mom.} = 500x - 5x^2$

****

$$\text{Shear} = \int_0^x -10dx = C_1 - 10x$$

$C_1 = 500$, Shr. $= 500 - 10x$

****

$w = -10$

$E = 10{,}000{,}000$
$I = 3.0 \text{ in}^4$

Solving for actual values

$EI\theta = -417{,}000$ max., $\theta = -417{,}000/10{,}000{,}000 \times 3 = .0139$ radians $= .79$ degrees

$EI\delta_{max.}$ (at center) $= (-417{,}000)(50) + (83.3)(50)^3 - (.416)(50)^4$
$x = 50''$ $= -20{,}800{,}000 + 10{,}400{,}000 - 2{,}600{,}000 = -13{,}000{,}000$

$\delta = -13{,}000{,}000/10{,}000{,}000 \times 3 = -.433''$ (Check)

Note that we put the load curve at the bottom of the page and build upward from it.

Now let us review the steps in the solution given on the preceding page. Up to and including the moment curve we have had no difficulty in determining the Constants of Integration, because in the Shear Curve $C_1 = 500$, the value of the shear just to the right of the left hand reaction point; and in the moment curve $C_2 = 0$ because the moment at the left hand support is zero.

Now when we integrate the equation of the moment curve we obtain the Inclination equation.....

$$EI\theta = C_3 + 250x^2 - 1.667x^3$$

The Constant of Integration, C_3 , must now be evaluated. In order to obtain the correct shape of the curve we may assume that $C_3 = 0$ and plot the curve with reference to the Trial Axis shown. Now if we can establish the point in the span where the axis of the bent beam is horizontal, we will have the true location where $EI\theta = 0$ and consequently $C_3 =$ zero. A horizontal line drawn through this point on the assumed curve for $EI\theta$ will be the true axis of reference for the $EI\theta$ curve and the value of the starting ordinate, C_3, may be computed from our original trial equation.

For the beam of uniform cross section loaded symmetrically with respect to the reactions, the bent beam axis is horizontal at the center of the span. Therefore if we substitute the x value for the span center, in this case 50 inches, set $EI\theta$ equal to zero and solve for C_3 we will obtain the value of the starting ordinate in the true equation. The true equation then becomes

$$EI\theta = -417{,}000 + 250x^2 - 1.667x^3$$

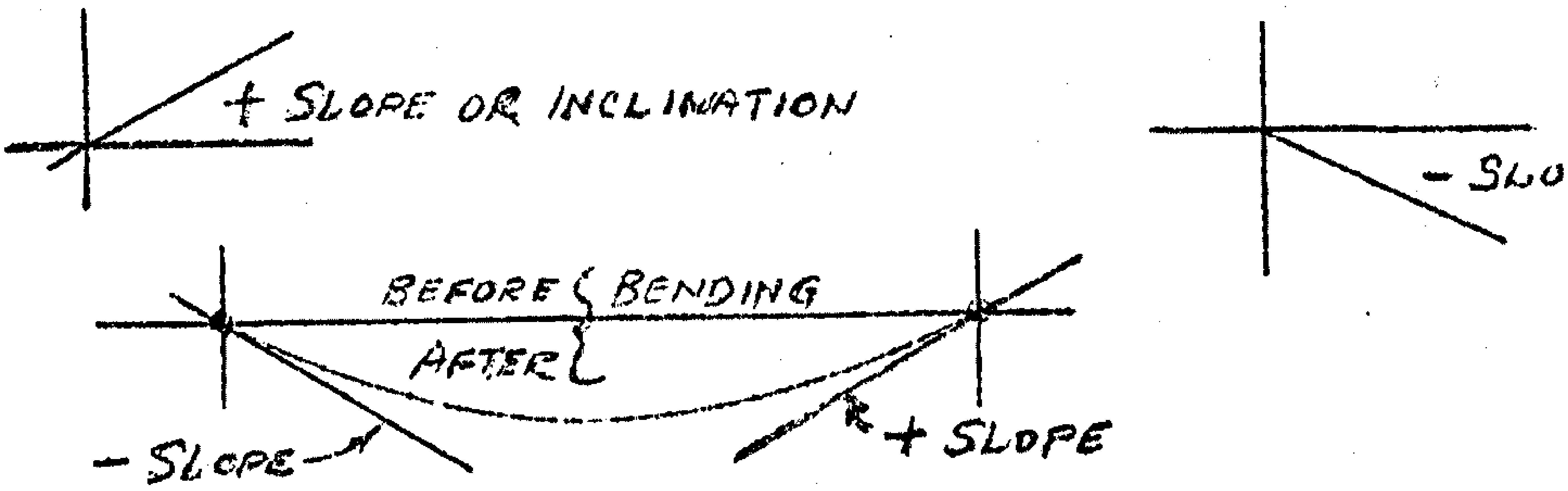
The equation of deflection is

$$EI\delta = -417{,}000x + 83.3x^3 - .416x^4$$

since $C_4 = 0$ from the physical fact that the deflection at the left hand support where $x = 0$ is zero.

In this case we have two natural checks of our mathematics against the physical facts.

1. If we substitute in our inclination equation the value of x for the right hand support, $x = 100$ and solve we should obtain the value $EI\Theta = +417{,}000$ from the symmetry of the beam load. The actual picture of the inclination must conform to our original sign convention (Assignment 7), which states that slopes are as shown in the figure below.



PHYSICAL BEAM

2. From our deflection curve if we substitute the values for $x = 100$ at the right hand support our deflection value, $EI\delta$, should equal zero, since deflection at the support is zero.

Both of the above checks should be carried through to check the accuracy of the work.

****

Deflection formulas are available in all the standard handbooks for a large number of typical cases of beam loading. For our example

$\delta_{max.}$ is at the center and equal to

$\delta_{max.} = \frac{5wL^4}{384EI}$, where w = unit running load and L = length of beam span between supports.

It will be noted that we must know the Modulus of Elasticity, E, and the Moment of Inertia of the cross-section, I, before the actual value of the deflection may be obtained. Assuming that $E = 10{,}000{,}000$ for aluminum alloy and that $I = 3.0$ inches to the fourth power

$$\delta_{max.} = \frac{(5)(-10)(100)^4}{(384)(10{,}000{,}000)(3)} = -.433 \text{ in.}$$

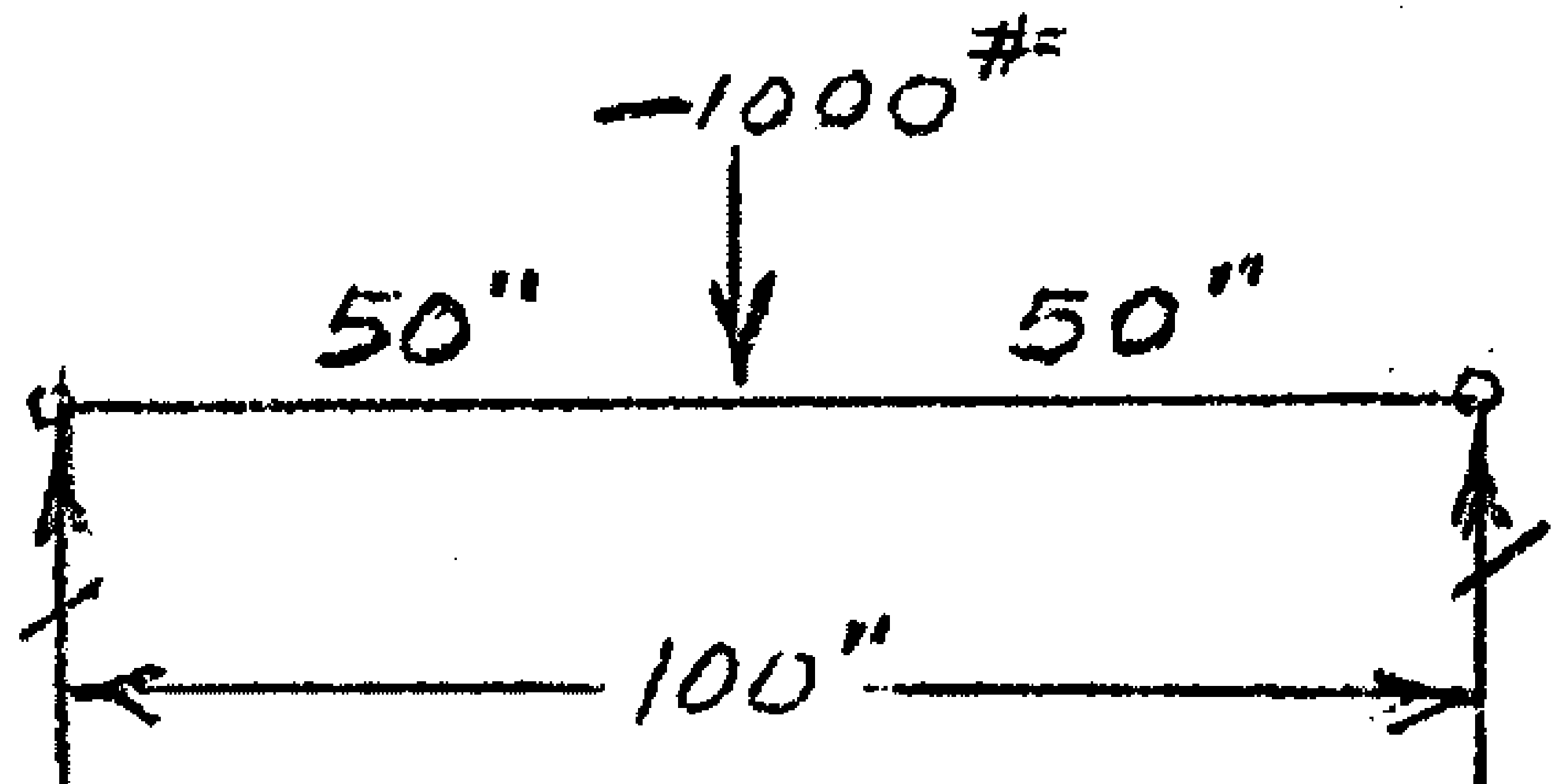
The minus sign indicates that the deflection is downward. Reference to the results from our Integral Curves will show that we obtain exactly the same result.

It should also be noted that the point of maximum deflection in the highest curve corresponds to the point of zero inclination. This agrees with our third law of Integral Curves and forms another natural check on the work.

Problem Assignment.

8:1 Obtain the maximum deflection of the beams shown below by using Integral Curves.

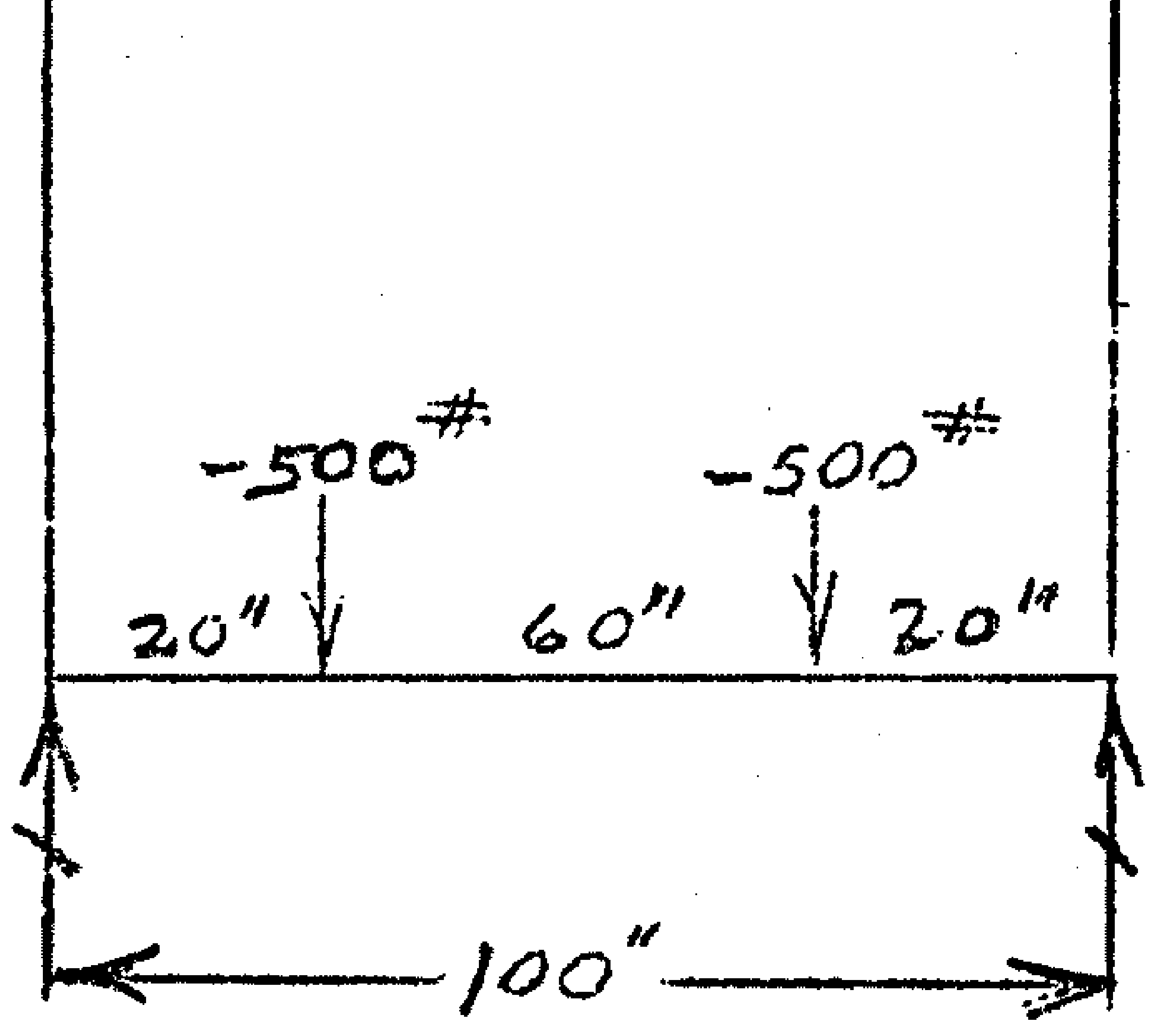
(a)



E = 10,000,000

$I = 3.0^{in^4}$

(b)



E = 10,000,000

$I = 3.0^{in^4}$

Hint: Where there is a discontinuity in the equation, as at a concentrated load, compute the value at that point and write a new equation for the next segment. (See also Assignment 7.)

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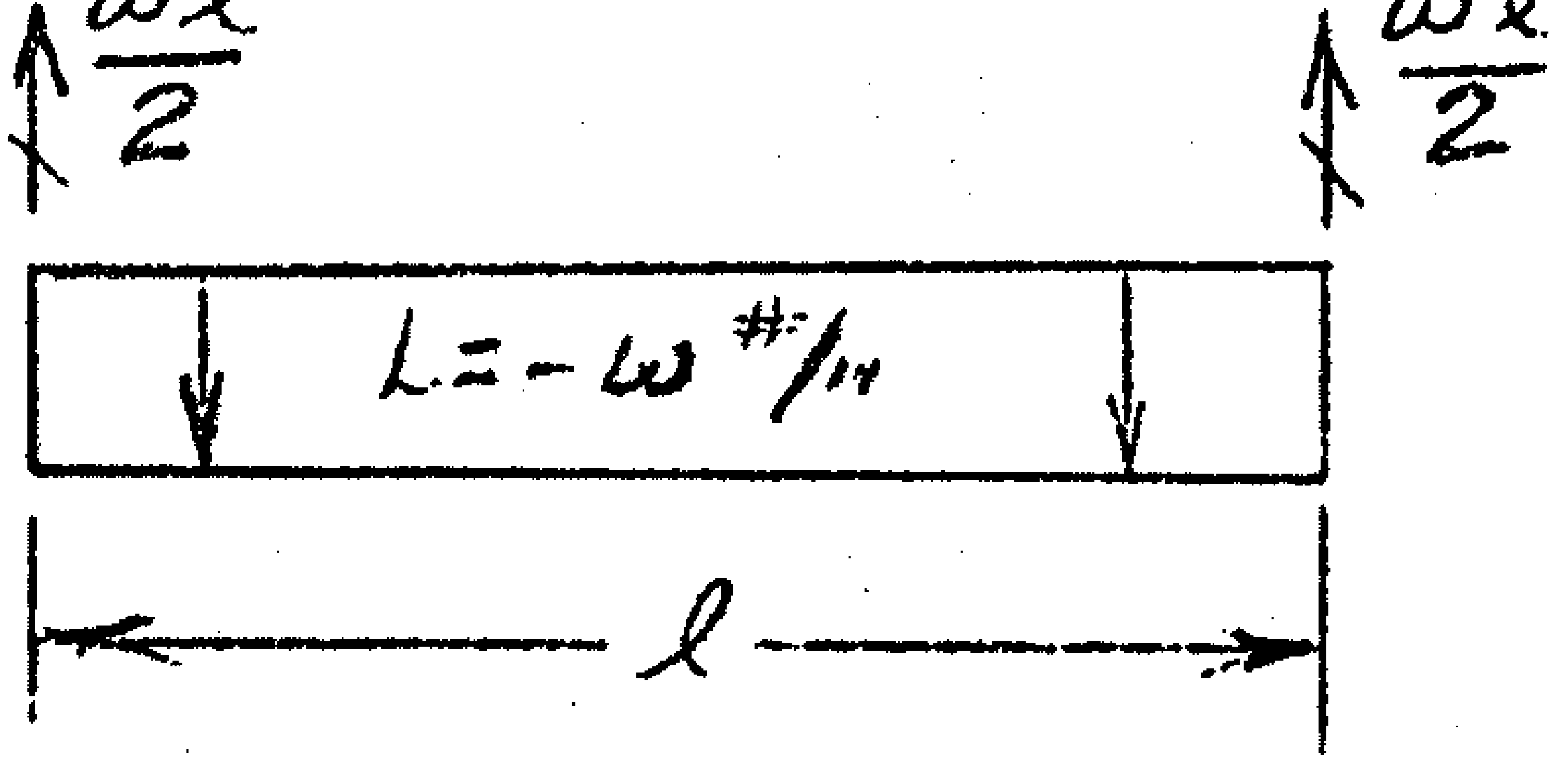
x is then referred to the new origin, (point of application of the concentrated load) where the equation of the next lower curve became discontinuous.

(c) Develop the general equation for maximum deflection at the center of a simple beam loaded as shown below.



BASIC STRESS ANALYSIS

ASSIGNMENT NINE

Assignment 9

BEAMS--SECTION PROPERTIES AND UNIT STRESSES

A section of a beam is a cross-section taken perpendicular to its longitudinal axis. Section properties define the character of a plane section so cut.

The Neutral Axis of a section is the line about which its area would balance. It is also the strata of fibers of the beam which retain their original length when the beam is bent.

The Statical Moment (first moment) of a plane section is the summation of the products of each little individual area by its distance to the neutral axis.

$$\underline{Q} = \sum Ah$$

The Moment of Inertia (second moment) of a plane section is the summation of the products of the small individual areas by the squares of their distances to the neutral axis.

$$I = \sum Ah^2$$

For any homogeneous beam section these values may be found as follows:

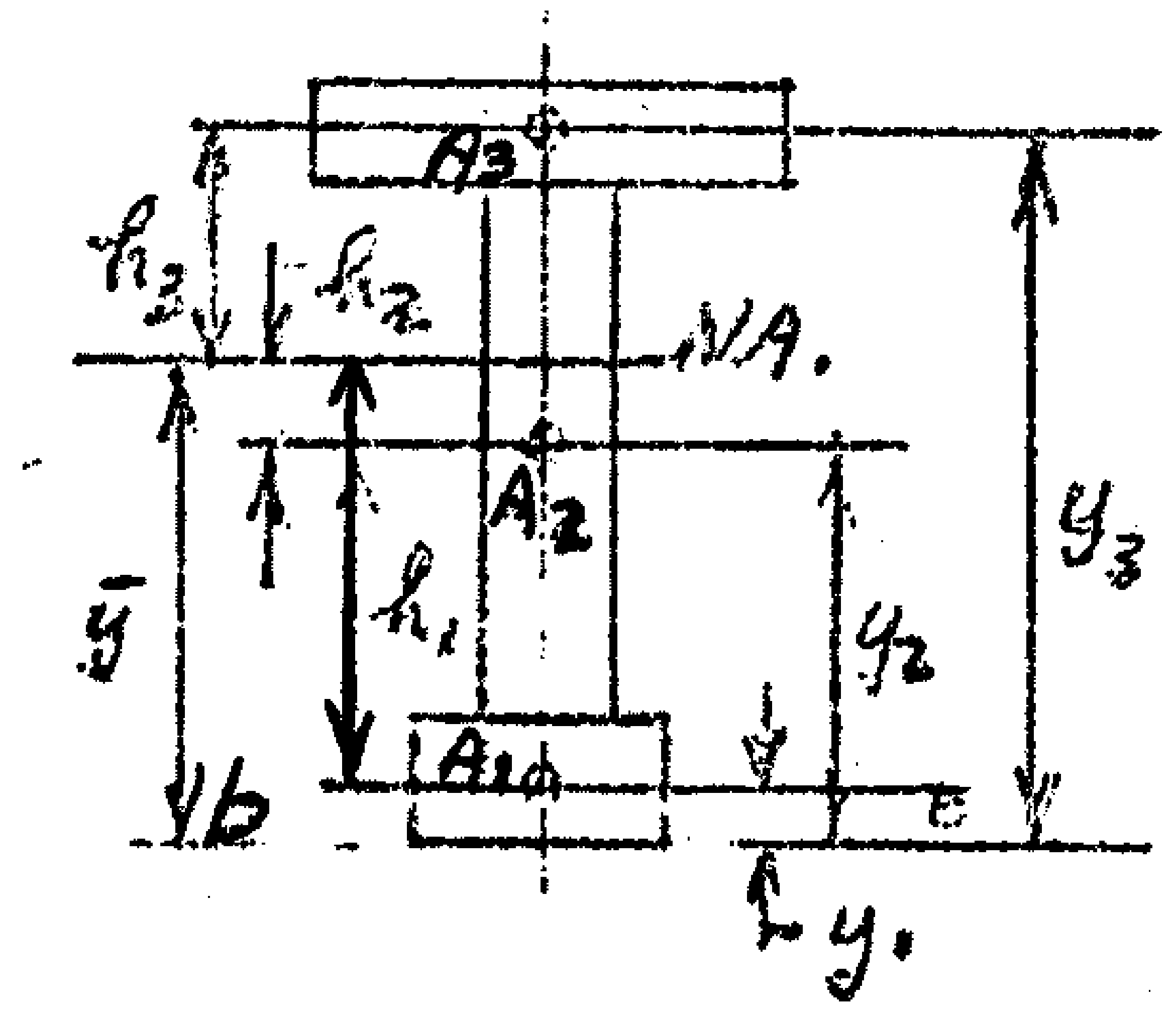


Fig. 1

1. Lay out the section divided into component parts which form regular shapes such as squares, rectangles, etc.
2. Determine the area of each segment.
3. Locate the center of gravity (c.g.) of each segment.
4. Take moments of the segments about the base and determine the location of the neutral axis.

Note: When holes exist in the tension side of the beam their cross-sectional area should be left out in calculation of section properties.

Distance to the neutral axis,

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

Tables of moments of inertia of regular plane sections are available in the steel company handbooks. They are developed by the actual calculus summation of the values mentioned in the first paragraph. For a rectangular section, the moment of inertia about its own gravity axis = $\frac{bh^3}{12}$ where b = width, h = height.

The moment of inertia of a section about its own gravity axis we term I_o.

Now the moment of inertia of a rectangle about any other axis parallel to its own gravity axis equals its moment of inertia about its own gravity axis plus the product of its area and the square of its distance to the new axis. This is expressed

$$I = I_o + Ad^2$$

Where I = Moment of Inertia about the new reference axis, I_o has its usual significance, A is the area of the segment, and d is the distance between the two axes. This principle is now applied to find the Moment of Inertia of the section of Fig. 1. The method is shown in tabular form below. It is convenient to calculate the static moment, Q, at the same time.

Area	y	Ay	h	h^2	Ah^2	I_o	Q
A_1	y_1	A_1y_1	h_1	h_1^2	$A_1h_1^2$	(A_1)	A_1h_1
A_2	y_2	A_2y_2	h_2	h_2^2	$A_2h_2^2$	(A_2)	A_2h_2
A_3	y_3	A_3y_3	h_3	h_3^2	$A_3h_3^2$	(A_3)	A_3h_3
ΣA		ΣAy			ΣAh^2	ΣI_o	ΣAh

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A}, \quad I = \Sigma Ah^2 + \Sigma I_o$$

For the section of Fig. 1 it is obvious that the neutral axis will lie somewhere above the half-height of the beam. From our assumption of straight line distribution of stress (Assignment 8), it is also apparent that the unit stresses at the top and bottom of the beam will not be equal, but will have a ratio exactly proportional to their distances to the neutral axis.

At any part of the beam cross section the unit stresses from bending,

$$f_b = \frac{Mh}{I}$$

where

M = moment on the section.
h = distance to the neutral axis from the part in question.
I = moment of inertia of the entire section (all units in inches or inch-pounds).

Also at any part of the beam a horizontal shear per inch of length exists of the magnitude

$$S_h = \frac{S_v Q}{I},$$

where

S_h = the horizontal shear in lbs. per inch.
Q = the static moment about the neutral axis of the parts of the section above or below the horizontal section investigated.
S_v = vertical shear load at the sta. along the beam where the section is taken.

If any small section of material subjected to shear is taken from a beam the forces holding it in equilibrium will be as shown in Fig. 2.

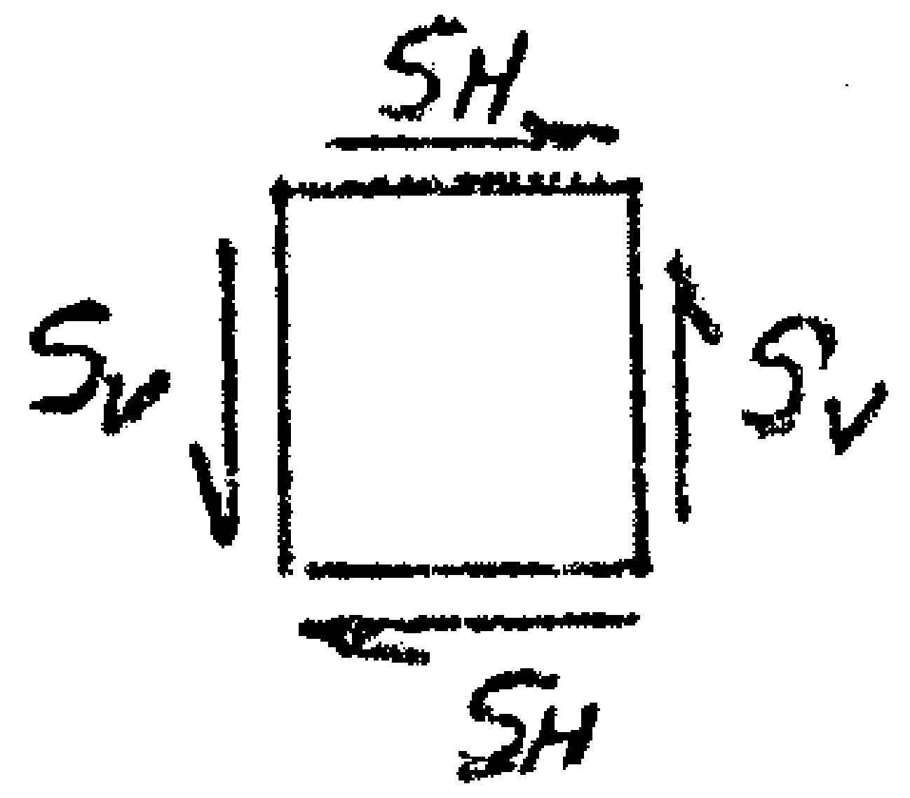


Fig. 2

It follows therefore that $S_v = S_h$ and the intensity of the unit longitudinal shear at any point equals the intensity of unit vertical shear at that point. This is a very useful law since it may be applied to find the maximum shear stresses on a beam of any cross-section.

A numerical example will be studied completely in order to illustrate all points.

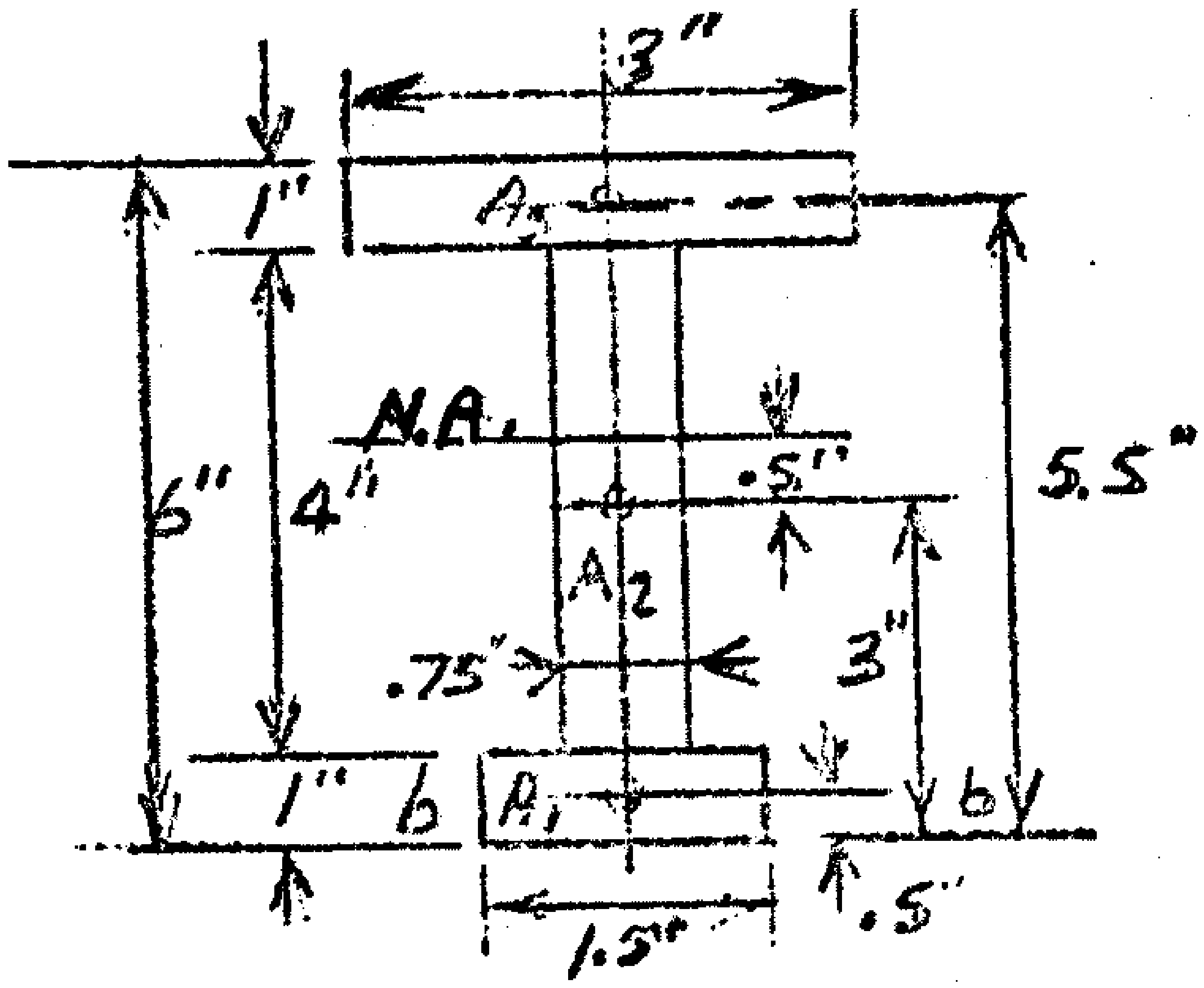


Fig. 3

area	y	Ay	h	h^2	Ah^2	I_o	Q=Ah
A_1=1.5	.5	.75	3.0	9.0	13.50	.125	4.5
A_2=3.0	3.0	9.00	.5	.25	.75	4.000	
A_3=3.0	5.5	16.50	2.0	4.0	12.00	.250	6.0
Σ=7.5"	$\bar{y}$=3.5	26.25			26.25	4.375	

$$I_o \text{ of } A_1 = \frac{bh^3}{12} = \frac{(1.5)(1)^3}{12} = .125$$

$$\text{of } A_2 = \frac{(.75)(4)^3}{12} = 4.0$$

$$\text{of } A_3 = \frac{(3)(1)^3}{12} = .25$$

$$\begin{array}{rl} 26.25 & = Ah^2 \\ + \underline{4.375} & = I_o \\ I = 30.625 & \end{array}$$

Check of static moment at neutral axis:

From bottom portion:

$$\begin{aligned} 3 \times 1.5 &= 4.50 \\ 2.5 \times .75 \times 1.25 &= \underline{2.35} \\ \Sigma &= 6.85 \end{aligned}$$

From top portion:

$$\begin{aligned} 3 \times 2 &= 6.00 \\ 1.5 \times .75 \times .75 &= \underline{.85} \\ &\ 6.85 \end{aligned}$$ Check.

Assume that this section has a shear of 20,000 lbs. (S) and a bending moment of 300,000"#: Then the maximum shear per inch at the neutral axis =

$$\frac{SQ}{I} = \frac{(20,000)(6.85)}{30.625} = 4460\#/\text{inch}.$$

The unit stress = $\frac{4460}{1 \times .75}$ = 5960#/sq. inch.

Maximum fiber stress in bending at the top = $\frac{My}{I} = \frac{(300,000)(2.5)}{30.625}$ = 24,400#/sq.in.

and at the bottom = $\frac{(300,000)(3.5)}{30.625}$ = 34,200#/sq.inch.

These are maximum values. We will next examine the load distribution and balance of forces across the entire cross-section.

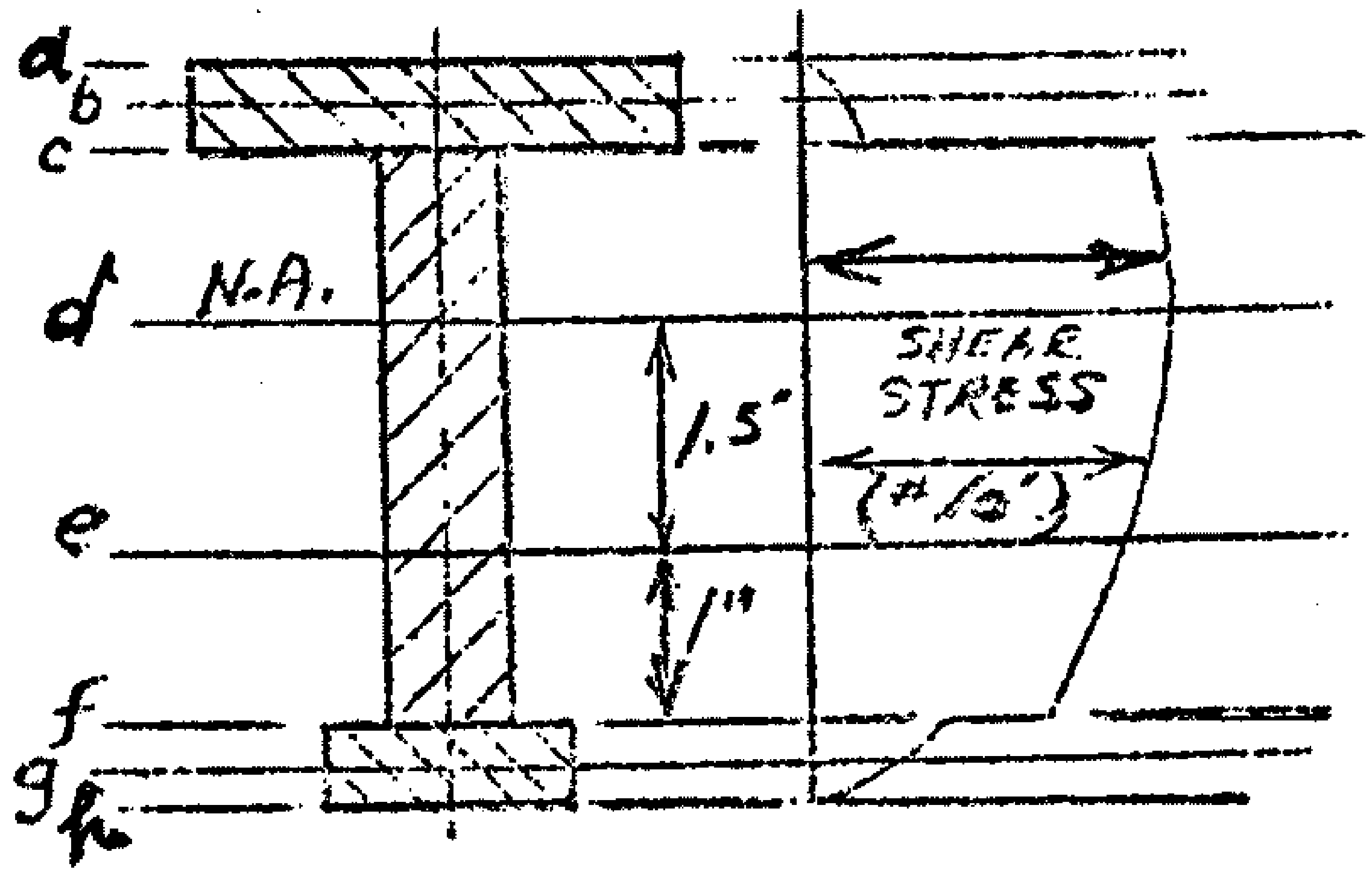


Fig. 4

At a and h top and bottom there is no material on the side away from the neutral axis and therefore there can be no shear transfer.

At b, area above section = 1.5 sq. in.

Static moment = 1.5 x 2.25 = 3.38

Shear per inch = $\frac{20,000 \times 3.38}{30.625}$ = 2200 lbs.

Shear per sq. in. = $\frac{2200}{1 \times 3.0}$ = 735 lbs. per sq.in.

At c,

Shear per inch = $\frac{20,000 \times 6.0}{30.625}$ = 3920 lbs.

Shear per sq. in. = $\frac{3920}{1 \times .75}$ = 5230 lbs. per sq.in.

At d, (N.A.) = 5960 lbs. per sq. in. (see preceding work)

At e, Q = 4.5 + (.75)(2) = 6.0 f_s = 5230 lbs. per sq. in.

At f, Q = 4.5 Shear = $\frac{20,000 \times 4.5}{30.625}$ = 2940 lbs. per . in.

fs = 2940/.75 = 3920 lbs. per sq. in.

At $\underline{g}$, $Q = .75 \times 3.25 = 2.44$

$$f_s = \frac{SQ}{bI} = \frac{20{,}000 \times 2.44}{1.5 \times 30{,}625} = 1{,}060 \text{ lbs. per sq. in.}$$

The diagram of unit shear stresses is plotted on Fig. 4.

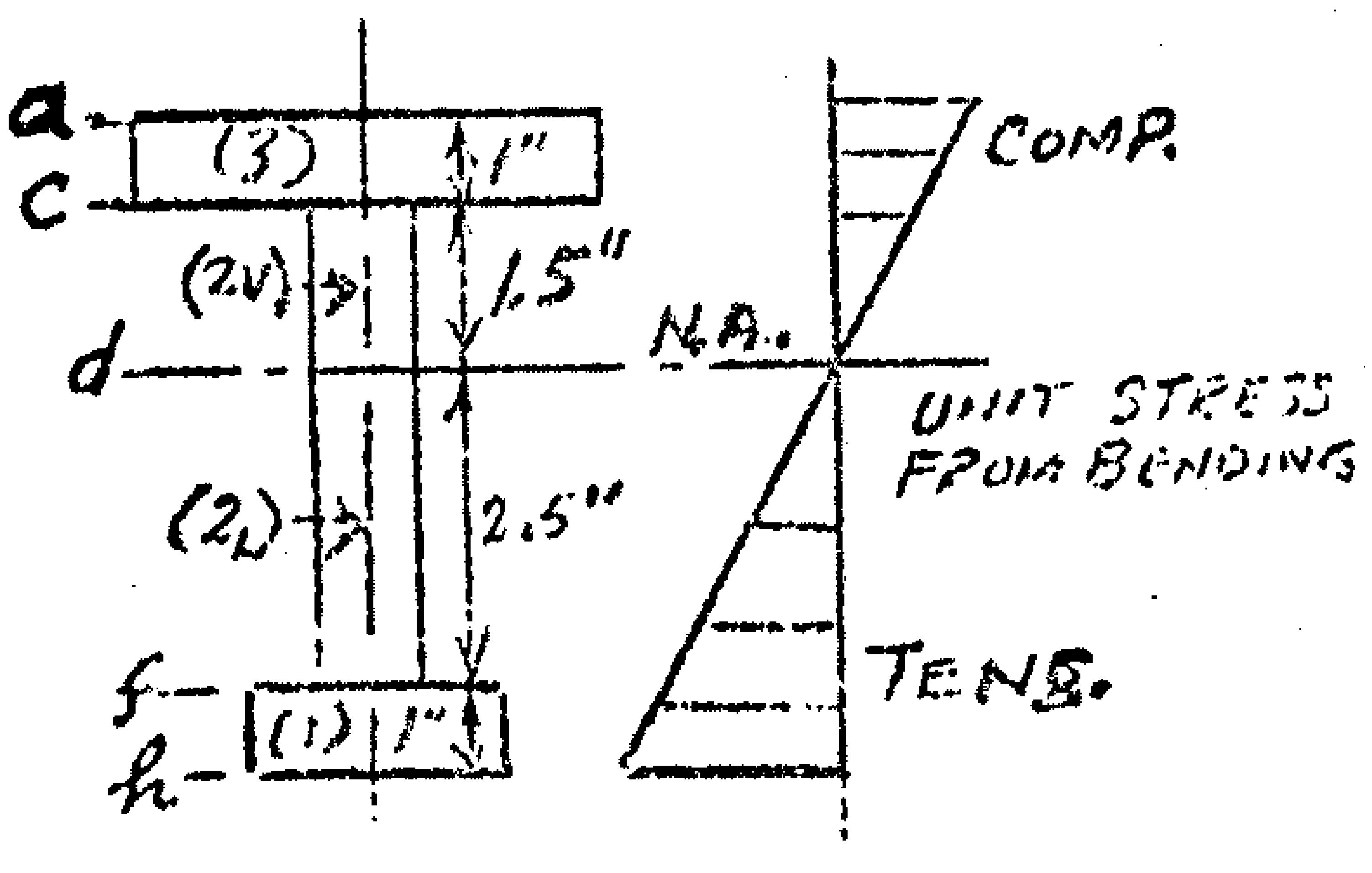


Fig. 5

Using the straight line variation of unit stress:

At $\underline{a}$ = 24,400 lbs. per sq. inch.

At $\underline{c} = \frac{1.5}{2.5} \times 24{,}400 = 14{,}600$

At $\underline{d} = 0$

At $\underline{f} = \frac{2.5}{3.5} \times 34{,}200 = 24{,}400$

At $\underline{h} = 34{,}200$

Consider the total loads on sections 1, 2_L, 2_u, and 3. From $\Sigma H = 0$, the total tension loads must balance the total compression loads. For tension on the bottom fibers

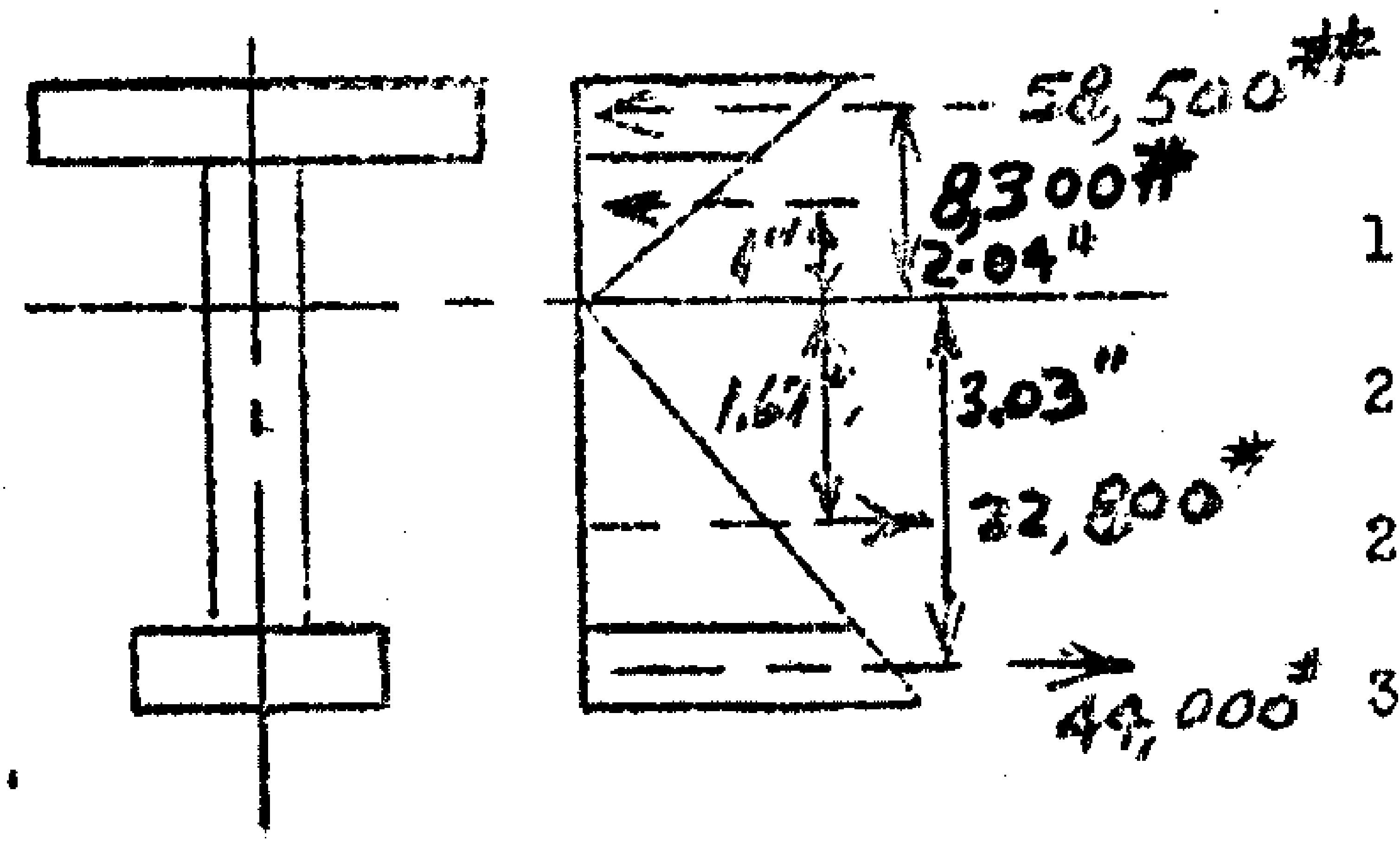


Fig. 5 (a)

Average Load on Segments

$1 = \frac{1}{2}(34{,}200 + 24{,}400) = 29{,}300 \ \#/\square''$

$2_L = \frac{1}{2}(24{,}400) = 12{,}200$

$2_u = \frac{1}{2}(14{,}600) = 7{,}300$

$3 = \frac{1}{2}(24{,}400 + 14{,}600) = 19{,}500$

Total loads on sections

$1 = 1.5 \times 29{,}300 = 44{,}000$

$2_L = (2.5)(.75)(12{,}200) = \underline{22{,}800}$

$\Sigma T = 66{,}800$ lbs.

$2_u = (1.5)(.75)(7300) = 8{,}300$

$3 = 3 \times 19{,}500 = \underline{58{,}500}$

Check $\Sigma C = 66{,}800$ lbs.

ΣM about N.A. must also equal the external moment.

$$\begin{aligned} 58{,}500 \times 2.04 &= 119{,}500 \\ 8{,}300 \times 1.0 &= 8{,}300 \\ 22{,}300 \times 1.67 &= 38{,}200 \\ 44{,}000 \times 3.03 &= \underline{133{,}400} \\ \Sigma M &= 299{,}400 \end{aligned}$$

against 300,000 applied
Error = 600"# = .2 of 1%.

The concept of horizontal shear may be further clarified by taking a section of beam 1 inch long out as a free body. Assuming that the moment at one end of the cut section is 300,000"# and the shear is 20,000# the moment at the other end will be 300,000 ± 1 x 20,000 depending upon whether we are increasing or reducing the moment in the direction taken. The example assumes no additional load on the segment of the beam, and that the moment is reducing toward the left.

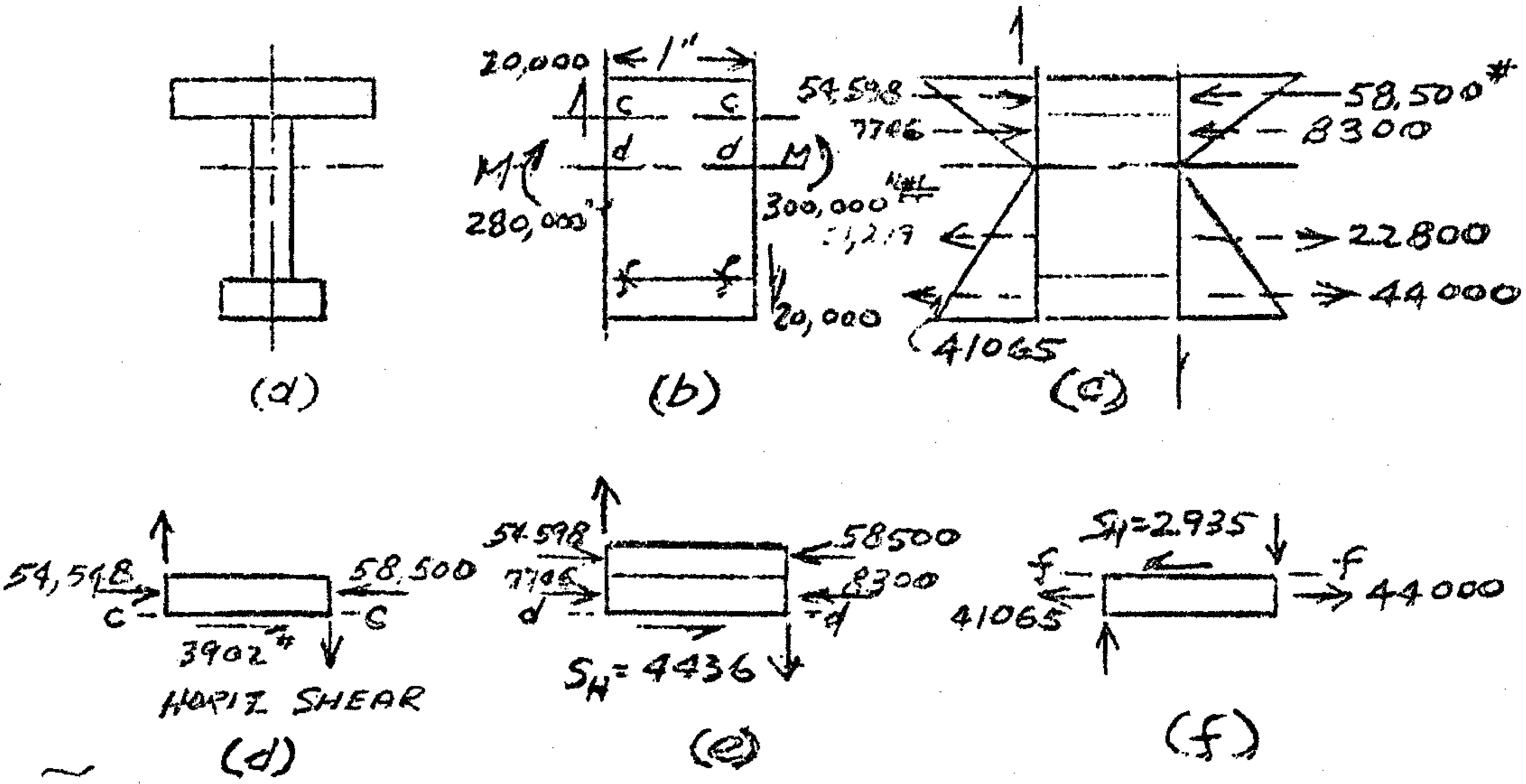


Fig. 6

The forces on the beam are plotted on Fig. 6 (b) and (c). Fig. 6 (d), (e), and (f) show longitudinal sections lifted out of the segment with the forces which hold the sections in equilibrium. The values of horizontal unit shearing stress thus obtained are as follows:

(1) Across section c-c, load = 3902 lbs., Area = 1 x .75 = .75 sq. in.

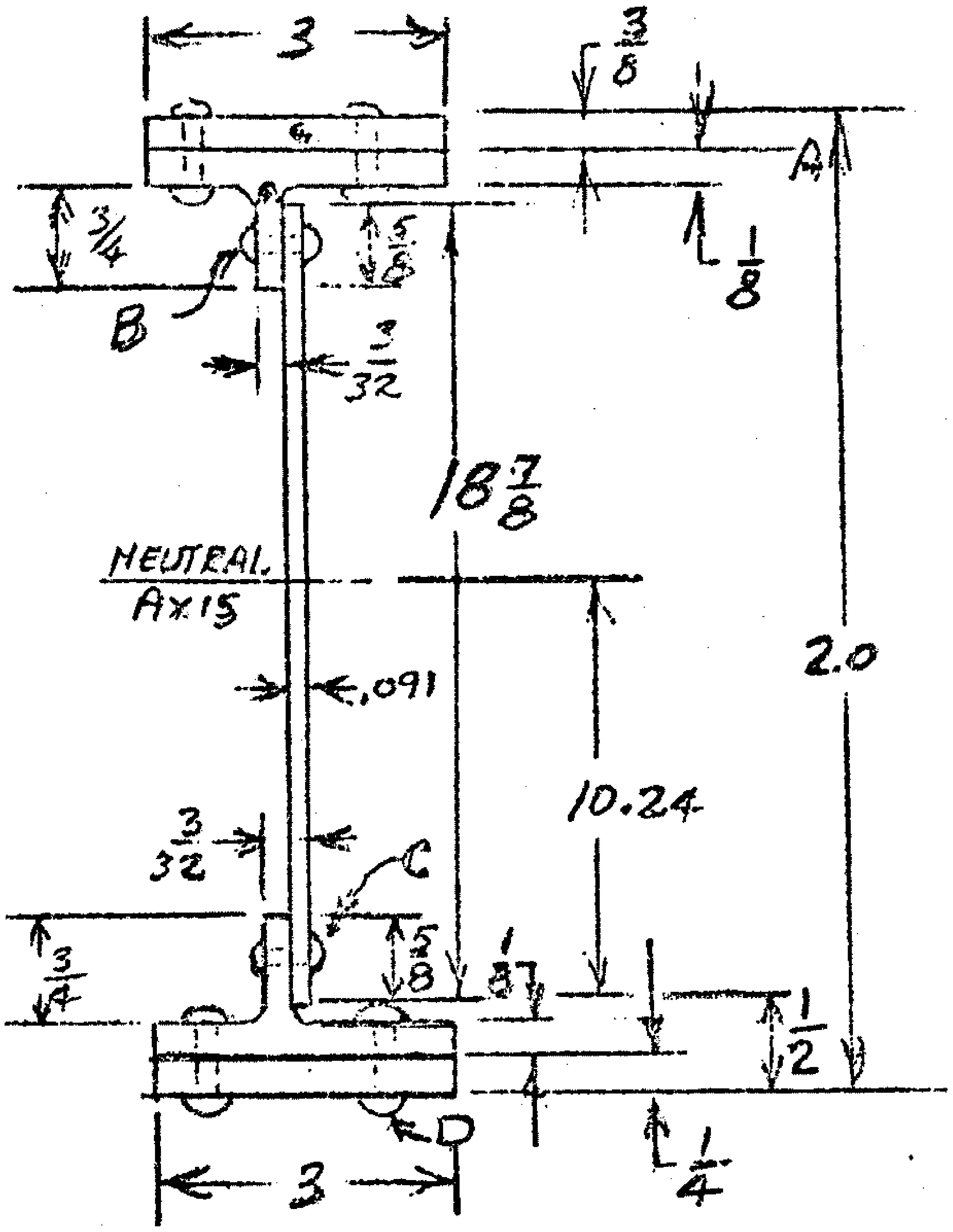
Longitudinal shear stress = $\frac{3902}{.75}$ = 5200 lbs. per sq. in. against 5230 obtained from formula $\frac{SQ}{I}$

(2) At neutral axis d-d = $\frac{4456}{.75}$ = 5940 against 5960 obtained by formula.

(3) Across f-f = $\frac{2935}{.75}$ = 3913 lbs. per sq. in. against 3920 from formula.

Problem Assignment.

9:1 Assume section of wing beam as shown.



(1) Compute section properties.

(2) Compute maximum bending stresses on both compression and tension sides for moment of +1,200,000"# (compression in top).

(3) Compute maximum shear stress in web for shear of 20,000 lbs.

(4) Determine shears per inch along lines A, B, C, and D. Compute allowable rivet spacing, assuming $\frac{5}{32}$ - 24ST rivets good for 672 lbs. each.

(5) Determine loads in elements and check balance of these loads against external moment.

(6) Determine changes in loads in elements of section in 10" assuming constant section and constant shear. Check shears along lines A, B, C, and D. Compare with (4) above.

Note: The change in moment where the shear is constant is equal to the shear times the distance through which it acts. In this case the difference in moment over a 10 inch length equals the shear, (20,000) times 10, or 200,000 in. lbs.

RYAN AERONAUTICAL INSTITUTE

STUDENT WORK-GRADE SHEET

INSTRUCTIONS: BEFORE BEGINNING YOUR STUDY, READ THIS CAREFULLY.

As you complete each portion of this assignment, you are to answer the problems listed for you. Each set of problems is to be completed in order.

Upon completion of all portions of this assignment, you are to mail them to us for correction and comment, and then proceed with the work involved in the next assignment. Send each assignment in for correction as completed.

Before sending your work to us, check the completed problems against the list below. Indicate by check mark in the margin that you have done so. DO NOT SEND IN INCOMPLETED ASSIGNMENTS. Title each problem correctly.

This sheet will be returned to you for a permanent work-grade record. Problem solutions to all assignments in part four will be included in part six of this course.

AIRPLANE STRESS ANALYSIS

PART FOUR

ASSIGNMENT 10, BASIC STRESS, (2 Problems) ________

ASSIGNMENT 11, BASIC STRESS, (1 Problem) ________

ASSIGNMENT 12, BASIC STRESS, (2 Problems) ________

COMPLETED GRADE: ________

Time spent on assignment, hours: ______

REMOVE THIS SHEET AND ATTACH TO YOUR WORK. This sheet must accompany your work so that you will have a grade record.

AIRPLANE
STRESS
ANALYSIS

PART FOUR

ASSIGNMENT 10

ASSIGNMENT 11

ASSIGNMENT 12

BASIC STRESS ANALYSIS

ASSIGNMENT TEN

Assignment 10

THE MOMENT AREA METHOD FOR CALCULATING DEFLECTIONS

The method shown in Assignment 8 for solving for beam deflections works very well where loading is symmetrical and points of maximum deflection (or zero inclination) can be chosen by inspection. The method may be further extended to include unsymmetrical cases as shown by the following example. The method is based upon the principle, THE DEFLECTION OF A POINT IN ANY BENT BEAM FROM A TANGENT AT SOME OTHER ARBITRARILY SELECTED POINT IS EQUAL TO THE STATICAL MOMENT OF THE M/EI AREA BETWEEN THE TWO POINTS WITH RESPECT TO A LINE (NORMAL TO THE REFERENCE TANGENT) THROUGH THE DEFLECTED POINT. The use of this principle does away with the necessity for locating a true axis for the inclination curve, or even with the curve itself if so desired. It is known as the moment-area method.

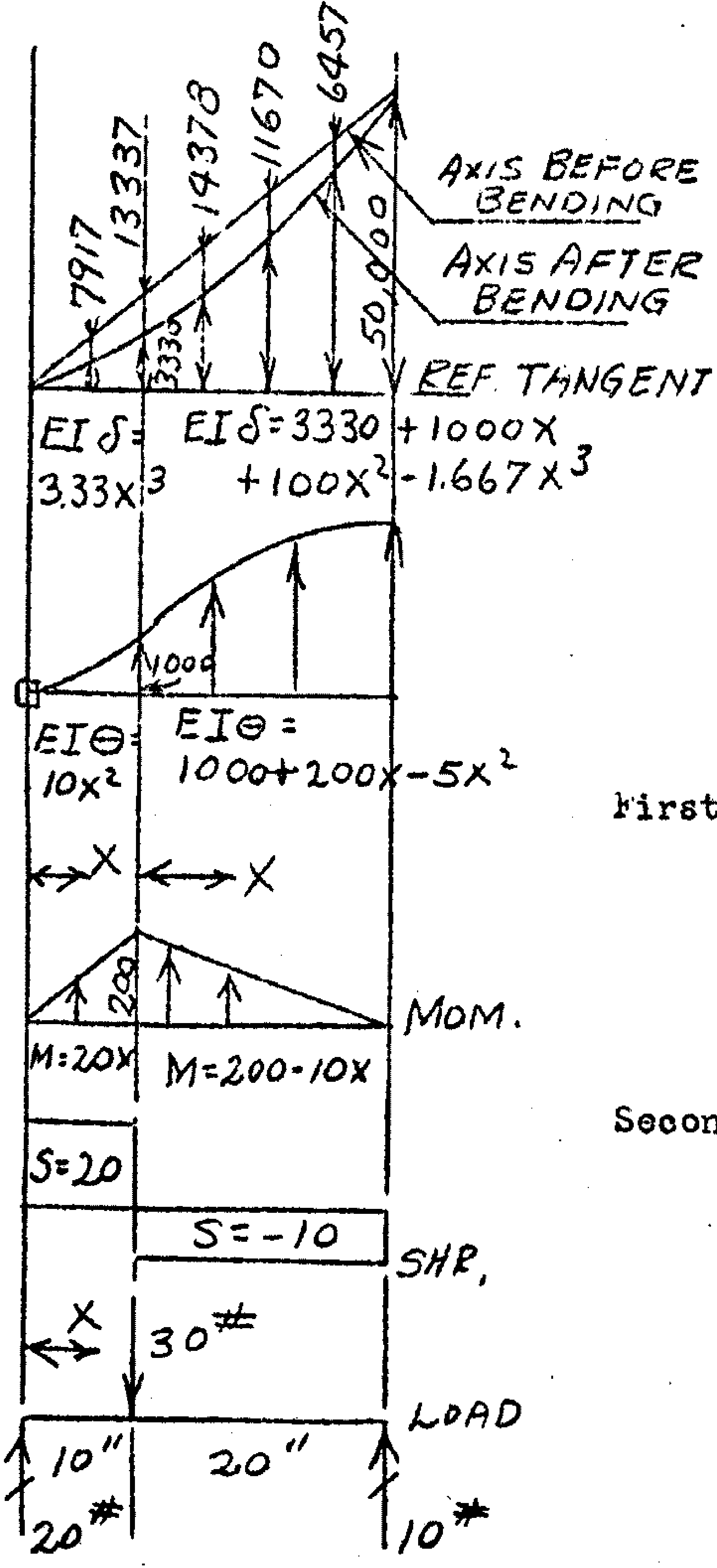


Fig. 1

Using the trial axes the equations of the deflection curve locate the axis after bending with respect to a horizontal reference tangent.

Substituting the value for "x" at the right hand reaction the deflection of reaction point from the reference tangent is obtained.

A straight line connecting the two reaction points locates the undeflected axis in space. The difference between the axis before bending and the axis after bending measured normal to the reference tangent is the true beam deflection.

First equation: $EI\delta = 3.33X^3$

where $X = 5$, $EI\delta = 3.33 \times 5^3 = 416$

true defl. $= \frac{5}{30} \times 50{,}000 - 416 = 7{,}917$

at $x = 10$, $EI\delta = 3{,}330$;

defl. $= \frac{10}{30} \times 50{,}000 - 3330 = 13{,}337$

Second equation: $EI\delta = 3330 + 1000X + 100X^2 - 1.667X^3$

$x = 5$, $EI\delta = 3330 + 1000 \times 5 + 100 \times 5^2 - 1.667 \times 5^3 = 10{,}622$

defl. $= \frac{15}{30} \times 50{,}000 - 10{,}622 = 14{,}378$

$x = 10$, $EI\delta = 3330 + 1000 \times 10 + 100 \times 10^2 - 1.667 \times 10^3 = 21{,}663$

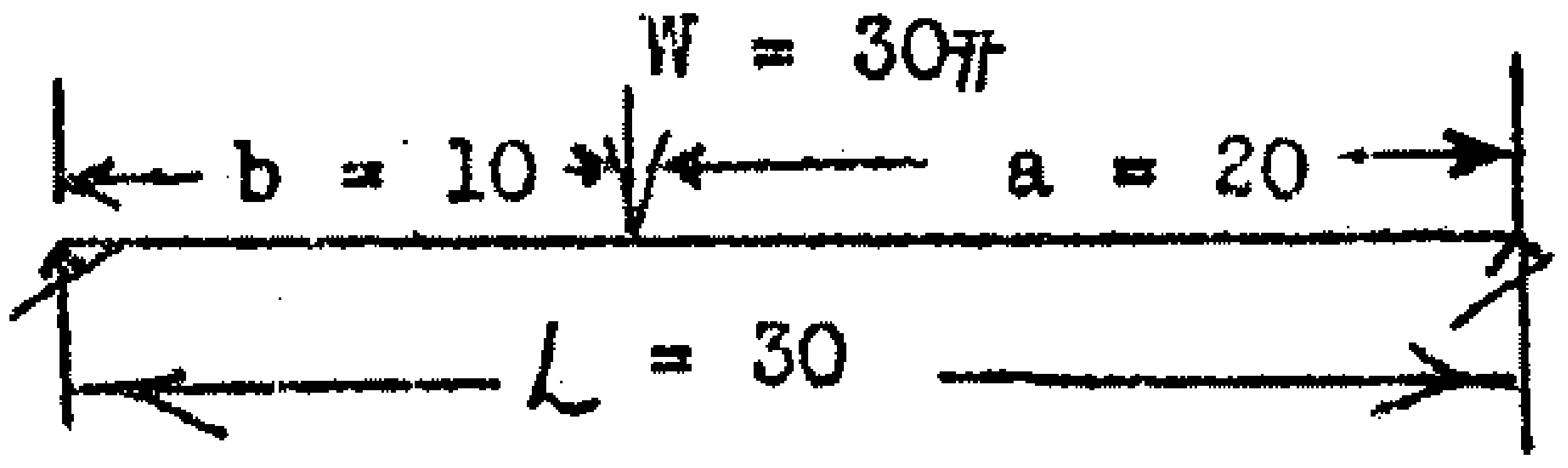
defl. $= \frac{20}{30} \times 50{,}000 - 21663 = 11670$

etc.

True deflections are found by dividing the values listed by EI.

The values may be checked by handbook formulas

$$EI\delta_{Max.} = \frac{Wab\,(a + 2b)\sqrt{3a\,(a + 2b)}}{27\ell}$$



Subst. Values:

$$EI_{\quad Max.} = \frac{(30)(20)(10)(20 + 20)\sqrt{(60)(40)}}{(27)(30)} = 14,520$$

This max. occurs at $X = \ell - \sqrt{\frac{a}{3}\,(a + 2b)}$ from the left hand support.

$X = 30 - \sqrt{\frac{20}{3}\,(20 + 20)} = 30 - 16.4 = 13.6''$

This corresponds to a value of X = 3.6" in the second equation.

Subst. in gen. eq., $EI\,\delta = 3330 + (1000)(3.6) + (100)(3.6)^2 - (1.667)(3.6)^3$
$= 3330 + 3600 + 1296 - 78 = 8148$

Defl. $= \frac{13.6}{30}\,(50,000) - 8148 = 14,519$ (checks above result)

- - - - - -

Moment Area Method, Leaving Out EI θ Curve

Difference between tangent at the origin and a point in the bent beam at the right hand reaction. EI values carried as constants:

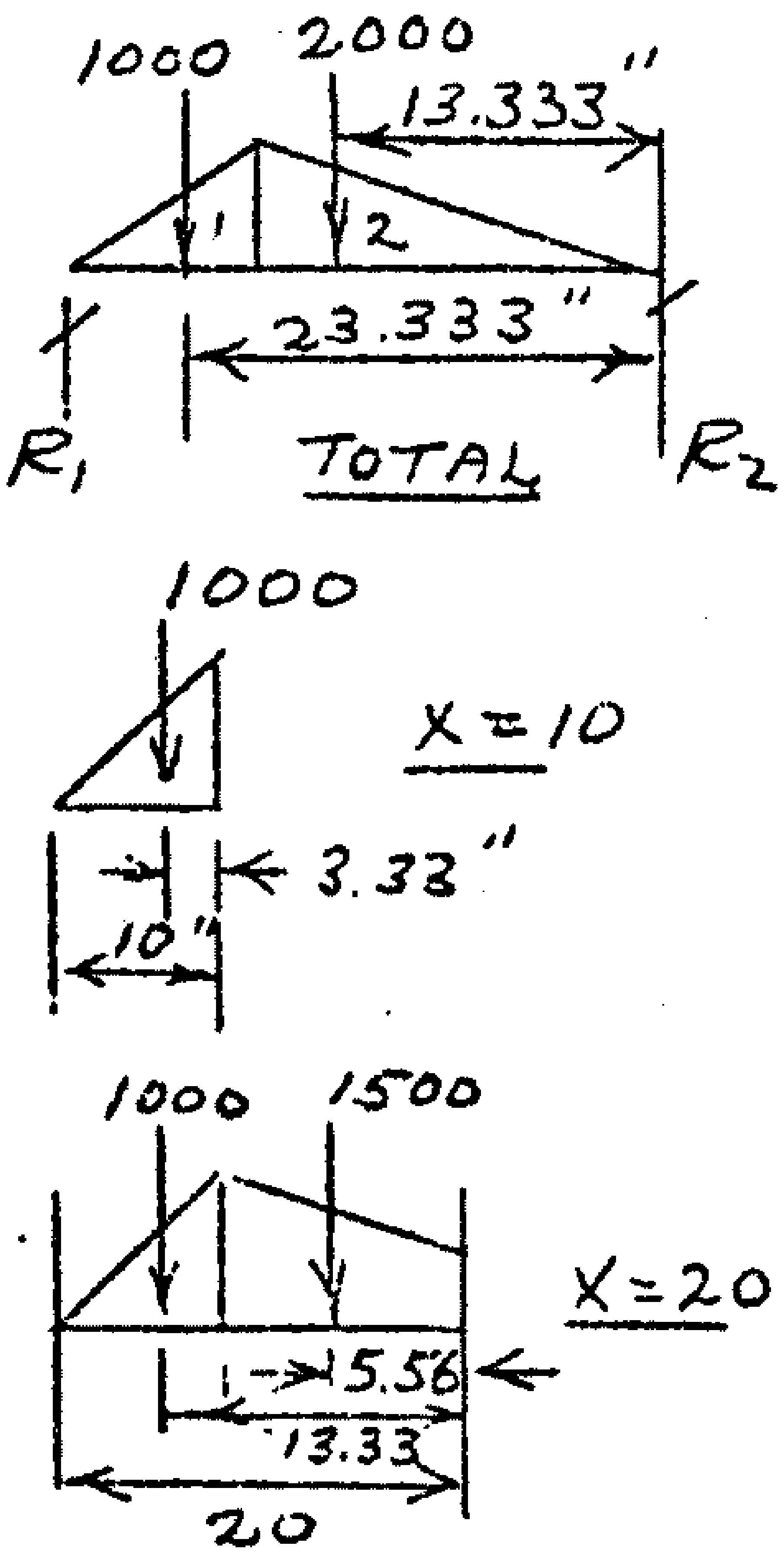


Fig. 2

Mom. area seg. (1) $= 200 \times \frac{10}{2} = 1000$

Mom. area seg. (2) $= 200 \times \frac{20}{2} = 2000$

Static mom. of mom. areas about $R_2 = (1000)(23.333) + (2000)(13.333) = 50,000$

At X = 10", static mom. = 1000 x 3.33 = 3,330

Defl. $= \frac{10}{30} \times 50,000 - 3,330 = 13,337$ checks curve, page 1.

At X = 20" from L. H. reaction

Mom. of areas = 1000 x 13.33 + 1500 x 5.56
= 13,330 + 8330 = 21,660

Defl. $= \frac{20}{30} \times 50,000 - 21,660 = 11,670$ checks

Properties of Moment Diagrams for Use with Moment Area Method

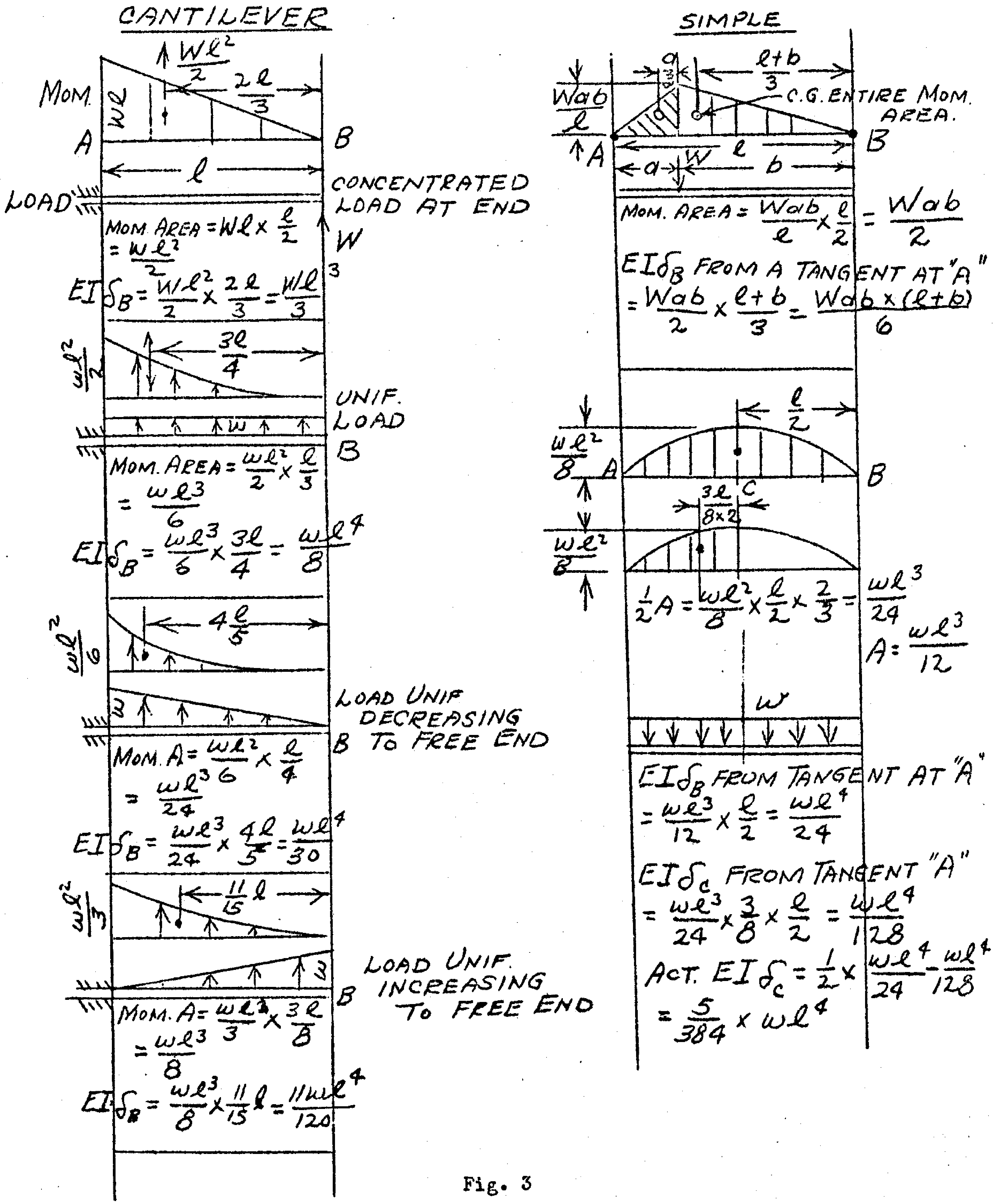
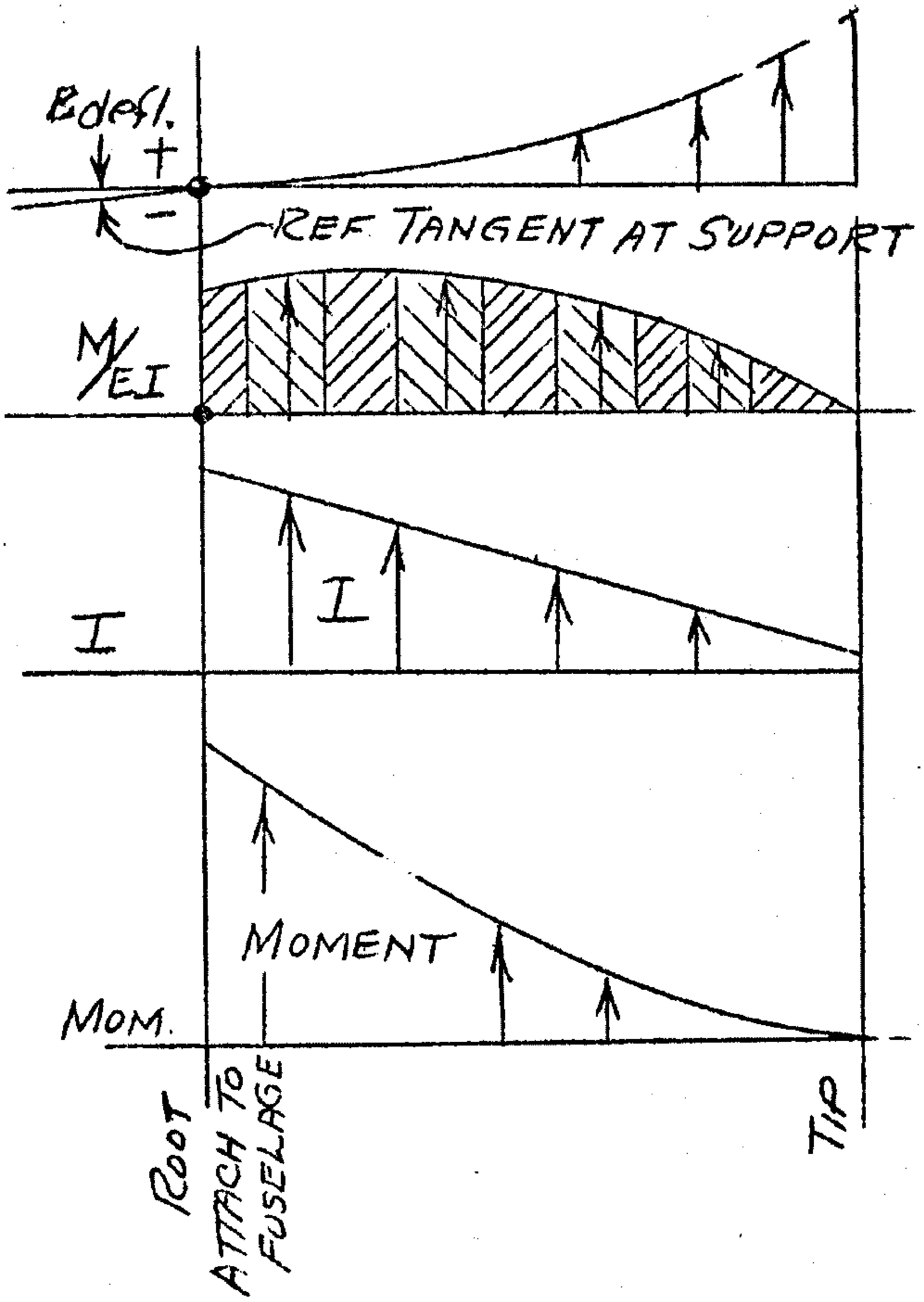

CANTILEVER
MOM.
$\frac{W\ell^2}{2}$
$W\ell$
$\frac{2\ell}{3}$
A
B
ℓ
LOAD
CONCENTRATED LOAD AT END
W
MOM. AREA $= W\ell \times \frac{\ell}{2} = \frac{W\ell^2}{2}$
$EI\delta_B = \frac{W\ell^2}{2} \times \frac{2\ell}{3} = \frac{W\ell^3}{3}$
$\frac{w\ell^2}{2}$
$\frac{3\ell}{4}$
UNIF. LOAD
w
B
MOM. AREA $= \frac{w\ell^2}{2} \times \frac{\ell}{3} = \frac{w\ell^3}{6}$
$EI\delta_B = \frac{w\ell^3}{6} \times \frac{3\ell}{4} = \frac{w\ell^4}{8}$
$\frac{w\ell^2}{6}$
$\frac{4\ell}{5}$
3
LOAD UNIF DECREASING TO FREE END
B
MOM. A $= \frac{w\ell^2}{6} \times \frac{\ell}{4} = \frac{w\ell^3}{24}$
$EI\delta_B = \frac{w\ell^3}{24} \times \frac{4\ell}{5} = \frac{w\ell^4}{30}$
$\frac{w\ell^2}{3}$
$\frac{11}{15}\ell$
3
LOAD UNIF. INCREASING TO FREE END
B
MOM. A $= \frac{w\ell^2}{3} \times \frac{3\ell}{8} = \frac{w\ell^3}{8}$
$EI\delta_B = \frac{w\ell^3}{8} \times \frac{11}{15}\ell = \frac{11w\ell^4}{120}$
SIMPLE
$\frac{Wab}{\ell}$
$\frac{a}{3}$
$\frac{\ell+b}{3}$
C.G. ENTIRE MOM. AREA.
A
B
ℓ
a
W
b
MOM. AREA $= \frac{Wab}{\ell} \times \frac{\ell}{2} = \frac{Wab}{2}$
$EI\delta_B$ FROM A TANGENT AT "A" $= \frac{Wab}{2} \times \frac{\ell+b}{3} = \frac{Wab \times (\ell+b)}{6}$
$\frac{\ell}{2}$
$\frac{w\ell^2}{8}$
A
B
$\frac{3\ell}{8\times2}$
C
$\frac{w\ell^2}{8}$
$\frac{1}{2}A = \frac{w\ell^2}{8} \times \frac{\ell}{2} \times \frac{2}{3} = \frac{w\ell^3}{24}$
$A = \frac{w\ell^3}{12}$
w
$EI\delta_B$ FROM TANGENT AT "A" $= \frac{w\ell^3}{12} \times \frac{\ell}{2} = \frac{w\ell^4}{24}$
$EI\delta_C$ FROM TANGENT "A" $= \frac{w\ell^3}{24} \times \frac{3}{8} \times \frac{\ell}{2} = \frac{w\ell^4}{128}$
ACT. $EI\delta_C = \frac{1}{2} \times \frac{w\ell^4}{24} - \frac{w\ell^4}{128}$
$= \frac{5}{384} \times w\ell^4$

Fig. 3

The moment area method is used extensively to calculate cantilever* wing deflections where very often neither the moment of inertia nor the moment curve can be expressed in the form of a single equation. Where such a condition exists the values of the moment on the wing and the moment of inertia of the bending material in the wing are calculated separately. Next a curve is drawn on which each ordinate represents the value $\frac{M}{EI}$ for a particular section of the wing. This curve is then treated as though it were a load curve on a wing. The moment curve corresponding to this imaginary $\frac{M}{EI}$ loading is then the deflection curve of the wing. Note carefully that units must be consistent. If moment is in inch pounds E in pounds per square inch and moment of inertia in inches to the fourth power, deflection will be in inches.



$$\frac{M}{EI} = \frac{\text{Ordinate in M curve}}{(\text{Ordinate in I curve})(E)}$$

I curve plotted by calculating

I values for several sections along the span

Moment calculated by moving inward from the tip.

Fig. 4

Note carefully that the deflection calculated by the moment-area method is with respect to a tangent at the reference point. True deflections in space are then obtained directly only when the reference point on the axis of the bent beam is horizontal as at a fixed support (see Assignment 7) or at a location about which the beam is symmetrical and loaded symmetrically. For example if a cantilever wing is attached at two locations along the sides of a fuselage and extends through the fuselage, the effect of deflection of the center portion will be to increase all of the deflections of the cantilever portion. (See Fig. 5)

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* Note: A cantilever beam is one which projects beyond a support. Cantilever wings are attached at the fuselage and project beyond their attachments on

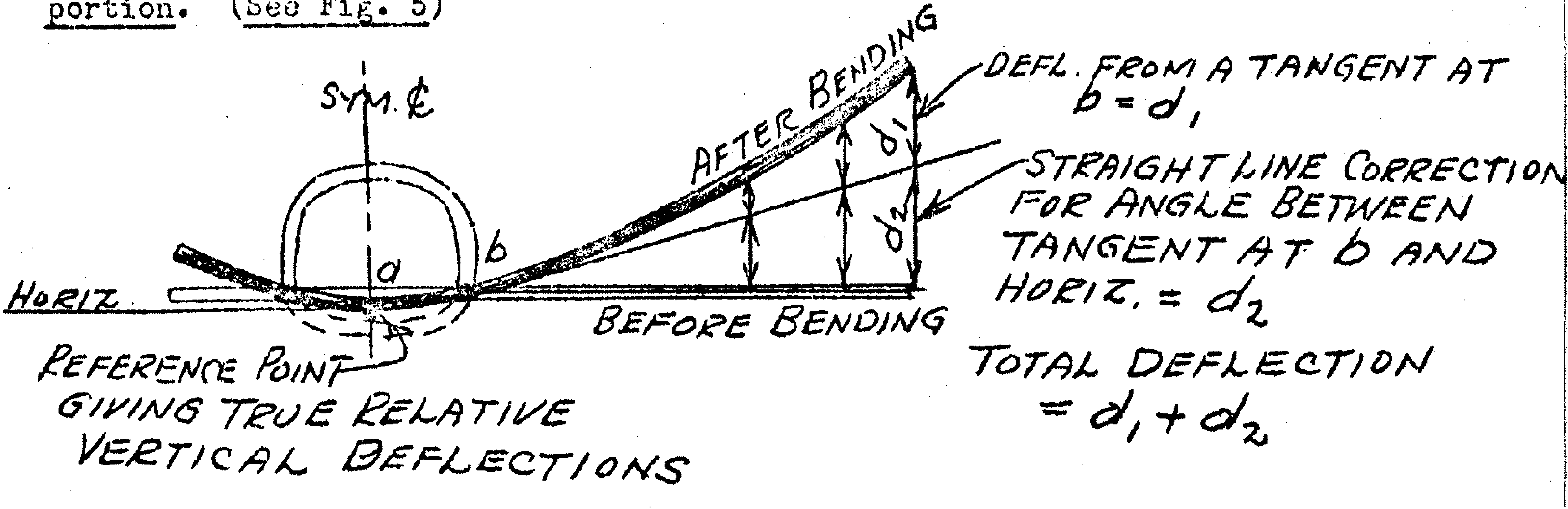

SYM. ℄
AFTER BENDING
DEFL. FROM A TANGENT AT b = d_1
STRAIGHT LINE CORRECTION FOR ANGLE BETWEEN TANGENT AT b AND HORIZ. = d_2
d_1
d_2
a
b
HORIZ.
BEFORE BENDING
TOTAL DEFLECTION = $d_1 + d_2$
REFERENCE POINT GIVING TRUE RELATIVE VERTICAL DEFLECTIONS

Fig. 5

Where the material in the beam does not change E is constant, and may be carried as a constant into the final, $E\delta$, curve. Then true values are obtained by dividing the ordinates of the $E\delta$ curve by E. Also for simplicity it is often convenient to introduce a constant multiplier or divider in the $\frac{M}{I}$ or $\frac{M}{EI}$ curve, and correct for it in the reduction of the final ordinates in the deflection curve. This value can be lumped in with the constant value of E.

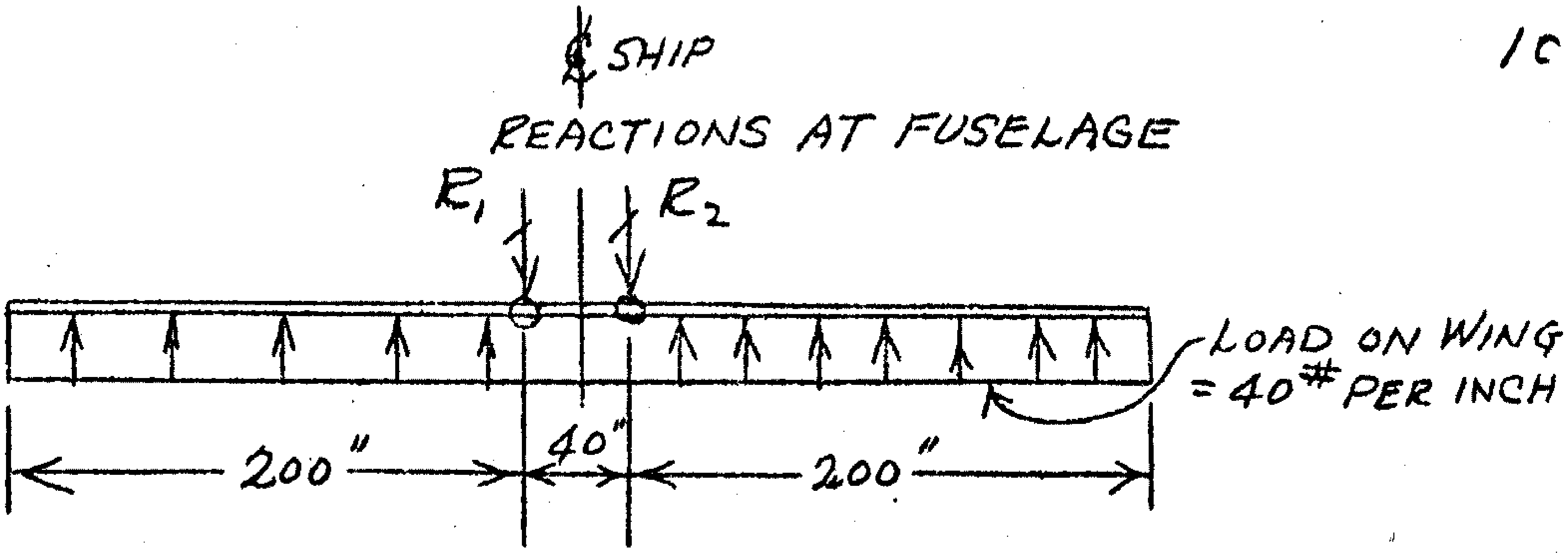
It is not always possible to use the full value of E for the material in computing the deflection of a fabricated (bolted or riveted up) wing. Some allowance is generally necessary for rivet slippage, etc. The above method accounts only for deflection due to bending and makes no allowance for deflection due to shear. In general close agreement with test results is obtained when it is assumed that E for the complete wing is .90 x E for the material used.

Note: E is obtained by tensile tests on pieces of the material.

$$E = \frac{\text{unit stress}}{\text{unit elongation}} = \frac{\text{lbs/sq. in.}}{\text{in./in.}}$$

The general case for cantilever wing deflection will be illustrated by an example. See Fig. 6.

The first step is to solve for the shears and moments. Then from symmetry the deflected wing axis is horizontal at the center line of the ship. Also $R_1 = R_2 = \frac{1}{2}$ the total load. It is possible to take one-half of the total wing out as a free body and solve for the moments and deflections.

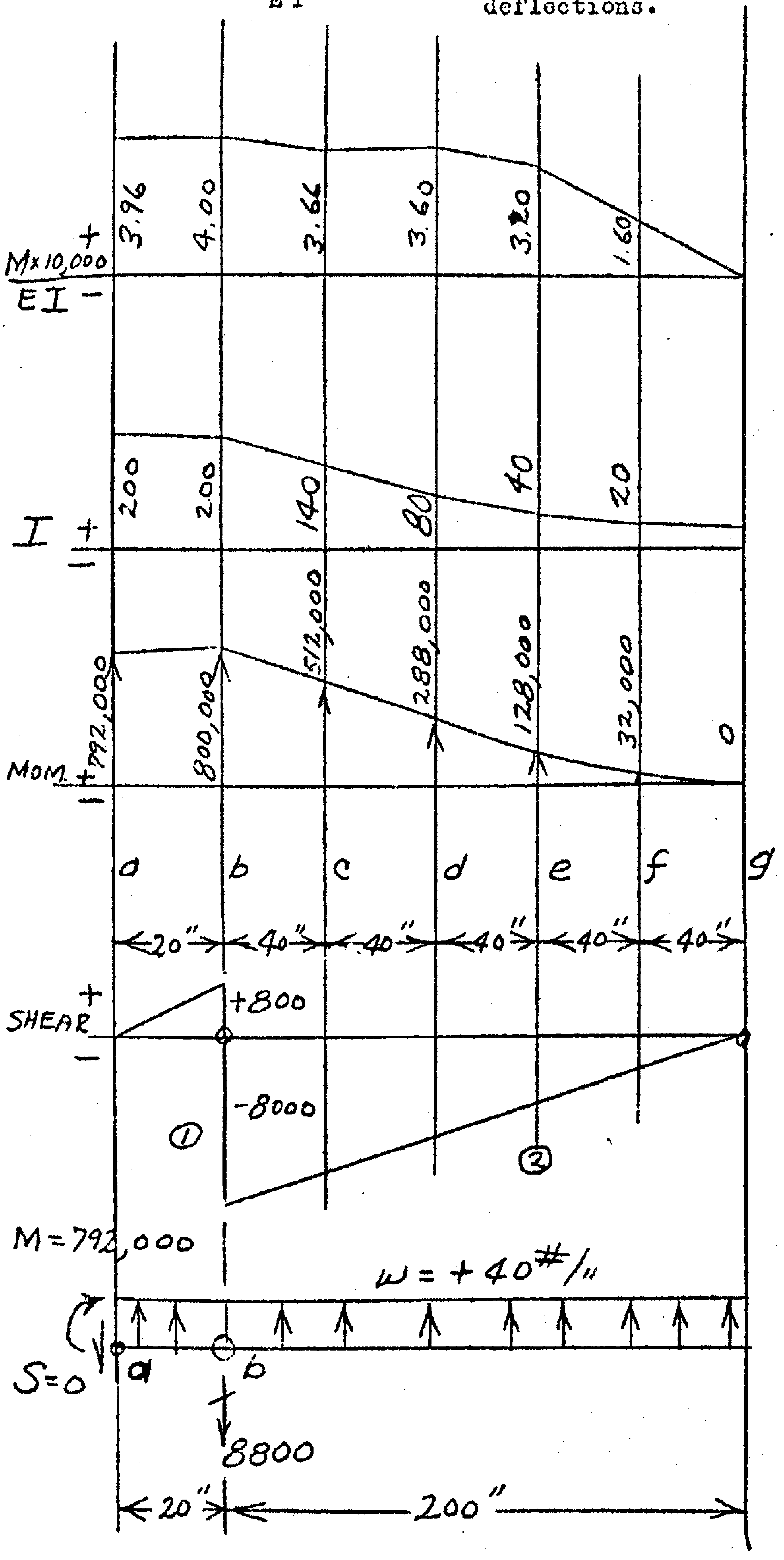


℄ SHIP
REACTIONS AT FUSELAGE
R_1
R_2
LOAD ON WING = 40# PER INCH
200"
40"
200"

Fig. 6

Use E = 10,000,000

Use curve $\frac{M}{EI}$ x 10,000. Divide final results by 10,000 to obtain true deflections.



Curve of $\frac{M}{EI}$ x 10,000

Assume straight line variation between stations shown.

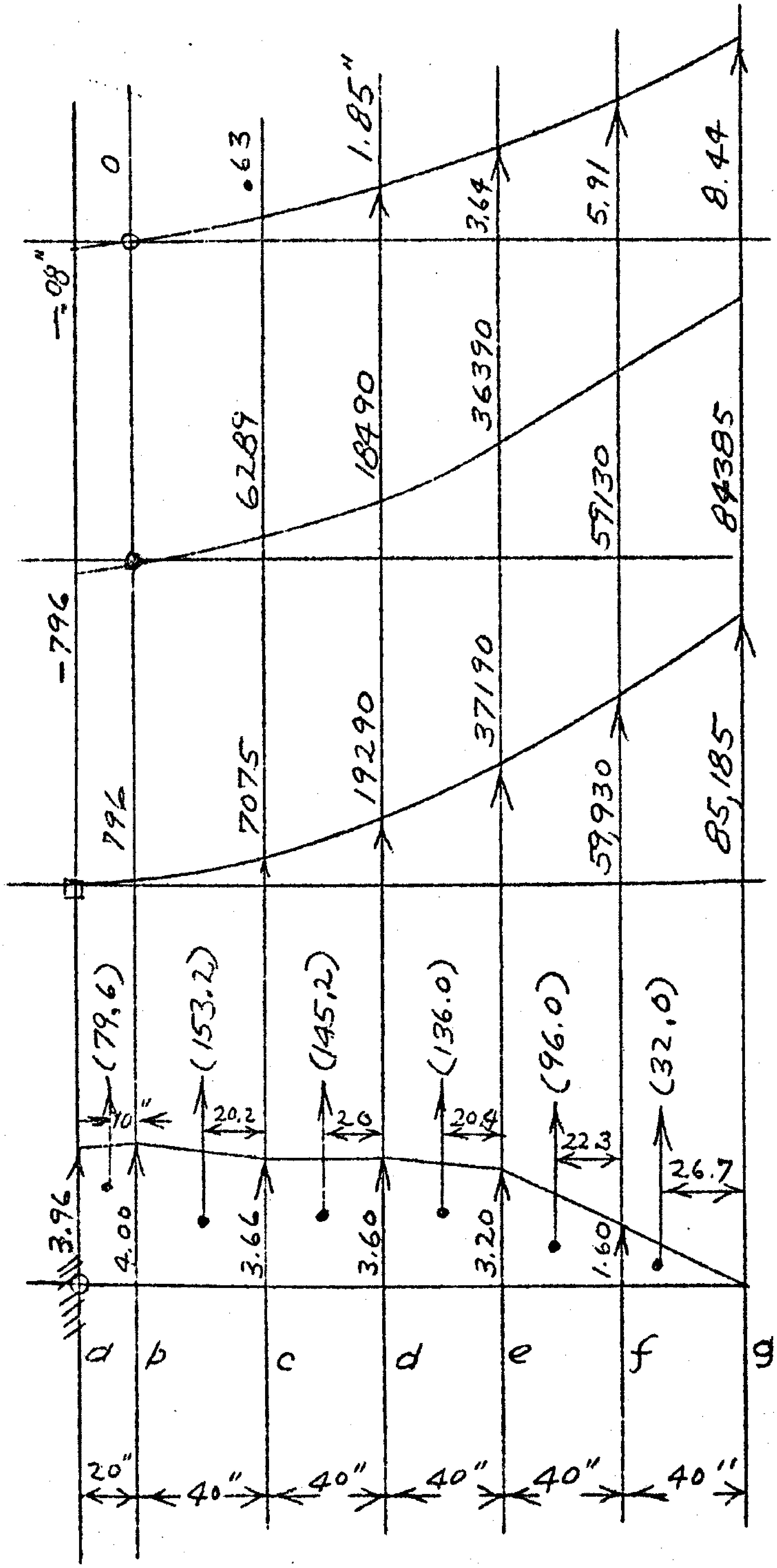
I curve

(1) $M = 792{,}000 + 20X^2$

(2) $M = 800{,}000 - 8000X + 20X^2$

(1) $S = 40X$

(2) $S = -8000 + 40X$



Deflection in inches from point "b"

Deflection from point "b" x 10,000

Deflection from tangent at "a" x 10,000

$\frac{M}{EI}$ x 10,000

Since axis of beam is horizontal at "a" it deflects with respect to "a" as though it were a cantilever beam with perfect fixity at "a".

Fig. 7

Moment areas

$$\underline{a-b} = (20)\frac{(3.96 + 4.0)}{2} = 79.6$$

arm to b = 10"

$$\underline{b-c} = (40)\frac{(4.00 + 3.66)}{2} = 153.2$$

arm to c = 20.2" (centroid of trapezoid)

$$\underline{c-d} = (40)\frac{(3.66 + 3.60)}{2} = 145.2$$

arm to d = 20"

$$\underline{d-e} = (40)\frac{(3.60 + 3.20)}{2} = 136.0$$

arm to e = 20.4"

$$\underline{e-f} = (40)\frac{(3.20 + 1.60)}{2} = 96.0$$

arm to f = 22.3"

$$\underline{f-g} = (40)\frac{(1.60)}{2} = 32.0$$

arm to g = 26.7" (triangle)

- - - - - -

Taking moments of the moment areas about b, c, d, e, f, and g in turn:

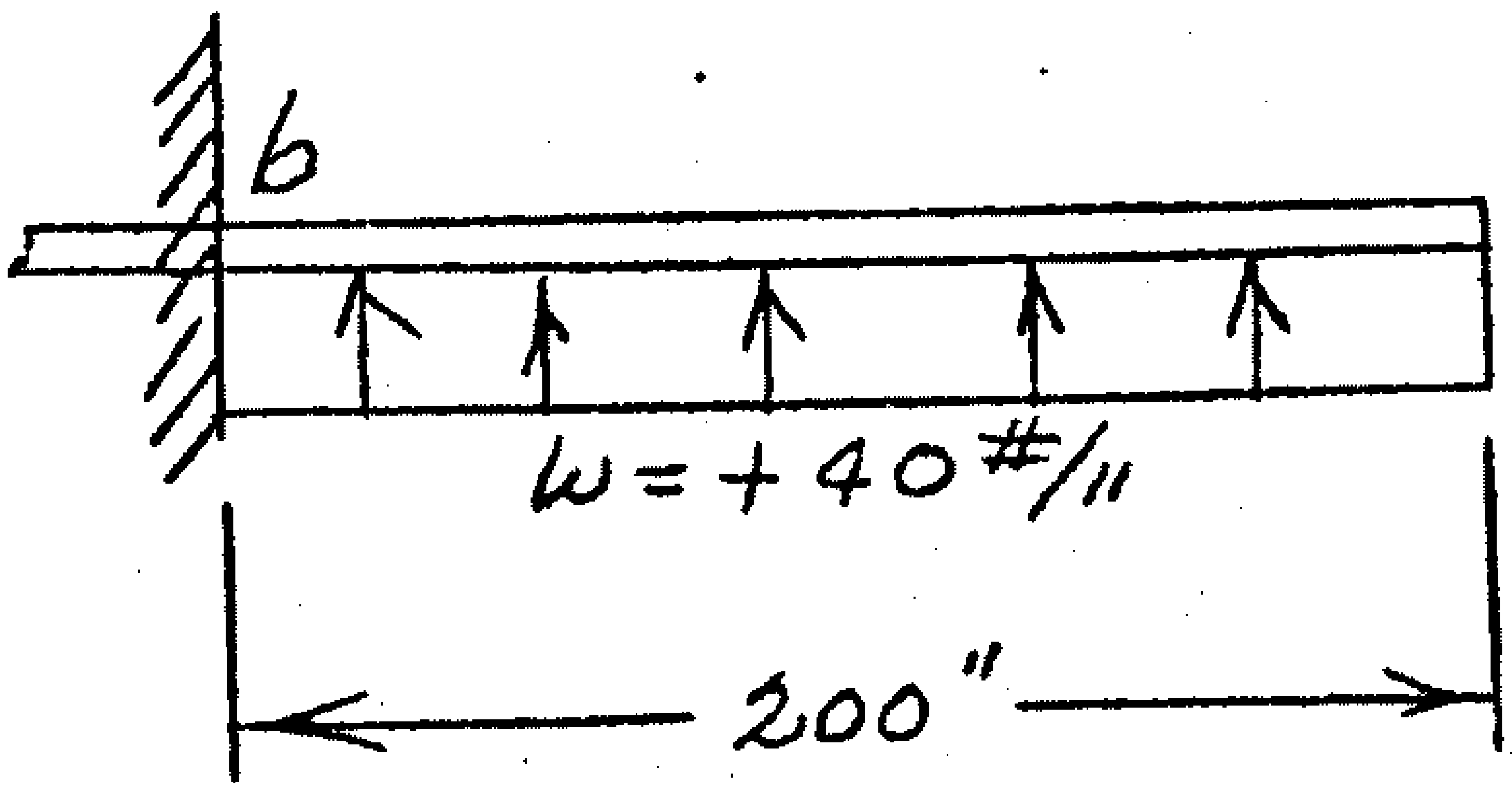
Mom. about b = $(79.6)(10) = 796$
Mom. about c = $(79.6)(10 + 40) + (153.2)(20.2)$
= $3980 + 3095 = 7075$
Mom. about d = $(79.6)(10 + 40 + 40) + (153.2)(20.2 + 40) + (145.2)(20)$
= $7164 + 9222 + 2904 = 19{,}290$

etc.

Problem Assignment.

10:1 Check the problems worked in the assignment, making sure that you understand each step.

10:2 Find the deflection of the cantilever wing illustrated, assuming that the wing is not continuous through the fuselage but is rigidly supported (full fixity) at the side of the fuselage. (Note: For this case the bent axis of the wing is horizontal at "b".)



BASIC STRESS ANALYSIS

ASSIGNMENT ELEVEN

Assignment 11

CONTINUOUS BEAMS

UNIT LOAD DEFLECTION METHOD OF FINDING REACTIONS

Continuous beams are those which are supported on more than two reaction points. For continuous beams the equations of equilibrium $\Sigma V = 0$, and $\Sigma M = 0$ are not sufficient to solve for the reactions. For example if a beam is continuous over three supports we have three unknown reactions, and only two equations. To solve this problem therefore involves the physical actions of the beam.

Where there are only three supports the beam may be made determinate by removing one reaction. Its effect is then solved for as follows. It is assumed that the reaction points do not move.

1. Remove one reaction and find the deflection of the beam at the point from which the reaction is removed using actual external loads on the beam.
2. Remove all external loads and calculate the deflection of the beam under a unit load only acting at the reaction point in the direction of the reaction. The ratio of the two deflections will give the true value of the reaction. (A unit load is an index load of 1 lb., 100 lbs., or 1000 lbs., etc. used as a measuring stick.)

In effect what we have done is to remove the reaction, let the beam sag, and applied the load at the reaction point necessary to deflect the beam back to its original position. The method is best illustrated by taking an actual problem. Assume that we place a support at the center of the beam shown on page 4, Assignment 8. Its effect will be to hold the deflected neutral axis of the beam in line with the end reactions. The deflection of the center under the external loads with no reaction at the center has been found to be .433 inches. If all other load is removed and a unit load is placed at the center reaction point the deflection of this point from handbook formula would be

$$\delta = \frac{wL^3}{48EI} = \frac{(1)(100)^3}{(48)(10,000,000)(3)} = \frac{1}{1440}$$

$$\Delta = \text{deflection with reaction removed} = .433 \text{ inches}$$

$$R_2 = \text{center reaction} = .433/{}^{1}/_{1440} = .433 \times 1440 = 625 \text{ lbs.}$$

By symmetry R_1 and R_3 are equal, or each equals $\frac{1}{2}(1000 - 625) = 187.5$ lbs. For this case of uniform loading and equal spans there are formula for the reactions:

$$R_1 = R_3 = 3/16 \times \text{total load}$$

$$R_2 = 5/8 \times \text{total load which check our result.}$$

Once the reactions are known the solution of shear, moment, inclination and deflection curves is possible. For this beam the three lower curves (load, shear, and moment) are shown on the following page.

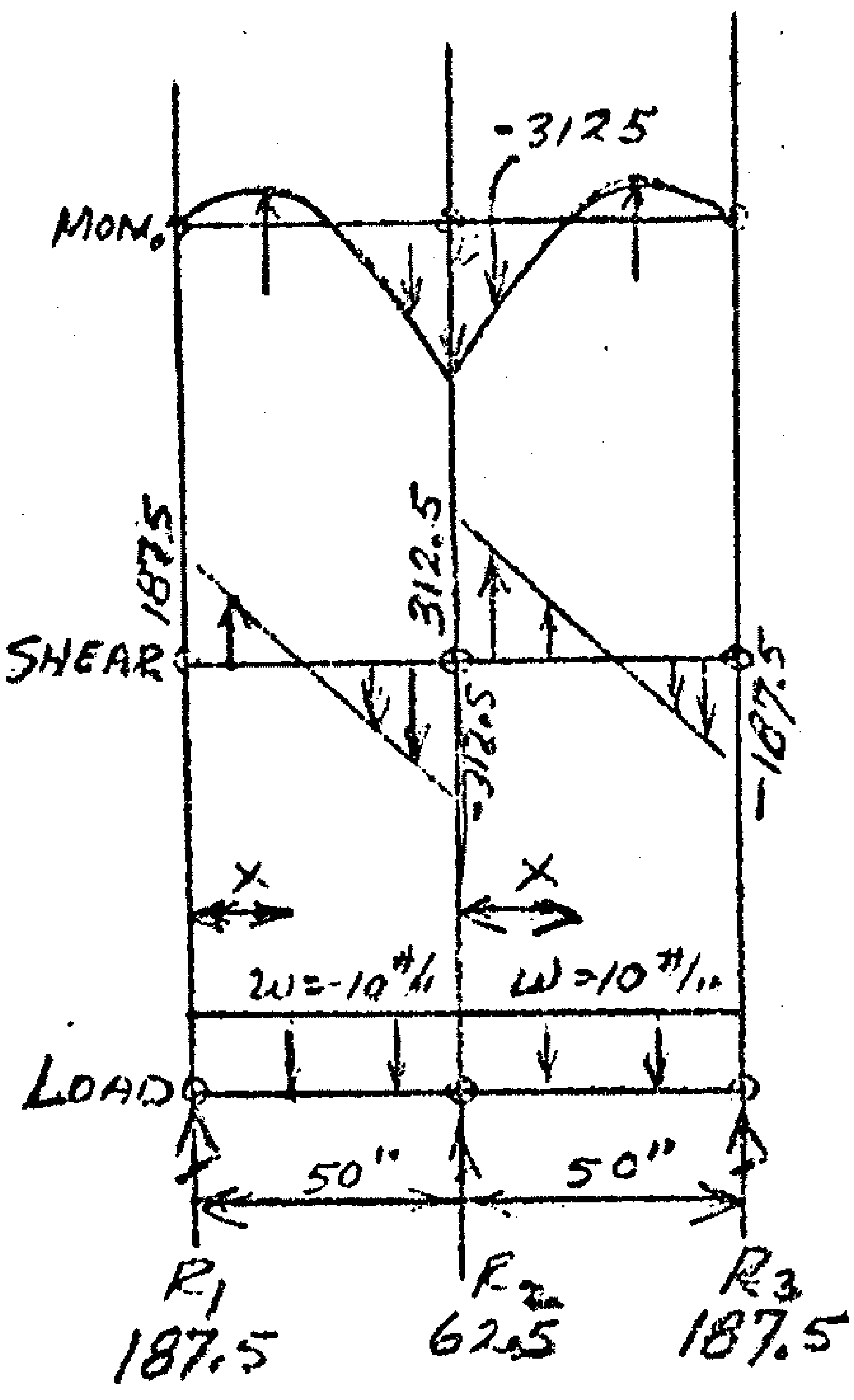


Fig. 1

Span 1-2, Mom. = $\int_0^x (187.5 - 10x)dx =$

$$187.5x - 5x^2$$

At x = 50", Mom. = $(187.5)(50) - (5)(50)^2$

$= 3,125"\#$

Span 2-3, Mom. = $\int_0^x (312.5 - 10x)dx =$

$$312.5x - \frac{10x^2}{2} + K$$

When x = 0, Mom. = K = -3,125

Then $\text{Mom.}_{2-3} = -3,125 + 312.5x - 5x^2$

Checking at R_3 where Mom. = 0, x = 50

$M = -3,125 + (312.5)(50) - (5)(50)^2$

$= -3,125 + 15,625 - 12,500 = 0$ Checks.

Span 1-2, Shear = 187.5 - 10x
Span 2-3, Shear = 312.5 - 10x

Note: When the functions are discontinuous as at a reaction or a concentrated load take a new origin for "x" and write new equations referring to the new origin.

****

Methods of Solution of Continuous Beams

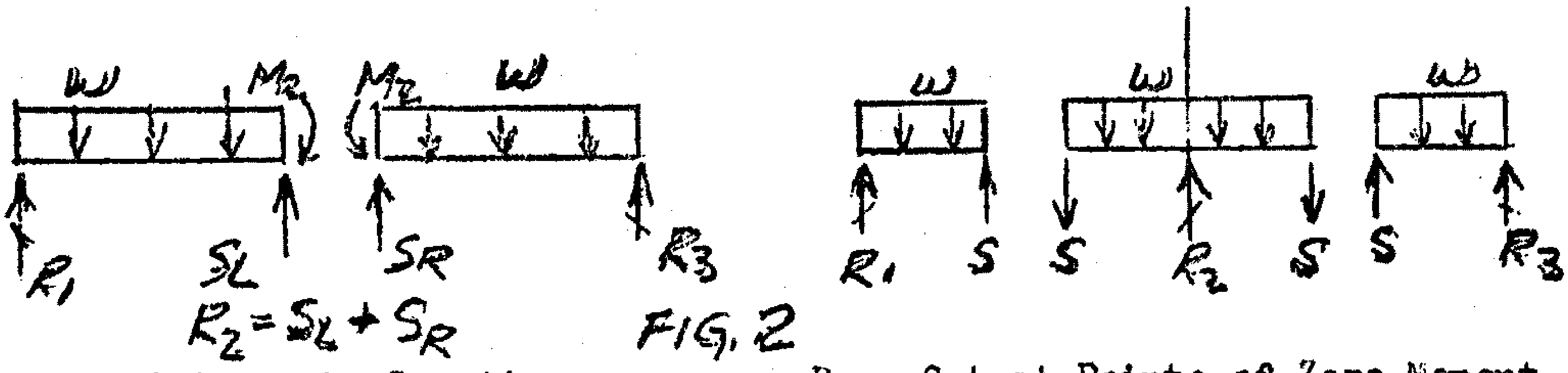


FIG. 2

Beam Cut at the Reaction

Beam Cut at Points of Zero Moment

Any span of a continuous beam may be taken out as a free body once the moments at the supports are known. Also if the points of zero moment in the beam can be found a continuous beam may be broken into a series of simple and cantilever beams. The force systems acting are shown on the preceding page. The method of solving directly for moments at supports will be given in the next assignment.

The Unit Load Deflection Method offers an easy solution for the problem of the restrained Cantilever Beam. Fig. 3

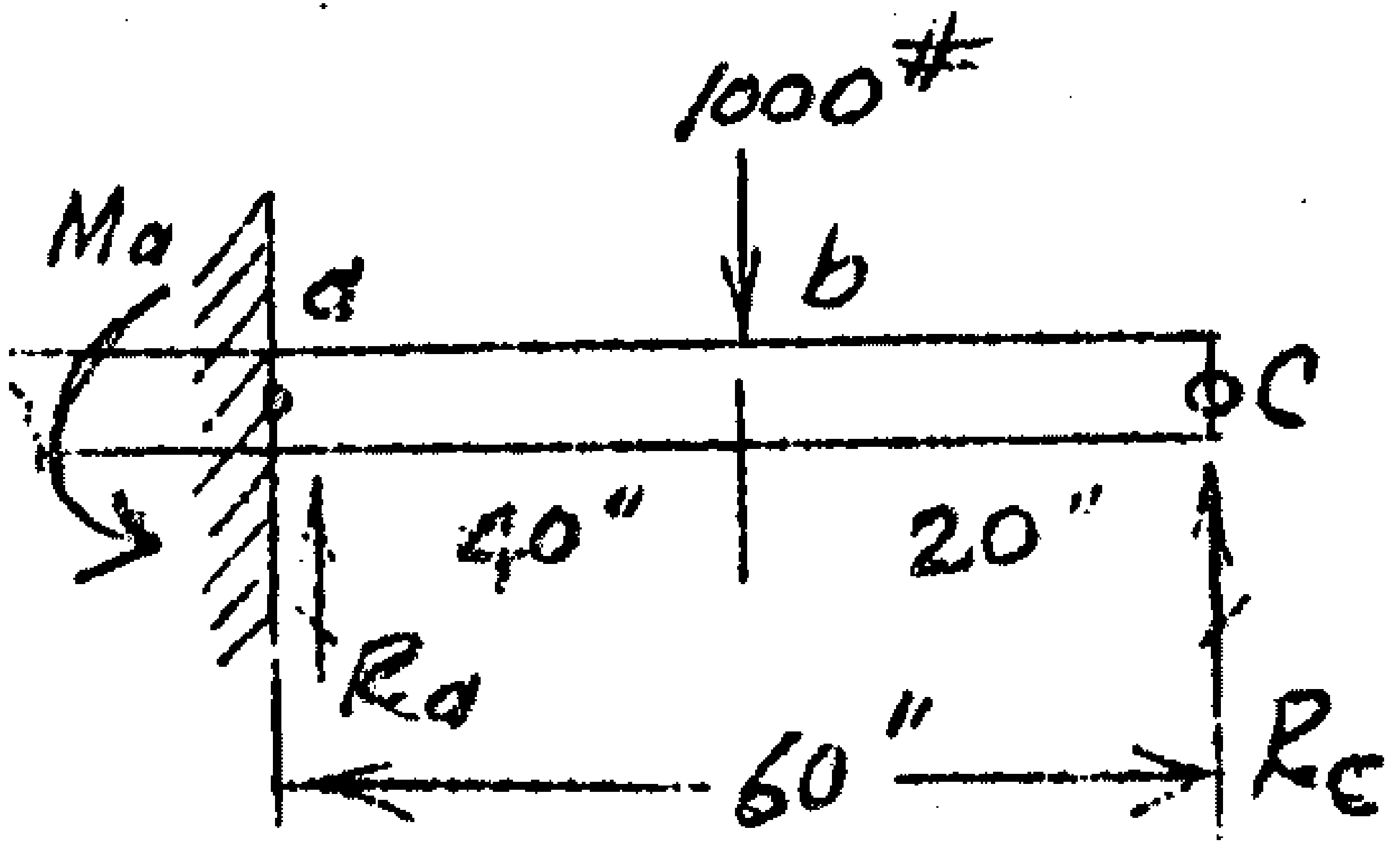


Fig. 3

This beam is statically indeterminate since the equations of equilibrium which we may apply consist of $\Sigma V = 0$ and $\Sigma M = 0$, while the unknown values are R_a, R_c and the moment, M_a, at the wall. With a fully fixed beam, the inclination of the neutral axis at the edge of the wall will be zero. (The axis remains horizontal.) Then the reference tangent for the moment area method of Assignment 10 is horizontal at "a", and deflections taken with respect to "a" will be true deflections.

First removing the reaction, R_c, and treating the beam as a simple cantilever with a load of 1000 lbs. acting at "b", the load shear and moment curves are as shown on Fig. 4. It is not necessary to use the values of E and I in the unit load deflection method since we are interested in the ratio of deflections under the actual loads with the reaction removed and under a unit load at the reaction point. Since E and I appear in the equation for each, their effect is cancelled when the ratio is set up. We omit them, therefore, in this case.

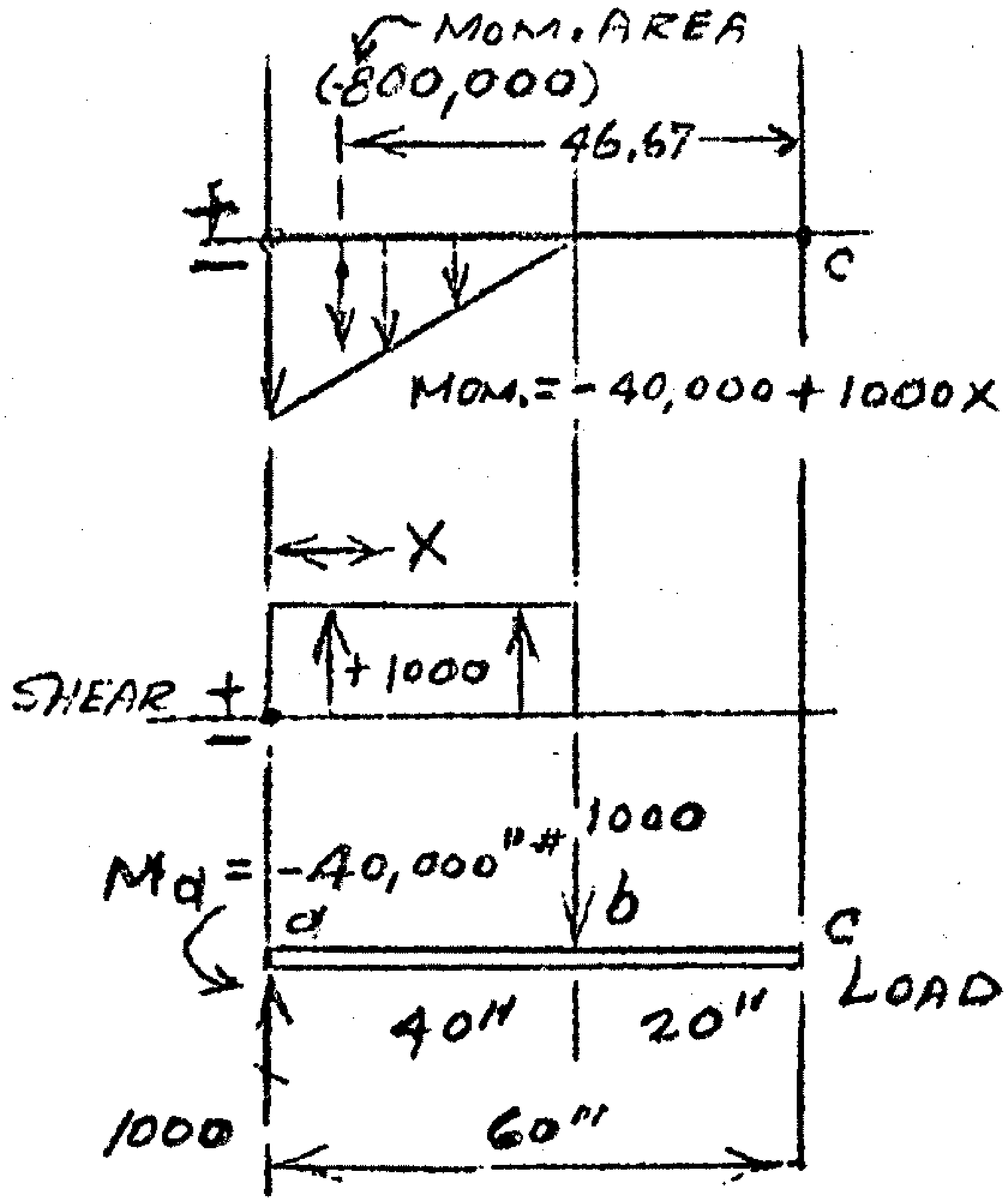


Fig. 4
Deflection of Point C with
Reaction Removed

Solution:

With reaction at "C" removed the moment at the wall is $-(1000)(40) = -40{,}000''\#$. The minus sign shows compression on the bottom side. Total moment area =

$$-(40{,}000)\left(\frac{40}{2}\right) = -800{,}000$$

$$EI\,\delta_c = (-800{,}000)(46.67) = -37{,}336{,}000$$

Deflection under actual load with support at c removed.

Next we remove the external load from the beam and in place of the reaction at C apply a unit load of 100 lbs. acting upward.

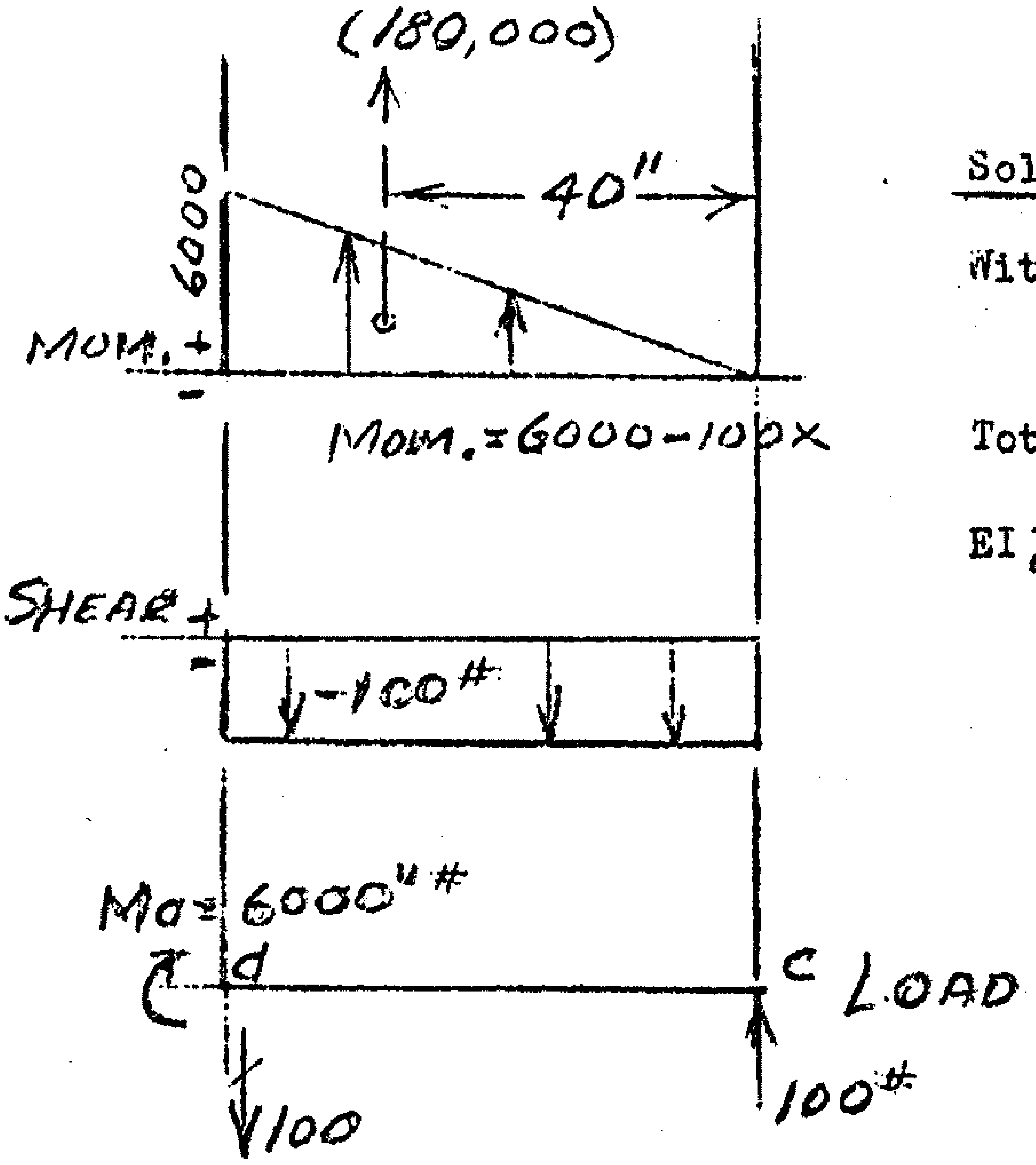


Fig. 5
Deflection of Point C with
100 lb. unit load

Solution:

With an upward reaction at C of 100 lbs.,

$$M_a = (100)(60) = 6000''\#$$

$$\text{Total Moment area} = (6000)\left(\frac{60}{2}\right) = 180{,}000$$

$$EI\,\bar{\delta}_c = (180{,}000)(40) = 7{,}200{,}000$$

The total upward reaction at C $= \left(\dfrac{37{,}336{,}000}{7{,}200{,}000}\right)(100) = 518$ lbs.

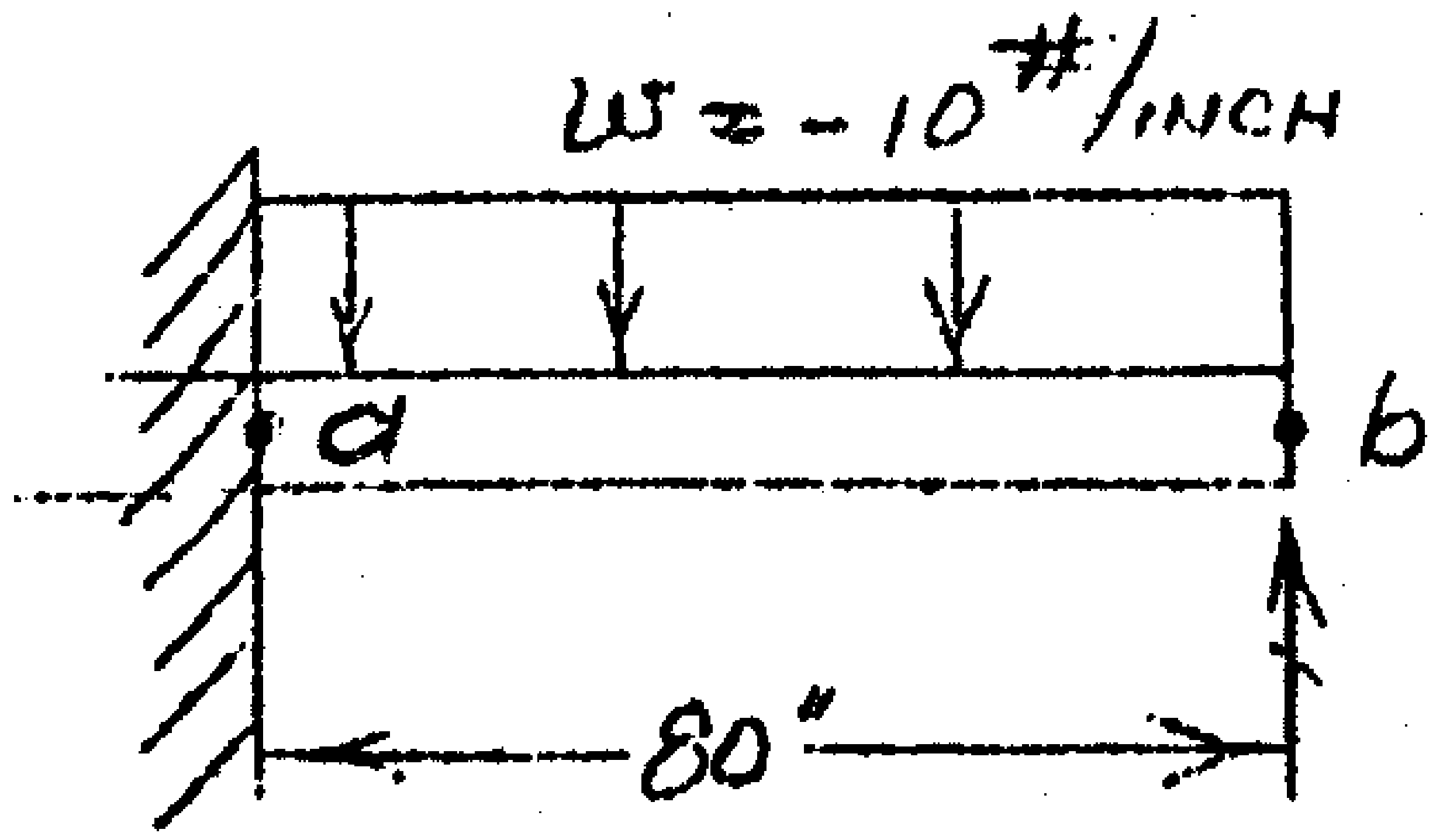
$R_c = 518$ lbs., $R_a = 1000 - 518 = 482$ lbs.

$M_a = (518)(60) - (1000)(40) = -8{,}920''\#$

Problem Assignment.

11:1 Solve for the reactions and find the loads, shears, and moments on the following indeterminate beams, using the unit load deflection method.

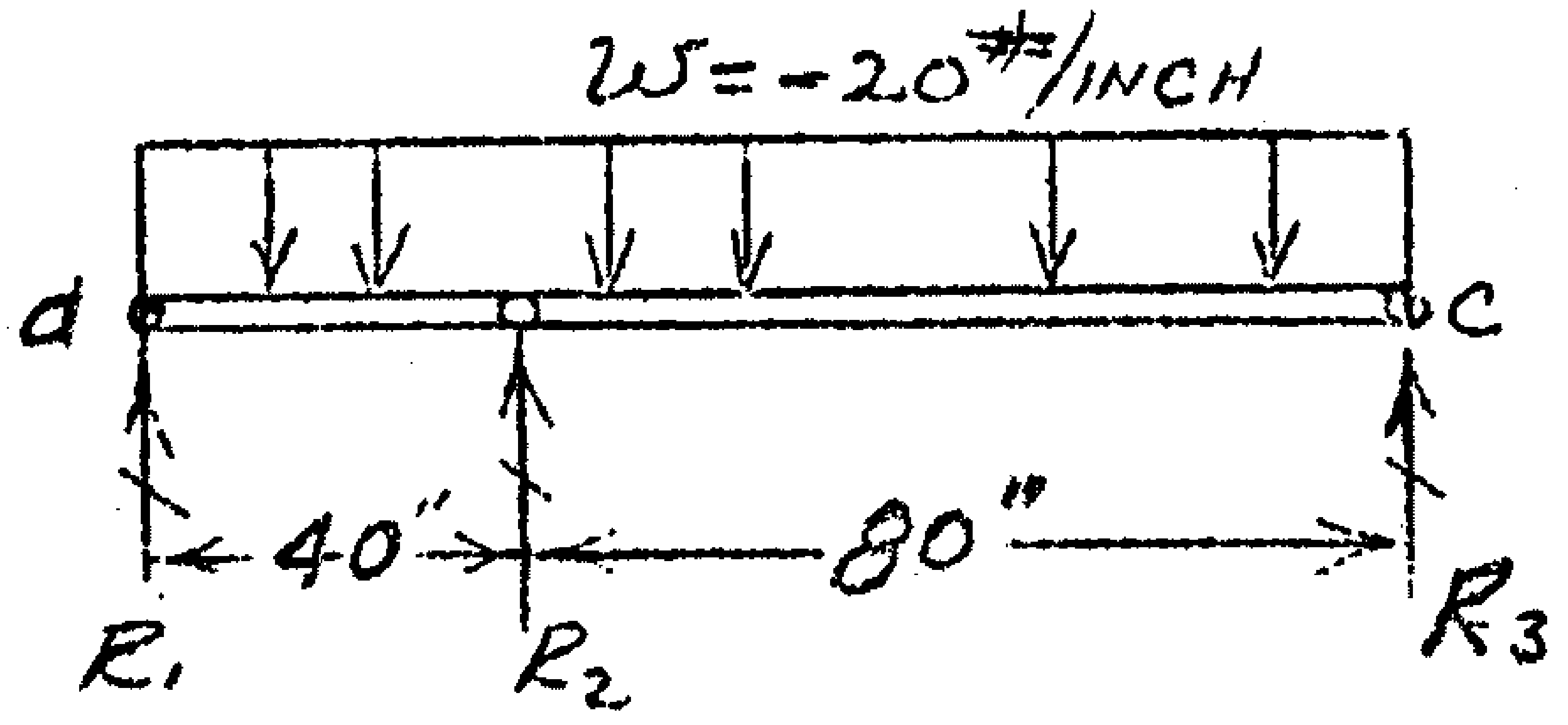
(a)



Restrained Cantilever Beam with uniformly distributed load.

(b)

Beam over 3-Supports uniformly loaded.



BASIC STRESS ANALYSIS

ASSIGNMENT TWELVE

Assignment 12

CONTINUOUS BEAMS

The Equation of 3-Moments.

We have seen in Assignment 11 that the Unit Load Deflection method for solving for reactions on a continuous beam can involve considerable work. A more general method, by which we solve for the moments at the support points, instead of the reactions, is available. This method which involves the equation of moments at any three consecutive supports in a continuous beam is known as the "Equation of 3-Moments". It is based upon the assumption that the slope of a continuous beam just to the left of a support equals the slope just to the right of that support. In other words it is based upon the physical action of a beam.

Equations of "Loading Terms" involving the various types of loads to be found in a span may be developed in general terms involving lengths, L , and loads w, or P. Use of these loading terms then facilitates the solution for support moments where spans carry any of the common types of loading. The method of development of the equation for uniform running loads in adjacent spans is given on the following pages.

Note the following points carefully:

1. The loading terms developed are based upon the convention that upward acting loads are plus.

2. A plus sign for a moment in the final result indicates tension on the bottom fibers of the beam at the point where the moment is taken.

3. Once the effect of the moment at the end of any span taken out as a free body is known, that is either to put tension or compression in the top fibers of the beam at the joint, the value of the plus or minus sign has been fulfilled. For example, if the moment at the right hand end of an isolated span is plus it puts tension on the bottom fibers. Therefore it turns counter clockwise around the joint and in as far as the balance of moments is concerned it is neither a plus or a minus moment but merely a moment of so many inch lbs. turning counter clockwise around the joint.

4. At any support in a continuous beam the moment just to the left of the support is equal to the moment just to the right of the support. However if the spans are isolated as free bodies the direction in which the moment turns will be opposite on the two spans. This is necessary in order to produce the same type of stress in the beam on either side of the joint. It is just as true of a beam cut at any section, not necessarily at a joint.

3-Moment Equation. Support Points do not move.

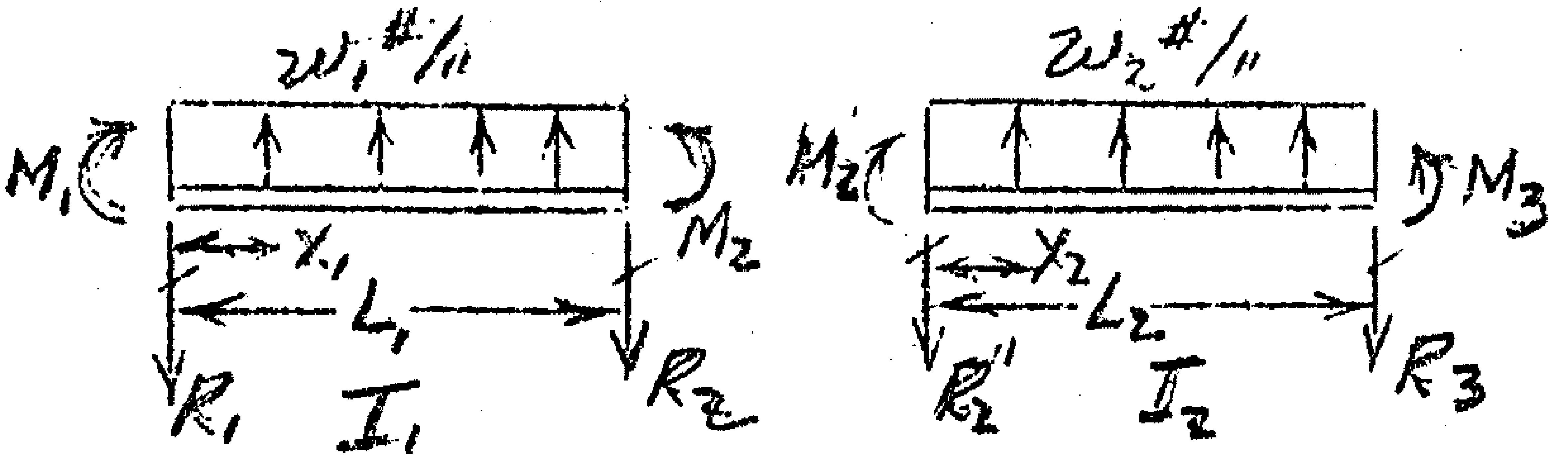


Fig. 1

Consider (1) and (2) as two spans of a continuous beam taken out as free bodies in space. Same value of "E" in each.

Span 1

$$\Sigma M_{R_2} = 0 \quad \curvearrowleft + \qquad R_1L_1 + M_2 - w_1L_1 \times \frac{L_1}{2} - M_1 = 0$$

$$R_1L_1 = M_1 - M_2 + \frac{w_1L_1^2}{2} \qquad \text{or}$$

$$R_1 = \frac{M_1 - M_2}{L_1} + \frac{w_1L_1}{2}$$

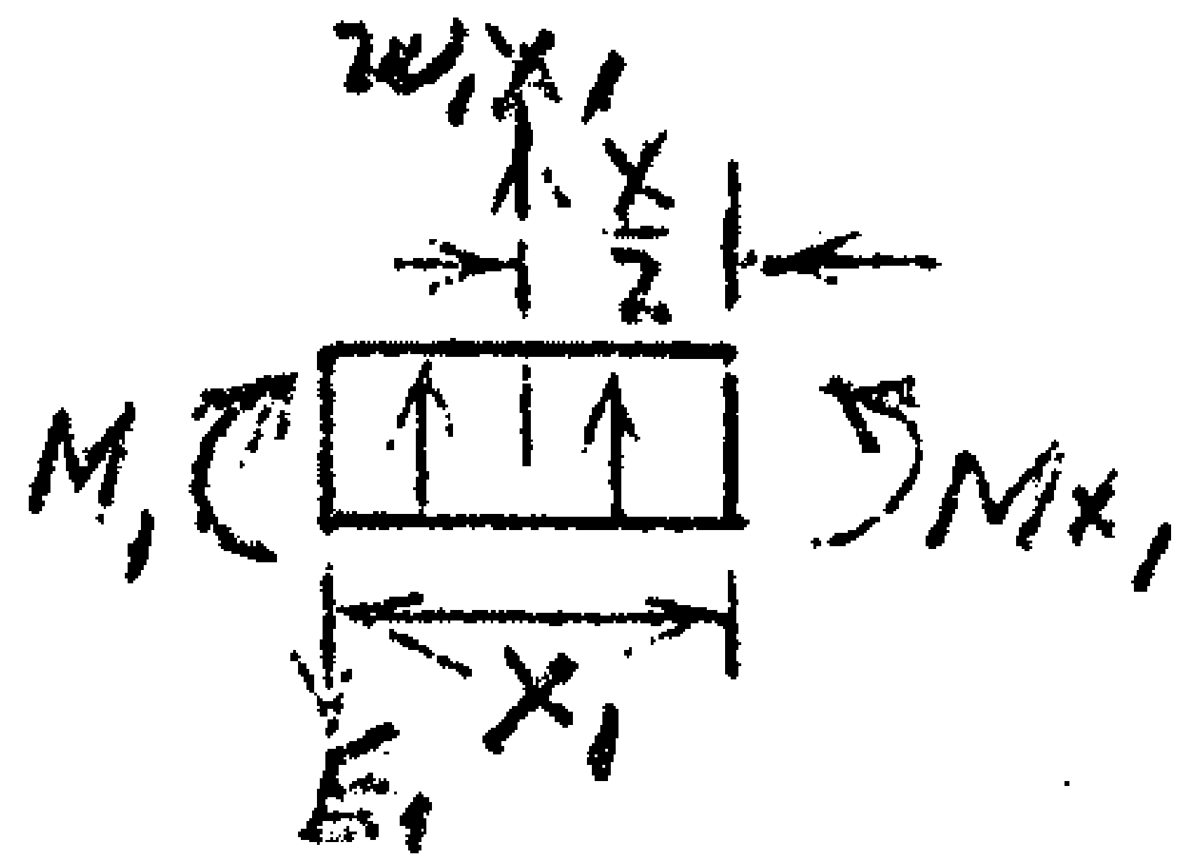


Fig. 2

$$\Sigma M_{x_1} = 0 \quad \curvearrowleft +$$

$$M_{x_1} = M_1 - w_1x_1\left(\frac{x}{2}\right) + \left(\frac{M_1 - M_2}{L_1} + \frac{w_1\ L_1}{2}\right)x_1 = 0$$

$$M_{x_1} = M_1 + \frac{w_1x_1^2}{2} - \frac{(M_1 - M_2)}{L_1}x_1 - \frac{w_1L_1x_1}{2} = M_1 + \frac{(M_2 - M_1)}{L_1}x_1 - \frac{w_1L_1x_1}{2} + \frac{w_1x_1^2}{2}$$

$$\theta_{x_1} = \frac{1}{EI_1}\left[M_1x_1 + \frac{(M_2 - M_1)}{L_1}\frac{x_1^2}{2} - \frac{w_1L_1x_1^2}{4} + \frac{w_1x_1^3}{6} + C_1\right]$$

$$\delta_{x_1} = \frac{1}{EI_1}\left[\frac{M_1x_1^2}{2} + \frac{(M_2 - M_1)}{L_1}\frac{x_1^3}{6} - \frac{w_1L_1x_1^3}{12} + \frac{w_1x_1^4}{24} + C_1x_1 + C_2\right]$$

When $x_1 = L_1$, $\theta_1 = \frac{1}{EI_1}\left[M_1L_1 + \frac{(M_2 - M_1)}{L_1}\frac{L_1^2}{2} - \frac{w_1L_1L_1^2}{4} + \frac{w_1L_1^3}{6} + C_1\right]$

$$= \frac{1}{EI_1}\left[M_1L_1 + (M_2 - M_1)\frac{L_1}{2} - \frac{w_1L_1^3}{12} + C_1\right] \qquad \longleftarrow \text{Eq. a}$$

Span 2

$$\Sigma M_{R_3} = 0 \quad \circlearrowleft + \qquad R^1_2 L_2 - M^1_2 - w_2L_2 \times \frac{L_2}{2} + M_3 = 0$$

$$R^1_2 L_2 = M^1_2 + \frac{w_2L_2^2}{2} - M_3$$

$$R^1_2 = \frac{M^1_2 - M_3}{L_2} + \frac{w_2L_2}{2}$$

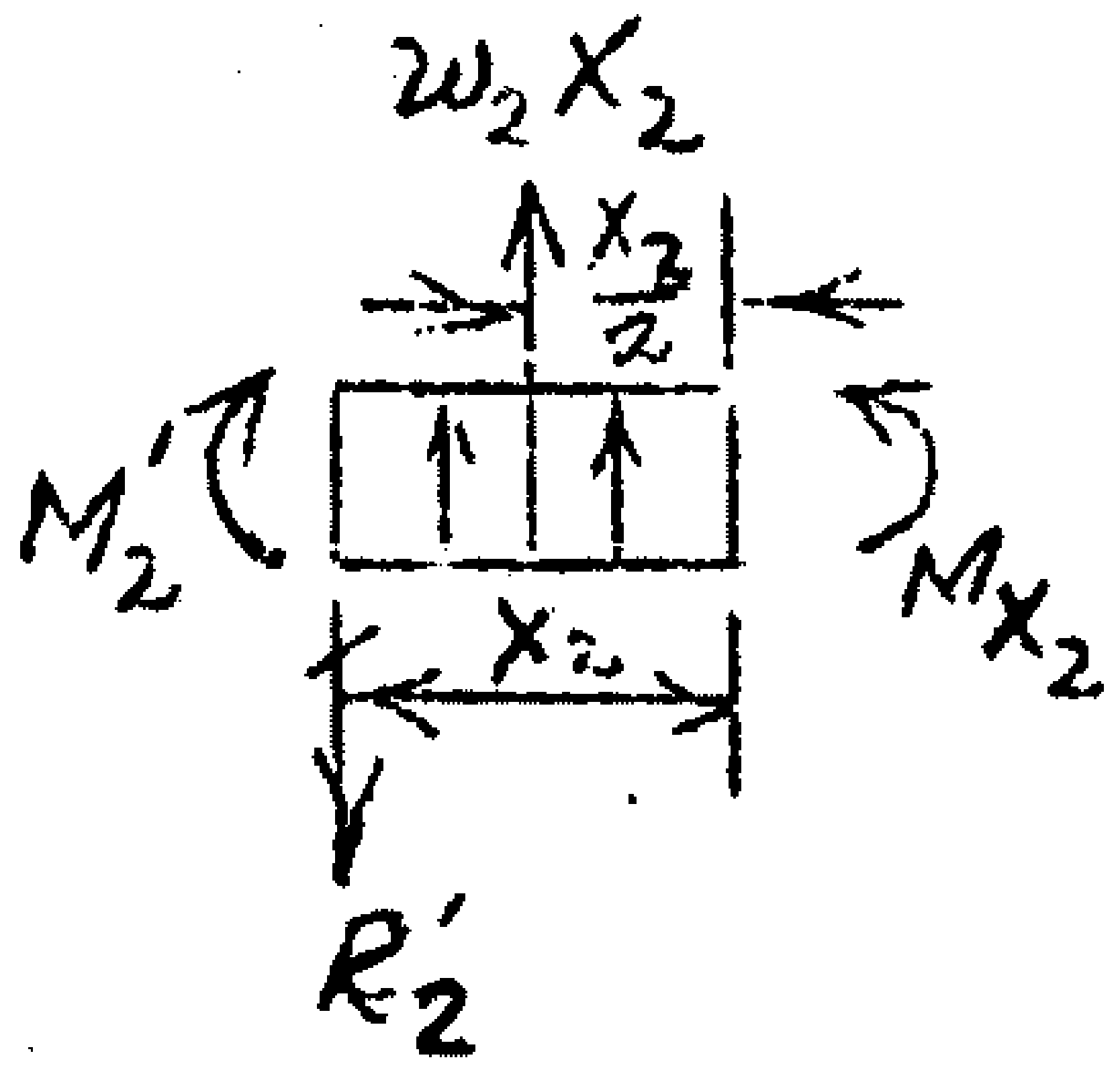


Fig. 3

$$\Sigma M_{x_2} = 0 \quad \circlearrowleft +$$

$$M_{x_2} - M^1_2 - w_2x_2\left(\frac{x_2}{2}\right) + \left(\frac{M^1_2 - M_3}{L_2} + \frac{w_2L_2}{2}\right)x_2 = 0$$

$$M_{x_2} = M_2' + \frac{w_2x_2^2}{2} - \left(\frac{M_2 - M_3}{L_2}\right)x_2 - \frac{w_2L_2x_2}{2}$$

$$M_{x_2} = M_2' + \frac{(M_3 - M_2')x_2}{L_2} - \frac{w_2L_2x_2}{2} + \frac{w_2x_2^2}{2}$$

$$\theta_2 = \frac{1}{EI_2}\left[M_2'x_2 + \frac{(M_3 - M_2')x_2^2}{2L_2} - \frac{w_2L_2x_2^2}{4} + \frac{w_2x_2^3}{6} + C_1'\right]$$

$$\delta_2 = \frac{1}{EI_2}\left[\frac{M_2'x_2^2}{2} + \frac{(M_3 - M_2')}{2L_2}\frac{(x_2)^3}{3} - \frac{w_2L_2x_2^3}{12} + \frac{w_2x_2^4}{24} + C_1'x_2 + C_2'\right]$$

Due to continuity at 2, θ_1 at $L_1 = \theta_2$ at $x_2 = 0$.

θ_2 at $x_2 = 0 = \frac{1}{EI_2}(C_1')$ $\longleftarrow$ Eq. b

Equating (a) and (b)

$$\frac{1}{EI_1}\left[M_1L_1 + \frac{(M_2 - M_1)L_1}{2} - \frac{w_1L_1^3}{12} + C_1\right] = \frac{1}{EI_2}(C_1') \qquad \text{or}$$

$$\frac{1}{I_1}\left[M_1L_1 + \frac{(M_2 - M_1)L_1}{2} - \frac{w_1L_1^3}{12} + C_1\right] - \frac{1}{I_2}(C_1') = 0 \longleftarrow \text{Eq. (1)}$$

If the support points do not move relative to each other $\bar{\delta}_1 = 0$ where $x_1 = 0$ and where $x_1 = L_1$, and $\delta_2 = 0$ .. where $x_2 = L_2$ or 0

Subst. in the deflection equations

$x_1 = L_1$
$$\delta_1 = \frac{1}{EI_1}\left[\frac{M_1L_1^2}{2} + \frac{(M_2 - M_1)}{L_1}\frac{L_1^3}{6} - \frac{w_1L_1L_1^3}{12} + \frac{w_1L_1^3}{24} + C_1L_1 + C_2\right] = 0$$

or
$$\frac{1}{EI_1}\left[\frac{M_1L_1^2}{2} + \frac{(M_2 - M_1)}{6}L_1^2 - \frac{w_1L_1^4}{24} + C_1L_1 + C_2\right] = 0 \longleftarrow \text{Eq. c}$$

where $x_1 = 0$ $\quad \delta_1 = C_2 = 0$

Therefore $\frac{1}{EI_1}\left[\frac{M_1L_1^2}{2} + \frac{(M_2 - M_1)L_1^2}{6} - \frac{w_1L_1^4}{24} + C_1L_1\right] = 0$

$$\frac{1}{EI_1}(C_1L_1) = \frac{1}{EI_1}\left[-\frac{M_1L_1^2}{2} + \frac{(M_1 - M_2)L_1^2}{6} + \frac{w_1L_1^4}{24}\right] \quad \text{and}$$

$$C_1 = \left[-\frac{M_1L_1}{2} + \frac{(M_1 - M_2)L_1}{6} + \frac{w_1L_1^3}{24}\right] \longleftarrow \text{Eq. (2)}$$

$\delta_2 = 0$ $\quad$ where $x_2 = L_2$

$$\frac{1}{EI_2}\left[\frac{M_2'L_2^2}{2} + \frac{(M_3 - M_2')}{2L_2}\frac{L_2^3}{3} - \frac{w_2L_2L_2^3}{12} + \frac{w_2L_2^4}{24} + C_1'L_2 + C_2'\right] = 0$$

or $\frac{1}{EI_2}\left[\frac{M_2'L_2^2}{2} + \frac{(M_3 - M_2')L_2^2}{6} - \frac{w_2L_2^4}{24} + C_1'L_2 + C_2'\right] = 0 \longleftarrow$ Eq. d

where $x_2 = 0$, $\delta_2 = C_2' = 0$ $\quad$ Therefore Eq. d reduces to

$$\frac{1}{EI_2}\left[\frac{M_2'L_2^2}{2} + \frac{(M_3 - M_2')}{6}L_2^2 - \frac{w_2L_2^4}{24} + C_1'L_2\right] = 0$$

$$C_1'L_2 = -\frac{M_2'L_2^2}{2} + \frac{(M_2' - M_3)}{6}L_2^2 + \frac{w_2L_2^4}{24}$$

$$C_1' = -\frac{M_2'L_2}{2} + \frac{(M_2' - M_3)}{6}L_2 + \frac{w_2L_2^3}{24} \longleftarrow \text{Eq. (3)}$$

Subst. C_1 and C_1' in Eq. (1) $\qquad M_2 = M_2'$ from continuity

$$\frac{1}{I_1}\left[M_1L_1 + (M_2 - M_1)\frac{L_1}{2} - \frac{w_1L_1^3}{12} - \frac{M_1L_1}{2} + (M_1 - M_2)\frac{L_1}{6} + \frac{w_1L_1^3}{24}\right]$$

$$+ \frac{1}{I_2}\left[\frac{M_2L_2}{2} - (M_2 - M_3)\frac{L_2}{6} - \frac{w_2L_2^3}{24}\right] = 0$$

$$\frac{M_1L_1}{I_1} + \frac{M_2L_1}{2I_1} - \frac{M_1L_1}{2I_1} - \frac{M_1L_1}{2I_1} + \frac{M_1L_1}{6I_1} - \frac{M_2L_1}{6I_1} + \frac{M_2L_2}{2I_2} - \frac{M_2L_2}{6I_2} + \frac{M_3L_2}{6I_2}$$

$$- \frac{w_1L_1^3}{12I_1} + \frac{w_1L_1^3}{24I_1} - \frac{w_2L_2^3}{24I_2} = 0$$

Multiplying by 6 --

$$\frac{6M_1L_1}{I_1} + \frac{3M_2L_1}{I_1} - \frac{3M_1L_1}{I_1} - \frac{3M_1L_1}{I_1} + \frac{M_1L_1}{I_1} - \frac{M_2L_1}{I_1} + \frac{3M_2L_2}{I_2} - \frac{M_2L_2}{I_2}$$

$$+ \frac{M_3L_2}{I_2} = 6(\frac{w_1L_1^3}{24I_1} + \frac{w_2L_2^3}{24I_2})$$

$$\frac{M_1L_1}{I_1} + \frac{2M_2L_1}{I_1} + \frac{2M_2L_2}{I_2} + \frac{M_3L_2}{I_2} = \frac{w_1L_1^3}{4I_1} + \frac{w_2L_2^3}{4I_2}$$

$$\boxed{\text{or} \quad \frac{M_1L_1}{I_1} + 2M_2(\frac{L_1}{I_1} + \frac{L_2}{I_2}) + \frac{M_3L_2}{I_2} = \frac{w_1L_1^3}{4I_1} + \frac{w_2L_2^3}{4I_2}}$$

Since the foregoing equations apply to any two isolated spans in a continuous beam, the solution for a continuous beam of any number of spans consists in isolating two adjacent spans at a time and obtaining the equation for the moments at the joints in those two spans. A series of equations result which must be solved simultaneously to obtain the values for the unknown moments at the joints. Since the left hand side of the 3-moment equation will be the same regardless of the loads in the spans, if tables of loading terms are available, the moments at the joints may be obtained quickly. A table of the common loading terms is attached. If an unusual type of loading is encountered which fits none of these it is sometimes possible to break the loading into two or more types and add the loading terms for each on the right hand side of the equation. Very irregular loadings may be reduced to a series of concentrated loads.

Important Note:

The 3-moment equation applies to beams whose Moments of Inertia, I, and Moduli of Elasticity, E, are constant over spans. Where either of these values varies in a span an average value is used.

Where a number of spans are involved the setup for the 3-moments equation will be as shown diagramatically below. (Fig. 4) Joints are 1 to 7 inclusive.

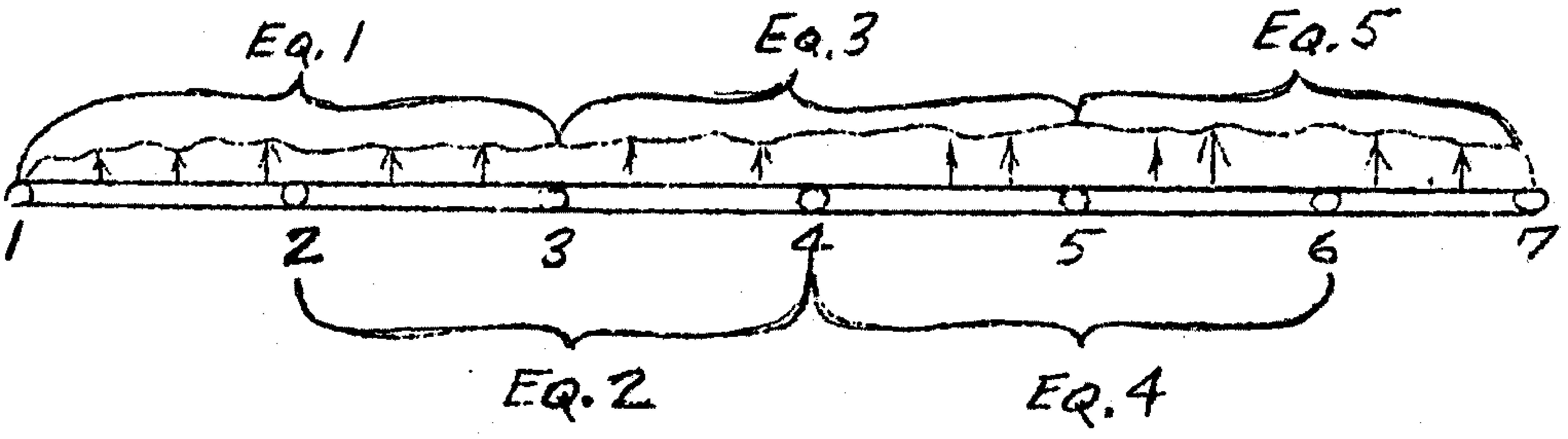


Fig. 4

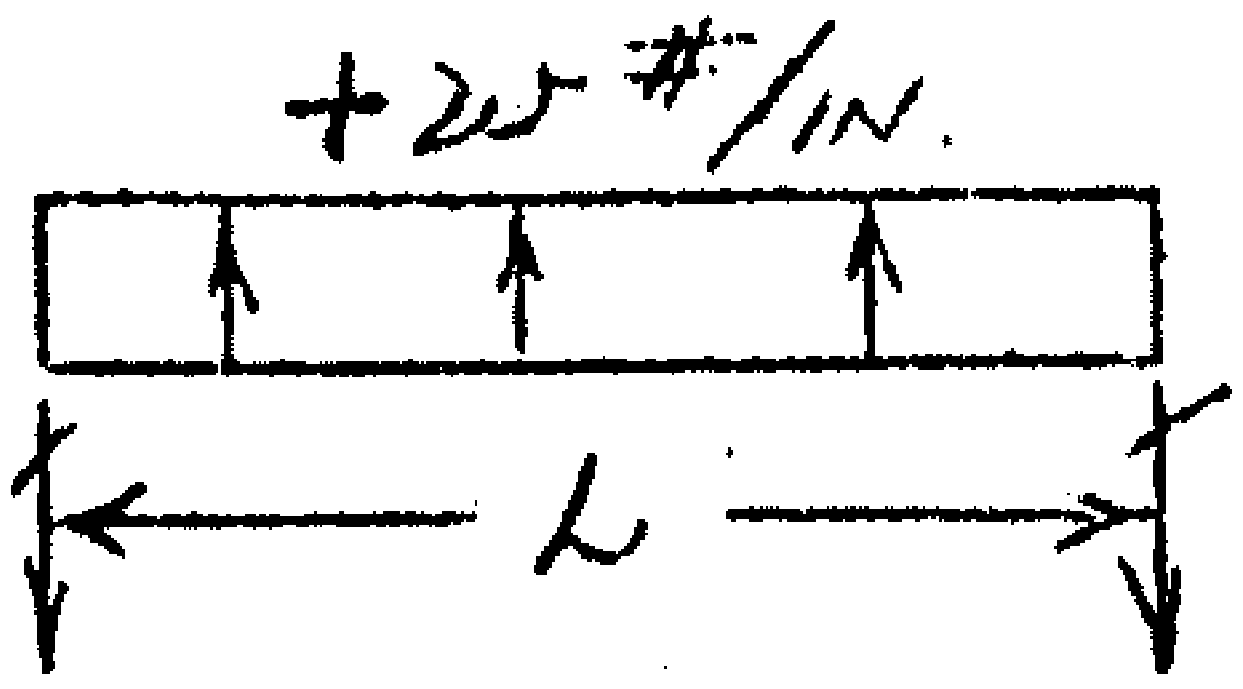
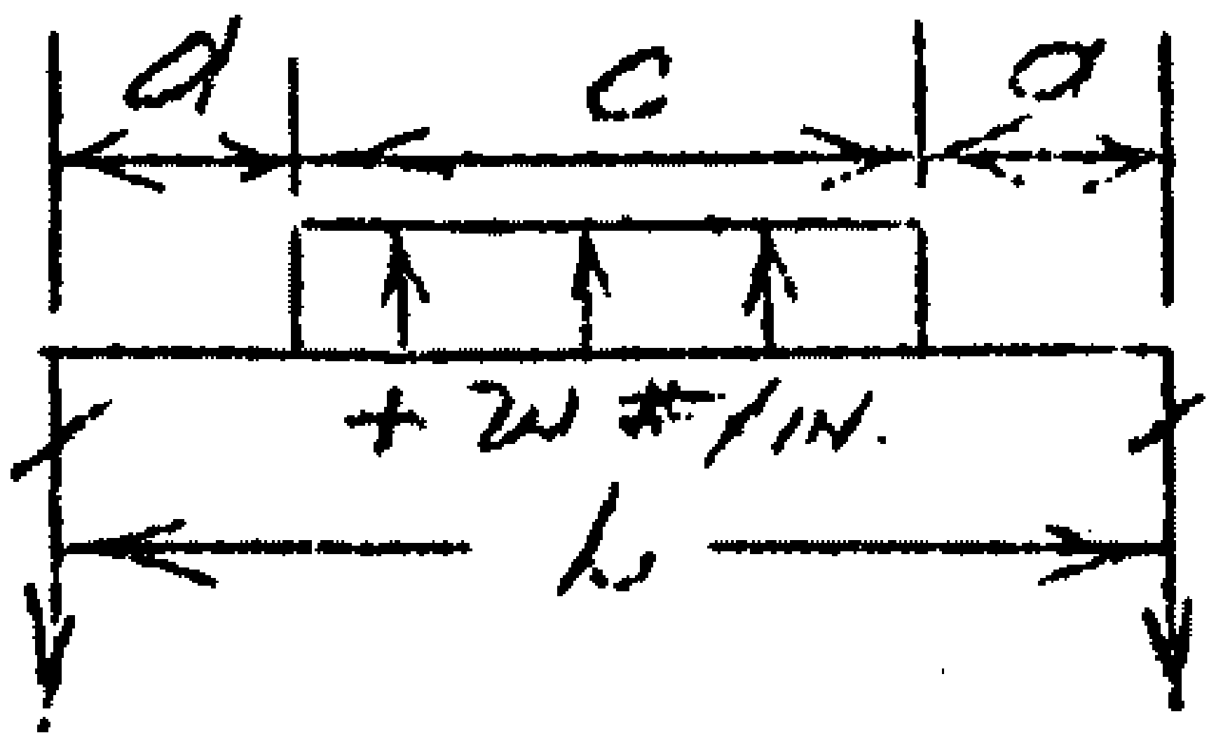
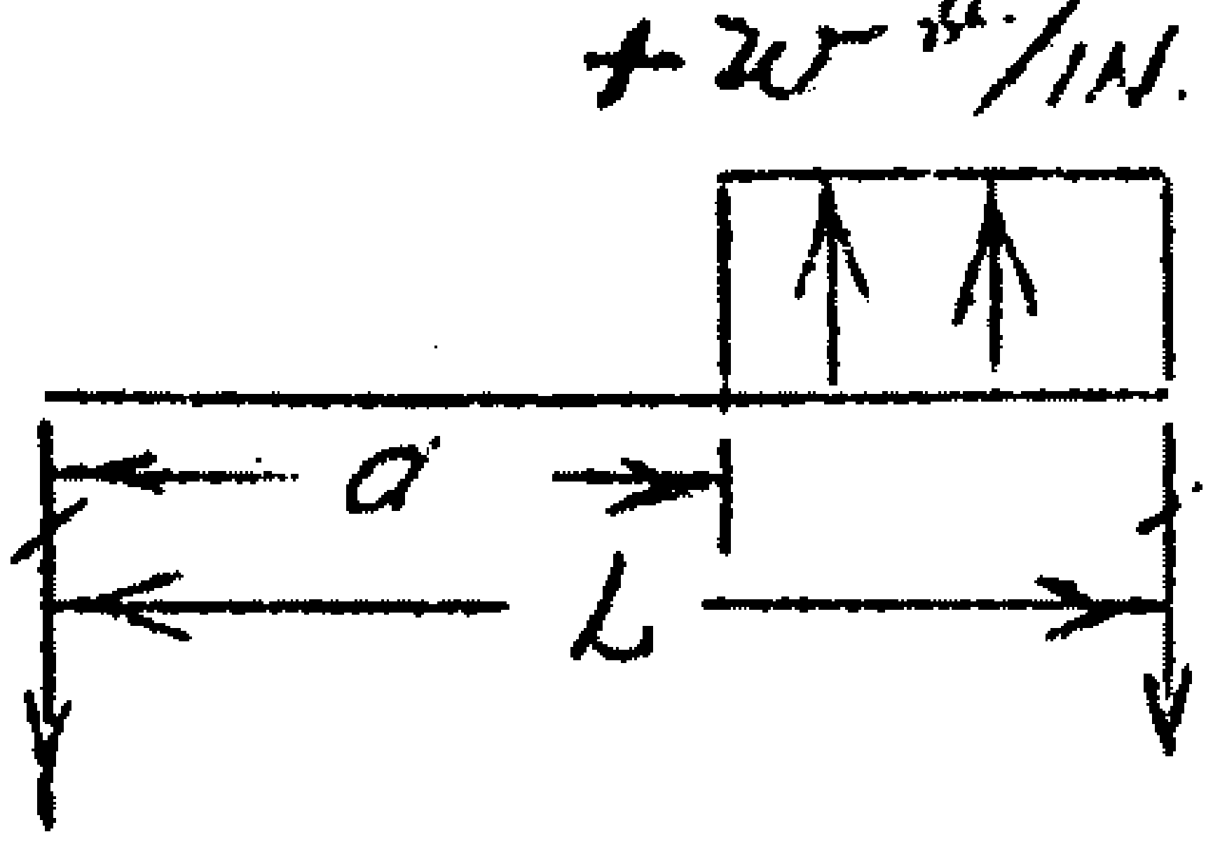
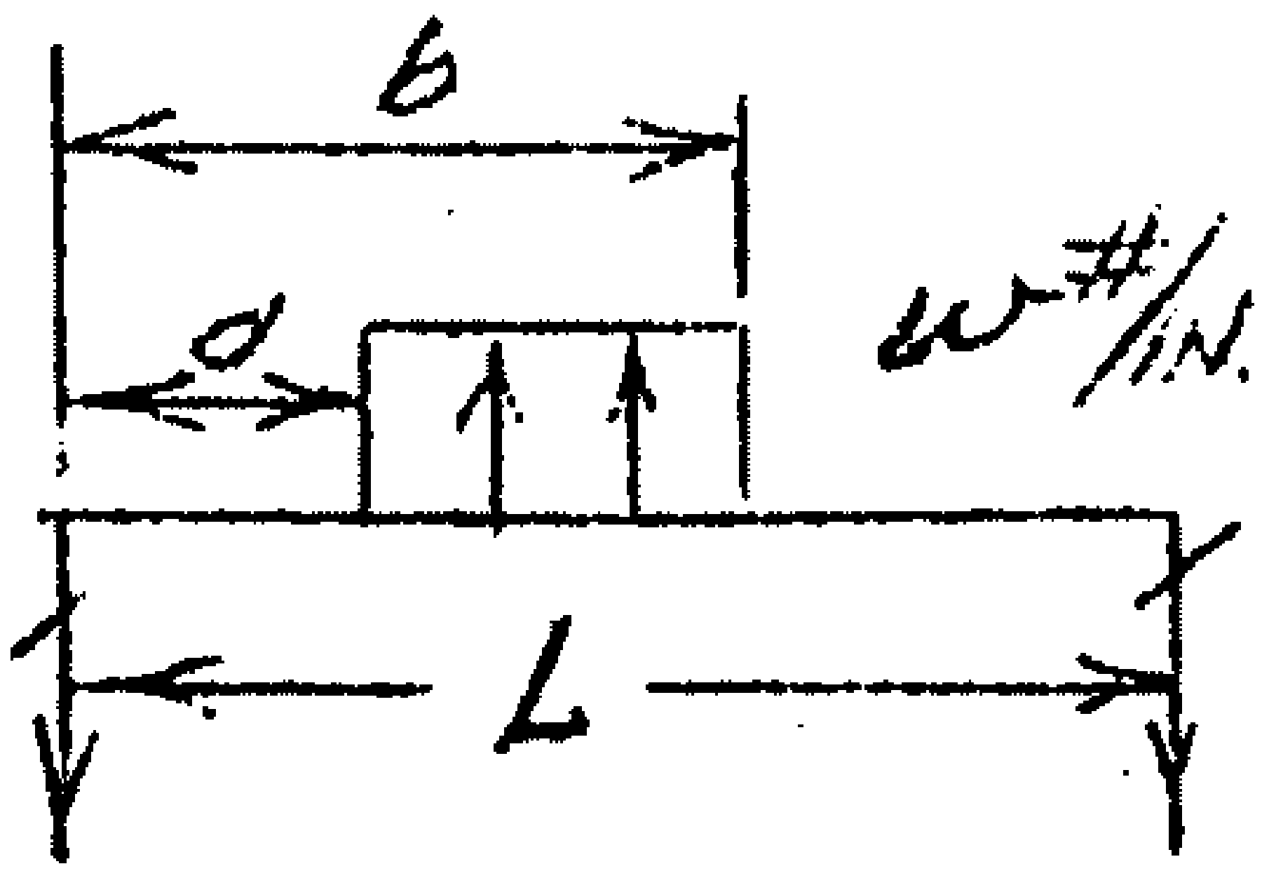
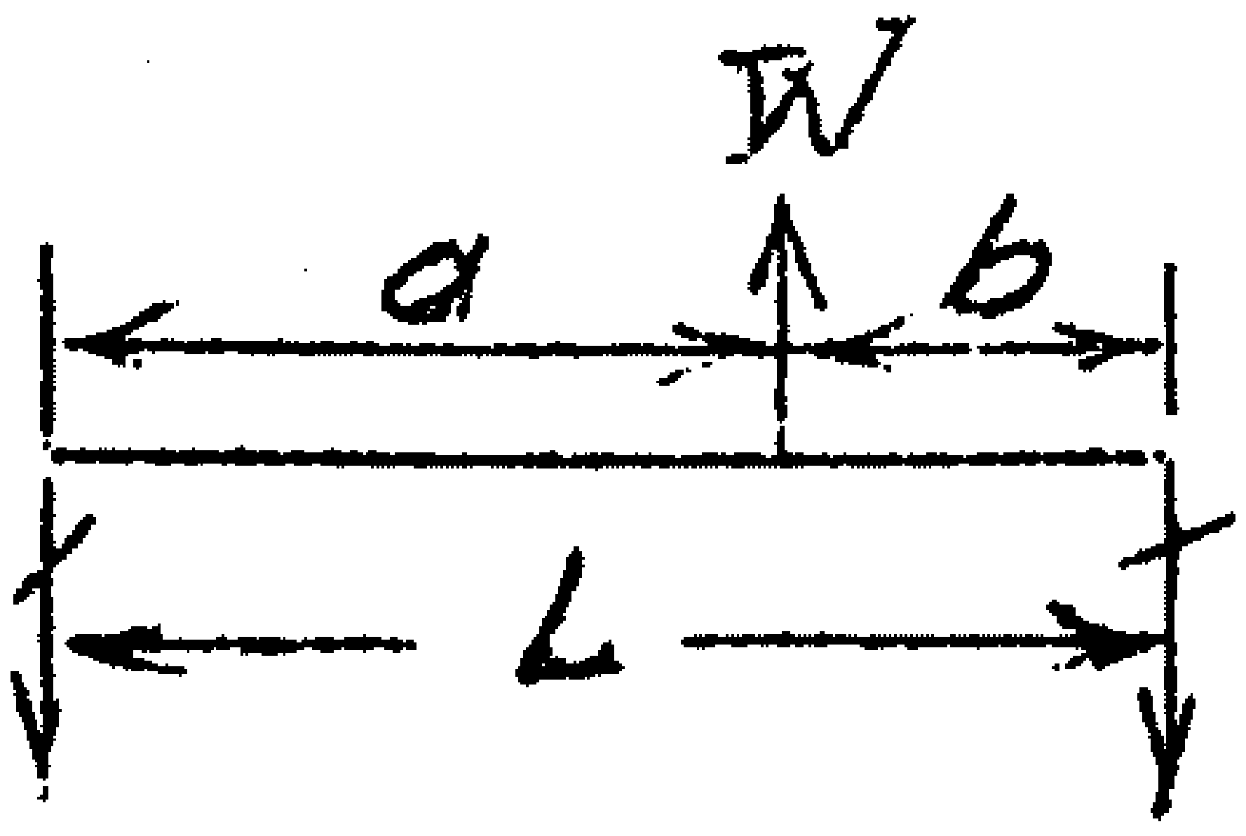
Type of Load	Loading Terms	
	in Left Hand Bay	in Right Hand Bay
UNIFORMLY DISTRIBUTED 	$+\frac{w_1 L_1^3}{4I_1}$	$+\frac{w_2 L_2^3}{4I_2}$
SYM. OVER PART SPAN 	$+\frac{w_1 c_1 (3L_1^2 - c_1^2)}{8I_1}$	$+\frac{w_2 c_2 (3L_2^2 - c_2^2)}{8I_2}$
PART SPAN 	$+\frac{w_1 (L_1^2 - a_1^2)^2}{4I_1 L_1}$	$+w_2\left[\frac{(L_2^2 - a_2^2)^2}{4I_2 L_2} + \frac{a_2^3}{I_2}\right]$
PART SPAN 	$+\frac{w_1\left[b_1^2(2L_1^2 - b_1^2) - a_1^2(2L_1^2 - a_1^2)\right]}{4I_1 L_1}$	$+\frac{w_2\left[b_2^2(2L_2^2 - b_2^2) - a_2^2(2L_2^2 - a_2^2)\right]}{4I_2 L_2}$
CONC. LOAD 	$+\frac{W_1 a_1 (L_1^2 - a_1^2)}{I_1 L_1}$	$+\frac{W_2 b_2 (L_2^2 - b_2^2)}{I_2 L_2}$

Table 1

Uniformly Varying Loads

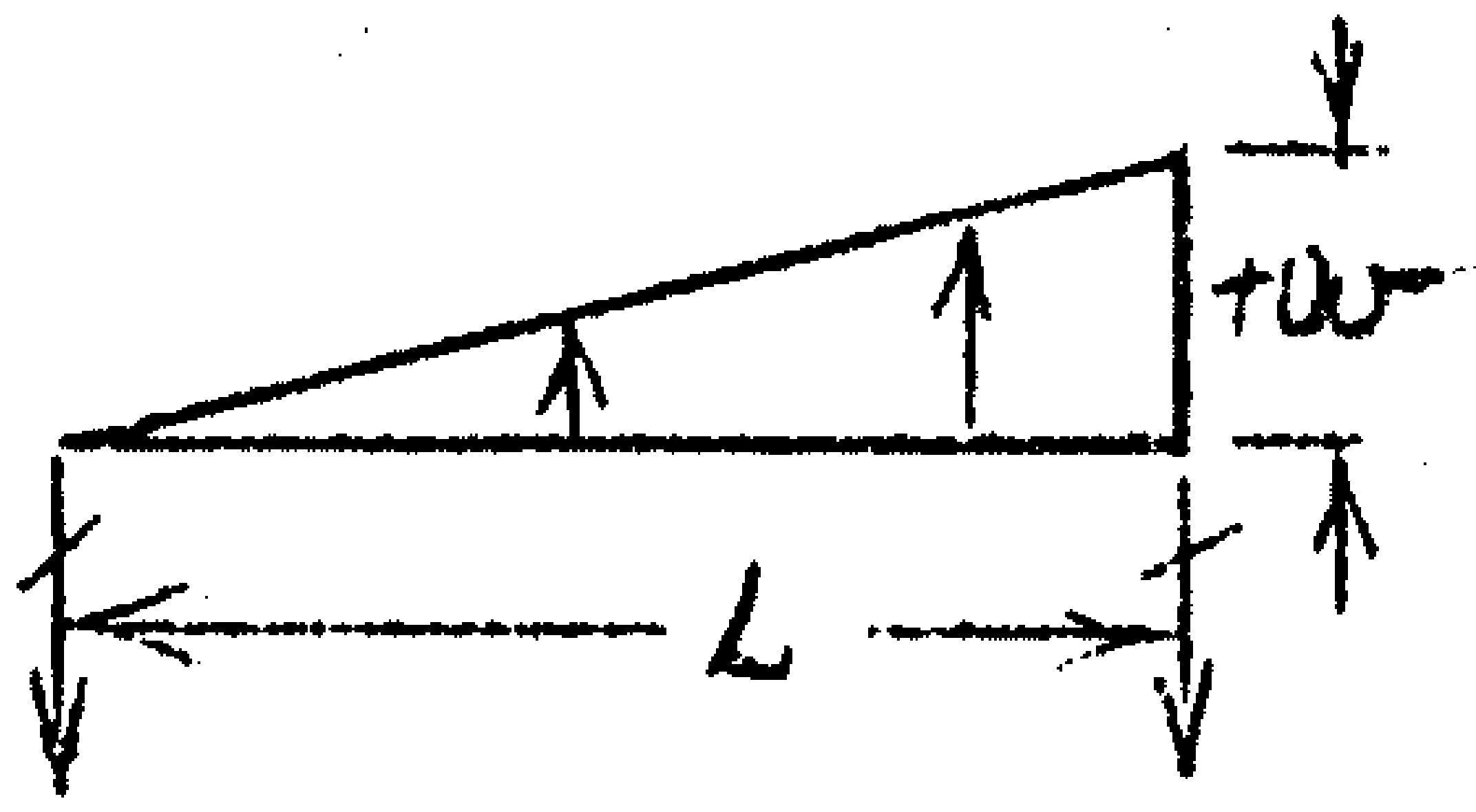
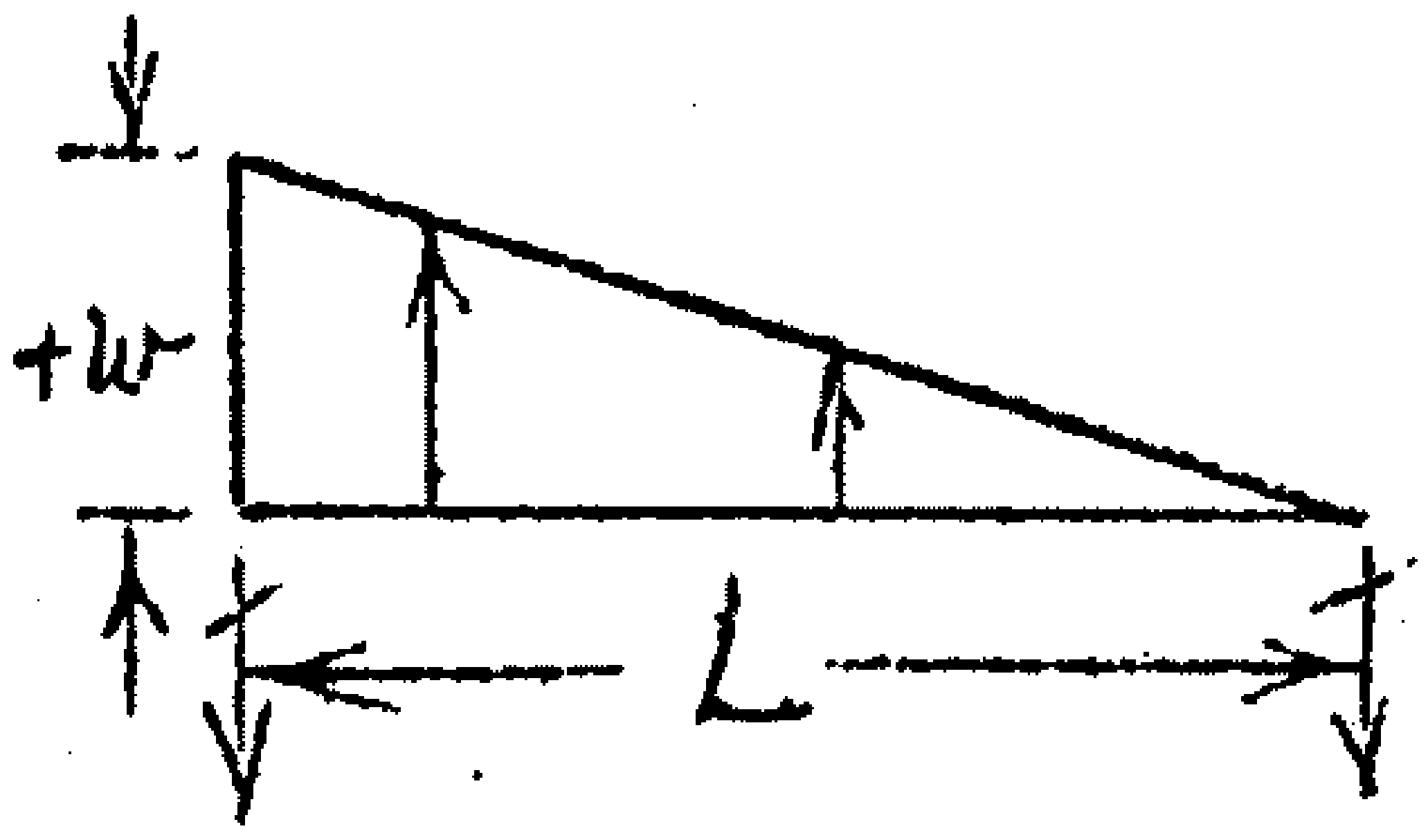
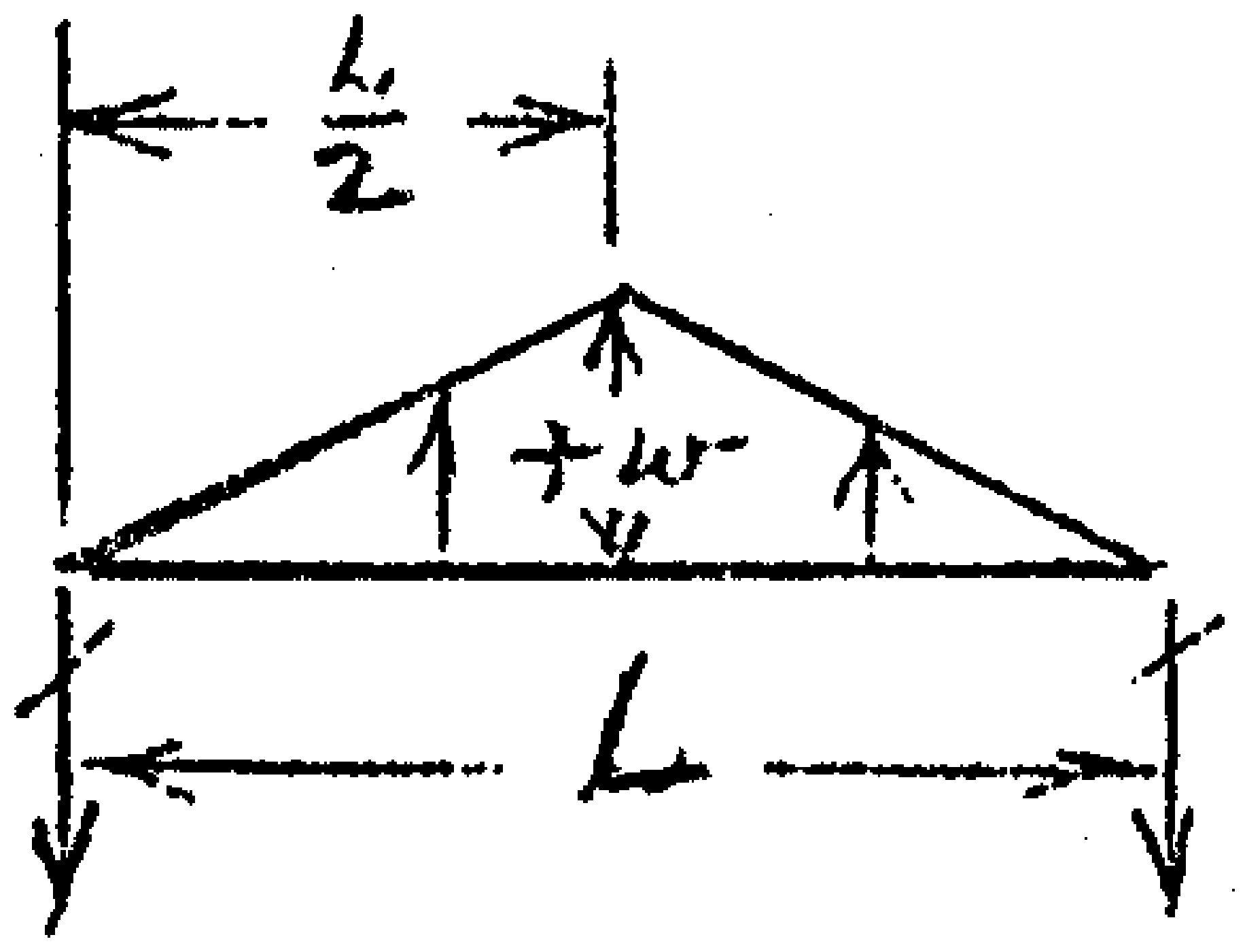
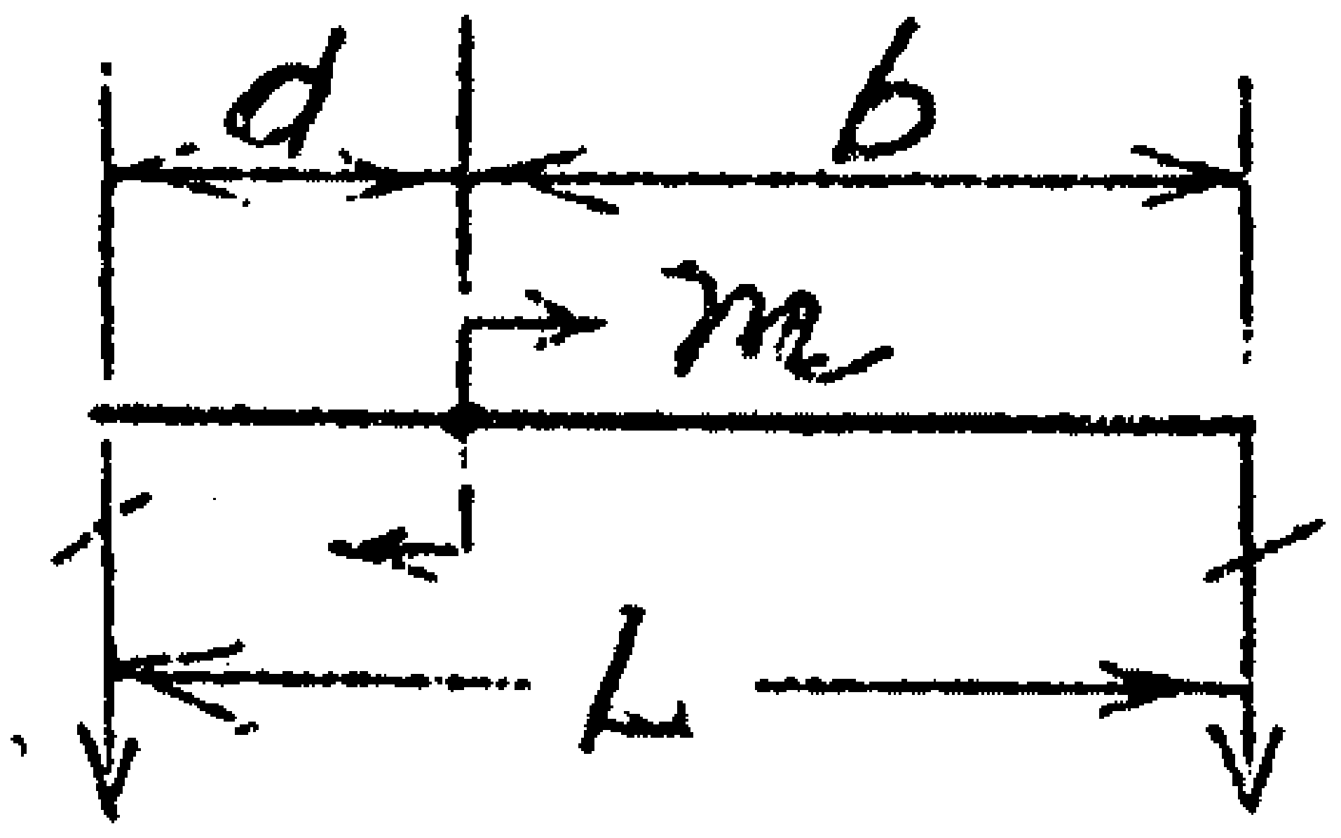
Type of Load	Loading Term in Left Bay	 in Right Bay
	$+\frac{2w_1L_1^3}{15I_1}$	$+\frac{7w_2L_2^3}{60I_2}$
	$+\frac{7w_1L_1^3}{60I_1}$	$+\frac{2w_2L_2^3}{15I_2}$
	$+\frac{5w_1L_1^3}{32I_1}$	$+\frac{5w_2L_2^3}{32I_2}$
EXTERNAL MOMENT 	$+\frac{m_1}{I_1}\left(\frac{3a_1^2}{L_1}-L_1\right)$	$+\frac{m_2}{I_2}\left(L_2-\frac{3b_2^2}{L_2}\right)$

Table 2

Assuming that the end joints 1 and 7 are pinned, (moments = 0) or are known, as from a cantilever overhang, the unknown values are the moments at joints 2, 3, 4, 5, and 6. These then require five applications of the equation of 3-moments for solution. Joints 1, 2, and 3 are taken for the first equation. Then span 1-2 is the left hand and span 2-3 the right hand in choosing the loading terms. Joints 2, 3, and 4 are taken for the second equation. For the second equation span 2-3 becomes the left hand and span 3-4 the right hand for choosing loading terms. In order to keep the nomenclature correct we would designate the unknown moments at the joints as

$$M_1 \ (=0), \quad M_2, M_3, M_4, M_5, M_6 \text{ and } M_7 \ (=0)$$

Then our first equation would read

$$\frac{M_1L_1}{I_1} + 2M_2\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + \frac{M_3L_2}{I_2} = \left[\begin{array}{l}\text{L.H. Load term +} \\ \text{R.H. Load term}\end{array}\right]$$

Equation 2 would read

$$\frac{M_2L_1}{I_2} + 2M_3\left(\frac{L_2}{I_2} + \frac{L_3}{I_3}\right) + \frac{M_4L_3}{I_3} = \left[\sum \text{Loading terms}\right]$$

etc.

If the end moments are not zero but are known as they would be for a cantilever overhang the terms involving M_1 and M_7 would be composed of constants only. If the ends of the beam are fixed, as into a wall, a special method of attack is used. This will be given in the next Assignment.

Problem Assignment.

12:1 Using the same attack as given for uniformly loaded spans in this assignment develop the loading term for a beam with a concentrated load in each span. Check your results against the loading terms given in this assignment.

12:2 Solve problem 11:1 (b) by the equation of 3-moments. Your results should check those obtained by the Unit Load Deflection Method.

AIRPLANE
STRESS
ANALYSIS

PART FIVE

ASSIGNMENT 13

ASSIGNMENT 14

ASSIGNMENT 15

BASIC STRESS ANALYSIS

ASSIGNMENT THIRTEEN

CONTINUOUS BEAMS
3-MOMENT EQUATION
MOMENTS AT A FIXED SUPPORT

The 3-moment equation may be used to solve for moment at a fixed support by assuming that a "phantom" span equal to that adjacent to the fixed support and similarly loaded extends beyond the support.

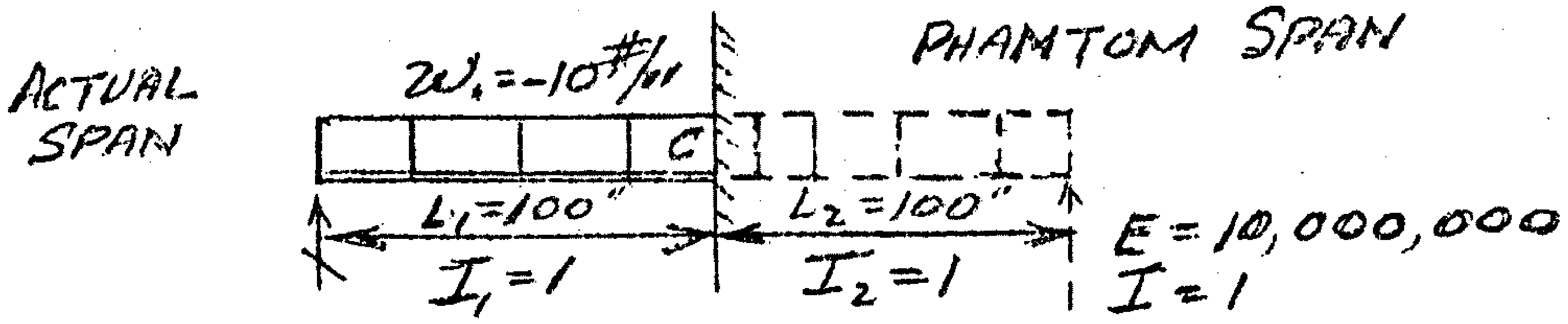


Fig. 1

By the unit load deflection method:

Deflection at R_1 with reaction removed

$$= \frac{wL^4}{8EI} = \frac{(10)(100,000,000)}{(8)(10,000,000)(1)} = 12.5 \text{ inch.}$$

Deflection at R_1 under 1 lb. load $= \frac{wL^3}{3EI} = \frac{(1)(1,000,000)}{(3)(10,000,000)(1)} = 1/30$ inch.

$$R_1 = 12.5/{}_{1/30} = (12.5)(30) = 375 \text{ lbs.}$$

$$\text{Moment at wall} = (375)(100) - (10)(100)(50) = 37,500 - 50,000$$

$$= -12,500''\# \;\ldots\ldots\; M_c$$

Checking by the 3-moment equation putting a phantom span L_2 in place of the wall: EI is constant and is dropped from the equation. Also $M_1 = M_3 = 0$, and $w_1 = w_2$, $L_1 = L_2$.

Under these conditions the 3-moment equation simplifies into

$$2M_2(L_1 + L_2) = 2\left(\frac{w_1L_1^3}{4}\right)$$

In the problem of Fig. 1

$$2M_2(100 + 100) = \frac{(2)(-10)(1,000,000)}{4}$$

$$400M_2 = -5,000,000. \quad M_2 = -12,500''\# \quad \text{(Checks)} = M_c.$$

The phantom span method may be used where a beam continuous over several spans fixes into a wall but only if on the phantom span we duplicate the conditions in the next adjacent span. What we are actually doing is to set up conditions which will bring the beam axis horizontal at the wall, which is our definition of full fixity. In effect the phantom span and the adjacent actual span balance exactly over the fixed support point (wall) as a fulcrum. Let us take problem 11:1 (b) and consider that instead of having pinned joints at a and c the beam is fixed at those points. Fig. 2.

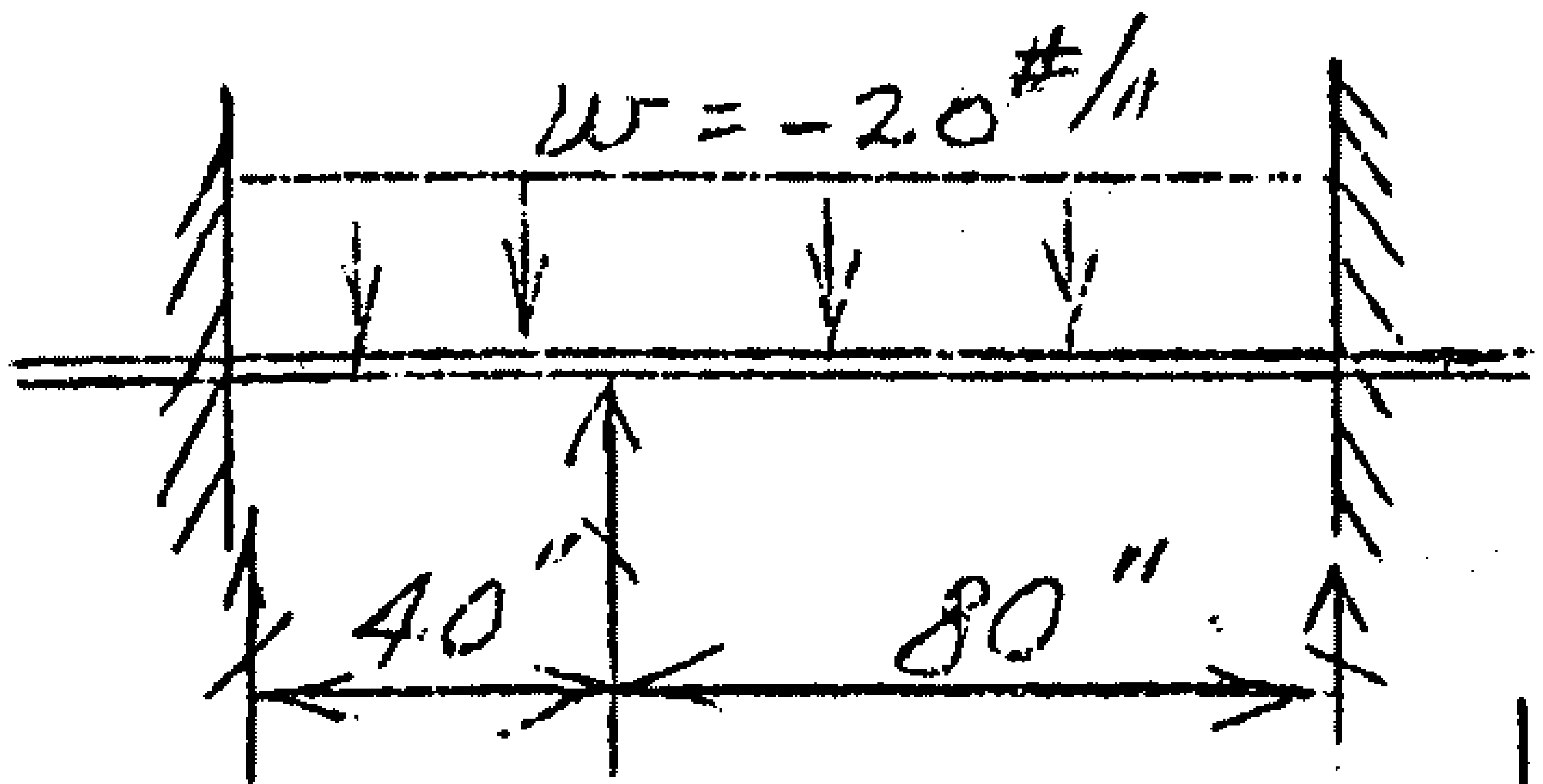


Fig. 2

E and I constant.

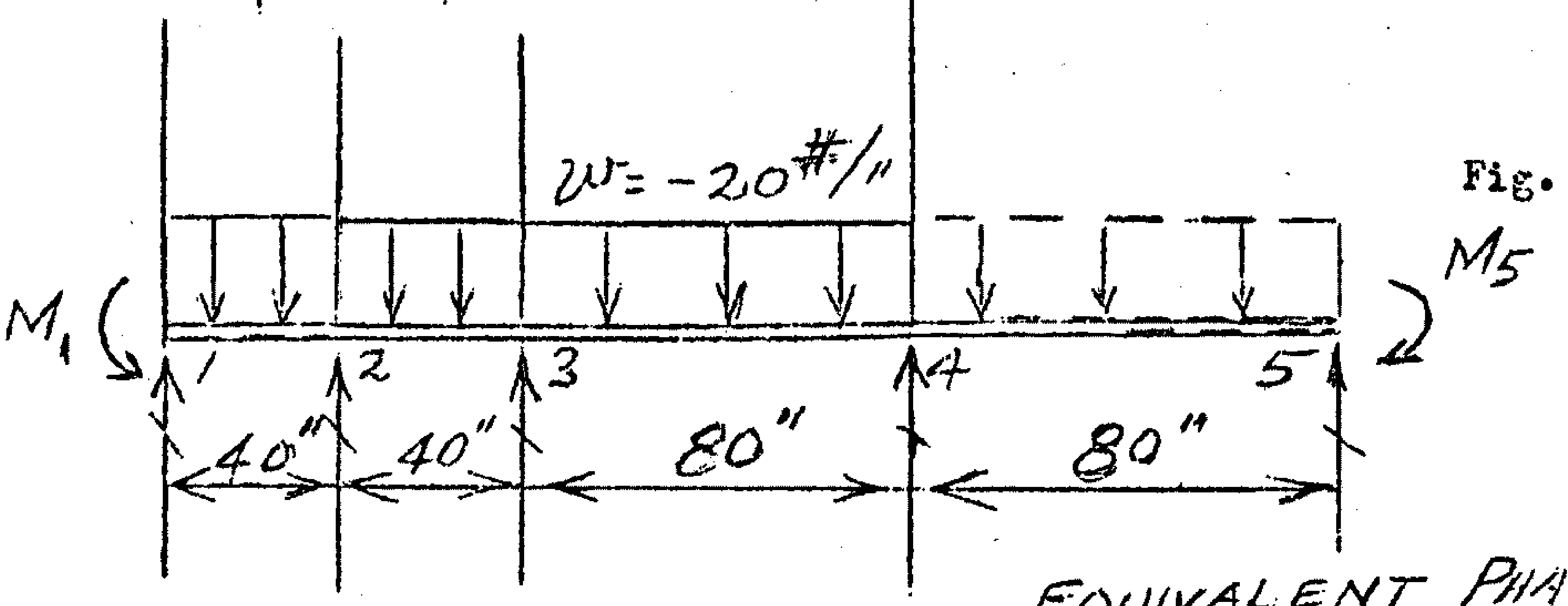


Fig. 3

EQUIVALENT PHANTOM SPANS

Fig. 3 shows the equivalent phantom spans. Phantom span 1-2 has the same length and loading as actual span 2-3 and phantom span 4-5 has the same length and loading as actual span 3-4. In addition if we are to separate out the left hand spans 1-2 and 2-3 and have them balance completely over point 2, (horizontal inclination of the beam axis at that point), M_1 must equal M_3. Similarly M_5 must also equal M_3. Then we start with the following relations

(1) $M_1 = M_3 = M_5$

(2) Loading terms for spans 1-2 and 2-3 equal

(3) Loading terms for spans 3-4 and 4-5 equal

The solution is as follows: (leaving out I which is constant).

$$(M_1)(40) + 2M_2(40 + 40) + (M_3)(40) = 2\frac{(-20)(40)^3}{4}$$

$$40M_1 + 160M_2 + 40M_3 = -640{,}000$$

$$M_1 + 4M_2 + M_3 = -16{,}000$$

But $M_1 = M_3$, so $4M_2 + 2M_3 = -16{,}000$

$$2M_2 + M_3 = -8{,}000 \qquad \longleftarrow (1)$$

$$(M_2)(40) + 2M_3(40 + 80) + M_4(80) = \frac{(-20)(40)^3}{4} + \frac{(-20)(80)^3}{4}$$

$$40M_2 + 240M_3 + 80M_4 = -320{,}000 - 2{,}560{,}000 = -2{,}880{,}000$$

$$M_2 + 6M_3 + 2M_4 = -72{,}000 \qquad \longleftarrow \quad (2)$$

$$(M_3)(80) + 2M_4(80 + 80) + M_5(80) = \frac{(2)(-20)(80)^3}{4}$$

$$80M_3 + 320M_4 + 80M_5 = -5{,}120{,}000$$

$$M_3 + 4M_4 + M_5 = -64{,}000$$

But $M_3 = M_5$

Then $2M_3 + 4M_4 = -64{,}000$

$$M_3 + 2M_4 = -32{,}000 \qquad (3)$$

Subtracting equation 3 from equation 2:

$$\begin{array}{rcrcrcl} M_2 & + & 6M_3 & + & 2M_4 & = & -72{,}000 \\ & & M_3 & + & 2M_4 & = & -32{,}000 \\ \hline M_2 & + & 5M_3 & + & 0 & = & -40{,}000 \end{array} \qquad (4)$$

$$\begin{array}{lrcrcl} \text{Mult. by 2} & 2M_2 & + & 10M_3 & = & -80{,}000 \\ \text{Subtr. eq. (1)} & 2M_2 & + & M_3 & = & -\ 8{,}000 \\ \cline{2-6} & 0 & + & 9M_3 & = & -72{,}000 \\ & & & M_3 & = & -\ 8{,}000''\# \\ & & & M_2 & = & 0 \\ & & & M_4 & = & -12{,}000''\# \end{array}$$

The Effect of Deflected Supports.

If the supports for a beam deflect the general equations of 3-moments are written:

$$\frac{M_1L_1}{I_1} + 2M_2\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + \frac{M_3L_2}{I_2} = \left[\begin{array}{c}\text{L.H. Load}\\ \text{Term}\end{array}\right] + \left[\begin{array}{c}\text{R.H. Load}\\ \text{Term}\end{array}\right]$$

$$+ \frac{6E\,(y_1-y_2)}{L_1} + \frac{6E\,(y_3-y_2)}{L_2}\ , \quad \text{where}$$

y_1, y_2, and y_3 are the deflections at supports 1, 2, and 3 + deflections are upward. Solution of a beam by this equation depends upon known or estimated deflections at the support points. Refer to notes of Assignment 12. When known deflected values are equated to the deflections at the supports there is no change in the general equation up to Eq. (1) but from this point...

where $x_1 = L_1$ $$\delta_1 = \frac{1}{EI_1}\left[\frac{M_1L_1^2}{2} + \frac{(M_2 - M_1)L_1^2}{6} - \frac{w_1L_1^4}{24} + C_1L_1 + C_2\right] = y_2$$

where $x_1 = 0$ $\delta_1 = \frac{C_2}{EI_1} = y_1$ from which

$$\frac{1}{EI_1}\left[\frac{M_1L_1^2}{2} + \frac{(M_2 - M_1)L_1^2}{6} - \frac{w_1L_1^4}{24} + C_1L_1\right] + \frac{C_2}{EI_1} = y_2$$

$$\frac{1}{EI_1}\left[\frac{M_1L_1^2}{2} + \frac{(M_2 - M_1)L_1^2}{6} - \frac{w_1L_1^4}{24} + C_1L_1\right] = y_2 - \frac{C_2}{EI} = y_2 - y_1$$

$$\frac{M_1L_1^2}{2} + \frac{(M_2 - M_1)L_1^2}{6} - \frac{w_1L_1^4}{24} + C_1L_1 = (y_2 - y_1)EI_1$$

$$C_1L_1 = -\frac{M_1L_1^2}{2} + \frac{(M_1 - M_2)L_1^2}{6} + \frac{w_1L_1^4}{24} + (y_2 - y_1)EI_1$$

$$C_1 = \left[-\frac{M_1L_1}{2} + \frac{(M_1 - M_2)L_1}{6} + \frac{w_1L_1^3}{24} + (y_2 - y_1)\frac{EI_1}{L_1}\right] \longleftarrow (2)$$

$\delta_2 = y_3$ where $x_2 = L_2$, and where $x_2 = 0$, $\delta_2 = \frac{C'_2}{EI_2} = y_2$

$$\frac{1}{EI_2}\left[\frac{M'_2L_2^2}{2} + \frac{(M_3 - M'_2)L_2^2}{6} - \frac{w_2L_2^4}{24} + C'_1L_2\right] + y_2 = y_3$$

$$\left[\frac{M'_2L_2^2}{2} + \frac{(M_3 - M'_2)}{6}L_2^2 - \frac{w_2L_2^4}{24} + C'_1L_2\right] = (y_3 - y_2)EI_2$$

$$C'_1L_2 = \left[-\frac{M_2L_2^2}{2} + \frac{(M'_2 - M_3)}{6}L_2^2 + \frac{w_2L_2^4}{24} + (y_3 - y_2)EI_2\right]$$

$$C'_1 = \left[-\frac{M_2L_2}{2} + \frac{(M'_2 - M_3)L_2}{6} + \frac{w_2L_2^3}{24} + (y_3 - y_2)\frac{EI_2}{L_2}\right] \longleftarrow (3)$$

Subst. C_1 and C'_1 in Eq. (1), $M_2 = M'_2$ from continuity

$$\frac{1}{I_1}\left[M_1L_1 + \frac{(M_2 - M_1)}{2}L_1 - \frac{w_1L_1^3}{12} - \frac{M_1L_1}{2} + \frac{(M_1 - M_2)}{6}L_1 + \frac{w_1L_1^3}{24} + (y_2 - y_1)\frac{EI_1}{L_1}\right] - \frac{1}{I_2}\left[-\frac{M_2L_2}{2} + \frac{(M'_2 - M_3)}{6} + \frac{w_2L_2^3}{24} + (y_3 - y_2)\frac{EI_2}{L_2}\right] = 0$$

$$\frac{M_1L_1}{I_1} + \frac{M_2L_1}{2I_1} - \frac{M_1L_1}{2I_1} - \frac{M_1L_1}{2I_1} + \frac{M_1L_1}{6I_1} - \frac{M_2L_1}{6I_1} + \frac{M_2L_2}{2I_2} - \frac{M_2L_2}{6I_2} + \frac{M_3L_2}{6I_2}$$

$$= \frac{w_1L_1^3}{12I_1} - \frac{w_1L_1^3}{24I_1} + \frac{w_2L_2^3}{24I_2} - \frac{(y_2 - y_1)EI_1}{L_1I_1} + \frac{(y_3 - y_2)EI_2}{L_2I_2}$$

Multiplying the equation through by 6.0

$$\frac{6M_1L_1}{I_1} + \frac{3M_2L_1}{I_1} - \frac{3M_1L_1}{I_1} - \frac{3M_1L_1}{I_1} + \frac{M_1L_1}{I_1} - \frac{M_2L_1}{I_1} + \frac{3M_2L_2}{I_2} - \frac{M_2L_2}{I_2} + \frac{M_3L_2}{I_2}$$

$$= \frac{w_1L_1^3}{4I_1} + \frac{w_2L_2^3}{4I_2} + \frac{6E(y_1 - y_2)}{L_1} + \frac{6E(y_3 - y_2)}{L_2}$$

$$\frac{M_1L_1}{I_1} + \frac{2M_2L_1}{I_1} + \frac{2M_2L_2}{I_2} + \frac{M_3L_2}{I_2} = \frac{w_1L_1^3}{4I_1} + \frac{w_2L_2^3}{4I_2} + \frac{6E(y_1 - y_2)}{L_1} + \frac{6E(y_3 - y_2)}{L_2}$$

$$\frac{M_1L_1}{I_1} + 2M_2\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + \frac{M_3L_2}{I_2} = \frac{w_1L_1^3}{4I_1} + \frac{w_2L_2^3}{4I_2} + \frac{6E(y_1 - y_2)}{L_1} + \frac{6E(y_3 - y_2)}{L_2}$$

Note: The above equation is based on the solution of Assignment 12 for uniform running load on the beam. For any other loading substitute the corresponding loading terms in the right hand side of the equation. The terms showing the effect of deflection do not change with changes in the type of loading.

The three moment equation with supports deflected is used to solve for the moments on control surfaces such as ailerons or wing flaps which are influenced by deflection of the surfaces supporting them. For example in Assignment 10 we used the moment area method to compute the deflection of a wing. Suppose an aileron is attached to this wing at three hinge points. The wing is so much stiffer in bending than the aileron that the effect of the aileron in restraining the wing from deflecting may be assumed to be negligible. It is good practice to attach ailerons to their supports by self aligning ball bearings which allow them to realign themselves without binding when the wing deflects. With such a method of support the aileron would deflect relative to the wing as shown in Fig. 4. If the two outer hinge points are assumed not to deflect relative to each other, the center hinge may be considered on a deflected support and the solution for the moments on the aileron beam may be made accordingly,

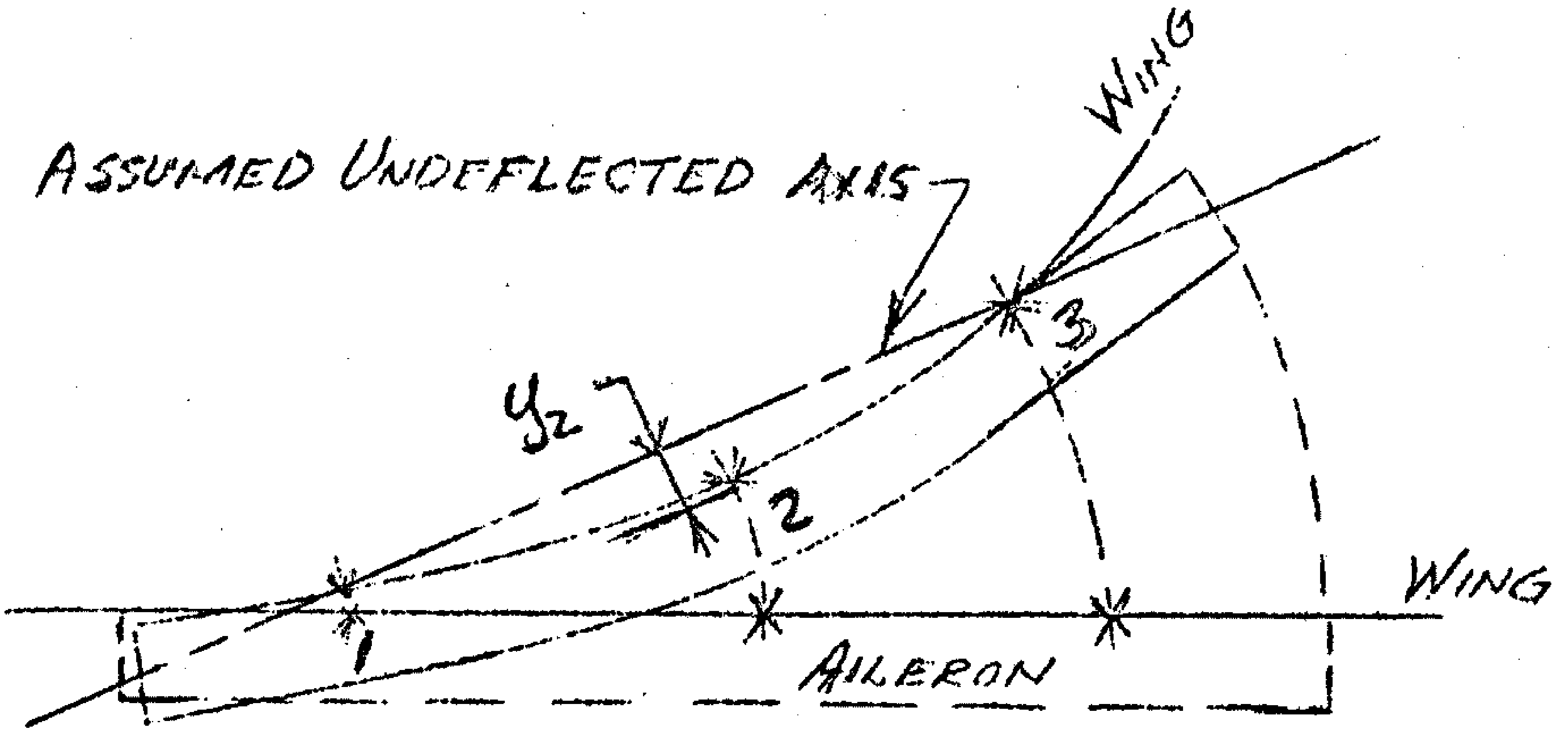
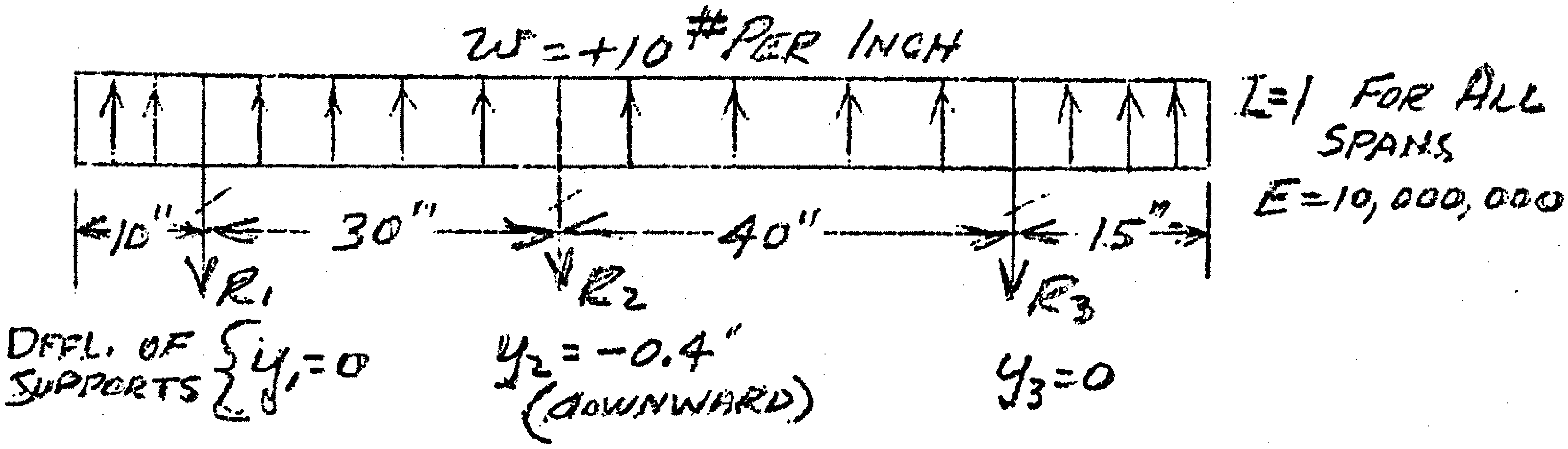


Fig. 4

Problem Assignment.

13:1 Solve for the moments and reactions and construct load, shear, and moment curves for the deflected aileron shown below. E = 10,000,000 (all material aluminum alloy); I = 1.0, all spans. Loading is upward on the aileron. The Wing is bent upward. The reference axis has been shifted in the sketch to show the original, unbent axis horizontal.
Note: M_1 and M_3 may be solved for directly.



13:2 Solve for the moments assuming no deflection of the supports and note the difference.

BASIC STRESS ANALYSIS

ASSIGNMENT FOURTEEN

Assignment 14

COMBINED AXIAL AND BENDING LOADS

When a beam carrying a bending load is subjected to an additional axial load the bending moment on the beam is modified. An axial tension load decreases the bending moment in a beam and an axial compression load increases it.

Primary moment is the moment in a beam before the axial load is taken into account.

Secondary moment is that due to the axial load. (The product of the axial load and the final deflection of the beam.)

Total moment equals primary moment plus secondary moment, when the beam has reached equilibrium under the side and axial loads; that is when deflection has ceased.

If the beam did not deflect under side loads there would be no secondary moment, for under this condition the effect of the axial load would be merely P/A distributed evenly over the cross section of the beam.

Axial Tension loads lower the moment in a beam. The secondary moment subtracts from the primary moment. This must always be true since the tension load is tending to keep the neutral axis in a straight line between support points. See Fig. 1.

Axial Compression loads increase the moment in a beam. The secondary moment adds to the primary moment. Furthermore the effect of axial compression is increasingly additive until equilibrium is reached (or failure occurs if the beam is under strength). Consider the beam of Fig. 2, deflected under the side load only. Its moment and deflection curves are shown by the full lines. Now consider the effect of the axial compression load. For the deflection shown it will increase the moment by an amount $P \times \delta$, where δ is the deflection under the primary load only, at the section shown. If the $P \times \delta$ values are added at each station a new moment curve is obtained which will have a correspondingly increased deflection curve. The increase in deflection at each station, δ', when multiplied by the end load gives a new increment of secondary moment, $P \times \delta'$. If the second increment of secondary bending moment is larger than the first the beam is unstable and deflection will continue until complete failure has taken place. If, however, the second increment is smaller than the first, each successive increment will be smaller than the one preceding it until the changes are infinitesimal; that is until equilibrium has been reached. Successive approximations will carry the work to any accuracy desired. If the deflection due to the first increment of moment is increased arbitrarily by a small percentage the second trial will give sufficient accuracy.

Fig. 3 shows a simple beam, bending under a uniform load, with an additional compression end load. Assume that the beam is 100 inches long, has a uniform load of 20 lbs. per inch, axial compression load of 5000 lbs., $I = 5.0$ and $E = 10,000,000$. Then from the vertical load only on the beam, the deflection at the center is

$$\delta_c = \frac{5wl^4}{384EI} = \frac{-5 \times 20 \times 100^4}{384 \times 10,000,000 \times 5} = -.521 \text{ inches}$$

Maximum moment at the center = $\frac{wl^2}{8}$ $= \frac{20 \times 100^2}{8} = 25{,}000''\#$ (Primary)

First increment of secondary moment = (5000)(.521) = 2605"#, an increase of 10.4%.

Now we can estimate that the second increment will be approximately 10% of the first.

The total of the first two increments will be 10.4 + 1.04 = 11.44% of the primary moment. This will be .1144 x 25,000 = 2860"#.
Total moment = 25,000 + 2860 = 27,860"#.
This value conforms to the moment at the center of a pinned-end beam loaded with a running load of 22.29 lbs. per inch., for which by proportion the deflection at the center would be $\frac{22.29}{20} \times .521 = .581$ inches.

The secondary moment divided by the end load should equal the deflection.

$$\frac{2860}{5000} = .572 \text{ inches}$$

The actual value will be .56 inches at the center and the maximum moment at the center will be 25,000 + 5000 x .56 = 27,800"#.

The exact solution of this case is as follows: (Niles and Newell, Airplane Structures, page 202, first edition)

When
P = axial compression load
E = Modulus of Elasticity
I = Moment of Inertia
L = length of span
w = running load + = upward

$$j = \sqrt{\frac{EI}{P}}$$

The maximum moment at the center of the beam is $M_{max} = wj^2\left[1 - \frac{1}{\cos\frac{L}{2j}}\right]$

In our problem $j = \sqrt{\frac{10{,}000{,}000 \times 5}{5{,}000}} = 100$ $\qquad w = -20 \qquad j^2 = 10{,}000$

$\frac{L}{2j} = \frac{100}{200} = .50$ $\qquad \cos\frac{L}{2j} = .878$ from table of natural trigonometric functions, page 193 in Niles and Newell.

$$M_{max} = -20 \times 10{,}000 \left(1 - \frac{1}{.878}\right)$$

$$= -200{,}000\ (1 - 1.139) = (-200{,}000)(-.139) = 27{,}800''\#$$

Checks

Now let us introduce a moment at the left end of -10,000"# (Fig. 4). The effect is to shorten the effective pinned end span to a shorter span determined by the right hand reaction point and the inflection point (point of zero moment) adjacent to the left hand support. The solution is given by both approximate and exact methods.

Axial Tension

Axial Compression

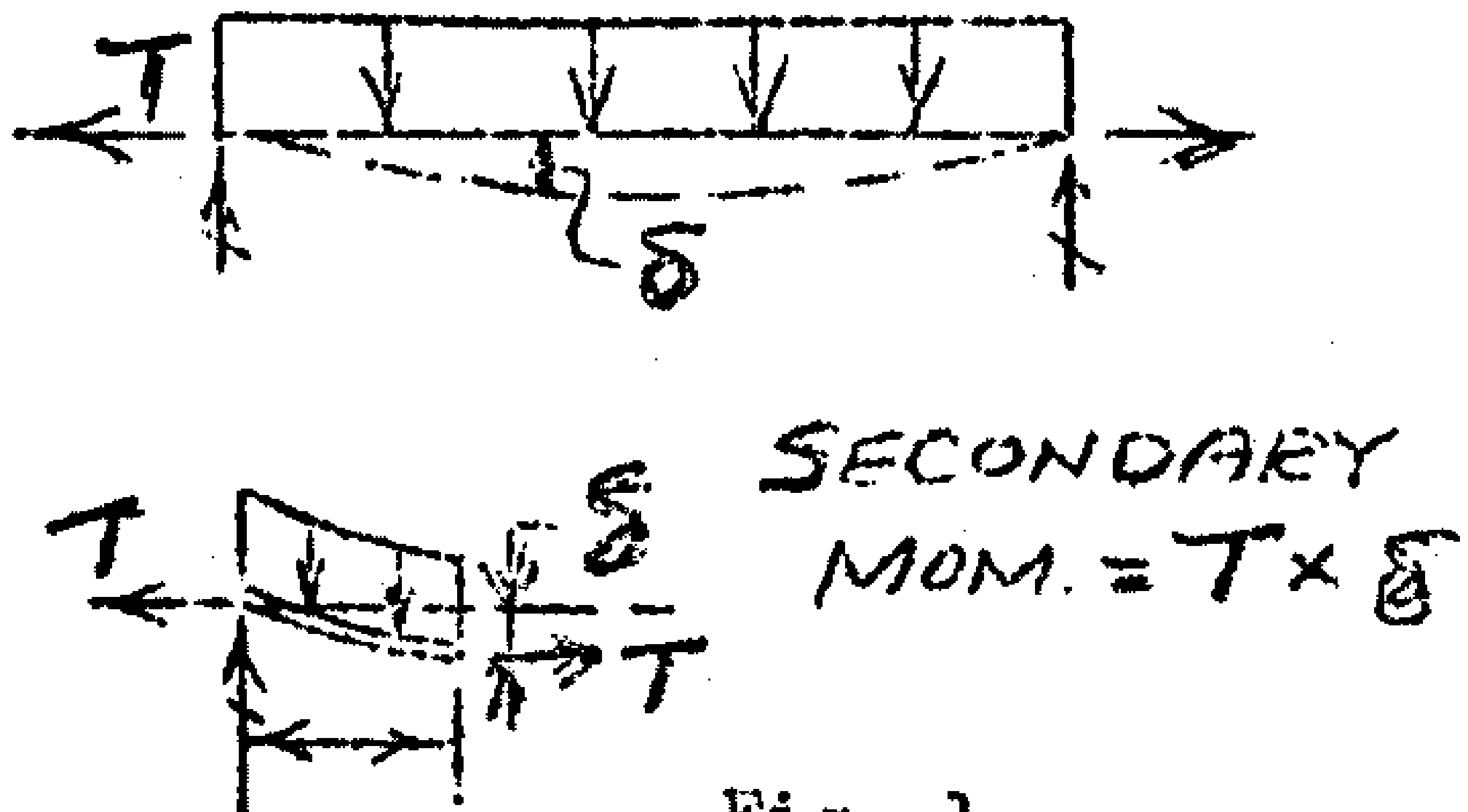


Fig. 1

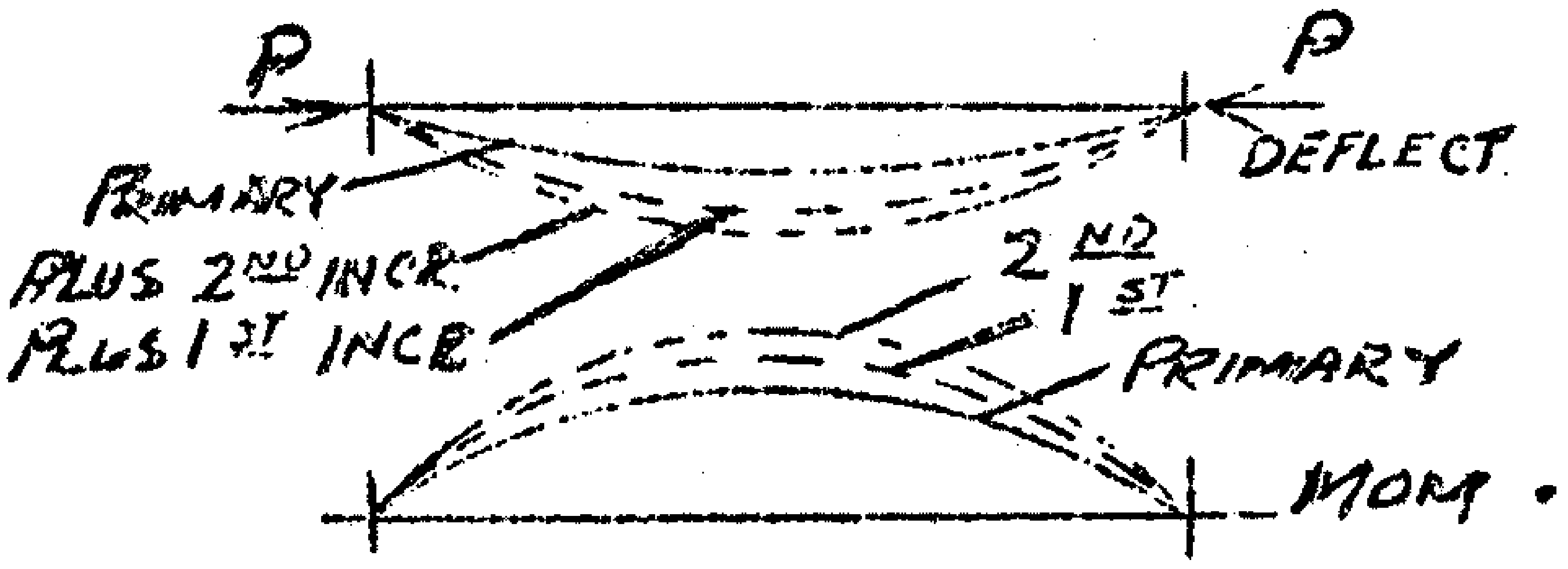


Fig. 2

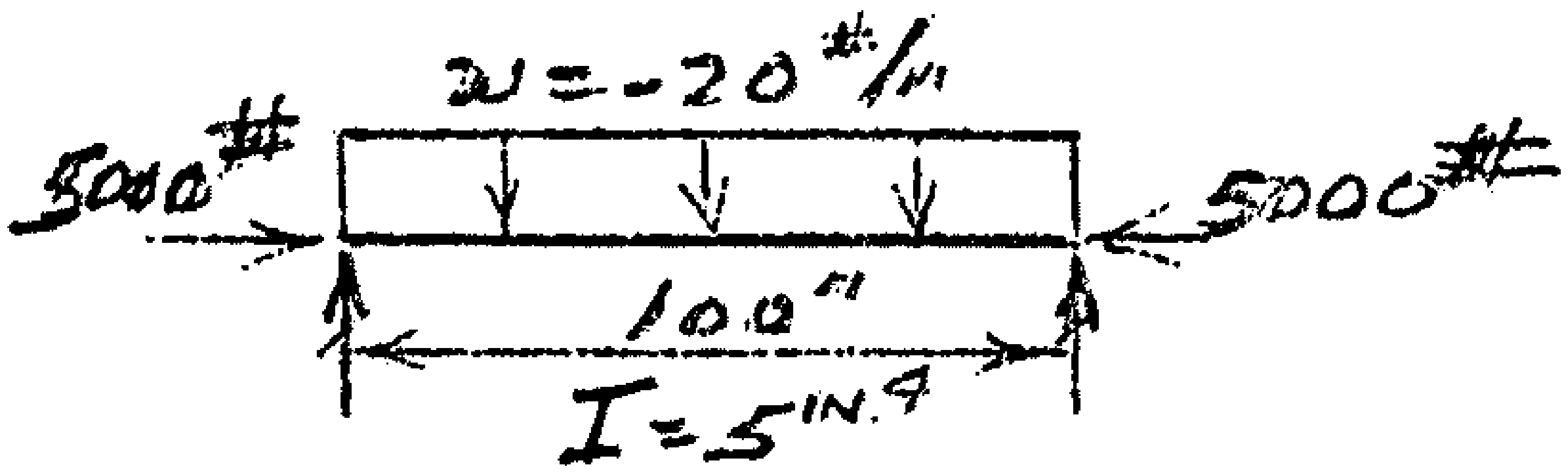


Fig. 3

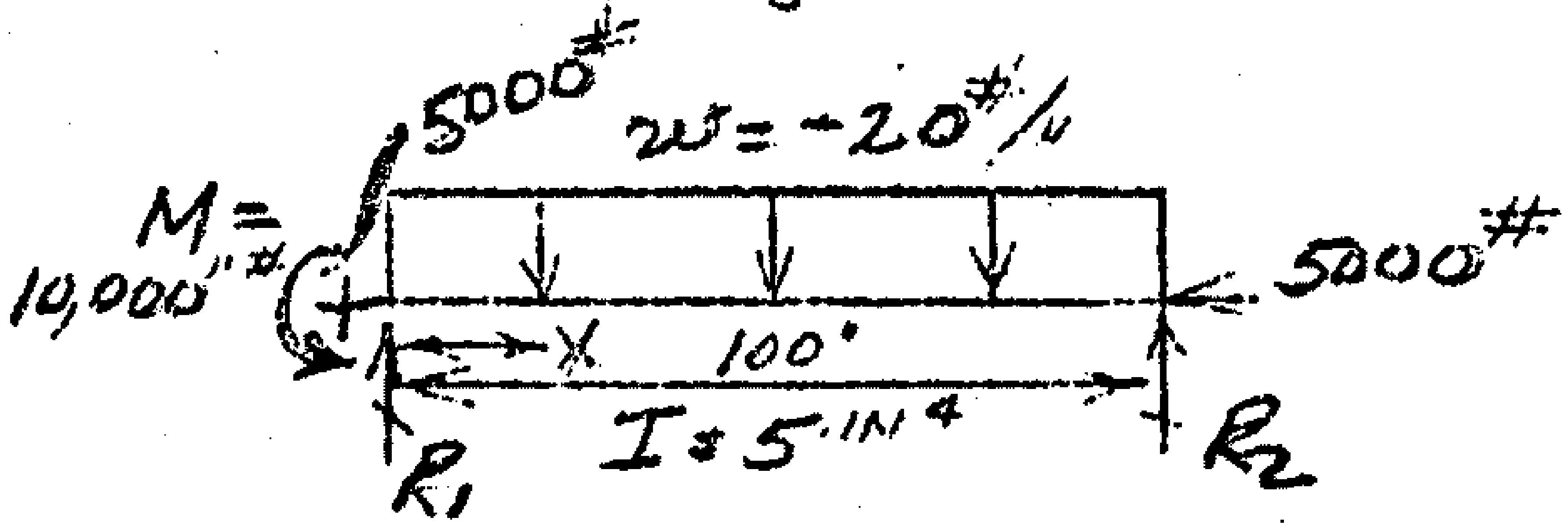


Fig. 4

Solution of Problem of Fig. 4

By exact method - "Niles and Newell", page 201 - Unequal End Moments.

Mom. is max. where $\tan \frac{x}{j} = \frac{D_2 - D_1 \cos L/j}{D_1 \sin L/j}$

$$M_{max} = \frac{D_1}{\cos \frac{x}{j}} + wj^2$$

$$D_1 = M_1 - wj^2 = -10{,}000 - (-20)(100)^2 = -10{,}000 + 200{,}000 = +190{,}000$$

$D_2 = M_2 - wj^2$, But $M_2 = 0$ Therefore $D_2 = -(-200{,}000) = +200{,}000$

$L/j = 100/100 = 1$ $\cos L/j = .540$ $\sin L/j = .841$

$$\tan x/j = \frac{200{,}000 - (190{,}000)(.540)}{(190{,}000)(.841)} = .610$$

From tables N. & N., page 193, $x/j = .55$ and $\cos x/j = .853$

Then $M_{max.} = \frac{190{,}000}{.853} + (-200{,}000) = 222{,}600 - 200{,}000$

$M_{max.} = 22{,}600$ lbs.

Approximate Solution.

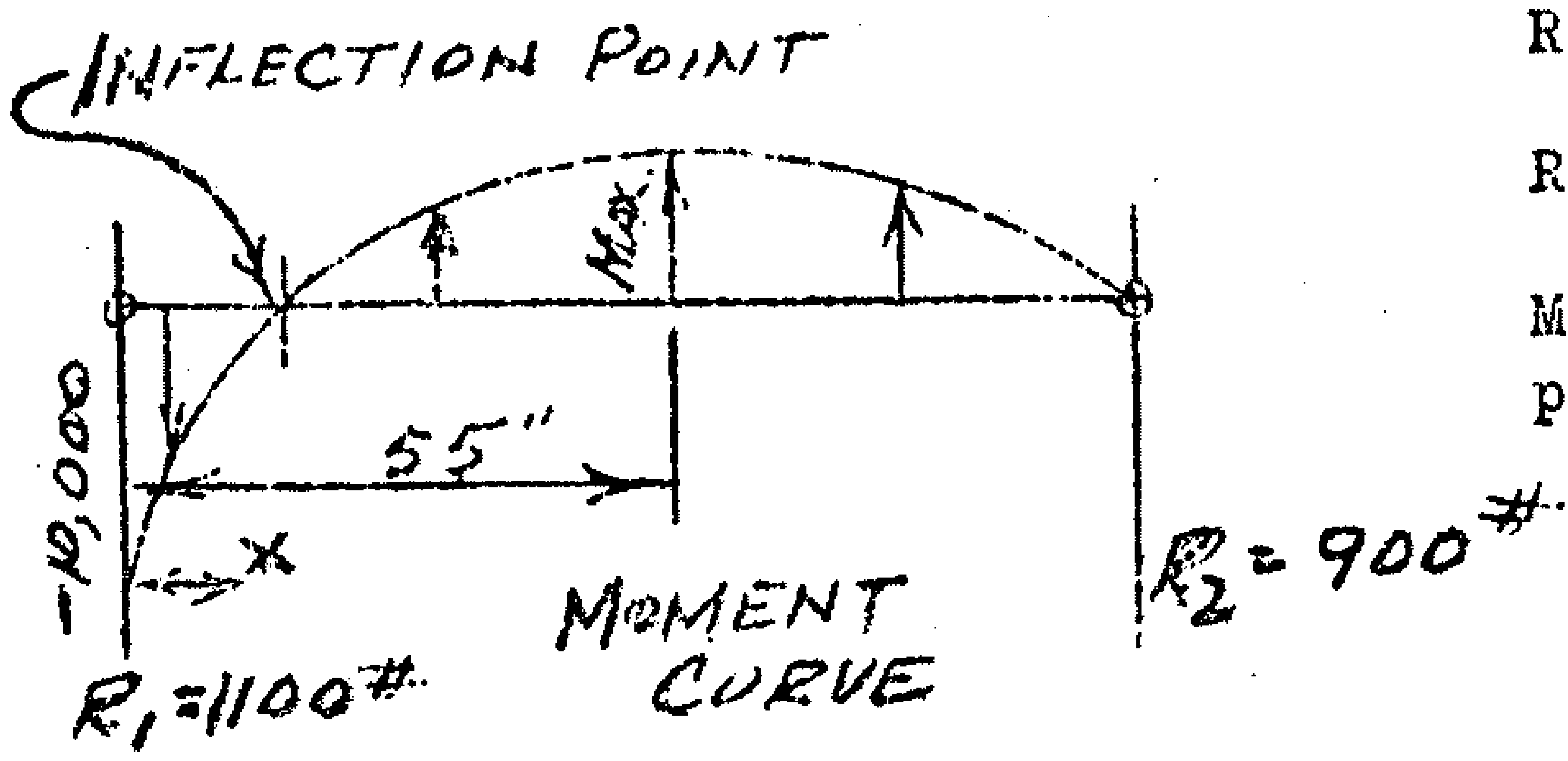


Fig. 5

$R_1 = \dfrac{20 \times 100}{2} + \dfrac{10,000}{100} = 1100$ lbs.

$R_2 = 900$ lbs.

Max. mom. in span where shear passes through zero. $x = 55$ in.

To find distance from R_1 to inflection point:

Mom. equation, $M_x = -10,000 + 1100x - 10x^2$

At inflection point $M_x = 0$, or $-10,000 + 1100x - 10x^2 = 0$

Solving for "x"

$x^2 - 110x + 1000 = 0, \quad x^2 - 110x = -1000$

$x^2 - 110x + 55^2 = -1000 + 55^2 = 2025$

$x - 55 = \sqrt{2025} = \pm 45$

$M_x = 0$ where $x = 10$ inches or 100 inches (checks mom. at R_2)

The end load will displace the inflection point toward R_1. Estimate a 2 inch shift. Main portion of beam now is pinned end with length = 92 inches. Max. primary mom. at $x = 55$ inches $= -10,000 + (1100)(55) - 10(55)^2$; $-10,000 + 60,500 - 30,250 = 20,250$"#. This value is also $\dfrac{wL^2}{8}$ for a uniformly loaded beam of 90 inch span. For such a beam with 92 inch span the max. deflection =

$$\frac{-(5)(20)(92)^4}{(384)(5)(10)^7} = .37 \text{ inches}$$

First increment of secondary mom. = 5000 x .37 = 1850
Mom. = 20,250 + 1850 = 22,100"#

Defl. for this mom. = $\dfrac{22,100}{20,250} \times (.37) = .40$ inches

First + Second increments of secondary mom. = 5000 x .40 = 2000

Total mom. = 20,250 + 2000 = 22,250"#

Successive approximations give the degree of accuracy desired.

Combined Axial and Bending Loads.

The development of the exact solution for beams under any combination of end loads, side loads, and end moments is based upon the principle that if we integrate the M/EI curve twice we have the deflection curve of the beam. This corresponds to the work we have already done in developing the 5-beam curves. If we are able to write an expression for the deflection of the beam after it has reached equilibrium, and differentiate this expression twice we will arrive at the equation for the moment curve corresponding to the moment after the beam has reached equilibrium under the loads. The solution of the general equation has been made for different combinations of loading so that by use of tables it is possible to make the exact solution without doing the entire development work. The exact solution involves the solution of a differential equation which is beyond the scope of this work. The general solution is included here to show the student the principles on which it is based. For a more complete study of the precise equations the student is referred to Niles and Newell's "Airplane Structures", first edition, or Vol. 2 of second edition.

The solution for the exact moment from side loads and end loads is essentially as follows:

1. The equation for the moment at any point 'x' is set up as in the derivation of the Three Moment Equation already given. Note that this equation was set up so that upward acting loads are + and + moments put tension on the lower fibers of the beam. The effect of an end load moment 'Py' may be added numerically if we observe the sign convention that for + deflection the deflected point is above its undeflected position. Then the equation for moment at any distance 'x' in the span becomes:

$$M = M_1 + \frac{M_2 x}{L_1} - \frac{M_1 x}{L_1} - \frac{w_1 L_1 x}{2} + \frac{w_1 x^2}{2} - Py \qquad 14:1$$

   Note that for downward deflection y is minus and -P(-y) becomes +Py putting additional tension on the lower fibers.

2. If we integrate the moment equation twice with respect to x we obtain the equation of the deflection curve, the actual values of deflection in inches being found by dividing the ordinates by EI. We can therefore set up an expression for y in terms of the moment equation and substitute this value for y in equation 14:1.

3. If we differentiate the moment equation twice with respect to x we obtain the equation of the load curve. For the case of equation 14:1 we obtain two parts for the load equation. The differentiation of that part referring to the primary moment gives the term for the side loading, w. The differentiation of the term representing the secondary moment gives an expression for an equivalent side loading on the beam which would replace the effect of the end load in producing bending. The essential portions of the solution follow. The simple rules to be remembered in differentiating are:
   (a) When differentiating with respect to x, x is the only variable and all other terms are considered as constants.
   (b) The differential of a constant is zero.
   (c) The differential of $x^n = Nx^{n-1}$. In other words decrease the power of the exponent by 1 and multiply by the original exponent.

Tables of natural trigonometric functions similar to those on page 193 of Niles and Newell are not required for the solution of the *Precise* Equations. When they are not available reduce the values of $L/_j$ which are given in radians to degrees by multiplying by 57.3 and use the ordinary tables for functions of angles in degrees.

Derivation of Formula for Combined Loading.

$$M = M_1 = \frac{M_2x}{L} - \frac{M_1x}{L} - \frac{wLx}{2} + \frac{wx^2}{2} - Py \qquad \text{(Uniform side load)}$$

Integrating the above equation twice we obtain

$$EI\delta = EIy = \frac{M_1x^2}{2} + \frac{M_2x^3}{6L} - \frac{M_1x^3}{6L} - \frac{wLx^3}{12} + \frac{wx^4}{24} - \frac{Pyx^2}{2} + C_1x + C_2$$

$$y = \frac{1}{EI}\left[\frac{M_1x^2}{2} + \frac{M_2x^3}{6L} - \frac{M_1x^3}{6L} - \frac{wLx^3}{12} + \frac{wx^4}{24} - \frac{Pyx^2}{2} + C_1x + C_2\right]$$

Substituting this value in the equation for moment in place of y and calling the expression inside the brackets "F"

$$M = \left[M_1 + \frac{M_2x}{L} - \frac{M_1x}{L} - \frac{wLx}{2} + \frac{wx^2}{2}\right] - \frac{P}{EI}\left[F\right]$$

Differentiating each term separately twice with respect to "x" we obtain the loading equation. For the first term

$$\frac{dM}{dx} = \frac{M_2}{L} - \frac{M_1}{L} - \frac{wL}{2} + wx \quad \text{and} \quad \frac{d^2M}{dx^2} = w$$

For the term $\frac{-P}{EI}\left[F\right]$ the second derivative is M since the term was obtained by integrating M twice.

Then $\frac{d^2M}{dx^2} = w - \frac{P}{EI}(M)$ or $\frac{d^2M}{dx^2} + \frac{P}{EI}M = w$

If for $\frac{P}{EI}$ we write $\frac{1}{j^2}$, $j = \sqrt{\frac{EI}{P}}$ and $\frac{d^2M}{dx^2} + \frac{1}{j^2}M = w$

This is a differential equation the solution for which is

$M = C_1 \sin\frac{x}{j} + C_2 \cos\frac{x}{j} + wj^2$ (C_1 and C_2 are constants of integration)

When x = 0, $M = M_1$ and when x = L, $M = M_2$

Substituting in the general equation

$$M_1 = C_1 \times 0 + C_2 \times 1 + wj^2, \quad \text{and } C_2 = M_1 - wj^2$$

$$M_2 = C_1 \sin\frac{L}{j} + C_2 \cos\frac{L}{j} + wj^2$$

$$C_1 \sin \frac{L}{j} = M_2 - C_2 \cos \frac{L}{j} - wj^2 = M_2 - wj^2 - (M_1 - wj^2) \cos \frac{L}{j}$$

$$C_1 = \frac{M_2 - wj^2 - (M_1 - wj^2) \cos \frac{L}{j}}{\text{Sin } \frac{L}{j}}$$

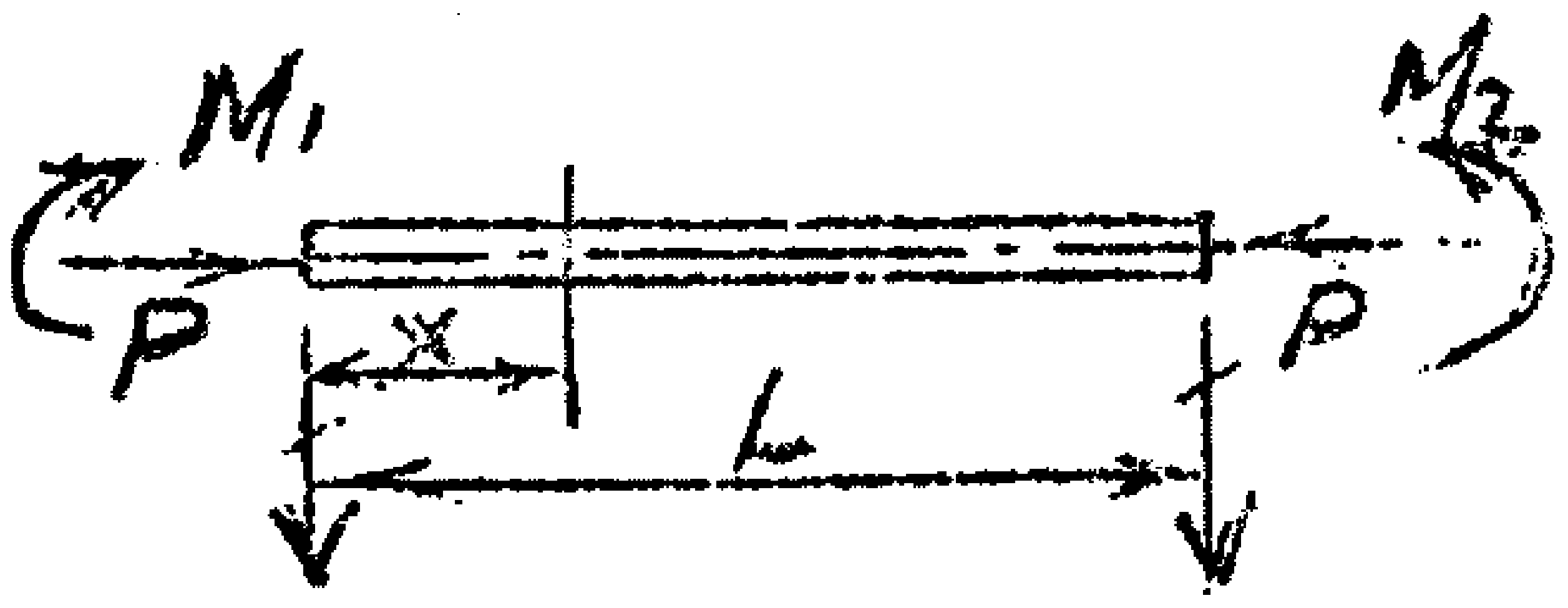
If $D_1 = M_1 - wj^2$ $\quad D_2 = M_2 - wj^2$ $\quad$ Then $C_2 = D_1$

$$C_1 = \frac{D_2 - D_1 \cos \frac{L}{j}}{\sin \frac{L}{j}}$$

And $M = \dfrac{D_2 - D_1 \cos \frac{L}{j}}{\sin \frac{L}{j}} \sin \dfrac{x}{j} + D_1 \cos \dfrac{x}{j} + wj^2$

1. No side Loads with Unequal End Moments

$$M_x = \frac{M_2 - M_1 \cos \frac{L}{j}}{\sin \frac{L}{j}} \sin \frac{x}{j} + M_1 \cos \frac{x}{j}$$



At point of max. moment

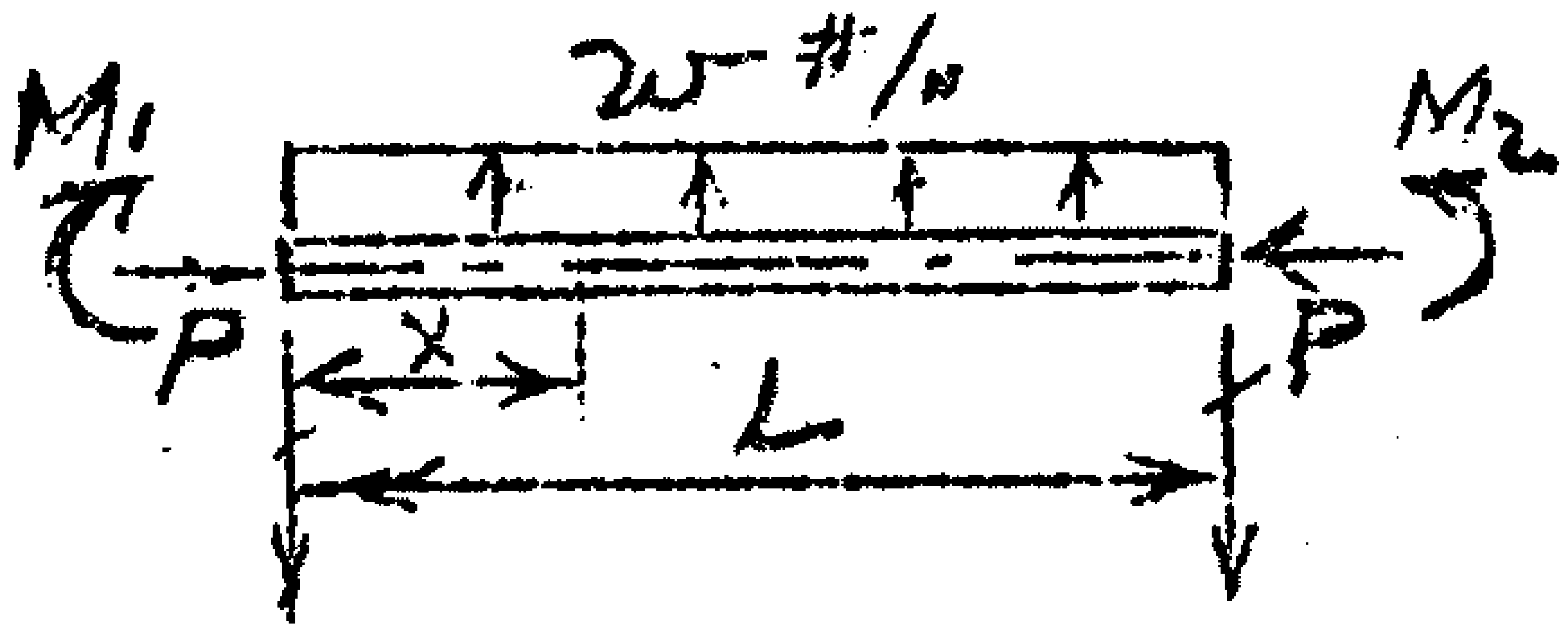
$$\text{Tan } \frac{x}{j} = \frac{M_2 - M_1 \cos \frac{L}{j}}{M_1 \sin \frac{L}{j}}$$

$$M_{max.} = \frac{M_1}{\cos \frac{x}{j}}$$

*****

2. Uniform Side Load with Unequal End Moments

$$D_1 = M_1 - wj^2 \qquad D_2 = M_2 - wj^2$$



$$M_x = \frac{D_2 - D_1 \cos \frac{L}{j}}{\sin \frac{L}{j}} \sin \frac{x}{j} + D_1 \cos \frac{x}{j} + wj^2$$

At point of max. moment

$$\text{Tan } \frac{x}{j} = \frac{D_2 - D_1 \cos \frac{L}{j}}{D_1 \sin \frac{L}{j}}$$

$$M_{max.} = \frac{D_1}{\cos \frac{x}{j}} + wj^2$$

*****

3. Uniform Side Load with Pinned Ends

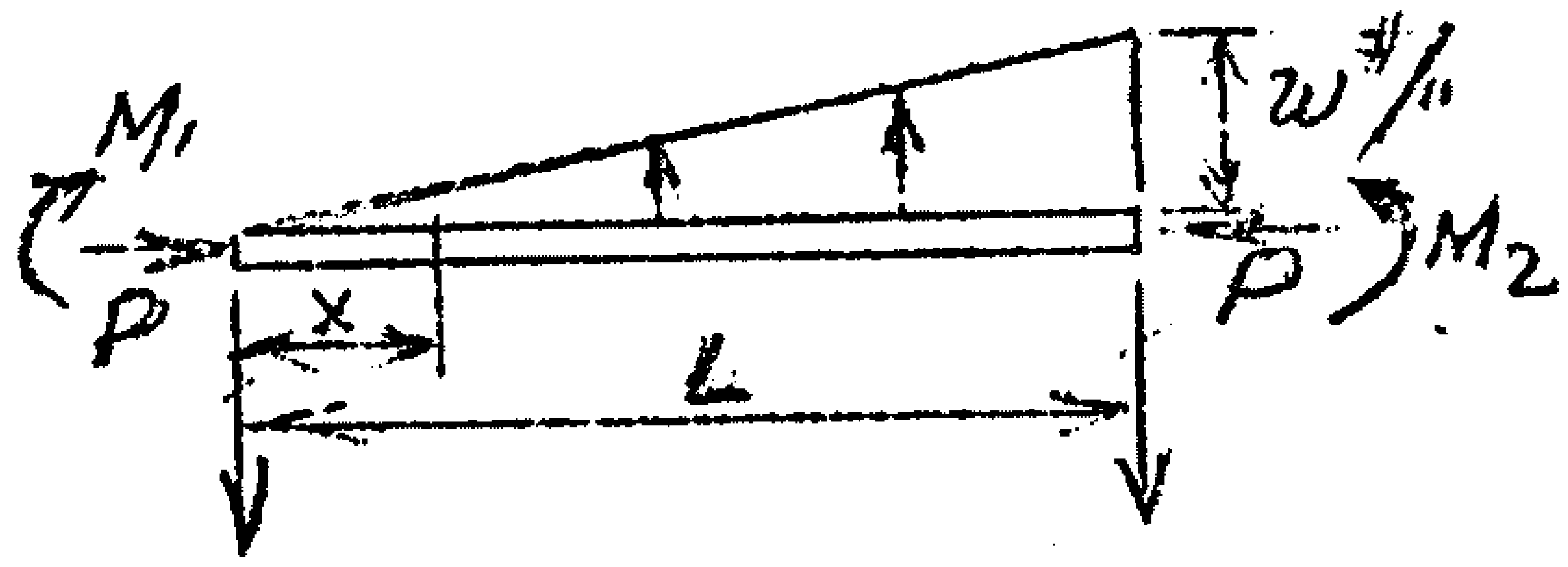
Same as (2) above except $M_1 = M_2 = 0$

$$M_{max} = wj^2 \left[1 - \frac{1}{\cos \frac{L}{2j}}\right] \quad \text{at } x = 0.5L$$

$$M_x = \frac{wj^2 \left(\cos \frac{L}{j} - 1\right)}{\sin \frac{L}{j}} \sin \frac{x}{j} + wj^2 \left(1 - \cos \frac{x}{j}\right)$$

*****

4. Uniformly Varying Load Increasing to the Right Unequal End Moments



$$C_1 = \frac{M_2 - wj^2 - M_1 \cos \frac{L}{j}}{\sin \frac{L}{j}}$$

and $C_2 = M_1$

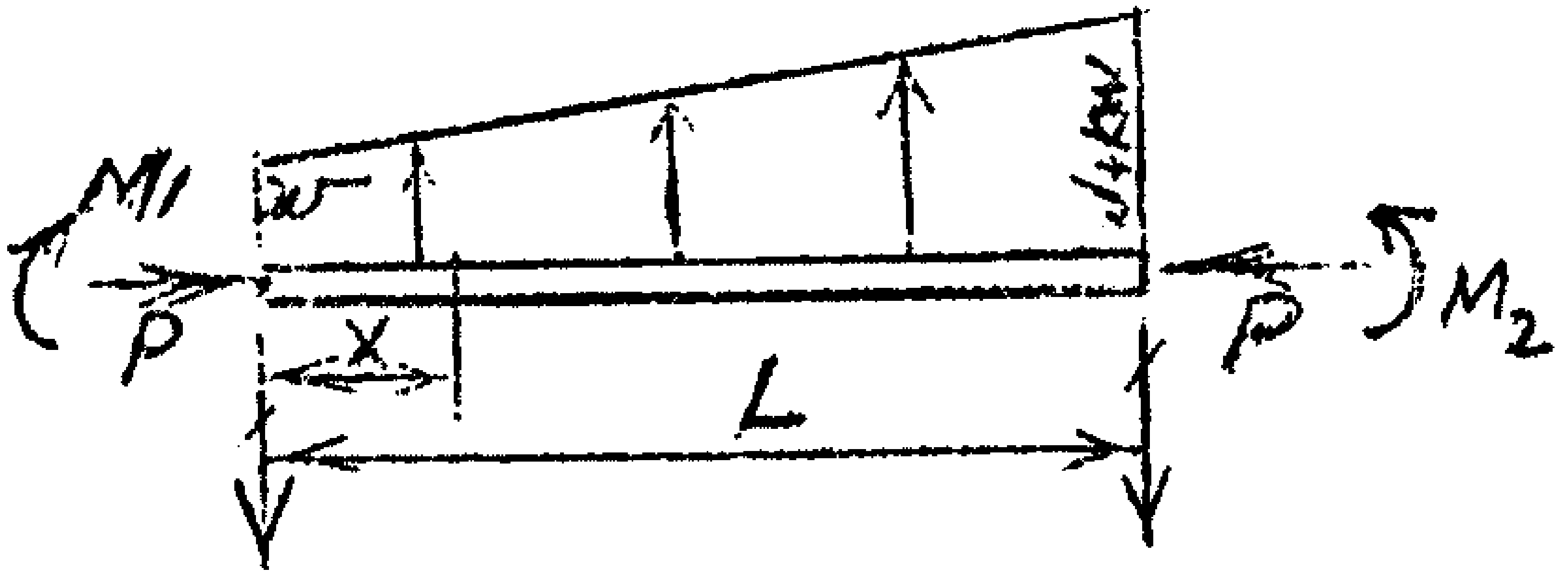
$$M_x = C_1 \sin \frac{x}{j} + C_2 \cos \frac{x}{j} + \frac{wj^2}{L} x$$

At point of maximum moment

$$\tan \frac{x}{j} = \frac{C_1}{C_2} - \frac{(C_1{}^2 + C_2{}^2)\, wj^3}{C_1 C_2 wj^3 \pm \sqrt{L^2 (C_1{}^2 + C_2{}^2) - (wj^3)^2}}$$

*****

5. Uniformly Varying Load Increasing from "w" at the left to "w + kw" at the Right. Unequal End Moments.



$$C_1 = \frac{M_2 - (1 + k)wj^2 - (M_1 - wj^2)\cos \frac{L}{j}}{\sin \frac{L}{j}}$$

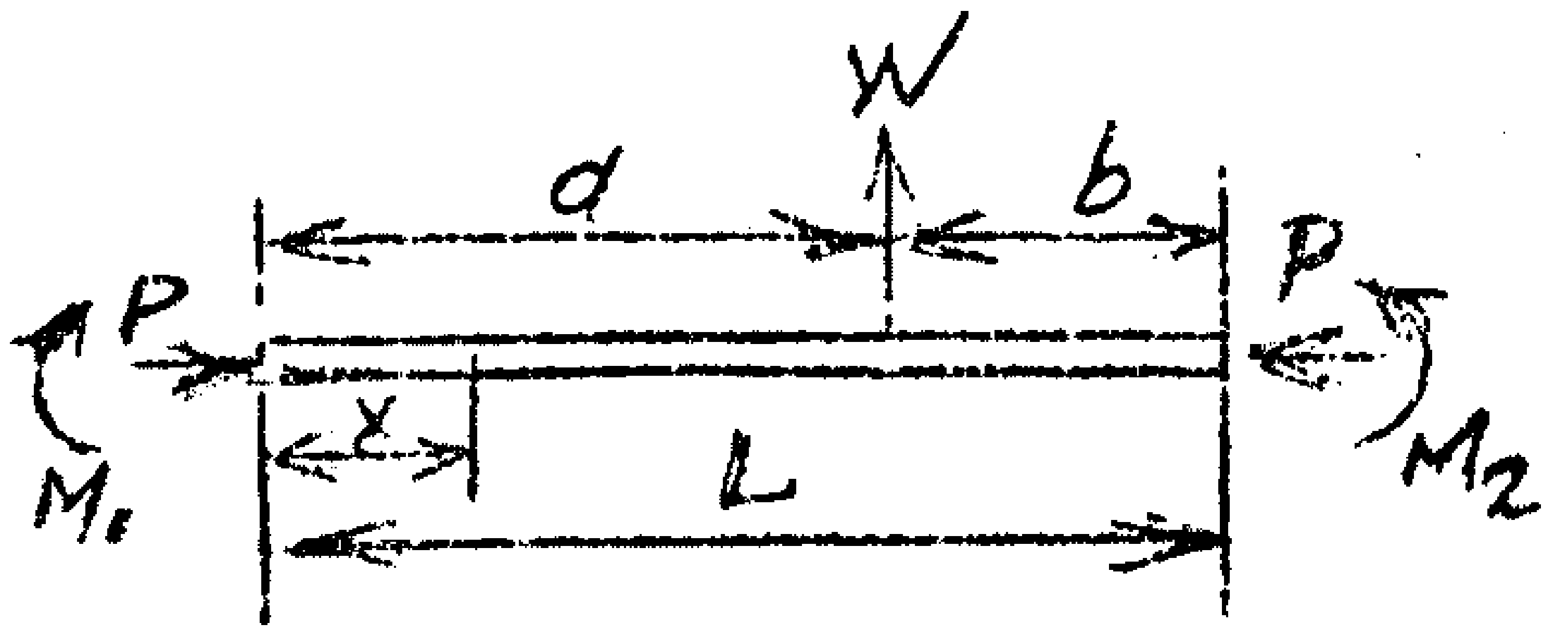
$$C_2 = M_1 - wj^2$$

$$M_x = C_1 \sin \frac{x}{j} + C_2 \cos \frac{x}{j} + wj^2 \left(1 + \frac{kx}{L}\right)$$

For this case the solution for point of maximum moment is so long that it is better to compute enough points to plot a faired moment curve and scale the maximum moment.

*****

6. Single Concentrated Load. Unequal End Moments.



$$C_1 = C_3 - Wj \cos \frac{a}{j}$$

$$C_2 = M_1$$

$$C_3 = \frac{M_2}{\sin \frac{L}{j}} - \frac{M_1 - Wj \sin \frac{a}{j}}{\tan \frac{L}{j}}$$

$$C_4 = M_1 - Wj \sin \frac{a}{j}$$

Where x is less than a:

$$M_x = C_1 \sin \frac{x}{j} + C_2 \cos \frac{x}{j}$$

Where x is more than a:

$$M_x = C_3 \sin \frac{x}{j} + C_4 \cos \frac{x}{j}$$

At point of max. moment $\tan \frac{x}{j} = \frac{C_1}{C_2}$ or $\frac{C_3}{C_4}$

$$M_{max.} = \frac{C_1^2 + C_2^2}{C_2} \cos \frac{x}{j} \quad \text{or} \quad \frac{C_3^2 + C_4^2}{C_4} \cos \frac{x}{j}$$

Note: To find max. moment--

(1) Compute x from the two expressions for $\tan \frac{x}{j}$.

(2) If $\frac{C_1}{C_2}$ gives a value of x less than a or $\frac{C_3}{C_4}$ gives a value of x between a and L, x is at the point of maximum moment and should be used in equations for M_{max}.

(3) If the x values fall outside the above limits the max. mom. is at the supports or at the load point. For mom. at the load point use the general equation for M_x.

*****

For any case in which the student has trouble in locating the point of maximum moment it is always possible to give values to x and plot the moment curve. It is then possible to find the maximum moment by scaling the diagram. It should be noted too that the functions of the angles shown do not represent actual angles but merely the solution of mathmatical series.

*****

If the tables of Niles and Newell are used the values of all angles will be found plotted against the value L/j in radians. This value should be shown "either L/j or x/j" since the values of the natural sines, cosines and tangents are the same whether the angle in radians is expressed in terms of L/j or x/j.

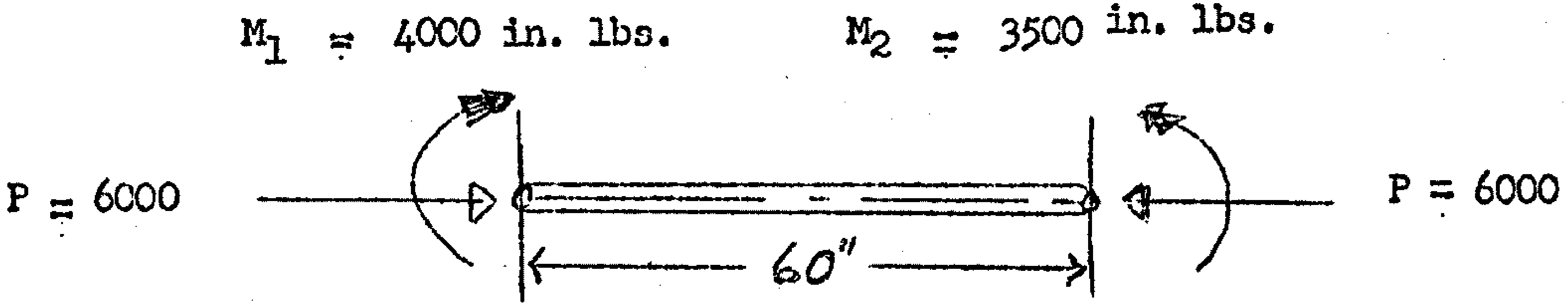
For example in our reference problem on page 3 the tangent of x/j is given as .610. If we look up this value in the tables we will find that it corresponds to an angle in radians of .55. Then $x/j = .55$ radians and the sine and cosine values for this angle are used.

****

Use of the precise equations with different types of loading will be illustrated by problems. Note that the size of the beam must be known before it can be solved by precise equations, since they depend upon the value of I. It becomes necessary to assume sizes and check the actual strength afterward.

Example Problem:

1. No side load, axial load, unequal end moments, Case I.



$E = 10,000,000$
$I = 3$ in.4

$$j = \left(\frac{EI}{P}\right)^{\frac{1}{2}} = \left(\frac{10,000,000 \times 3}{6,000}\right)^{\frac{1}{2}} = 70.7$$

$$L/j = 60/70.7 = .848 \text{ radians}$$

$$\sin L/j = .75 \qquad \cos L/j = .66$$

at point of maximum moment

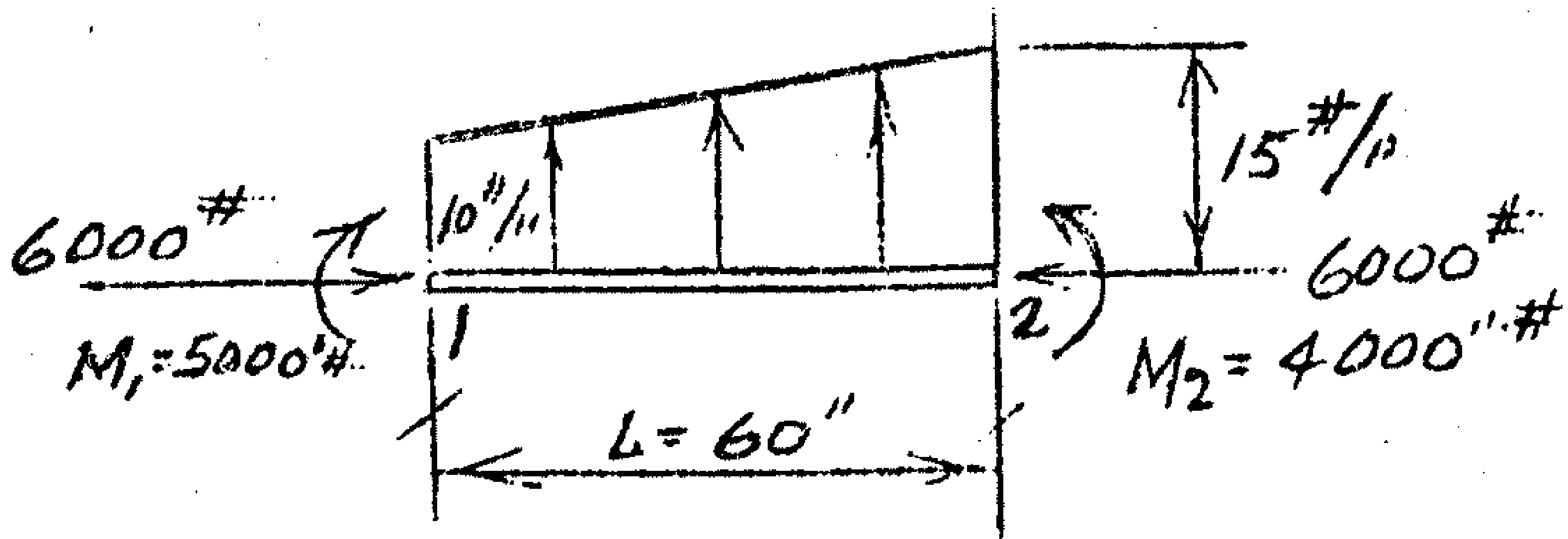
$$\tan\frac{x}{j} = \frac{M_2 - M_1 \cos\frac{L}{j}}{M_1 \sin\frac{L}{j}} = \frac{3500 - (4000)(.66)}{(4000)(.75)} = \frac{860}{3000} = .287$$

$\frac{x}{j} = .28$, point of max. moment, $x = .28j$

$= (.28)(70.7) = 19.8$ inches to right of 1.

$$M_{max.} = \frac{M_1}{\cos\frac{x}{j}} = \frac{4000}{.961} = 4160 \text{ inch lbs.}$$

2. Trapezoidal loading, unequal end moments, axial load. Case V.



$w = 10$ lbs. per inch.

$w + kw = 15$ lbs. per inch.

$k = 0.5$

$E = 29,000,000$ (steel)

$I = 10^{in^4}$

$$j = \sqrt{\frac{(29,000,000)(10)}{6000}} = \sqrt{48,300} = 220$$

$$\frac{L}{j} = \frac{60}{220} = .272 \text{ radians } (= 15.65 \text{ degrees})$$

$$\sin\frac{L}{j} = .269, \quad \cos\frac{L}{j} = .963$$

$$C_1 = \frac{M_2 - (1 + k)wj^2 - (M_1 - wj^2)(\cos\frac{L}{j})}{\sin\frac{L}{j}}$$

$$C_1 = \frac{4000 - (1 + 0.5)(10)(48,300) - \left[5000 - (10)(48,300)\right](.963)}{.269}$$

$$= \frac{4000 - 724,500 + 460,300}{.269} = -967,300 \qquad \leftarrow C_1$$

$$C_2 = M_1 - wj^2 = 5000 - (10)(48,300)$$
$$= 5000 - 483,000 = -478,000 \qquad \leftarrow C_2$$

The equation for moment at any point, x, along the span is

$$M_x = C_1 \sin\frac{x}{j} + C_2 \cos\frac{x}{j} + wj^2\left(1 + \frac{kx}{L}\right)$$

This equation is conveniently solved in tabular form. Values of x are chosen at 10 inch intervals.

Constants: $C_1 = -967,300$

$C_2 = -478,000$

$wj^2 = +483,000$

$k = 0.5$

$j = 220$

x (1)	$\frac{x}{j}$ (2)	sin $\frac{x}{j}$ (3)	cos $\frac{x}{j}$ (4)	C_1 sin $\frac{x}{j}$ (5)	C_2 cos $\frac{x}{j}$ (6)	$1 + k\frac{x}{L}$ (7)	$wj^2(1+k\frac{x}{L})$ (8)
10	.045	.045	.999	- 43,500	-477,500	1.083	+523,200
20	.091	.091	.996	- 88,000	-476,100	1.167	+563,500
30	.136	.136	.991	-131,600	-473,700	1.25	+603,800
40	.182	.181	.983	-175,100	-469,900	1.333	+644,000
50	.227	.224	.974	-216,700	-465,600	1.417	+684,300

Moment at any station = (5) + (6) + (8)
Numbers in ( ) refer to columns in the table above.

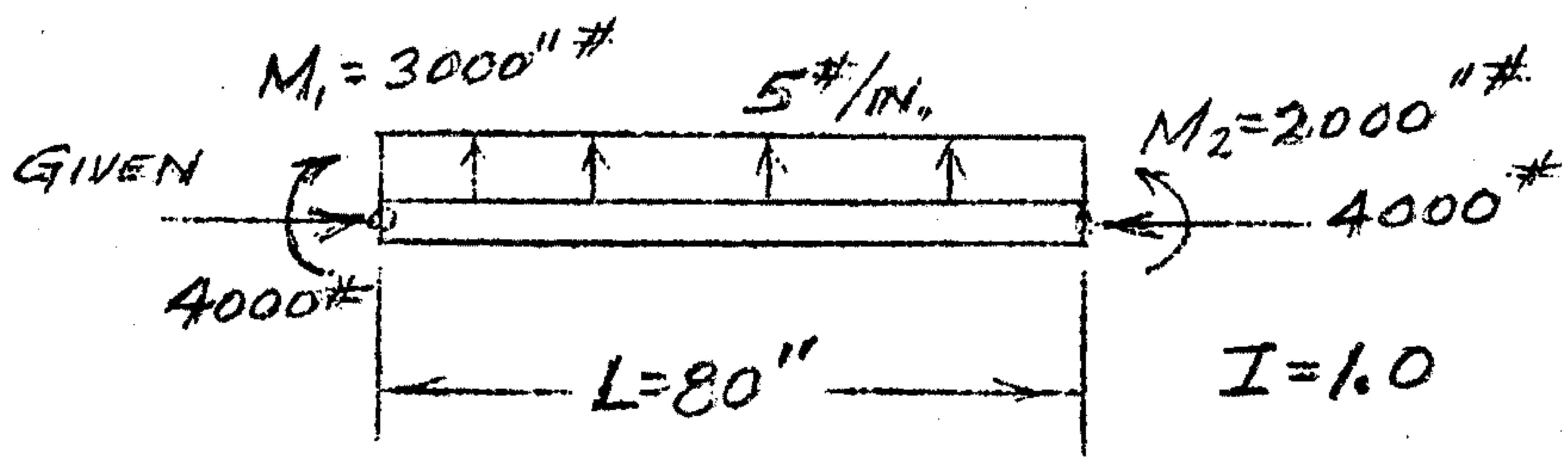
x

10 Mom. = - 43,500 - 477,500 + 523,200 = +2200
20 " = - 88,000 - 476,100 + 563,500 = - 600
30 " = -131,600 - 473,700 + 603,800 = -1500
40 " = -175,100 - 469,900 + 644,000 = -1000
50 " = -216,700 - 465,600 + 684,300 = +2000

Moments may then be plotted to obtain the maximum value. In this case the maximum value is next the supports.

Problem Assignment.

14:1



Find the maximum moment in the span when
(a) material is Aluminum Alloy (E = 10,000,000)
(b) material is steel (E = 29,000,000)

14:2 Solve the second example problem in the text assuming that I has been reduced to 1^{in^4} (was 10).

14:3 Solve the second example problem in the text assuming that the side load has been increased to 20 lbs. per inch at 1 and 30 lbs. per inch at 2, leaving I = 10^{in^4}, and the end moments, M_1 = 5000 in. lbs.
M_2 = 4000 in. lbs.

BASIC STRESS ANALYSIS

ASSIGNMENT FIFTEEN

Assignment 15

COMBINED STRESSES

In Assignment 14 we introduced the effect of combining axial and bending loads. Our work was carried to the point of finding the final bending moments in beams subjected to end loads as well as side loads. In order to find whether or not a beam so loaded is sufficiently strong it is necessary to find the combined unit stresses in the beam and to compare them with known allowable values. The combined unit stresses on an axially loaded beam are found as follows:

$$\text{Unit stress due to axial load} = \frac{P}{A} = \frac{\text{End Load}}{\text{Area}}$$

This assumes equal distribution of unit stress, that is that the end load acts at the centroid of area.

$$\text{Unit stress due to bending} = f_b = \frac{My}{I} \quad \text{where}$$

M is the total moment on the section and y is the distance from the neutral axis to the fiber being investigated. If the beam has an axial tension load the unit tension will add to the unit stress due to bending on the tension side of the beam and subtract from it on the compression side. Similarly if the axial load in the beam is compression, the unit stresses will add to the bending stresses on the compression side and subtract from them on the tension side.

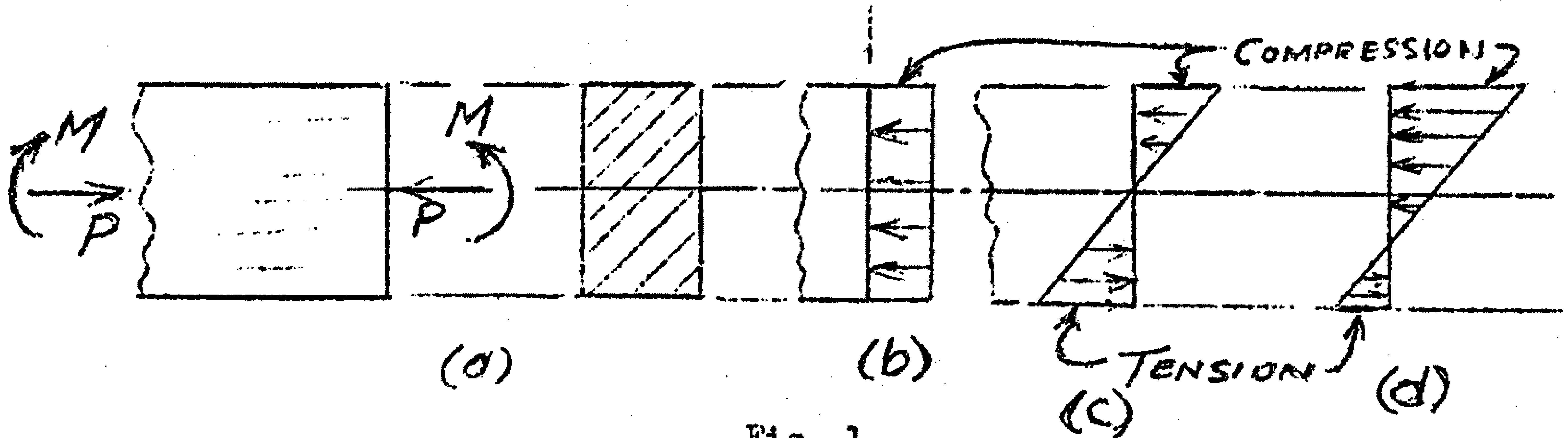


Fig. 1

The unit stresses for a beam carrying compression end loads, and subjected to a bending moment which puts tension on the bottom fibers is shown in Fig. 1. Fig. 1 (b) shows the distribution of the unit stress due to end load, P/A. Fig. 1 (c) shows the distribution of the unit stress due to bending, $f_b = \frac{My}{I}$. Fig. 1 (d) shows the distribution of the combined unit stresses.

ANC-5, Strength of Aircraft Elements, gives the approved methods of obtaining allowable combined stresses for a combination of loads. The methods given there are further explained in the June, 1937 issue of Aviation magazine.

The combinations treated include

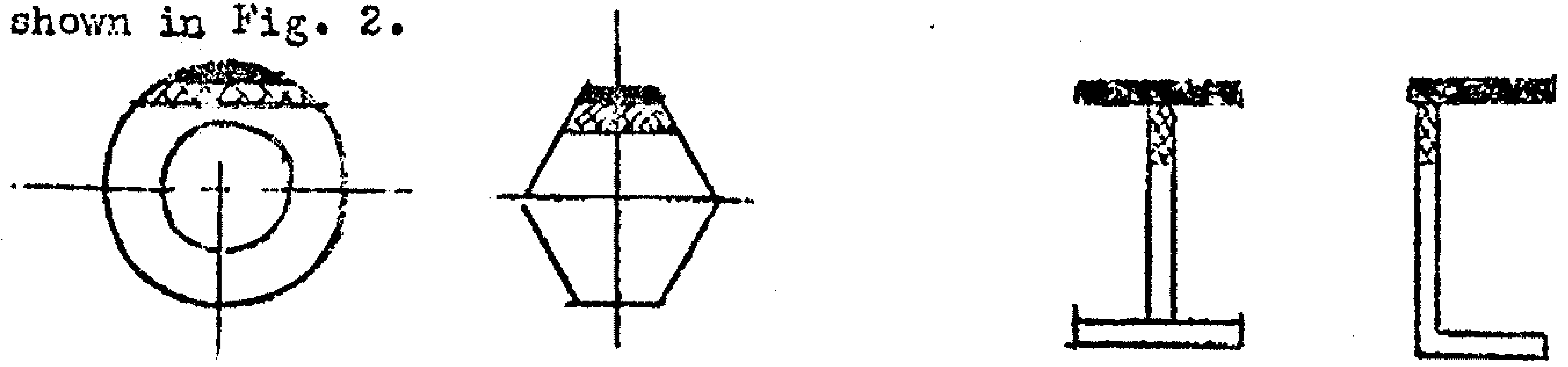
(a) bending combined with axial tension or compression
(b) bending combined with torsion
(c) bending combined with transverse shear
(d) compression combined with torsion.

The system used to obtain the values for combined stresses is known as the stress ratio method. The stress ratio method is founded upon the assumption that if two or more types of unit stress exist together in a member the unit stress that can be developed for one type of stress depends upon the residual strength remaining after the other type (or types) of stress have been taken care of. The actual stress ratio for any type of stress is taken as the value obtained when the unit stress existing in the member is divided by the allowable unit stress which could be used if that type of stress only were induced in the member. (See ANC-5, pages 1-16 to 1-20 and 4-3 and 4-4.)

No attempt will be made to introduce allowable combined stresses for cases involving torsion until after this subject is covered in a later assignment. Note that the allowable values for all types of loading are based on actual tests.

Combined Bending and Axial Tension.

It is conservative for this case to assume that the actual value of the combined tension allowable stress is the ultimate tensile value of the material. Actually, for metals, when most of the stress is due to bending, the allowable stress on the tension side of the beam approaches the Modulus of Rupture. The Modulus of Rupture is the theoretical unit stress, from the beam theory, at the extreme fiber when rupture of the beam occurs. It may be 50 per cent, or even higher, above the tension value of the material. Actually the outer fiber of the beam can not carry a tension load greater than that which would be shown by a test coupon taken from the beam. The fact that the beam will apparently carry a higher tensile stress at the extreme fiber is explained by saying that the outer fiber yields and transfers its load to the less stressed fibers underneath it. The shape of the cross-section then has considerable bearing on the Modulus of Rupture values. A bar or a tube has a high Modulus of Rupture because the extreme fibers are reinforced by other layers of fibers containing greater area than is represented by the extreme fibers. Types of beam cross-sections which have high and low Moduli of Rupture values are shown in Fig. 2.



High Moduli of Rupture

Lower Moduli of Rupture

Fig. 2

The layer of highly stressed extreme fibers is shown in solid section on Fig. 2, while the supporting fibers into which they unload are shown cross-hatched. It immediately appears logical that round bars or heavy walled tubes should have higher Moduli of Rupture than thin walled tubes. This is the actual case. Page 4-22 of ANC-5 shows the curves for Moduli of Rupture of Chrome Molybdenum steel tubing. It will be noted that the values of F_b, the allowable unit bending stress from the beam theory at the extreme fiber are plotted against the ratio D/t where D is the outside diameter of the tube and t is its thickness.

ANC-5 states that the allowable bending stresses for shapes other than tubes or round bars must be based upon tests. Two effects are being sought in such a test.

1. If the cross-section of the beam is such that local instability on the compression side is not present (that is no thin edges are left unsupported to buckle locally and thus cripple the beam) the beam will have a usable strength higher than its tensile strength and hence an increase for Modulus of Rupture.

2. If the distribution of area in the cross-section is such that local crippling can occur on the compression side of the beam, a lower value of bending stress is likely to be developed than the ultimate tensile stress allowable. In this case the allowable bending stress is said to be reduced for Form Factor. The Form Factor is the ratio of the maximum unit bending stress developed on test to the allowable tensile stress for the material in the beam.

It would have been possible to express both the Modulus of Rupture and the Form Factor in terms of the ratio of unit stress developed in a bending test, to the ultimate tensile stress. However, it is accepted practice to express the Modulus of Rupture in lbs. per sq. inch and the Form Factor as a ratio, some decimal less than one. For example suppose that the ultimate tensile stress for the material in beam (a) is 150,000 psi and that on a bending test the load carried corresponded to a calculated outer fiber stress at rupture of 200,000 psi. We could have said that the beam had a form factor of 200,000/150,000 = 1.33. Instead we say that it has a Modulus of Rupture in bending of 200,000 psi. Now suppose that beam (b) is made of material with a tensile value of 150,000 psi but that it has flanges on the compression side so thin that they fail by local buckling at a unit stress calculated by the beam theory to be only 120,000. We use 120,000 psi as the allowable stress in bending and express our reduction in allowable below the tensile value of the material by saying that the beam has a Form Factor of 120,000/150,000 = 0.80. Then if we used the same shape beam and changed the material say to aluminum alloy with a tensile value of 50,000 psi our allowable value for the compression stress in bending would be (50,000)(Form Factor) = (50,000)(0.80) = 40,000 psi.

Modulus of Rupture values from tests are secured on simply supported beams loaded at the centers or at the third points in the span. For the steels used in aircraft work the values may generally be used directly. For more plastic materials such as the aluminum alloys which do not have a well defined yield point, the Moduli of Rupture indicated by tests will be very high but the deflections will be excessive. For these materials an arbitrary yield point is established where the permanent set becomes .002 inches per inch on the extreme fiber. Then the Modulus of Rupture is taken as 1.5 times the stress at which this amount of permanent set occurs. The factor of 1.5 is the usual factor of safety between yield point loads at which the structure takes a permanent set and ultimate loads at which it fails.

An example of combined tension and bending loads follows:

Suppose we have a steel tube pinned at the ends and loaded as shown in Fig. 3. The tube size is 2 inches outside diameter by .065 wall thickness for which the

Area, $A = .3951$ $\qquad$ $I = .1851$

$D/t = 31$ $\qquad$ See table 8-1 of ANC-5.

Heat treatment is 160,000 psi.

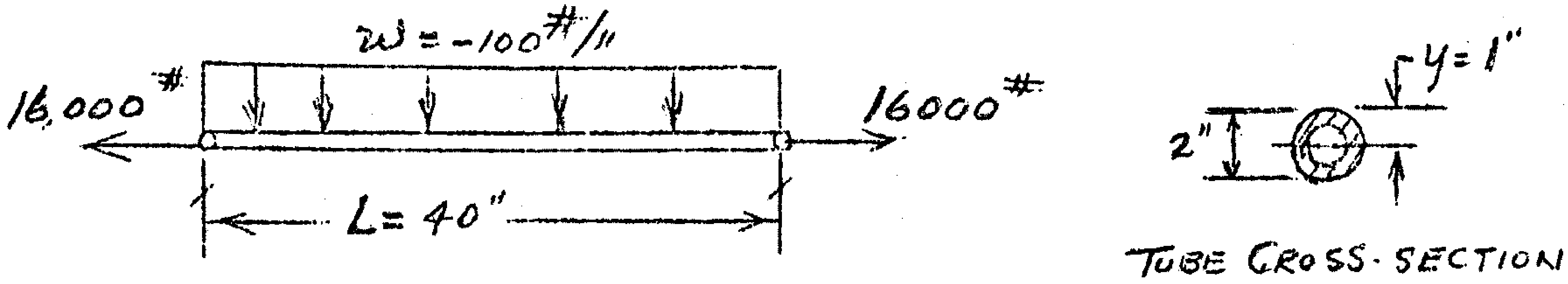


Fig. 3

The unit tension load, $f_t = \frac{16000}{.3951} = 40{,}500$ psi

The allowable unit tension load, $F_t = 160{,}000$

Neglecting secondary bending which subtracts from the primary bending when the axial load is tension in a straight beam, the maximum moment occurs at the center of the span and is equal to

$$\frac{wL^2}{8} = \frac{(100)(40)^2}{8} = 20{,}000 \text{ in lbs.}$$

$$f_b = \frac{My}{I} = \pm\frac{(20{,}000)(1)}{.1851} = \pm\, 108{,}000 \text{ psi. (Tens. or compr.)}$$

f_b = Mod. of Rupture = 173,000 psi. from Fig. 4-10, page 4-22 in ANC-5 (for $D/t = 31$, Heat treatment = 160,000).

$$\frac{f_t}{F_t} = \frac{40{,}500}{160{,}000} = .252 \qquad \frac{f_b}{F_b} = \frac{108{,}000}{173{,}000} = .624$$

The sum of the stress ratios is .252 + .624 = .876 which is less than 1.0. Therefore the design is satisfactory.

Combined Bending and Axial Compression.

Suppose that the axial load in the tube had been compression as shown in Fig. 4. Then the secondary moment would add to the primary moment and the final moment would have to be calculated by the precise equations of Assignment 14. This is case 3 where M_{max} is at the center and equal to

$$wj^2\left[1-\frac{1}{\cos\frac{L}{2j}}\right]$$

Assume the conditions shown in Fig. 4

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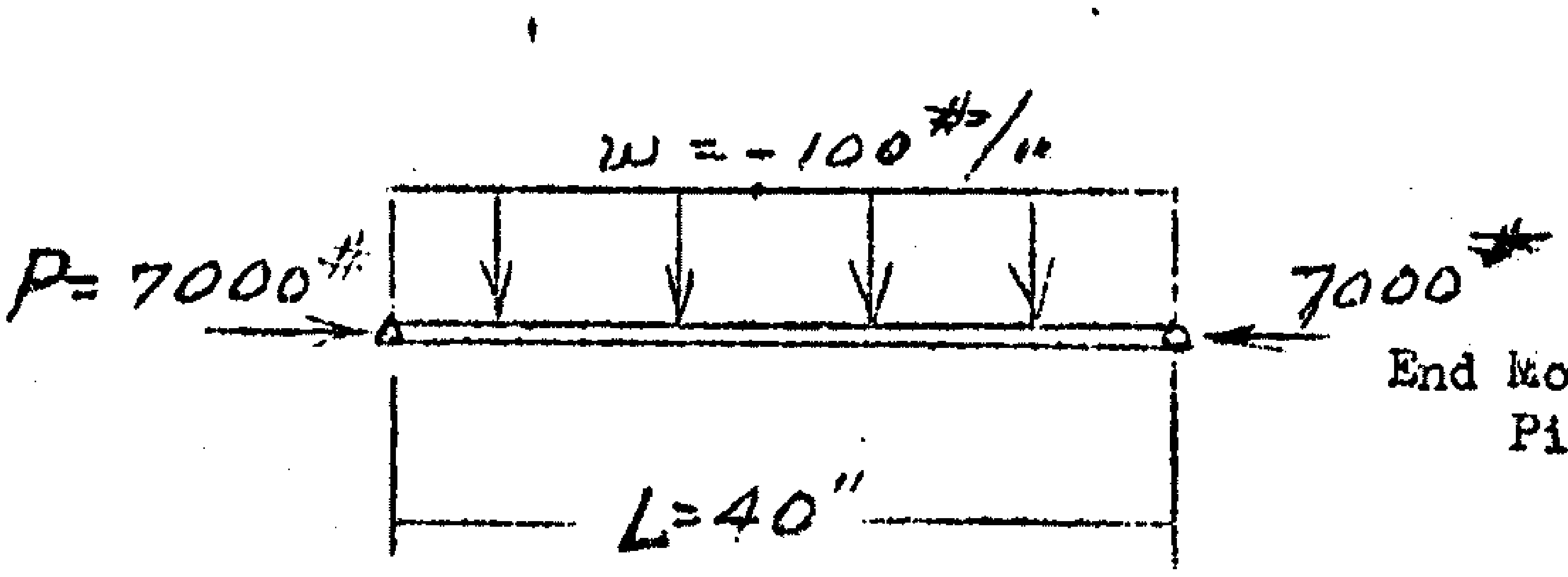


End Moments are zero.
Pinned ends.

Fig. 4

The tube is 2 inches O.D. x .065 wall and for this case we will raise the heat treatment to 180,000 psi. $E = 29,000,000$ $I = .1851$
Calculating the maximum moment at the center:

$$j=\sqrt{\frac{EI}{P}}=\sqrt{\frac{(29)(10)^6(.1851)}{7000}}=\sqrt{768}=27.7$$

$$\frac{L}{2j}=\frac{40}{(2)(27.7)}=.722 \qquad \cos\frac{L}{2j}=.750$$

$$wj^2=(-100)(768)=-76,800$$

$$M_{max}=(-76,800)\left(1-\frac{1}{.750}\right)=(-76,800)(-.333)=+25,600$$

This represents an increase of 5,600 in. lbs. over the primary moment as calculated in the preceding example.

Note: The deflection at the center under the combined loads could be calculated from the secondary moment which is equal to $(P)(\delta)$. In this case

$$(P)(\delta)=5600; \quad \delta=\frac{5600}{7000}=0.80 \text{ inches}$$

The bending stress at the center $=\pm\frac{My}{I}=\pm\frac{(25,600)(1)}{.1851}=\pm 138,200.$

From Fig. 4-20 of ANC-5 the allowable bending Modulus of Rupture is 197,000 psi.

$$\text{Then } \frac{f_b}{F_b}=\frac{138,200}{197,000}=.702$$

To find the allowable compression stress for an axial load we must check the tube as a column. From ANC-5, page 8-1, $\rho = .6845$.

Then $\frac{L}{\rho} = \frac{40}{.6845} = 58.5$ which is a short column at this heat treatment.

From Table 4-1, page 4-2 of ANC-5 the allowable column $\frac{P}{A}$ for a heat treatment of 180,000 and column yield stress, $F_{co} = 165,000$ (Table 4-5, ANC-5) is given by the formula:

$$\text{Allowable } \frac{P}{A} = 165,000 - (23.78)\left(\frac{L}{\rho}\right)^2$$

$$F_c = 165,000 - (23.78)(58.5)^2 = 83,700 \text{ psi.}$$

$$\text{Actual} \quad \frac{P}{A} = f_c = \frac{7000}{.3951} = 17,700 \text{ psi.}$$

$$\frac{f_c}{F_c} = \frac{17,700}{83,700} = .212$$

$$\frac{f_b}{F_b} + \frac{f_c}{F_c} = .702 + .212 = .914$$

Since the sum is less than 1.0 this design is strong enough.

Problem Assignment.

15:1 Assuming that you are limited in diameter to $3\frac{1}{2}$ inches, design a round tube in 17ST aluminum alloy with yield point of 40,000 psi. to sustain the loading shown in the example problem of Fig. 3. Use Fig. 5-6, page 5-17 in ANC-5 to obtain Modulus of Rupture values.

16:2 Assuming that you are limited in diameter to $3\frac{1}{2}$ inches, design a round tube in 17ST aluminum alloy with yield point of 40,000 psi. to sustain the loading shown in the example problem of Fig. 4. Use Fig. 5-6 and Table 5-1, page 5-2, from ANC-5 to obtain the allowable values F_b and F_c.

Note: Use only the tube sizes listed on Table 8-1 of ANC-5.
Use E = 10,000,000 for aluminum alloy.

AIRPLANE
STRESS
ANALYSIS

PART SIX

ASSIGNMENT 16

ASSIGNMENT 17

ASSIGNMENT 18

BASIC STRESS ANALYSIS

ASSIGNMENT SIXTEEN

Assignment 16

INDETERMINATE TRUSSES - LEAST WORK METHODS - WORK OF AXIAL LOADS

A statically determinate truss is one which contains just enough members to keep it stable. Its joints are restrained against motion in space so that it cannot collapse, and its panels consist of a series of triangles. The test for a statically determinate truss may be made thus:

Let j = the number of joints
m = the number of members
Then $m = 2j-3$

If m is more than 2j-3, the structure is redundant, and if m is less than 2j-3, it is unstable; that is, it will collapse when a load is applied on it. Note that the above criterion applies only when members can carry either tension or compression. If shear is carried through a panel by tension rods then two rods replace one diagonal shear strut. Two cross wires in a bay would therefore be counted as one member in the above equation. Two cross struts would create a redundancy since either one could transfer shear across a panel, no matter in which direction the shear was acting.

The method of least work is based upon the following assumptions:

1. Loads are applied to a structure so that the average load is equal to half the final load, when the structure comes to equilibrium.

2. The internal work in the members of a structure sum up to equal the external work done on the structure. That is, internal work equals external work. (Work = Force x distance which it moves in the direction of the Force.)

3. In a redundant structure, the total work done by an external load system will be a minimum consistent with the relative deflections of the members, as necessitated by their physical properties. (This is another way of stating that a structure will so work that the total internal work done is a minimum.)

For our first illustration we will consider a truss which has only one redundant (extra) member. All work in the truss will be done by axial tension or compression.

1. The first step in the solution is to remove the redundant member and solve the determinate framework remaining for the given system of external loads. The axial loads in the members for this condition we will call P_0.
   Then P_0 = load due to the system of external loads on the determinate truss, with the redundant member considered out of action.

2. Next consider that a unit tension load acts in the redundant member, and that all other loads on the truss are removed. This is done by removing the redundant member and substituting in its place unit loads at its ends pulling away from the joints in the direction of the redundant member. Loads due to this unit tension we will call P_a.
   Then P_a = load due to unit load in redundant member, "a".

3. If X_a is the actual (but unknown) load in redundant member, "a", and P is the total load in any other member of the truss, then

$$P = P_o + X_a P_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

P_o and P_a are determined from the principles of statics. It remains then to determine X_a by the principle of Least Work.

By Hooke's Law the change in length produced by axial load in a member is

$$\delta = \frac{PL}{AE} \quad \text{where}$$

P = end load
L = length
A = area of cross-section
E = Modulus of Elasticity

The average load producing the deflection is one-half the maximum (final) load in the member, or $P/2$.

$$\Sigma \text{ Internal Work} = \Sigma \text{ External Work} = \Sigma (\text{Force} \times \text{Distance})$$

The internal work done in a member by the axial load in it is (average load) x (distance) $= \frac{P}{2} \times \delta = \frac{P}{2} \times \frac{PL}{AE} = \frac{P^2L}{2AE}$; and for the entire structure

$$W = \Sigma \frac{P^2L}{2AE} = \Sigma \frac{(P_o + X_aP_a)^2L}{2AE} \quad \ldots\ldots \quad (2)$$

Since from the least work principle the total work done will be a minimum we may differentiate the work equation with respect to X_a and set the result equal to zero. In effect we are saying that load X_a when applied on the truss as an external load does no work. This is true since its point of application does not move when the structure is in equilibrium. (The cut faces of the redundant member do not move relative to each other.) To differentiate with respect to X_a, X_a is the only term in the work equation considered a variable and all other terms are considered constants. The partial derivative of the internal work with respect to X_a is expressed

$$\frac{\delta W}{\delta X_a} = (\quad\quad) = 0$$

If we expand equation (2) by squaring the term inside the parentheses it becomes

$$W = \Sigma \left[(P_o + X_aP_a)(P_o) + (P_o + X_aP_a)(X_aP_a) \right] \frac{L}{2AE}$$

$$= \Sigma \left[P_o^2 + X_aP_aP_o + P_oX_aP_a + X_a^2P_a^2 \right] \frac{L}{2AE}$$

$$W = \Sigma \left[\frac{P_o^2}{2} + P_oX_aP_a + \frac{X_a^2\ P_a^2}{2} \right] \frac{L}{AE}$$

Taking the partial derivative of work with respect to X_a and remembering that the derivative of a constant is equal to zero

$$\frac{\delta W}{\delta X_a} = \Sigma\left[P_oP_a + \frac{\cancel{2}X_aP_a^2}{\cancel{2}}\right]\frac{L}{AE} = \Sigma\left[P_o + X_aP_a\right]\frac{P_aL}{AE} = 0$$

For convenience let $\frac{L}{AE} = Q$

Then $\Sigma\,(P_o + X_aP_a)P_aQ = 0$

$$\Sigma P_oP_aQ + X_a\Sigma P_a^2Q = 0$$

from which $X_a = -\dfrac{\Sigma P_oP_aQ}{\Sigma P_a^2Q}$ .... (3)

Thus the load in the redundant member, "a", may be solved for directly in terms of P_o and P_a (which are determined by statics), and Q which is determined by the lengths, areas and Moduli of Elasticity of the members.

Several Redundant Members.

If we have several redundant members in a structure our solution proceeds along the same lines as for one redundant member except that the actual computations are greatly increased. Suppose that we have three redundant members, a, b, and c. The solution would be made as follows:

1. Remove all redundant members, leaving a statically determinate truss and solve for the P_o loads.

2. Consider a unit tension load P_a, P_b, P_c etc. acting in each of the redundant members in turn, solving for only one unit load at a time.

3. Then if X_a, X_b, and X_c are the actual (but unknown) loads in the redundant members the total load in any other member of the truss

$$P = P_o + X_aP_a + X_bP_b + X_cP_c \;........\quad (4)$$

For this case equation (2) would expand to

$$W = \Sigma\frac{P^2L}{2AE} = \Sigma\frac{(P_o + X_aP_a + X_bP_b + X_cP_c)^2L}{2AE}$$

When the above term is expanded it will be found to consist of terms in which each of the unknown loads appears as the square alone and to the first power in combination with each of the other unknown loads and with constants. When we take the partial derivative with respect to each of the unknown loads in turn, all other unknown values are considered as Constants. Taking them in turn $\frac{\delta W}{\delta X_a}$, $\frac{\delta W}{\delta X_b}$, and $\frac{\delta W}{\delta X_c}$ and equating them to zero we secure a series of three equations involving the three unknowns X_a, X_b, and X_c. Solution of these equations simultaneously gives the loads in the members.

A solution in tabular form will be given for the case of one redundant member. Solution for several redundant members can be made in tabular form by greatly expanding the tables.

One Redundant Member.

Solution is simplified by tabulating the work as follows: (Numbers in parentheses refer to columns in a table).

(1) Member
(2) Size
(3) Length, L
(4) Area, A
(5) $L/_{AE} = Q$
(6) P_o
(7) P_a
(8) P_oP_aQ
(9) $P_a{}^2Q$

$$\text{Then } X_a = -\frac{\Sigma(8)}{\Sigma(9)}$$

Forces are entered in a table (+) for tension and (-) for compression. The load in the redundant member is always assumed to be tension. If the final result is negative then X_a is actually compression.

Since the work done in the joints, etc. is not accounted for the effect of load in the redundant member is discounted 25%. A tabulation for the final design load in each member would then be as follows:

(1) Member
(2) P_o
(3) P_a
(4) P_aX_a
(5) $.25\ P_aX_a$
(6) $P_o + P_aX_a$
(7) (6) ± (5) depending on which sign gives the maximum value

The method will be illustrated on the redundant space frame work shown in Fig. 1. This figure represents a system of tubes supported at points 1 and 3 and loaded at 4 with an external load of 10,000 lbs. Point 1 can resist side load but is slotted in the vertical direction so that all vertical reaction must be taken at 3.

Fig. 2 shows the loads in the statically determinate structure which results when redundant member 1-4 is removed. Note that either 2-3 or 1-4 could have been considered the redundant member.

Fig. 3 shows the loads in the members when all loads on the structure are removed except those resulting from a 1 lb. tension load in the redundant member. The solution for the truss illustrated appears below.

Member	Size	Length	Area	Q	P_o	P_a	P_oP_aQ	$P_a{}^2Q$
1-2	1-.049	30	.146	2.05	+10,000	-.707	-14,500	1.025
1-3	1-.058	30	.172	1.74	0	-.707	0	.87
1-4	1-.049	42.4	.146	2.90	0	+1.0	0	2.90
2-3	1½-.083	42.4	.370	1.15	-14,140	+1.0	-16,250	1.15
2-4	1-.065	30	.191	1.57	+10,000	-.707	-11,100	.785
3-4	1-.058	30	.172	1.74	0	-.707	0	.87
Σ							-41,850	7.600

Since E is constant it is omitted in the solution for Q. Then

$$Q = \frac{L}{A} \left(x \frac{1}{100} \text{ for convenience}\right)$$

$$X_a = - \frac{\Sigma P_o P_a Q}{\Sigma P_a^2 Q} = - \frac{(-41,850)}{7.600} = +5,500 \text{ lbs. (Tension)}$$

Design Loads for Members

Member (1)	P_o (2)	P_a (3)	$P_a X_a$ (4)	$.25\ P_a X_a$ (5)	$P_o + P_a X_a$ (6)	(6) ± (5) (7)
1-2	+10,000	-.707	-3890	-970	+6110	+7080
1-3	0	-.707	-3890	-970	-3890	-4860
1-4	0	+1.0	+5500	+1375	+5500	+6875
2-3	-14,140	+1.0	+5500	+1375	-8640	-10,015
2-4	+10,000	-.707	-3890	-970	+6110	+7080
3-4	0	-.707	-3890	-970	-3890	-4860

X_a = +5,500 lbs.

Note: Column (6) gives the actual computed load in the member. Column (7) includes the 25% allowance for error in the least work solution required in obtaining values for design. Values are tabulated on Fig. 4.

Problem Assignment.

16:1 Check the solution given by using 2-3 as the redundant member.

16:2 Using member 1-4 as the redundant member change the tube sizes as follows:

Make 2-3 $1\frac{1}{2}$ - .065
Make 1-4 1 - .065

Find the margins of safety for the design loads, using C = 1 (pin end), and all tubes C.M. steel, heat treated to 95,000 psi. Use ANC-5 values.

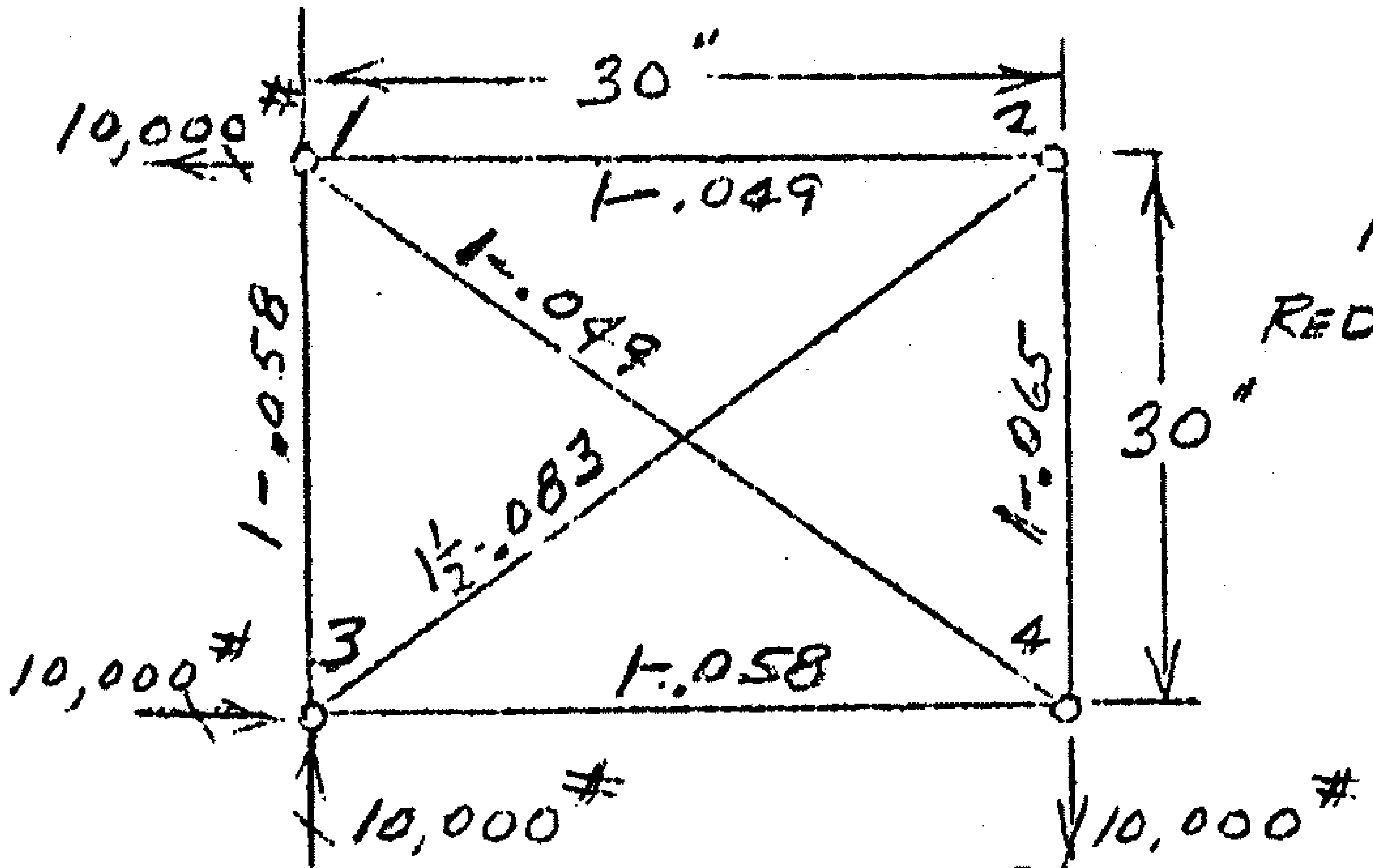


FIG. 1
REDUNDANT
STRUCTURE

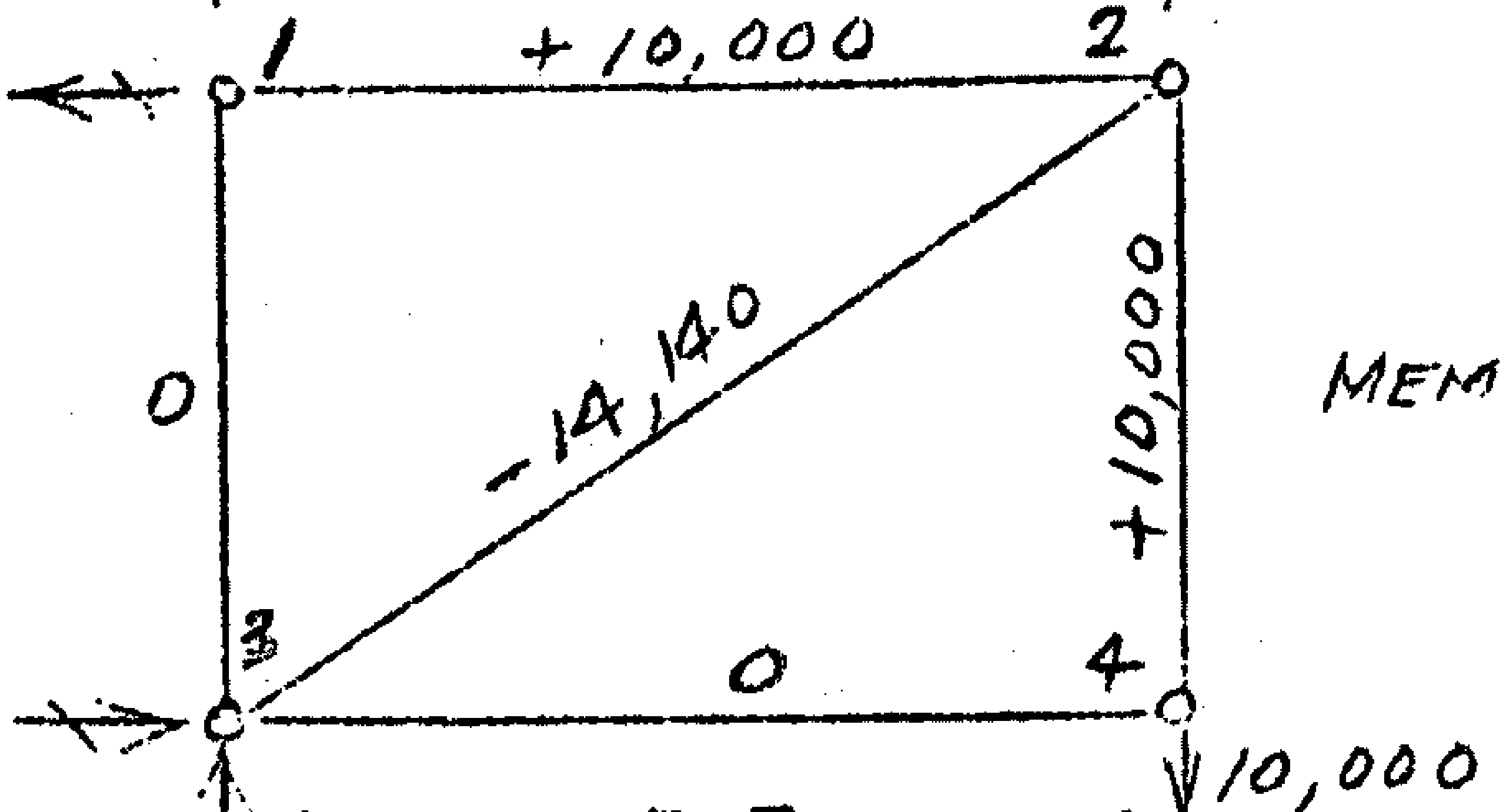


FIG. 2
MEMBER 1-4 REMOVED

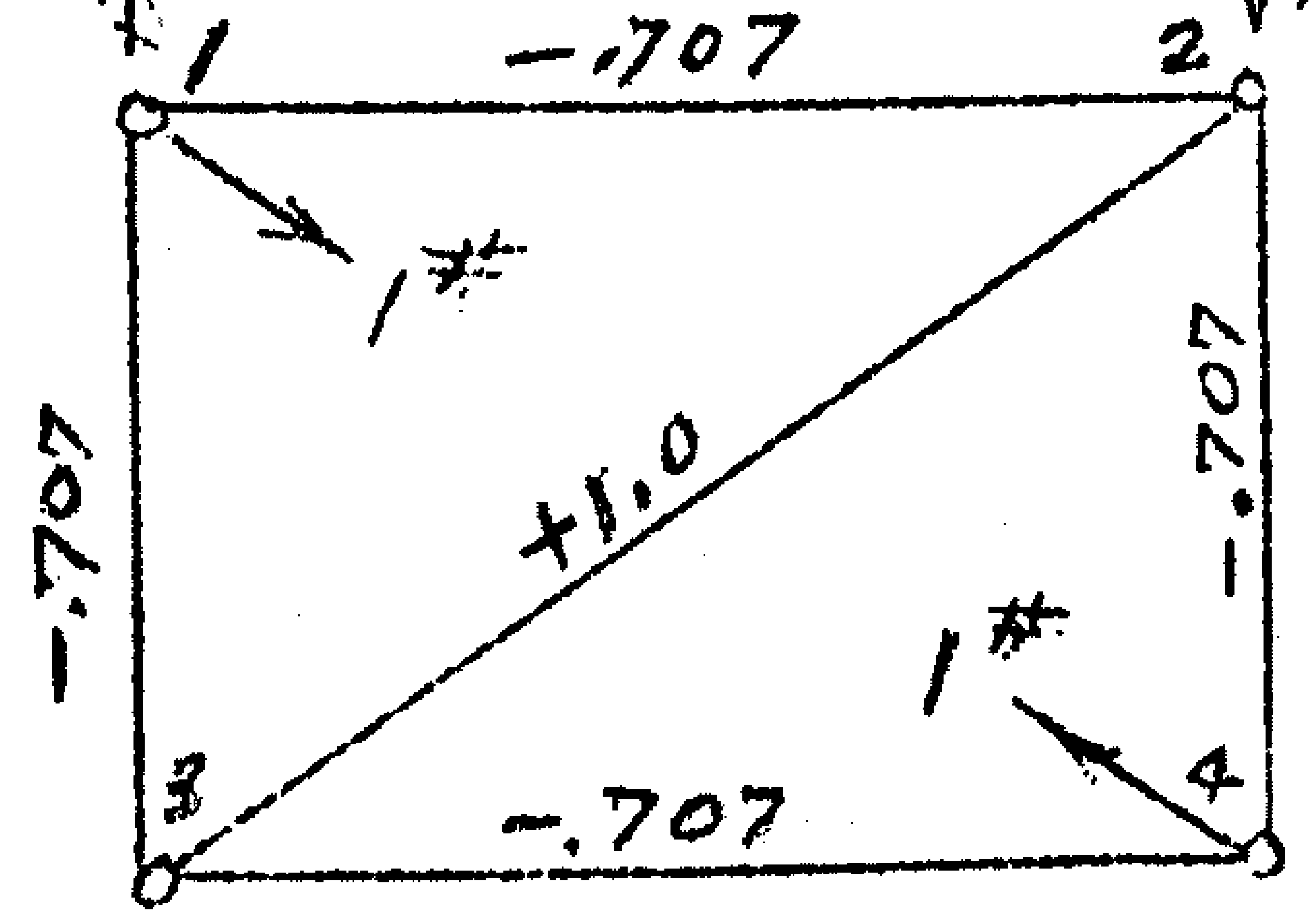


FIG. 3
UNIT LOAD IN
MEMBER 1-4

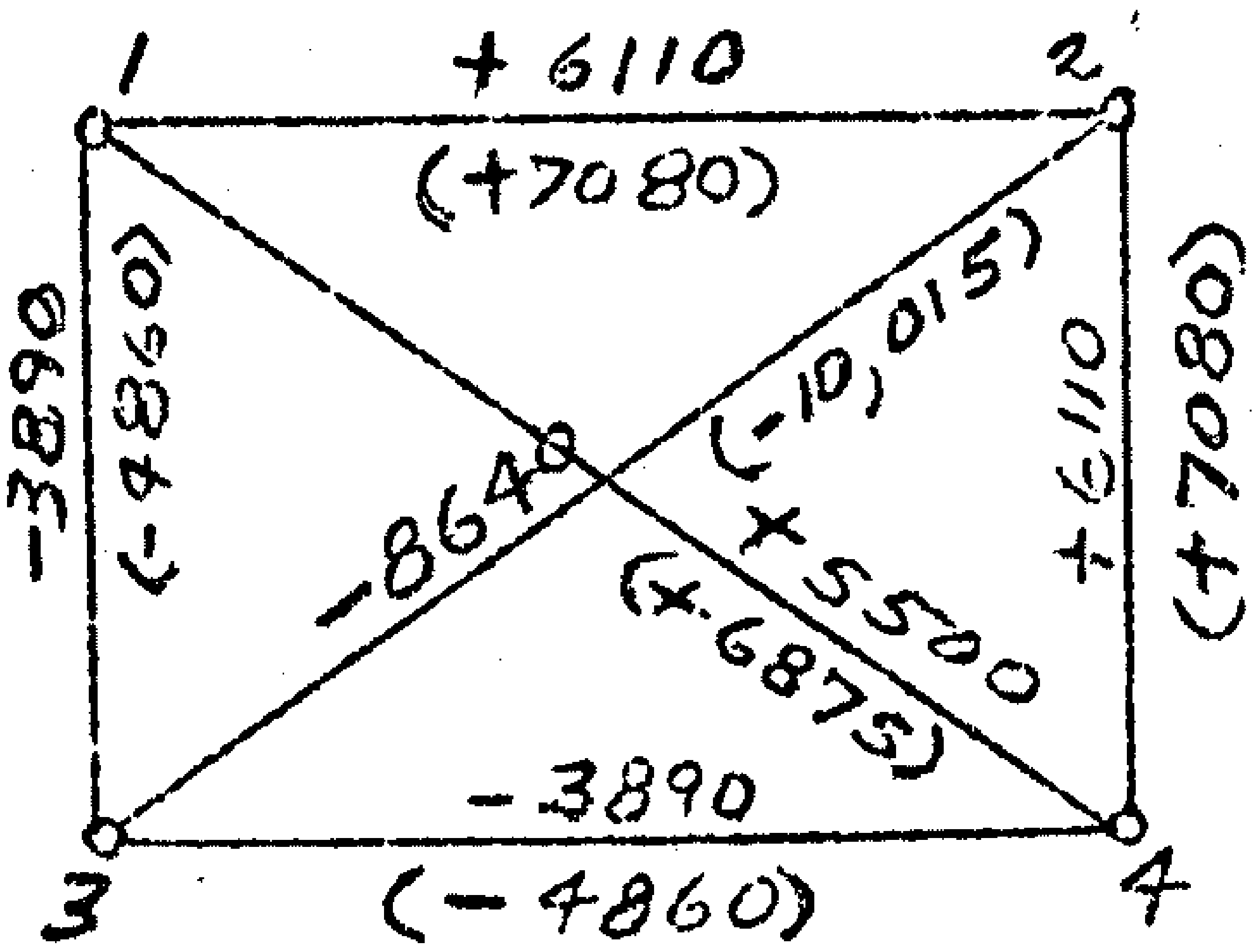


FIG. 4
CALC. LOADS AND
DESIGN LOADS
(DESIGN LOADS
ARE IN PARENTHESES)

BASIC STRESS ANALYSIS

ASSIGNMENT SEVENTEEN

Assignment 17

THE INTERNAL WORK OF BENDING

In Assignment 16 we developed an expression for the internal work in an axially loaded member, $W = \frac{P^2L}{2AE}$. This expression was founded upon the assumptions:

1. The load in the member increased linearly from zero to its final value, so that the average value was one-half the final value.

2. The cross-sectional area of the member, A, was constant throughout its length

The final expression for work can always be reduced to W = (Force)(distance through which it moves in the direction of the Force).

Now to develop a closer analogy to the expression for the work of bending, let us consider a member loaded axially in tension, for which the area, A, varies at different sections. For this member the expression for the work of axial load would not be written over the entire length but would be written over partial lengths for which the area could be considered constant, and the total internal work would be the summation of the internal work in the separate segments. This is still equal to the external work, which is equal to the average force times the deflection in the entire length.

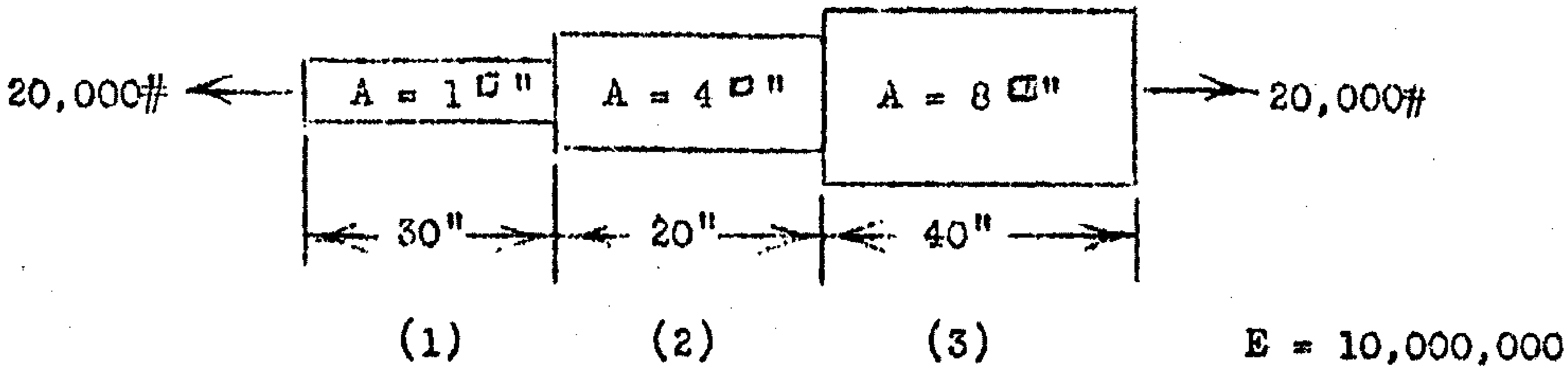


Fig. 1

Fig. 1 shows an aluminum alloy tension member for which the areas vary as shown.

In segment (1), $\delta_1 = \frac{PL}{AE} = \frac{(20,000(30)}{(1)(10,000,000)} = .06"$

In segment (2), $\delta_2 = \frac{(20,000)(20)}{(4)(10,000,000)} = .01"$

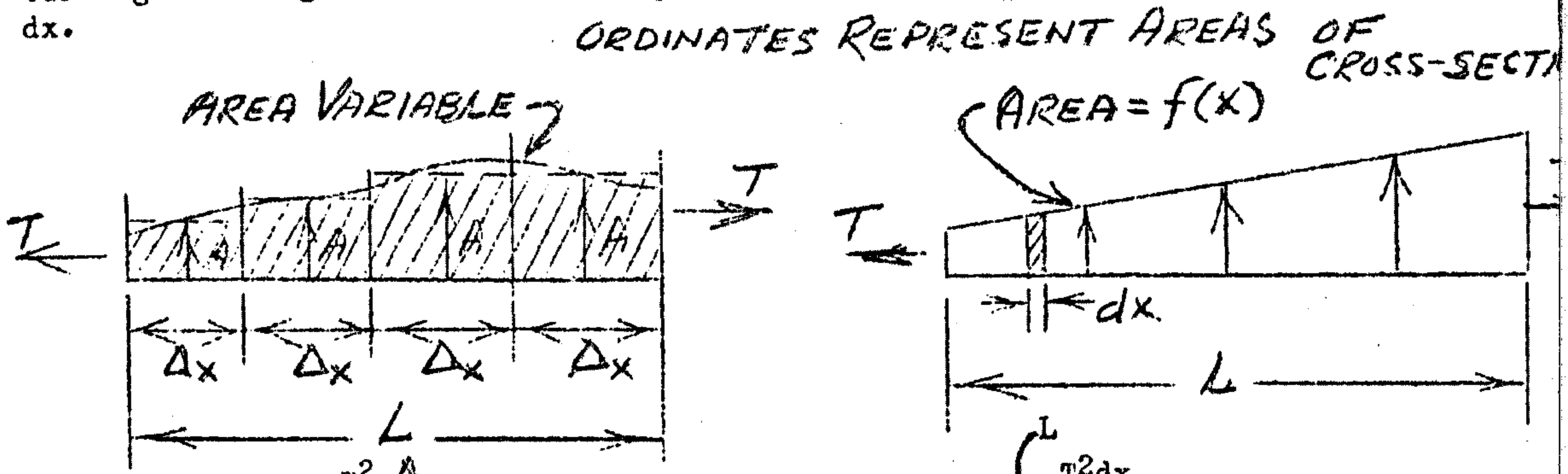
In segment (3), $\delta_3 = \frac{(20,000)(40)}{(8)(10,000,000)} = .01"$

Internal work, $W = \frac{P}{2}(\delta_1 + \delta_2 + \delta_3) = \frac{20,000}{2}(.06 + .01 + .01) = 800$ in. lbs.

****

(Note: The expression given above is the summation of the internal work for the three segments.)

Now if the area were not constant over any portion but varied from one section to another we would divide the length into small segments, Δ_x, over which the area was considered to be constant at the average value for the segment. If an expression could be written for the variation in area with respect to x we could resort to integration to solve the work equation; that is our segment length instead of being a finite value Δ_x, would be the very small, dx.



$$W = \sum \frac{T^2 \Delta_x}{2AE}$$

(a)

$$W = \int_0^L \frac{T^2 dx}{2AE}$$, where A is in terms of x.

(b)

Fig. 2

****

In evaluating the internal work of bending it is necessary to determine the work done on a cross-section of a segment of a beam of small length, and then to sum up the effects of the work done on all the segments which make up the length of the beam. Let us consider a small length of beam, dx. This length is taken so small that over it we assume:

1. A constant value of the moment.
2. A constant value of the section; that is a constant value of I.

(This is analogous on an axially loaded member to using a constant value of end load and a constant value for Area, over a given segment.)

Then considering the deflection due to bending for the small segment only: Fig. 3.

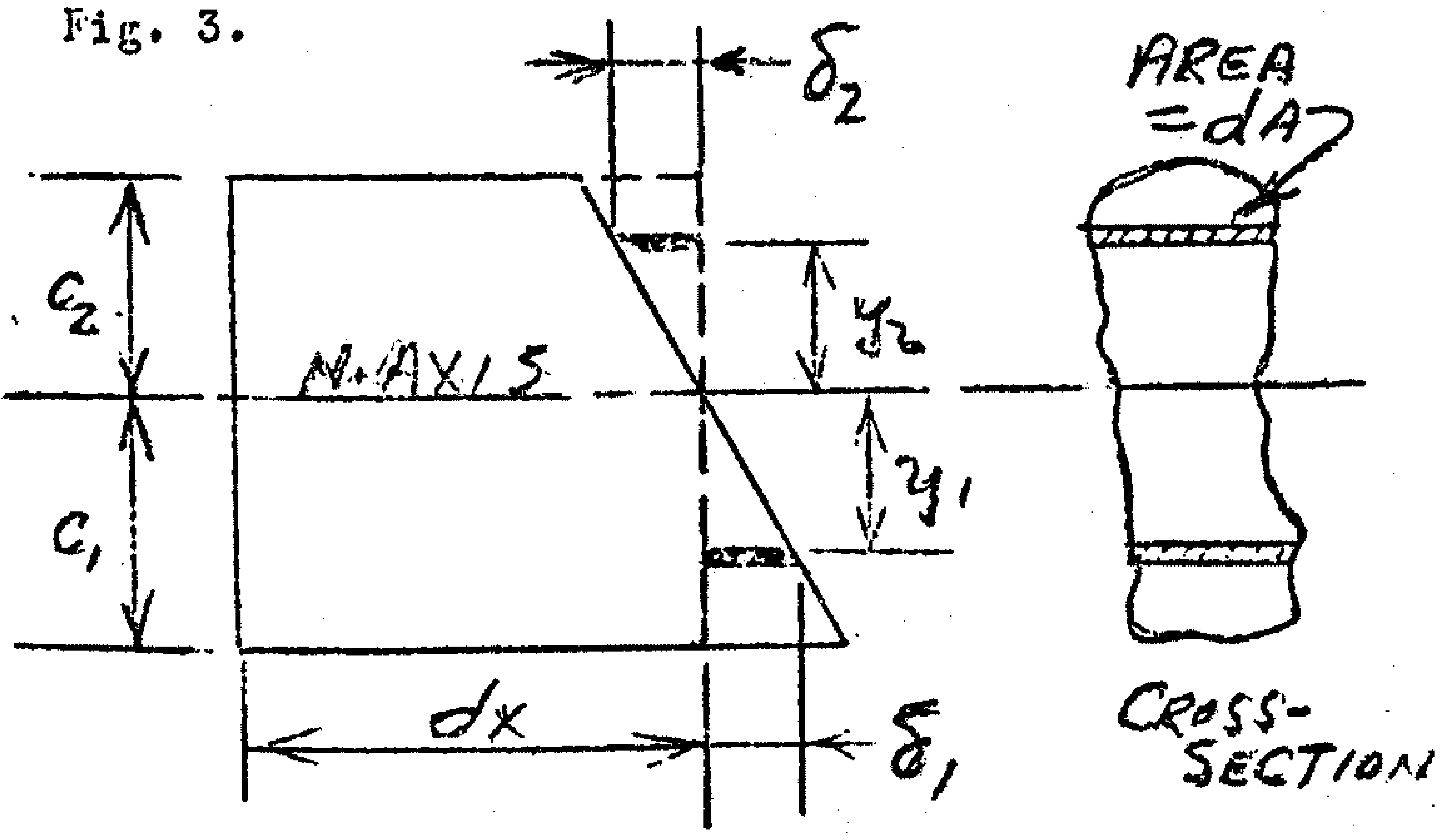


Fig. 3

Due to bending:

The unit stress on any strata of fibers on the beam is $\frac{My}{I}$, where M is the bending moment and y the distance from the neutral axis to the strata of fibers being investigated. Considering the work done by one strata of fibers only, say that at a distance y_2 from the neutral axis. The unit fiber stress in compression for the beam segment of Fig. 3 is $\frac{My_2}{I} = (\ )$ lbs/sq.in. compression. This value corresponds to $\frac{P}{A}$ in the axial load equations. Applying the equation for deflection, $\delta = \frac{PL}{AE}$, the deflection of this strata of fibers in length dx is

$$\delta_2 = (\frac{P}{A})(\frac{L}{E}) = (\frac{My_2}{I})(\frac{dx}{E});$$ the length in this case being dx. The area of the strata of fibers we will term dA. The load per sq. inch on this small portion of the area is $\frac{My_2}{I}$ from which the total load on the strata is $(\frac{My_2}{I})(dA)$. The average value, as the bending load comes on the beam, is one-half the final value. The work done in the strata of fibers is then (Average Force)(distance)

$$= (\frac{1}{2})(\frac{My_2}{I})(dA)(\frac{My_2}{I})(\frac{dx}{E}) = \frac{1}{2}\ (\frac{M^2dx}{EI^2})(y_2^2\ dA)$$

This expression represents the work in the one layer of fibers only. If we sum up the work in all the layers of fibers both above and below the neutral axis, using the calculus form and the variable distance y to any layer of fibers instead of the specific distance y_2 to a specific layer of fibers, the total work across the beam section for the segment length dx would be written

$$W = \frac{1}{2}\int_{C_1}^{C_2} \frac{M^2dx}{EI^2}\ y^2dA$$; but M, dx, E and I are constants and the only variable is y which varies between C_2 and C_1. Then the expression may be written

$$W = \frac{M^2dx}{2EI^2}\int_{C_1}^{C_2} y^2dA$$

If we refer back to our calculation of the section properties of beams we find that

$$\int_{C_1}^{C_2} y^2dA$$ is the expression for the moment of inertia of a beam cross-section about the x-x axis. Then $W = (\frac{M^2dx}{2EI^2})(I) = \frac{M^2dx}{2EI}$. This is the internal work of bending in length dx. If we sum up all the work in segments of length dx across the entire length, L, of the beam, the total internal work of bending in the beam is

$$W = \int_0^L \frac{M^2dx}{2EI}$$

The foregoing equation is valid only across sections of a beam where the moment and the moment of inertia may be expressed as functions of x. Where this is not possible the beam must be divided into segments for which the moments may be expressed in terms of x, and for which the moments of inertia may either be expressed in terms of x or taken as having a constant value.

It is perfectly valid to choose starting ordinates at either end of a beam or a segment in order to simplify the expression for the bending moment. This point will be illustrated in the examples to follow. Suppose a beam to be acted upon by a moment, M_2, at one end which is resisted by oppositely directed forces of a shear couple. Fig. 4.

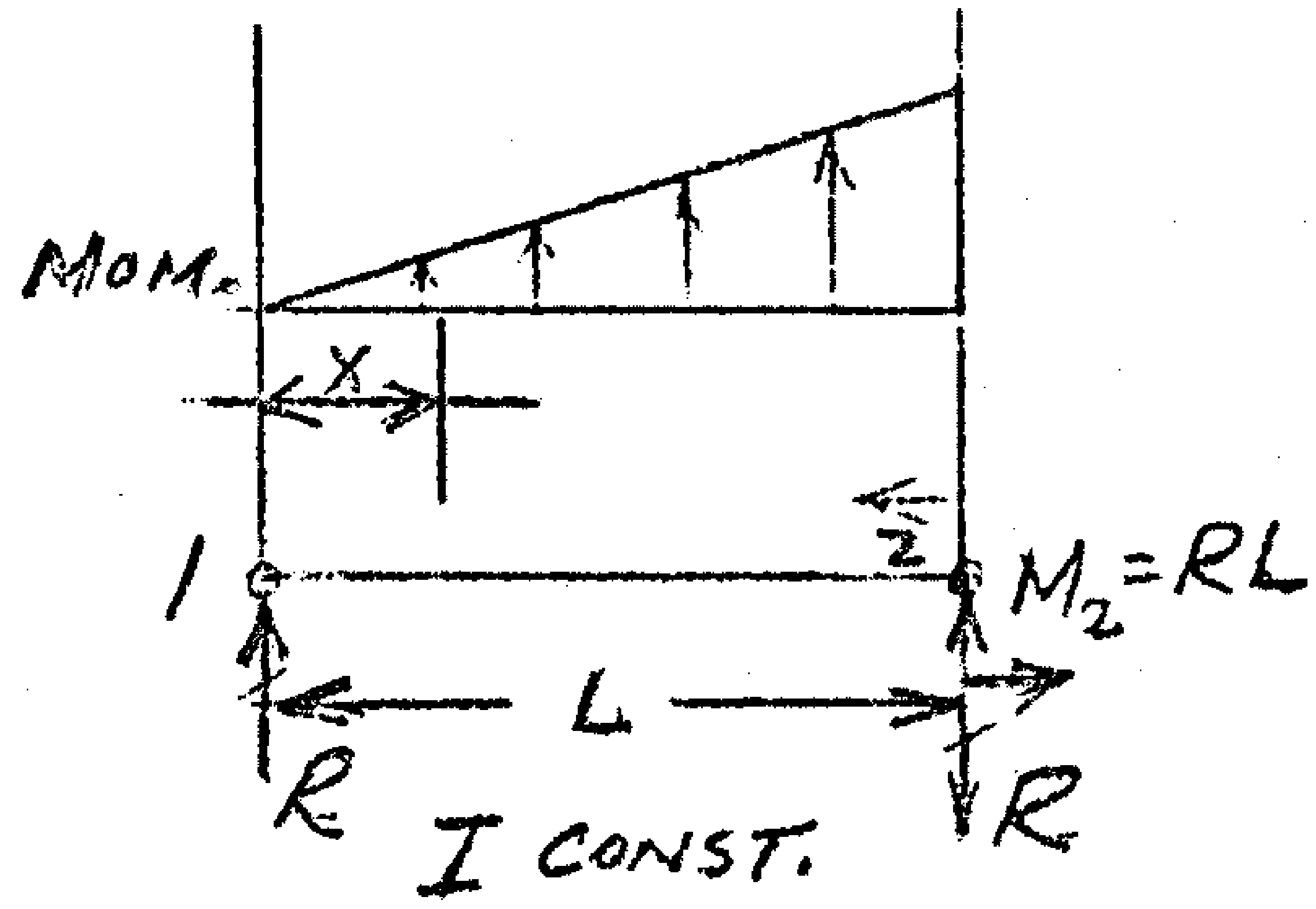


Fig. 4

Choosing joint 1 as the origin in writing the moment equation $M = Rx$

$$W = \int_0^L \frac{M^2 dx}{2EI} = \int_0^L \frac{R^2 x^2 dx}{2EI}$$

$$W = \frac{R^2}{2EI} \int_0^L x^2 dx =$$

$$\frac{R^2}{2EI} \left[\frac{x^3}{3} \right]_0^L = \left(\frac{R^2}{2EI}\right)\left(\frac{L^3}{3}\right)$$

$W = \left(\frac{R^2 L^2}{2EI}\right)\left(\frac{L}{3}\right)$; but $M_2 = RL$ from which $R^2 L^2 = M_2^2$

Then $W = \boxed{\frac{M_2^2 L}{6EI}}$; where M_2 = value of the end moment.

It would have been just as valid, but more complicated to have taken the origin for moments at 2. If this had been done the expression for the moment at any point working from 2 toward 1 would have been

$$M = M_2 - Rx = RL - Rx = R(L - x)$$

$$W = \int_0^L \frac{\left[R(L - x)\right]^2 dx}{2EI}$$ which reduces to

$$W = \frac{M_2^2 L}{6EI}$$. The advantage of the first choice of an origin is obvious.

For a pinned end beam under uniform load the internal work of bending would be expressed as follows: Fig. 5

$$\text{Mom.} = \frac{wLx}{2} - \frac{wx^2}{2} = \frac{wx}{2}(L - x)$$

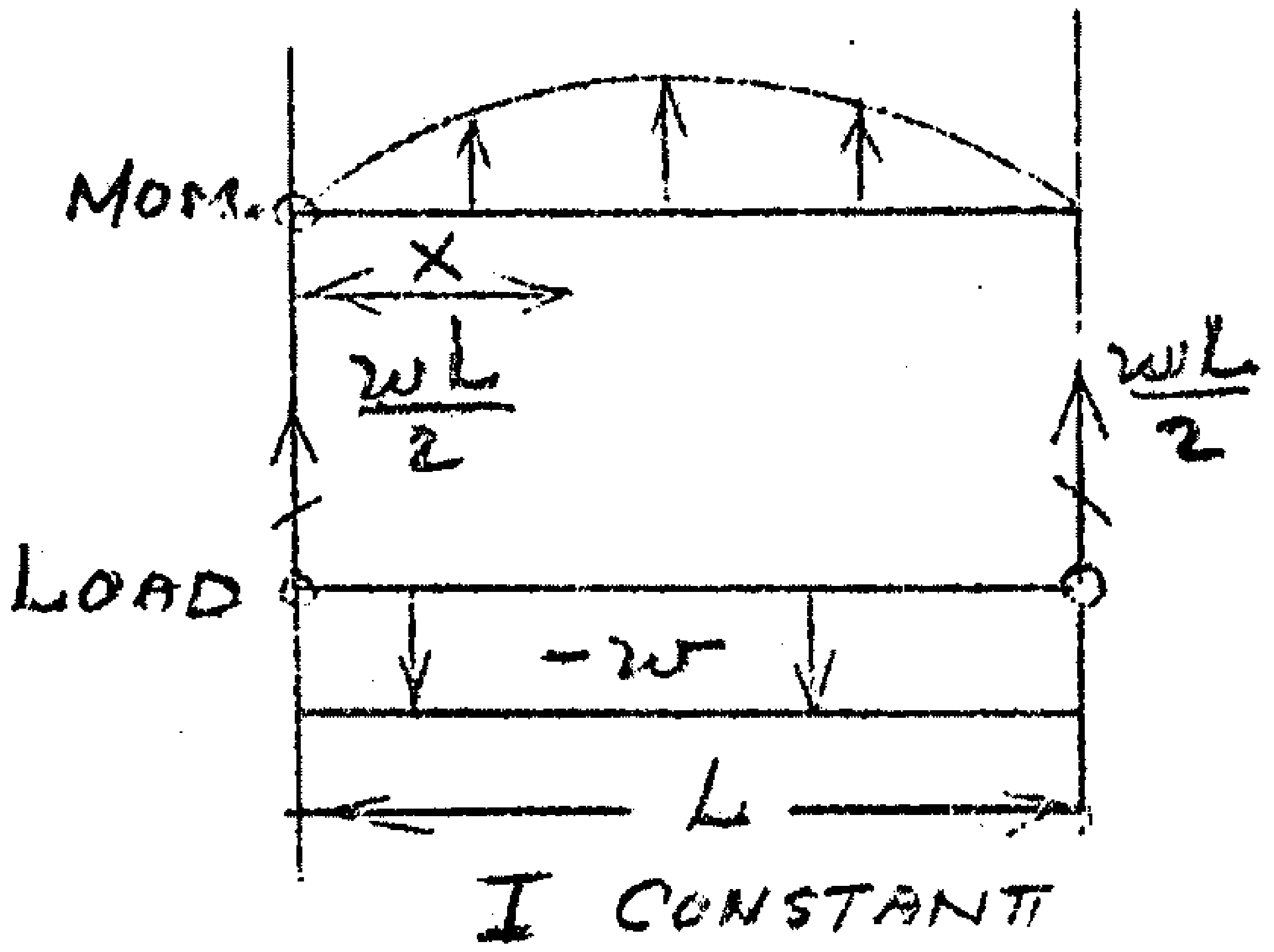


Fig. 5

$$W = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \int_0^L \frac{\left[\frac{wx}{2}(L - x)\right]^2 dx}{2EI}$$

$$= \frac{1}{2EI}\int_0^L \left[\frac{w^2x^2}{4}(L^2 - 2Lx + x^2)\right] dx$$

$$= \frac{1}{2EI}\int_0^L \left[\frac{w^2L^2x^2}{4} - \frac{w^2Lx^3}{2} + \frac{w^2x^4}{4}\right] dx$$

$$= \frac{1}{2EI}\left[\frac{w^2L^2x^3}{(3)(4)} - \frac{w^2Lx^4}{(4)(2)} + \frac{w^2x^5}{(5)(4)}\right]_0^L$$

$$= \frac{1}{2EI}\left[\frac{w^2L^5}{12} - \frac{w^2L^5}{8} + \frac{w^2L^5}{20}\right]$$

$$= \frac{1}{2EI}\left(\frac{w^2L^5}{120}\right) = \frac{w^2L^5}{240EI}$$

Note: The student who wishes to broaden his understanding may put values into the problem given on Fig. 5 and check his results by calculating the deflections of the beam he assumes. The calculation for the internal work may then be checked by summing up the external work. The external work is the summation of the products of the average loads on the beam times the distance through which they move in the direction of the load. Thus if we let $w = -10$, $L = 100$, $I = 1.0 \text{in}^4$, $E = 10{,}000{,}000$ the equation for internal Work gives

$$W = \frac{w^2L^5}{240EI} = \frac{(-10)^2(100)^5}{(240)(10)^7(1)} = 420 \text{ in. lbs.}$$

****

Dividing the beam into 5 segments of 20 inches length each and calculating the deflection at the center of each segment - Fig. 6.

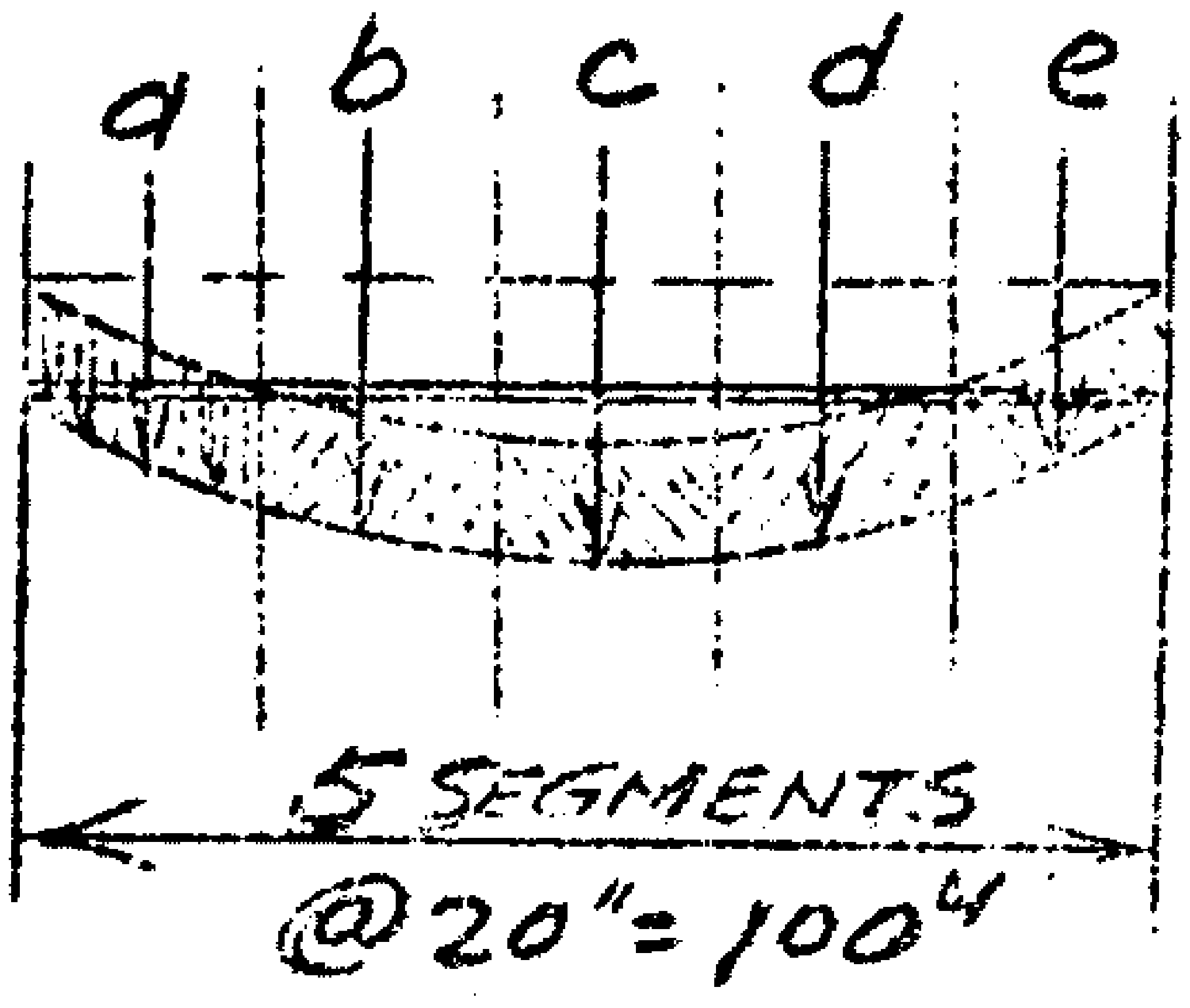


Fig. 6

The actual loads are replaced by concentrated loads of (20)(10) = 200 lbs. each.

From integrated curves or from the moment-area method the deflections are found to be as follows:

a and e = .60 inches
b and d = 1.04 inches
c = 1.20 inches

The average load causing deflection is one-half the final load, from which W (external) = (100)(1.20 + 2.08 + 1.20) = 448 in. lbs.

The value of the Method of Internal Work lies in the solution of statically indeterminate problems. For example it could be used to solve for the indeterminate reaction at the end of a restrained cantilever beam by writing the equation of work of bending on the beam in terms of the unknown reaction. The work equation would then be differentiated with respect to the unknown reaction (the partial derivative with respect to the unknown being taken considering the unknown as the only variable). The partial derivative is set equal to zero since the unknown reaction can do no work because its point of application is fixed. If we consider the beam shown in Fig. 7. The reaction R could be solved for by the unit load deflection method. It could also be solved for by the equation of 3-moments using the method of "phantom spans". The third solution by the method of internal work proceeds as follows.

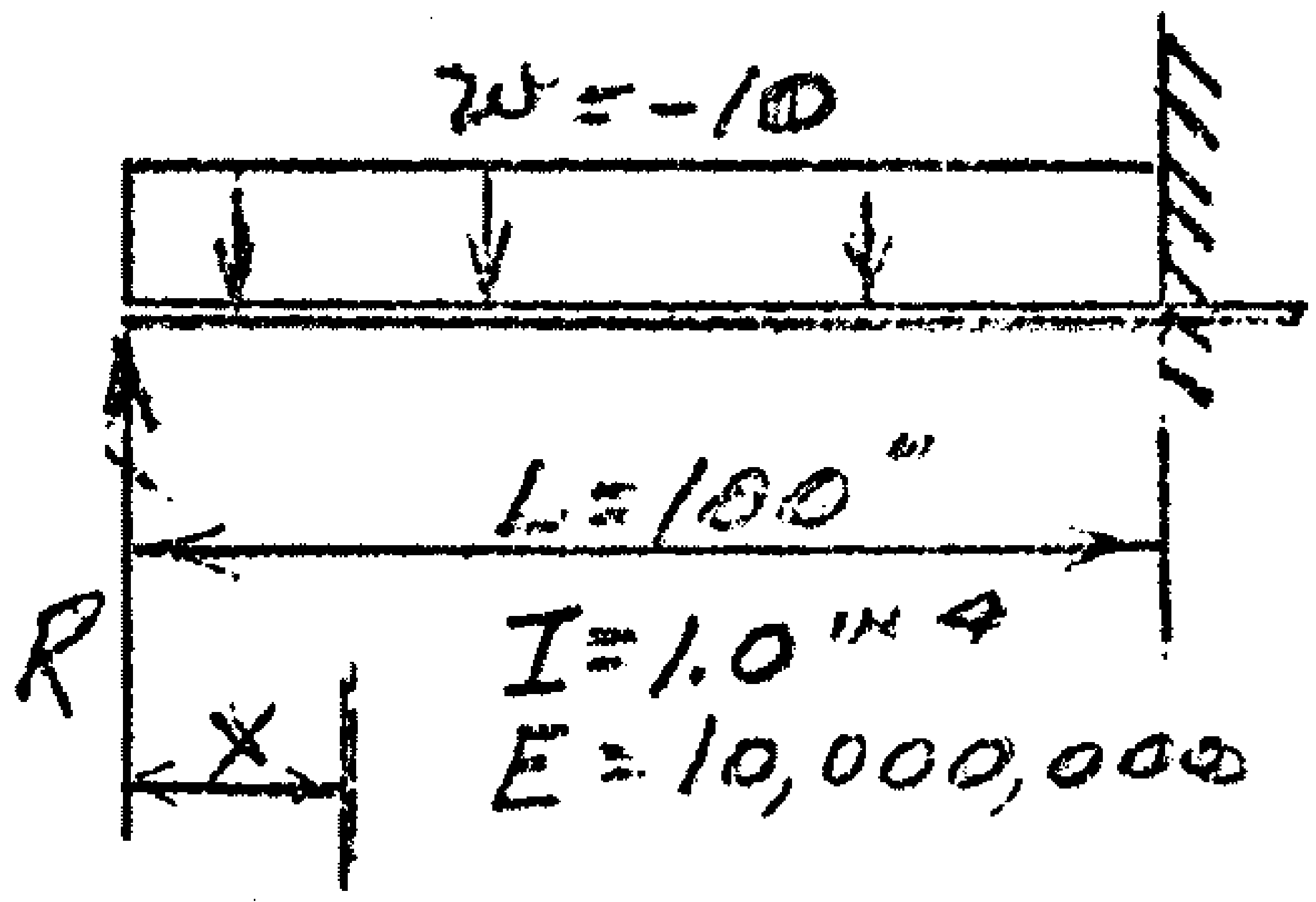


Fig. 7

The equation of moment in the span, proceeding from R to the right is $M = Rx - 5x^2$

$$\text{The work of bending, } W = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \int_0^{100} \frac{(Rx - 5x^2)^2 dx}{2EI} = \int_0^{100} \frac{(R^2x^2 - 10Rx^3 + 25x^4)\, dx}{2EI}$$

$$W = \frac{1}{2EI}\left[\frac{R^2x^3}{3} - \frac{10Rx^4}{4} + \frac{25x^5}{5} \right]_0^{100}$$

$$= \frac{1}{(2)(10,000,000)(1)} \left[\frac{R^2(100)^3}{3} - \frac{10R(100)^4}{4} + 5\,(100)^5 \right]$$

$$W = \frac{R^2}{60} - \frac{50R}{4} + 2500$$

$$\frac{\delta W}{\delta R} = \frac{2R}{60} - \frac{50}{4} = 0$$

$$\frac{2R}{60} = \frac{50}{4} ; \quad 8R = (60)(50) = 3000$$
$$R = 375 \text{ lbs.}$$

This result should be checked by one of the other methods you have been given.

****

The internal work of shear in beams is generally unimportant and is almost always neglected in problems involving internal work. The work done is again the summation of the products of the average shear loads times the shear deflections over the entire beam. The average shear loads are assumed to be one-half the maximum shear loads. The work of shear is important in problems involving twisting or torsion which will be treated in Assignment 18.

Problem Assignment.

17:1 Given the continuous beam shown in Fig. 8.

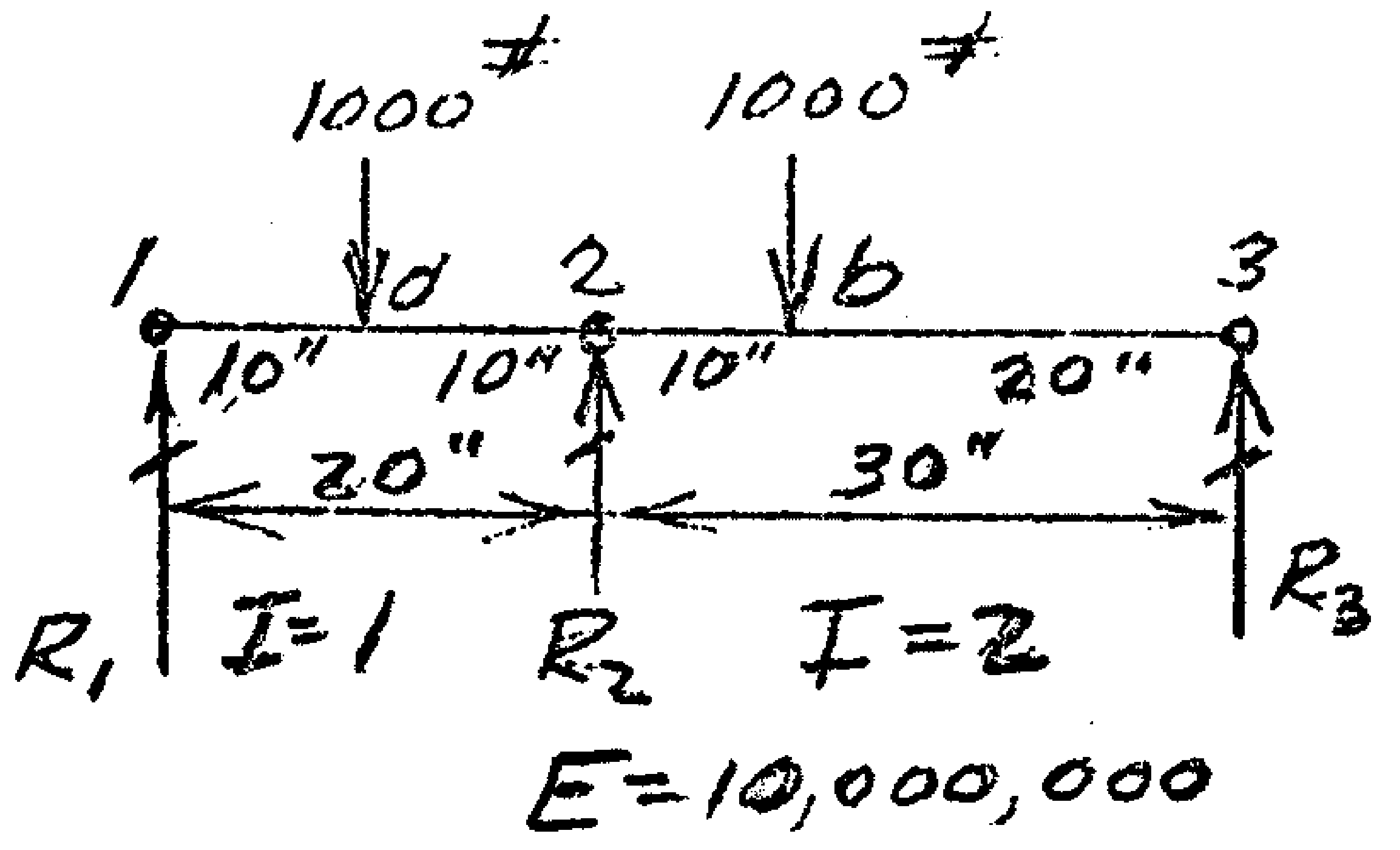


Fig. 8

Find the reactions and moments by the method of Internal Work.

Suggested Method of Attack.

(1) Assume that R_1 is the unknown reaction and solve for R_2 and R_3 in terms of R_1 and the external loads.

(2) Write the shear and moment equations for the beam in terms of the loads and the reactions from (1) and find the internal work of bending. In summing up over this continuous beam you will find it necessary to use 4 segments, 1-a, a-2, 2-b and b-3. It will be simpler to write your moment equation from R_1 toward 2 and from R_3 toward 2. It is perfectly valid to take your origin for moments at either end of a span so long as your work is consistent. It is generally possible to simplify the work by choice of the origin. When such a simplification is made it is advisable to check your moment at R_2 by using the equation from each span before proceeding with the evaluation of the internal work of bending.

17:2 Check the results obtained in 17:1 by using the equation of 3-moments.

BASIC STRESS ANALYSIS

ASSIGNMENT EIGHTEEN

Assignment 18

INTERNAL WORK OF SHEAR AND TORSION

So far our study of internal work has been concerned only with the work of axial stress and bending stress. In both types of internal work the deflection (and therefore the work done which is the summation of the products of the average forces times the distances through which they deflect their portion of the structure) has depended upon the Modulus of Elasticity. We now introduce a new concept, the Modulus of Shear, which is the determining factor in the work done by Shear or Torsion.

The Modulus of Shear is defined as the unit shear stress divided by the unit deformation in shear.

$$G = \frac{f_s}{e_s} = \frac{V}{Ae_s}. \qquad e_s = \frac{V}{AG}$$

The value G may be taken as 12,000,000 for steel and 3,700,000 for aluminum alloy. The work of shear is expressed

$$W = \int \frac{V}{2} \cdot \frac{V}{AG}\,dx = \int \frac{V^2dx}{2AG}.$$

The work of direct shear is nearly always neglected.

Torsion.

Torsion indicates twisting and pure torsion exists only when a member is subjected to equal and opposite couples in parallel planes at right angles to the axis as shown in Fig. 1. Torsion generally does not exist alone, but in combination with bending and shear. To illustrate this point let us consider a wing beam which is bent at an angle at some point along its length. Fig. 2. Suppose the beam is cantilever and subjected to distributed loads. If the outer leg of the beam is 30 inches long the total load on it is 300 lbs. which may be considered concentrated at the halfway point. Up to the joint, j, all of the load on the outer portion may be carried by straight shear and bending, but inboard of that point the beam would carry its load in shear, bending and torsion. The solution could be made in two ways. First calculate the moment at the joint, taking the 15 inch lever arm along the beam. $M_{outer} = (300)(15) = 4500$ in. lbs. This is the true moment just outboard of joint, j. Just inboard of joint j, the moment is converted to bending along the straight axis of the beam, and torsion around it. The amount of each may be determined by a vector summation thus

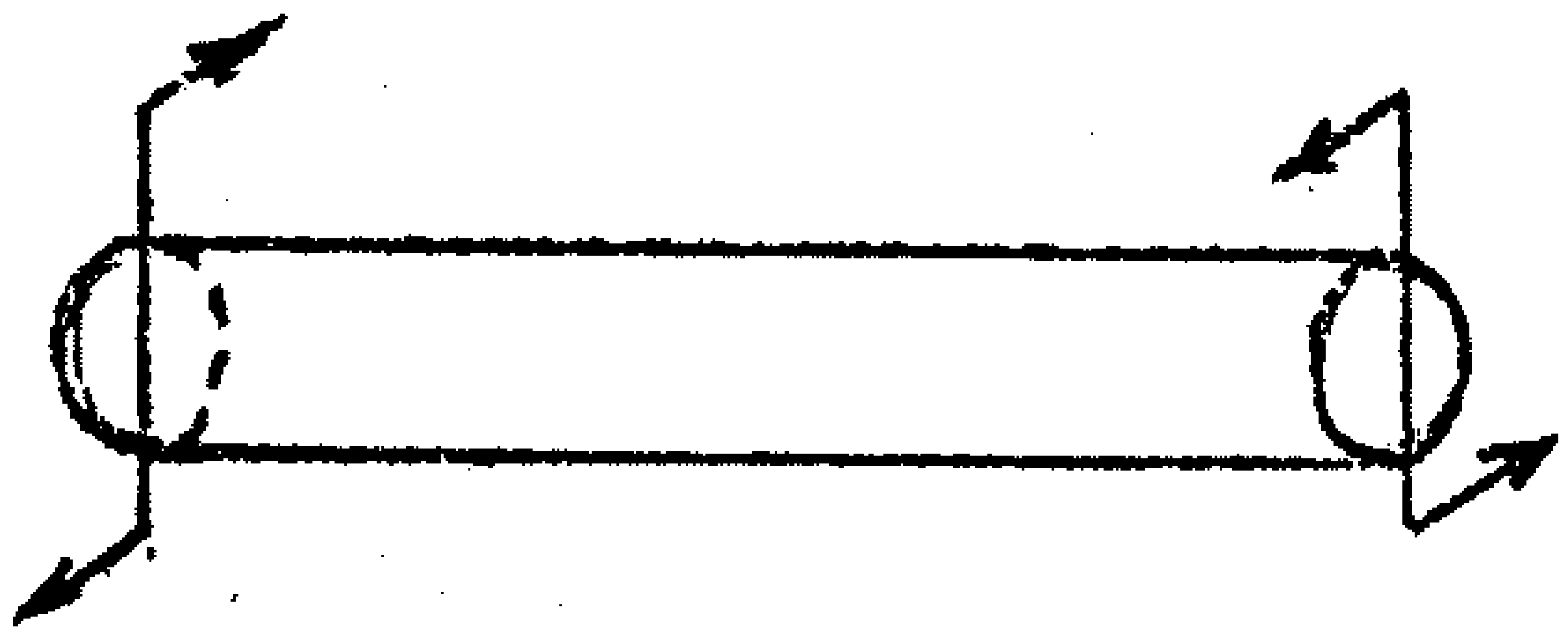
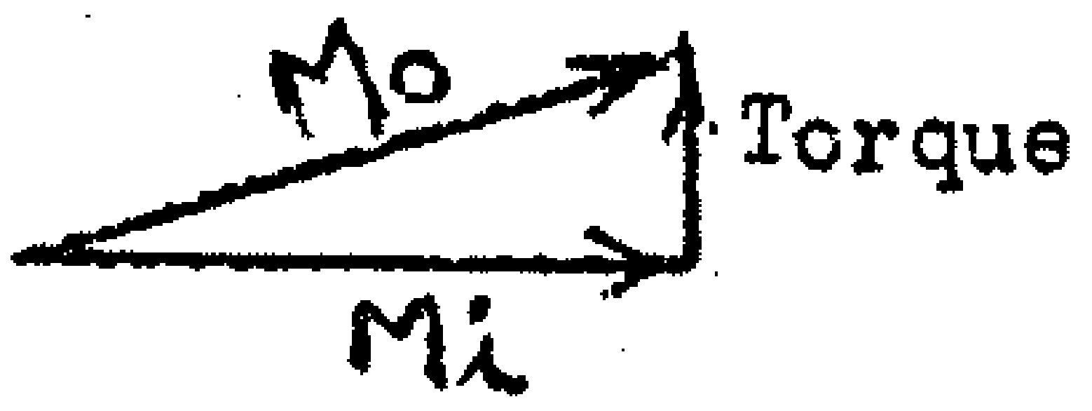
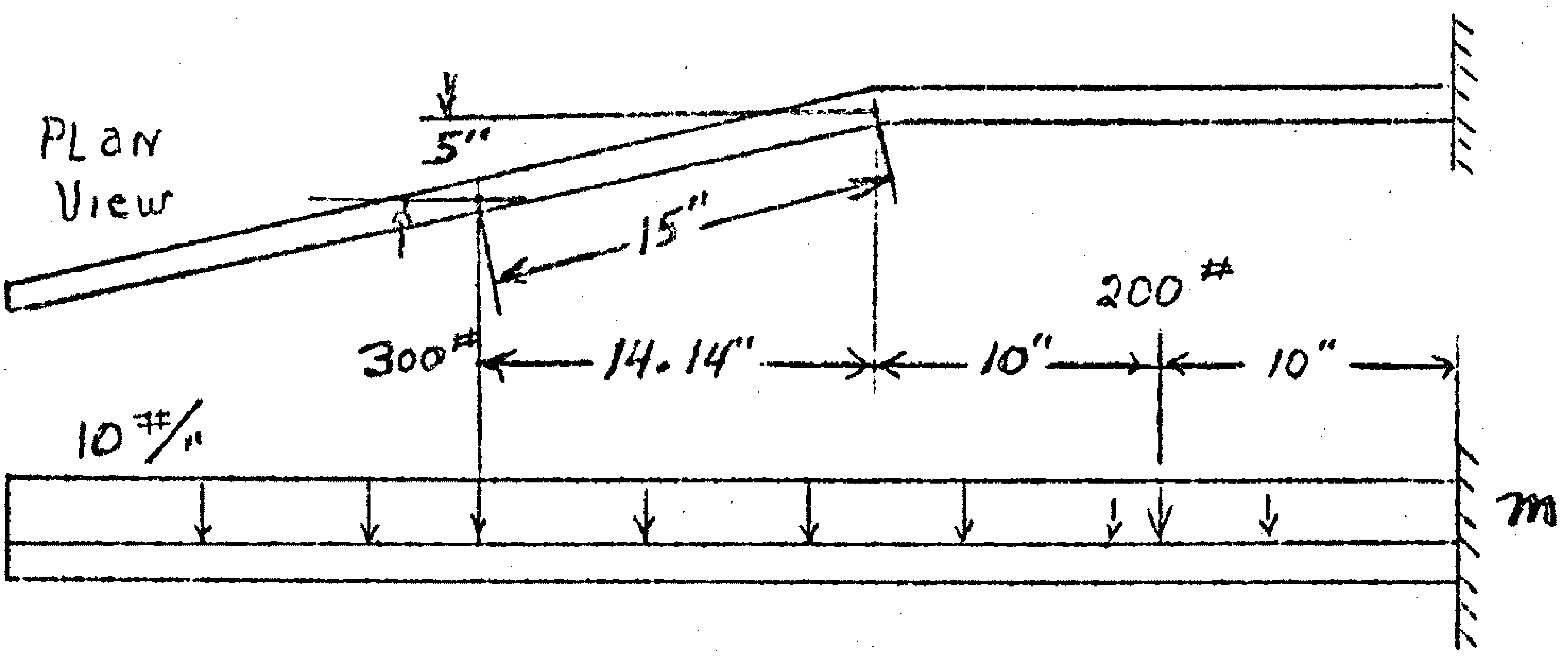


Fig. 1





Front View

Fig. 2

In this case the components are as follows:

$$M_{inner} = \frac{14.14}{15}(4500) = 4240 \text{ in. lbs.}$$

$$\text{Torque} = \frac{5}{15}(4500) = 1500 \text{ in. lbs.}$$

Shear at j = 300 lbs.

The vector resolution of outer Moment into moment and torque on the inner beam could have been obtained by taking our lever arms parallel and perpendicular to the axis, in plan view, of the inner beam.

Moment = $(300)(14.14) = 4240$ in. lbs.
Torsion (twist) = $(300)(5) = 1500$ in. lbs.

The torsion is constant from j to m while the shear and moment increase.

At m, Shear = $300 + 200 = 500$ lbs.
Moment = $(300)(14.14 + 20) + (200)(10) = 10{,}240 + 2{,}000 = 12{,}240$ in. lbs.
Torsion = $(300)(5) + (200)(0) = 1500$ in. lbs.

Torsion is transferred by shearing stresses in a member, that is it tends to shear one plane of the member past another, by twisting. The amount of the shear stress produced is assumed to be proportional to the distance from the torsional centroid and the shear deflection is assumed to vary in the same way. This is analagous to the distribution of bending stress on a cross-section of a beam except that the deformation of material is at right angles to the axis of the member, whereas in bending the internal deformation is parallel to the axis of the member. Fig. 3 shows a plot of unit shear stress against distance to the center for a cross-section of a rod subjected to torsion. Since the relative displacement of a ring of material at radius "r" is proportional to its distance from the center and the unit stress is also proportional to the distance from the center the ratio of unit stress to displacement is a constant for all rings in a given cross-section. This is analagous to the beam theory.

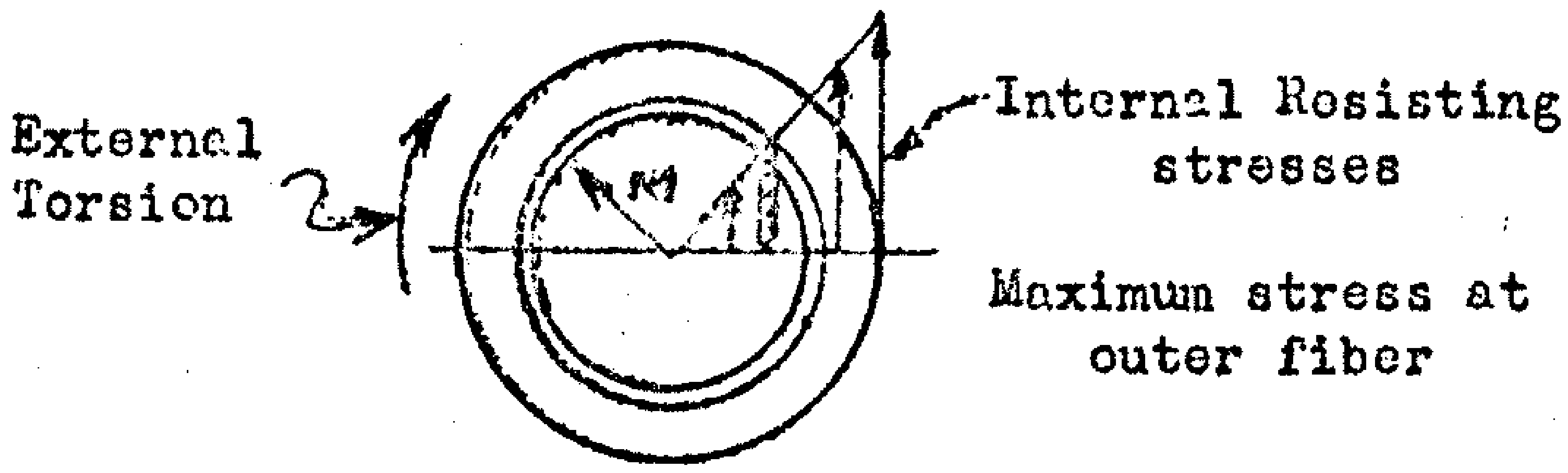


Fig. 3

The Polar Moment of Inertia.

For a beam the index of the resistance to bending of a cross-section is the Moment of Inertia. For a member subjected to torsion, the index of the resistance to twisting of the cross-section is the Polar Moment of Inertia.

The Polar Moment of Inertia referred to any set of rectangular axes is the summation of the products of the small elementary areas, dA, and the squares of their distances to the origin. It may be expressed,

$$I_{polar} = \int r^2 dA$$

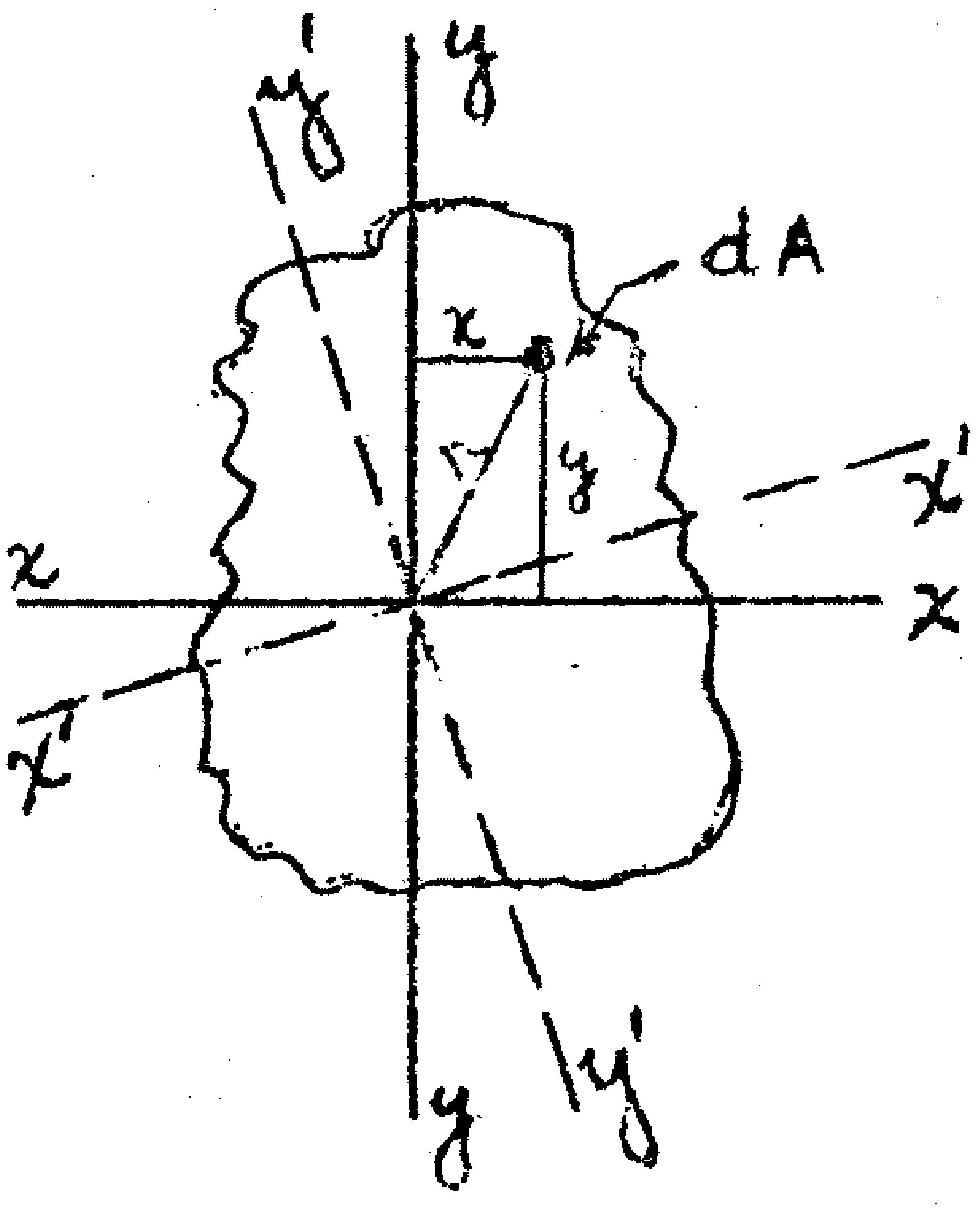


Fig. 4

But $r^2 = x^2 + y^2$ from which

$$\int r^2 dA = \int (x^2 + y^2) dA$$

$$= \int x^2 dA + \int y^2 dA.$$

From the Beam Theory:

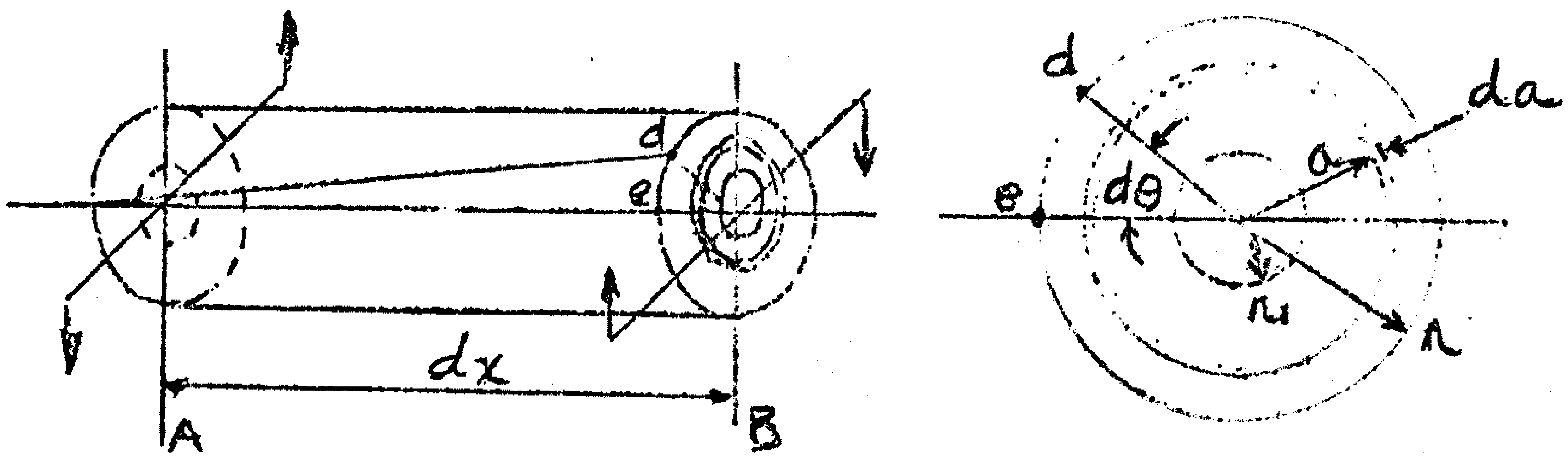
$\int x^2 dA$ = the Moment of Inertia about the y-y axis

$\int y^2 dA$ = the Moment of Inertia about the x-x axis,

from which it follows that the Polar Moment of Inertia is the sum of the moments of Inertia about the x-x and y-y axes. Since the Polar Moment of Inertia does not depend upon the direction of the coordinate axes it follows that it is the sum of the Bending Moments of Inertia about any two rectangular axes passing through the centroid of the section. Since the Polar Moment of Inertia is a constant it also follows that the sum of the moments of Inertia about any two sets of coordinate axes through the centroid of a section is a constant. Then

$$I_x + I_y = I_{x'} + I_{y'} \text{ in Fig. 4.}$$

For a section symmetrical about both coordinate axes, such as a circular bar or tube the Polar Moment of Inertia equals twice the bending Moment of Inertia. Thus for a tube $I_p = 2I_{bending}$.



(a) (b)

Fig. 5

Fig. 5 (a) shows a short length, dx, of a tube subjected to torsion. The twist between the faces at A and B is assumed sufficient to displace B with respect to A until a point originally at e is moved around to d. Fig. 5 (b) is an enlarged view of the end of the tube at B, showing the relative displacements. $d\Theta$ is shown exaggerated. Actually it will be very small. $d\Theta$ is an angle in radians.

From page 1, $G = \frac{f_s}{e_s} = \frac{\text{Unit shear stress}}{\text{Unit shear displacement}}$

For the tube of Fig. 5 the shear displacement at the circumference in length $dx = \overline{de} = r d\Theta$. Then the unit shear displacement $e_s = \frac{r d\Theta}{dx}$ and

$$G = \frac{f_{s_{max}}}{e_s} = \frac{f_{s_{max}}}{\frac{r d\Theta}{dx}} \qquad \text{or} \qquad f_{s_{max}} = G\,r\,\frac{d\Theta}{dx}$$

In the preceding equation $f_{s_{max}}$ is the stress at the outer circumference, r. However we have seen that the unit shearing stress on this cross-section is proportional to its distance from the centroid from which, if we take f_s as the unit stress at any variable radius, a, from the center we may write $f_s = G\,a\,\frac{d\Theta}{dx}$. (We have merely stated that the unit stress at "a" is to the unit stress at "r" as "a" is to "r".)

Now let us consider the resisting torque on an elementary ring at a distance "a" from the center. The width of this ring is the small increment of radius, da, and its perimeter is $2\pi a$ from which the area $dA = (2\pi a)(da)$.

The unit stress acting $f_s = G\ a\ \frac{d\Theta}{dx}$

Its lever arm is "a". Therefore the internal resisting torque is $(2\pi a)(da)(f_s)(a)$ on the elemental ring and for the entire cross-section the internal torque

$$Tq_i = \int_{r_1}^{r} (f_s)(2\pi a)(da)(a) = \text{the summation of the effects of all rings between the outer and inner walls of the tube.}$$

$$Tq = \int_{r_1}^{r} \left(G\ a\ \frac{d\Theta}{dx}\right)(2\pi a^2 da) = G\ \frac{d\Theta}{dx} \int_{r_1}^{r} 2\pi a\ da\ a^2$$

But the Polar Moment of Inertia

$$I_p = \int_{r_1}^{r} 2\pi a\ da\ a^2 = \left[\text{the summation of products of elemental areas times the squares of their distances to the centroid.}\right]$$

Then $Tq = \left(G\ \frac{d\Theta}{dx}\right)(I_p)$

If $d\Theta$ is the twist of the tube in length dx the total twist in length L will be Θ, from which $\frac{d\Theta}{dx} = \frac{\Theta}{L}$

Then we may write

$$Tq = (G)\left(\frac{\Theta}{L}\right)(I_p) \quad \text{or} \quad \Theta = \frac{(Tq)(L)}{(G)(I_p)}.$$ Θ is an angle in radians.

The internal work of torsion is equal to the external work = (Average Torque) (Angle of twist).

$$\text{Work} = \left(\frac{Tq}{2}\right)(\Theta) = \left(\frac{Tq}{2}\right)\frac{(Tq)(L)}{(G)(I_p)} = \frac{(Tq)^2(L)}{(2G)(I_p)}$$

For a tube or bar $I_p = 2I$ Therefore $\text{Work} = \frac{(Tq)^2(L)}{4GI}$

*****

Having developed this tool we will apply it in the solution of a chassis which is indeterminate with regard to the distribution of torque from the brakes. The method we will use is as follows:

1. The components of all members are computed in the three reference planes for use in the equations of static equilibrium.

2. The ways in which the members can carry brake torque load are considered (bending and torsion) and the equations of equilibrium are written for the reference planes, expressing all values eventually in terms of one which is unknown, such as the bending in the axle.

3. The equations of work in the members are then written and summed up for the total work. These equations will contain only one unknown in our example problem but are capable of solution with any number of unknowns.

4. The internal work in the members equals the external work and holds the force causing the external work in equilibrium. That is, the structure is held static and does not rotate under the action of the external torque. Castigliano's Theorem of Least Work states

   "The first partial derivative of the total internal work with respect to any external force equals the displacement of the point of application of the force in the direction of the force."

   If the point of application of the force does not move then the first partial derivative of the total internal work is equal to zero. This is expressed

   $$\frac{\delta W}{\delta X} = 0$$ where X is the only term in the work equation considered a variable and all other terms are considered constants.

5. Partial derivatives are taken with respect to as many unknowns as exist in the work equation and the resulting equations are solved by placing each derivative in turn equal to zero and solving for the unknown values. The method is illustrated in the following example.

Note: The example given herewith does not intend to illustrate a recommended type of landing gear. It merely serves as a medium to illustrate the method by which Internal Work may be used to solve this indeterminate problem. The points of attachment of the landing gear on the fuselage are considered fixed.

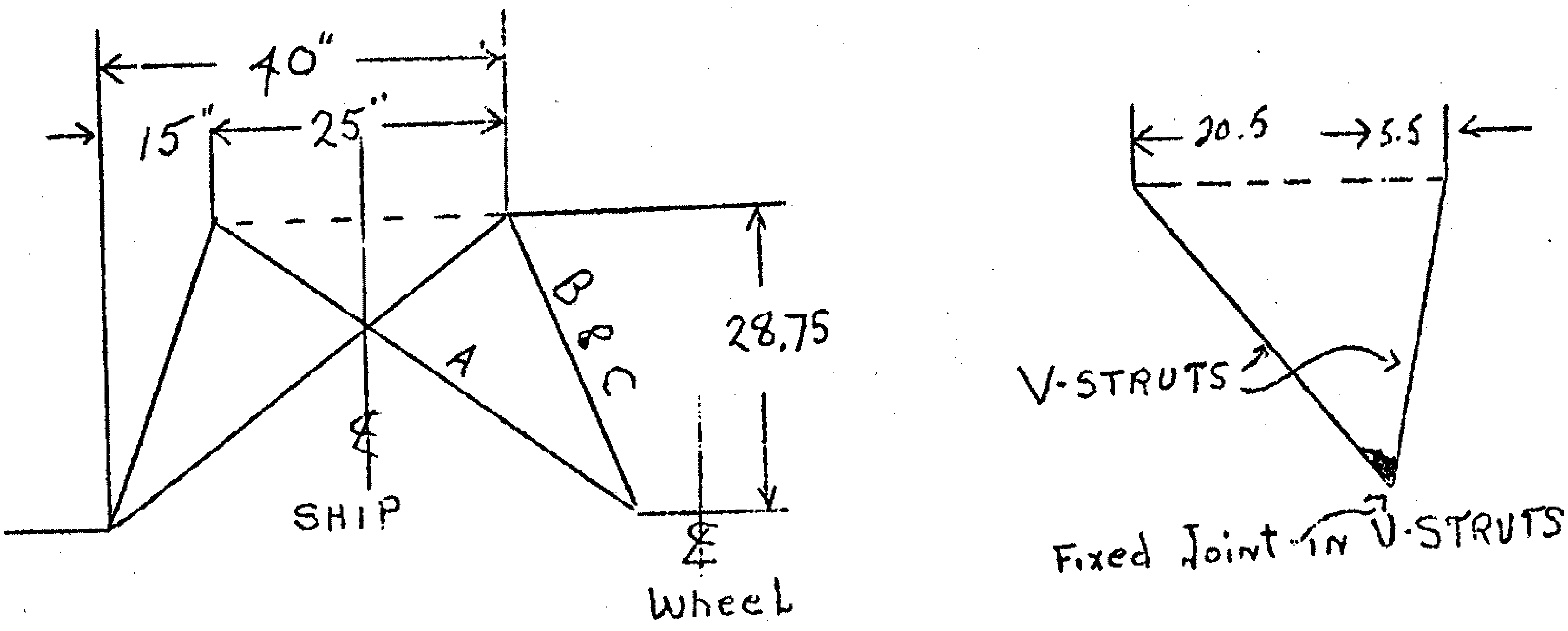
The vector diagram for torque and bending shown on Fig. 7 is not plotted according to the conventional rule. It is plotted as follows:

Bending in a member is shown paralleling the axis of that member.
Torsion (torque) in a member is shown perpendicular to the axis of that member.

(The conventional, "Right Hand", rule is stated as follows: (1) For torque on a member if the right hand is thought of as being placed around the member with the fingers extending in the direction in which the torque acts, the thumb will extend in the direction of a vector, parallel to the axis of the member, representing the torque.)

Distribution of Torque to an Indeterminate Chassis.

Example:



Front View

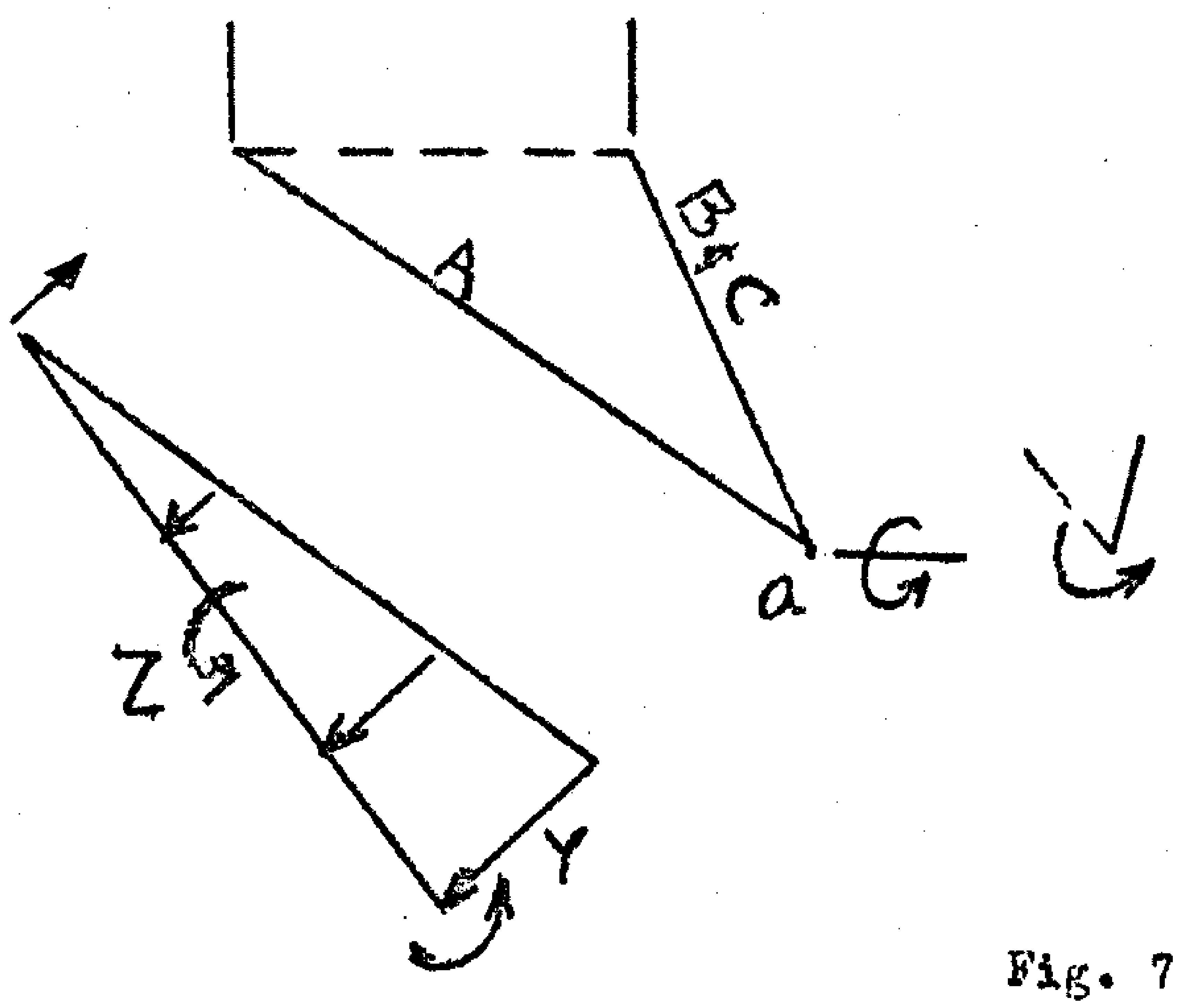
Side View

Fig. 6

Member	Length	Length^2	Component	
A Axle	28.75 5.5 40.0 49.57	826.6 30.2 1,600.0 2,456.8	= .5800 = .1110 = .8069	V H S
B Strut	28.75 20.5 15.0 38.36	826.6 420.2 225.0 1,471.8	= .7495 = .5344 = .3910	V H S
C Strut	28.75 5.5 15.0 32.89	826.6 30.2 225.0 1,081.8	= .8741 = .1672 = .456	V H S

Components of "B" and "C" in Front View

B And C	28.75 15.0 32.43	826.6 225.0 1,051.6	$\frac{28.75}{32.43} = .8865$ $\frac{15}{32.43} = .4625$	V S



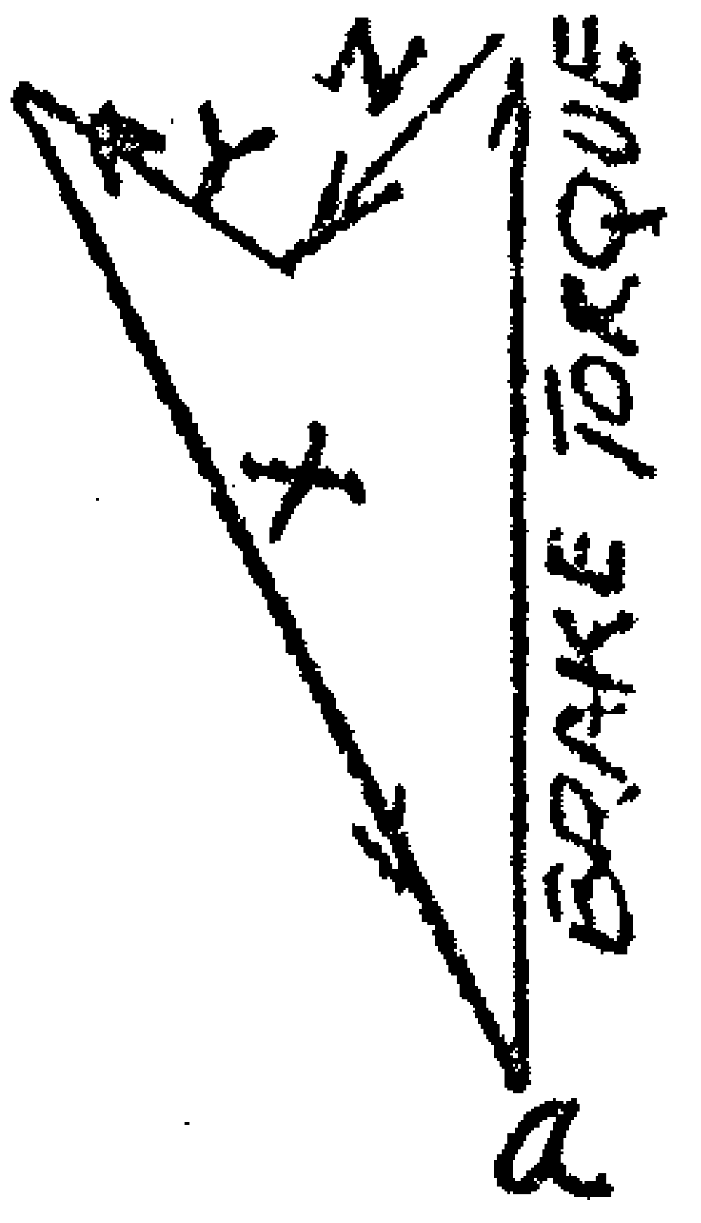


Diagram with Torque and bending shown in the plane in which they act.

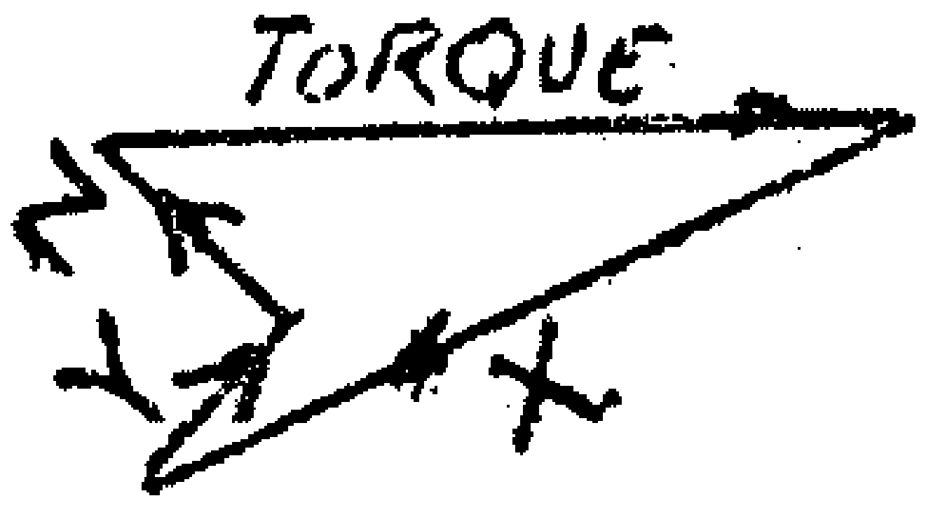


Diagram by Conventional R.H. Rule

Fig. 7

The torque due to braked landing is assumed to be taken in three ways:

(1) By bending in the axle, Y.
(2) By torsion in the axle, Z.
(3) By bending in the V struts, X.

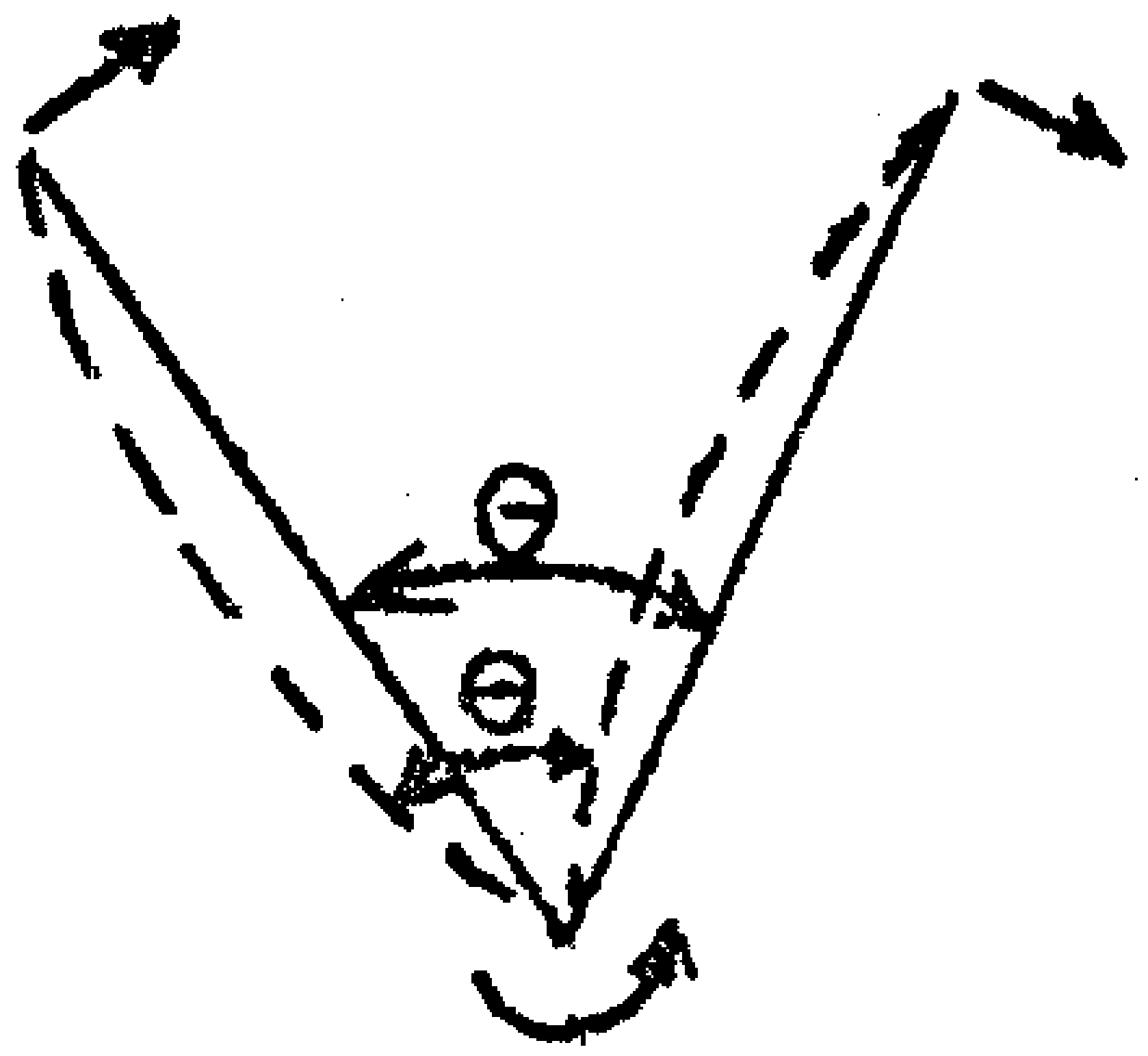


Fig. 8
V-Struts Bending

The torsion in the V-Struts is considered negligible. The distribution of torque load is calculated by Least Work. The work of bending and direct stress in the fuselage members, due to torsion in the axle is not considered, this quantity being too small to materially affect the result.

In the solution, the two V-Struts are replaced by an "equivalent strut" which is vertical in the side view and in the plane of the V-Struts in the front view. Later the bending is apportioned to the struts according to their rigidity ratios. (I/L ratios)

Y and Z are expressed in terms of X, the bending in the V-Struts, by applying the equations of static equilibrium at joint a.

(1) $\sum M$ in the vertical plane = 0
(2) $\sum M$ in the side plane = 0

Referring to Vector Diagram Fig. 7,

$\sum V = 0$

(X)(Vert. Comp. of V Struts in front view) - (Y)(Vert. Comp. of Axle) + (Z)(Side Comp. of Axle) = applied torque

$\sum S = 0$

(X)(Side Comp. of V-Struts in front view) - (Y)(Side Comp. of Axle) - (Z)(Vert. Comp. of Axle) = 0

Solution of Example - Unit load at wheel circumference = 15 in. lbs. torque load.

$\Sigma V = 0$

$$.8865X - .5800Y + .8069Z = 15$$

$$Z + 1.0986X - .7188Y = 18.5897 \qquad (1)$$

$\Sigma S = 0$

$$.4625X - .8069Y - .5800Z = 0$$

$$-Z + .7974X - 1.3912Y = 0 \qquad (2)$$

Adding (1) and (2)

$$1.8960X - 2.1100Y = 18.5897$$

$$Y = .8986X - 8.8103$$

Subst. in (2)

$$Z = .7974X - (1.3912)(.8986X - 8.8103)$$

$$Z = -.4527X + 12.2569$$

The sizes of members must be assumed in order to apply the least work solution. Use the following C.M. steel tubes:

Axle	A	1 7/8 x .125 wall	$I = .2644$
Strut	B	1 3/4 x .058	$I = .1105$
Strut	C	2 1/4 x .095	$I = .3741$

Bending in "B" and "C"

$$\text{Bending in B} = \frac{X}{1 + \frac{(.3741)}{(.1105)}\frac{(38.36)}{(32.89)}} = .2021X$$

$$\text{Bending in C} = \frac{X}{1 + \frac{(.1105)}{(.3741)}\frac{(32.89)}{(38.36)}} = .7979X$$

For this case:

$$\text{Work of Bending} = \frac{M^2L}{6EI} \qquad E = 29{,}000{,}000$$

$$\text{Work of Torsion} = \frac{T^2L}{2GI_p} = \frac{T^2L}{4GI} \qquad G = 12{,}000{,}000$$

Then the work in the members may be expressed as follows:

$$\text{Bending in B} = \frac{(.2021X)^2(38.36)}{6EI} = \frac{.2611X^2}{EI}$$

$$\text{Bending in C} = \frac{(.7979X)^2(32.89)}{6EI} = \frac{3.4899X^2}{EI}$$

$$\text{Bending in Axle} = \frac{(.8986X - 8.8103)^2(49.57)}{6EI} = \frac{6.6173X^2 - 130.8136X + C}{EI}$$

$$\text{Torsion in Axle} = \frac{(-.4527X + 12.2569)^2(49.57)}{4GI} = \frac{2.5392X^2 - 137.5245X + C}{GI}$$

(Constant terms are dropped since they do not appear in the first derivative.)

Values of EI

$\underline{B}$, $EI = (2.9)(10)^7(.1105) = (.3205)(10)^7$ $\qquad \frac{1}{EI} = (3.1201)(10)^{-7}$

$\underline{C}$, $EI = (2.9)(10)^7(.3741) = (1.0849)(10)^7$ $\qquad \frac{1}{EI} = (.9217)(10)^{-7}$

$\underline{A}$, $EI = (2.9)(10)^7(.2644) = (.7668)(10)^7$ $\qquad \frac{1}{EI} = (1.3041)(10)^{-7}$

$\underline{A}$, $GI = (1.2)(10)^7(.2644) = (.3173)(10)^7$ $\qquad \frac{1}{GI} = (3.1516)(10)^{-7}$

(10^{-7}) being a common term is dropped)

Total Work

Bending in B = $(.2611X^2)(3.1201) = .8147X^2$

Bending in C = $(3.4899X^2)(.9217) = 3.2166X^2$

Bending in A = $(6.6713X^2 - 130.8136X)(1.3041) = 8.7000X^2 - 170.594X$

Torsion in A = $(2.5392X^2 - 137.5245X)(3.1516) = \underline{8.0025X^2 - 433.4222X}$

Total Work $= 20.7338X^2 - 604.0162X$

Differentiating and equating to zero

$$41.4676X = 604.0162$$
$$X = 14.566$$

Bending in B = 2.9438
Bending in C = 11.6222

Problem Assignment.

18:1 Do the braked landing condition for the indeterminate chassis from this Assignment using

Axle = 1 7/8 x .125 wall C.M. Steel Tube
Strut B = 2" x .083 round dural tube
Strut C = 2 1/2 x 5/32 round dural tube

and find bending in the V struts and bending and torsion in the axle for a brake torque of 15 in. lbs.

Note: Use E = 10,400,000 for dural.
E = 29,000,000 for steel.
G = 12,000,000 for steel.

Follow the method shown in the example and check your own results by substituting the values obtained for the members in a vector summation similar to Fig. 7.

AIRPLANE
STRESS
ANALYSIS

PART SEVEN

ASSIGNMENT 19

ASSIGNMENT 20

BASIC STRESS ANALYSIS

ASSIGNMENT NINETEEN

Assignment 19

FITTINGS AND JOINTS

(SPECIAL CONSIDERATIONS IN ANALYSIS)

Any part which connects one member of a structure to another may be termed a fitting. Those which connect members of the primary structure are termed Primary fittings and receive the most careful treatment in design. Secondary fittings are those which connect members whose failure would not jeopardize the safety of the airplane as a whole, such as seats to floor fittings, etc. Design requirements recognize a difference in degree of importance of fittings in most cases although this is not always the case.

Fittings must maintain extra strength over that of the members they connect. The purpose of this extra margin is to allow for manufacturing tolerances, and for workmanship. The extra margins required are set at 15% for the army, 20% for the Department of Commerce and different values ranging from 15% to 100% for Navy designs, the amount of extra margin for the latter being dependent upon the importance of the fitting to the overall safety of the airplane. For example a wing to fuselage fitting might require a 50% margin of safety while a floor connection would require 15%. In general for Navy work the extra fitting factors are given in detail specifications, so that an exact statement of their value cannot be given here.

Welded Fittings and Joints.

The use of welding for structural joints and fittings is limited to steel parts except in the case of aluminum alloy tanks. When welded joints are used the material built up in the actual weld is cast structure, while that adjacent to the weld becomes annealed. It is therefore necessary to reduce the allowable unit stresses in the welds and in the adjacent structure. For the adjacent structure the allowable unit stresses are reduced by 20% for the effect of welding. This means that if a member has an allowable tensile strength of 100,000 psi. before welding, the material adjacent to the weld will have an allowable of only 80,000 psi. after welding. The amount of material affected by this limitation is different for different types of welding being greater for torch than for electric arc welding because of the greater heat involved in the torch welding process. Torch welding is allowed for all types of connections of steel to steel. Arc welding is allowed by the army and is preferred for joining heavy pieces together. Arc welding is permissible on commercial aircraft only by special permission of the Bureau of Air Commerce and is not allowed at all on Naval Aircraft. Thermite or resistance contact welding are not used in aircraft work to date, except in fabrication of stainless steel.

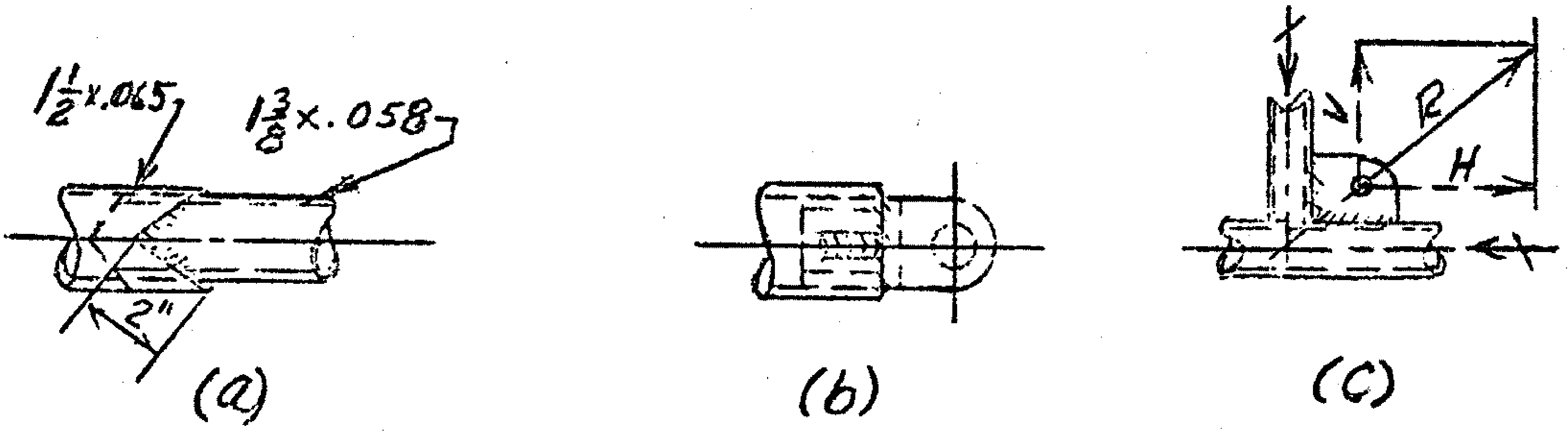
When members are joined by welding, the cross-sections through the welds themselves are greater than those through the adjacent members. This accounts in part for the fact that, on test loads, weld assembles rarely break in the welds themselves, but fail in the partially annealed structure just outside the weld. Because of the uncertainty as to the structure in the weld, the use of welds in tension is avoided. Welds should always be used in shear, or in such a fashion that the resultant loads on them may be broken up into shear components parallelling the lines of welding. In calculation of the shear strength available the following formula from ANC-5 is used. (For chrome-molybdenum steels)

Weld strength = $P = .48\ Lts$

where P = allowable load in lbs.
L = length of welded seam in inches.
t = thickness of thinnest material joined by the weld in the case of lap welds between two steel plates or between plates and tubes, in inches.
t = average thickness in inches of the weld metal in the case of tube assemblies (but may not be assumed greater than 1.25 times the thickness of the thinnest welded stock.)
s = 90,000 psi for material not heat-treated after welding.
s = ultimate tensile stress of material heat-treated after welding, but not to exceed 150,000 psi.

It is generally necessary to use gussets to assist in transferring the loads in a joint. Tension members in particular are apt to have insufficient weld to develop the strengths necessary. Where tubes are joined in a cluster fitting which includes an assembly of "fishmouthed" sleeves joined by gussets it is not necessary to calculate the strength of the weld between tubes and sleeves if standard fishmouth angles of 30° to 37° are maintained. In this case the weld will always develop at least 80% of the strength of the original tube before it was welded. Where more than six tubes join in a weld cluster the allowable weld values are reduced by half to account for the large amount of heating necessary in joining so many members. Fishmouth sleeves in clusters should be at least equal in guage to the members which they connect. Typical welded connections are shown in Fig. 1. The non-standard fishmouth weld of Fig. 1 (a) has the following strength:

Length of weld = 4 x 2 = 8 inches = L. Member is not heat-treated. (Normalized only)
t assumed .058 = thickness of thinnest tube. Tube Area = .240 sq. in.
$P = 8 \times .058 \times 90{,}000 \times .48 = 20{,}000$ lbs.
Tensile strength of tube at 80,000 psi = 19,200 lbs. = (.240)(80,000)
Efficiency of welded joint = 100% of rated tube strength.



(a) (b) (c)

Fig. 1

Where tubes attach to fittings machined from bar stock it is possible to secure a shorter fitting if the weld pattern of Fig. 1 (b) is used.

Machined fittings should be drilled out so that the ratio of thicknesses of the parts welded together does not exceed 2 to 1. Otherwise the thinner metal is burned away before the thicker is heated sufficiently to insure a good weld.

Corner gussets which are frequently used as fittings where the cross bracing consists of a pair of wires are assumed to carry their load in shear along the welds and not in tension. This assumption requires that the lug be strong enough to prevent buckling in compression. The strength of the weld will nearly always be determined by the guages of the adjacent tubes. See Fig. 1 (c).

Bolts Attaching Engine Mount to Main Structure.

The bolts attaching the engine mount to the main structure must allow close adjustment to prevent pounding from reverse loads and vibrations. If the bolts are primarily in shear, taper pins are used so that a very close fit is assured and takeup is available if there is any wear. Tension bolts allow a pre-stressed joint. When they are used provision should be made in the fitting design to take out the shear loads without stressing the bolts in shear, or large margins of safety should be used to allow for an increase in the maximum tension stress due to the effect of shear. Values are available in "Machinery Handbook" from which the maximum tension and shear unit stresses may be calculated for any combination of shear and tension loads in a bolt. Where the bracing is such that the shear loads may be taken out on the compression side of the truss it is possible to relieve the tension bolts of all shear loads by providing oversize holes. Note that where shear and tension exist together in a bolt the effect of each is to increase the unit stress due to the other. The overall strength could also be examined by using the stress ratio method described in ANC-5.

Bending in Joints and Members.

Where items such as batteries and oil coolers are attached to the engine mount structure, their effect in bending the members and joints must be taken into account. For the members to which the parts are attached it is conservative to consider pin ends and to design for the combined loads from bending and end loads on that basis.

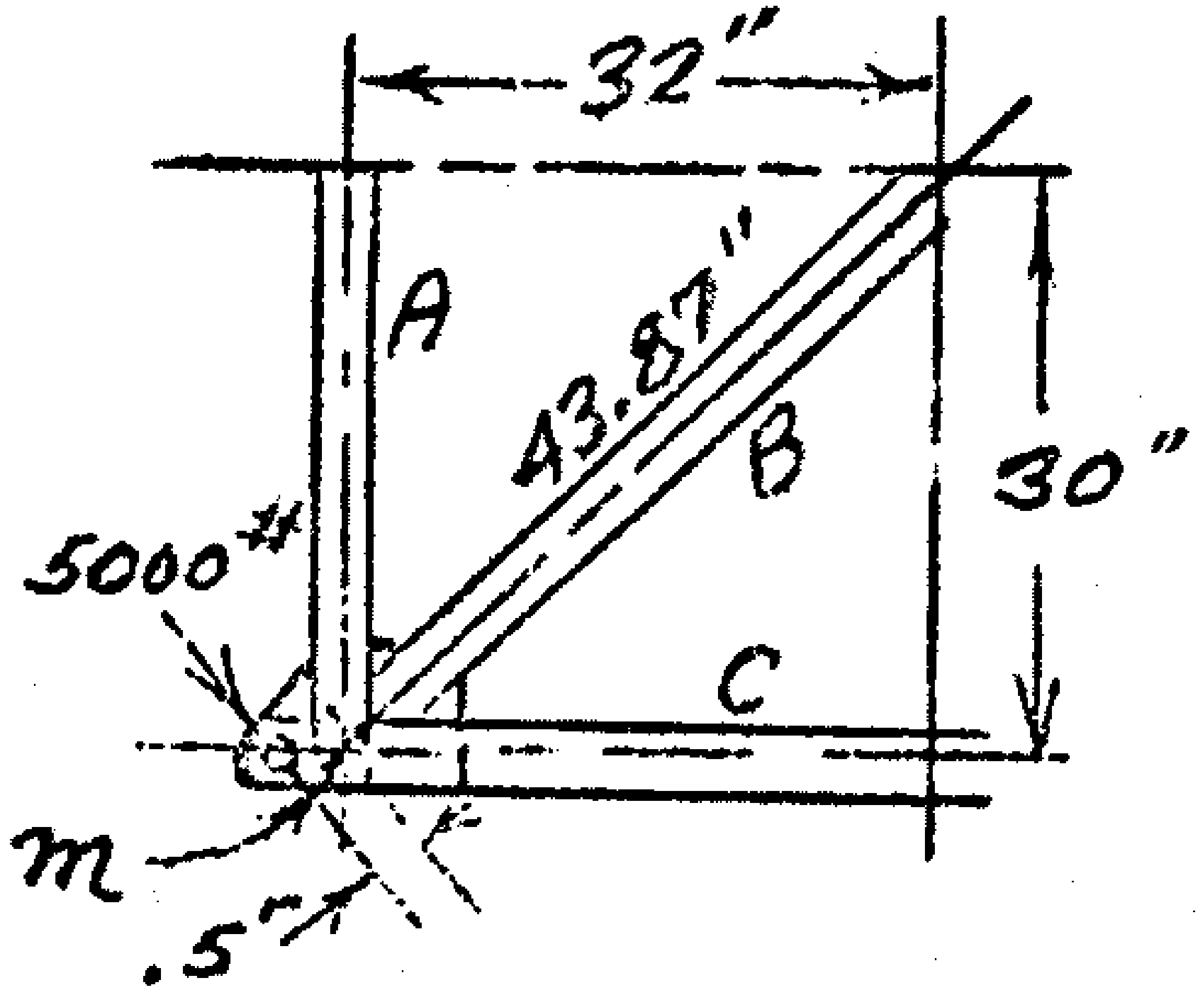


Fig. 2

For the adjacent members this system is non-conservative, since moments are actually set up at the joints. Also there are conditions in which large external moments are applied from braking action on landing gear which is attached to the fuselage, etc. or from eccentricities in fittings. Consider a side supporting truss in which moment is introduced by an eccentric load from the engine mount, Fig. 2.

The moment at "m" is equal to 5000 x .5 = 2500"#.

It is divided between tubes A, B, and C in proportion to their Rigidity Ratios. The Rigidity Ratio = I/L which means that the moment will be distributed to the tubes in proportion to their Moments of Inertia and inversely proportional to their lengths. Assume the following conditions:

Tube	Size	I	L	R = I/L x 10^3	R/ΣR
A	1½ x .065	.0756	30.0	2.52	.613
B	1 x .049	.0166	43.87	.36	.092
C	$1\frac{1}{4}$ x .058	.0387	32.0	1.21	.294
				4.11	.999

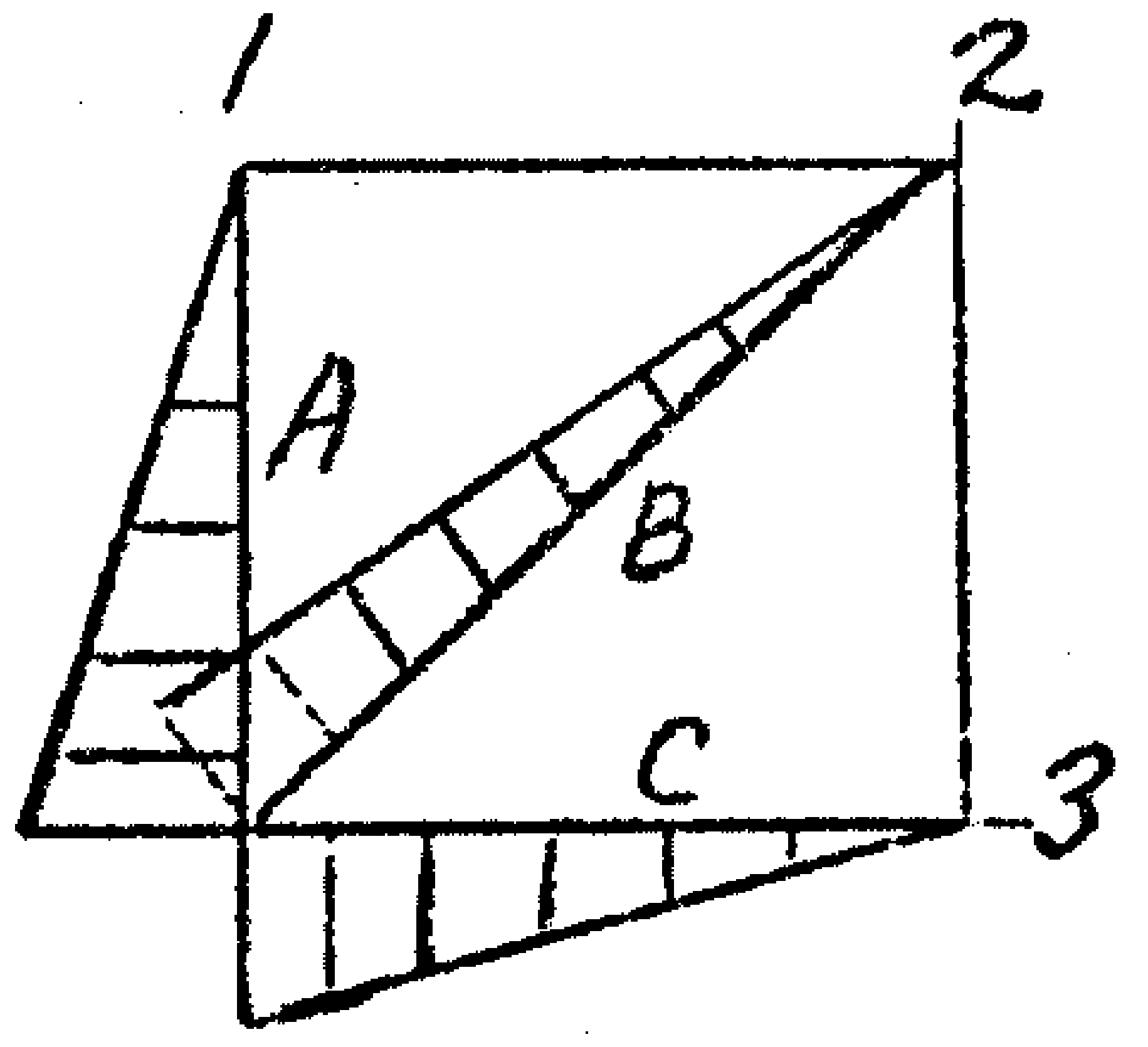


Fig. 3

The actual moment to each tube is as follows:

A = .613 x 2500 = 1534"#
B = .092 x 2500 = 230
C = .294 x 2500 = 736
Σ = 2500"#

Fig. 3 shows the moment distribution.* Design of the tubes should be made with the precise equations using end load plus bending. The method is illustrated in the following example.

(*Note: The moment diagram of Fig. 3 is based on the assumption that there is no fixing moment at the ends of the members at 1, 2, and 3.)

Assume that member "C" of Fig. 2 is carrying a compression load of 6000 lbs. in addition to the end moment of 736"# due to eccentric application of the fitting load. Then the loading condition on the tube is as shown on Fig. 4.

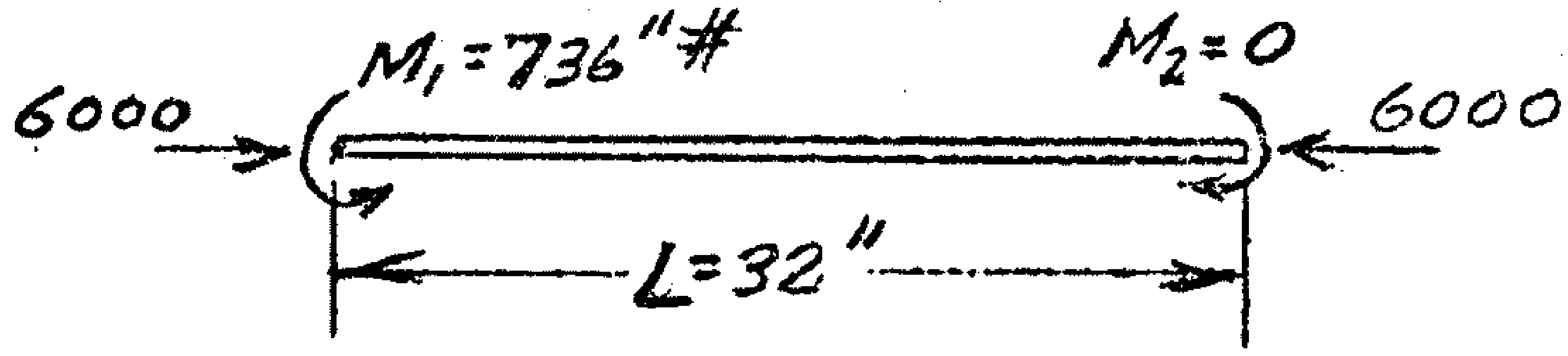


Fig. 4

From our work with the precise equations for combined end load and bending at point of maximum moment

$$\tan\frac{x}{j} = \frac{M_2 - M_1\cos\frac{L}{j}}{M_1\sin\frac{L}{j}} = -\frac{\cos\frac{L}{j}}{\sin\frac{L}{j}}$$

$$M_{max} = \frac{M_1}{\cos\frac{x}{j}}$$

$$j = \left(\frac{EI}{P}\right)^{\frac{1}{2}} = \left(\frac{29,000,000 \times .0387}{6,000}\right)^{\frac{1}{2}} = (187)^{\frac{1}{2}} = 13.7$$

$\frac{L}{j} = \frac{32}{13.7} = 2.34$ radians $\qquad \cos\frac{L}{j} = -0.696$

$\sin\frac{L}{j} = +0.718$

from tables of functions of angles in radians.

$\tan\frac{x}{j} = -\frac{-0.696}{+0.718} = +0.97, \ \cos\frac{x}{j} = 0.718$

$M_{max} = \frac{736}{0.718} = 1025$ in. lbs.

For the strut under compression alone the allowable $\frac{P}{A} = F_c = 45,000$ (ANC-5)

$F_c = 79,500 - (51.9)\left(\frac{L^1}{P}\right)^{1.5} = 45,000$ psi.

$f_c =$ actual $\frac{P}{A} = \frac{6000}{.217} = 27,600$ psi.

$R_c = \frac{27,600}{45,000} = .613 \qquad F_b = \frac{My}{I} = \frac{1025 \times .625}{.0387} = 16,150$ psi.

F_b allowable = 102,000

Modulus of Rupture from ANC-5, page 4-35.

$D/t = 21.6$

$R_b = f_b/F_b = \frac{16,150}{102,000} = .158$

$R_c + R_b = .613 + .158 = .771$

Since this value is less than 1.0 the tube is adequate.

****

Continuous welded joints in structures or continuous riveted joints in monocoque structures are not required to show the extra margins of safety required for fittings.

Riveted Connections.

Concentric riveted connections which carry no moment are assumed to be loaded evenly; that is the load is distributed equally to the rivets. This is approximately true, even when the rivets are in a single line, the end rivets not being overloaded as might be expected. Riveted joints are checked for the shear value of the rivets and for the bearing value of the rivet in the attached sheets. According to ANC-5 the bearing value of the sheet itself may be used up to a maximum value of 90,000 psi, where the rivets used are A-17ST aluminum alloy or better. This represents a considerable increase over the value of 60,000 psi. in bearing previously allowed for the rivets themselves. Where rivet clusters must carry bending loads as well as end loads in members they must be investigated for combined loads on the rivets as follows: See Fig. 5.

Assume a symmetrical rivet pattern, of 5 rivets, carrying a shear load of 1000 lbs. upward and a moment of 1200 in. lbs. acting as shown. For this case the centroid of the rivet joint about which bending will take place, is by inspection at the center rivet. The resistance of the rivet pattern to bending may be calculated just as if a moment of inertia of a cross section were being computed neglecting the values of I_o and using the moment arms for the "areas" represented by the rivets as being the radii from the centroid of the rivet group to the centers of the rivets. In short a load on a given rivet is proportional to its distance from the centroid, its arm is also proportional to the distance from the centroid and the moment it can resist is proportional to the square of its distance from the centroid. For the simple symmetrical pattern of Fig. 5, the center rivet cannot take any bending load because it has no lever arm, being at the centroid. The four outer rivets are equidistant from the centroid and take equal bending loads. The forces on the rivets are shown. The resultant forces on all rivets are different, being the resultants of the forces due to shear and bending loads.

For the condition shown rivets 1 and 4 carry the greatest loads and would be critical. This can be proved by constructing the vector diagrams for the loads on all rivets.

Where different sizes of rivets are used in a joint the calculation of bending loads should be made twice, once for shear and once for bearing since the shear value varies as the square of the diameter while bearing value varies as the diameter. One size of rivet is taken as standard and the remaining rivets are expressed as a proportion of its strength. Suppose the rivets in a joint are 1/8" diameter, 5/32 diameter, and 3/16 diameter. Taking the 1/8 diameter as 1.0 unit for shear, the 5/32 rivet is 1.56 and the 3/16 rivet is 2.25. For bearing (in the same guage sheet) the respective values would be 1.0, 1.25, and 1.5.

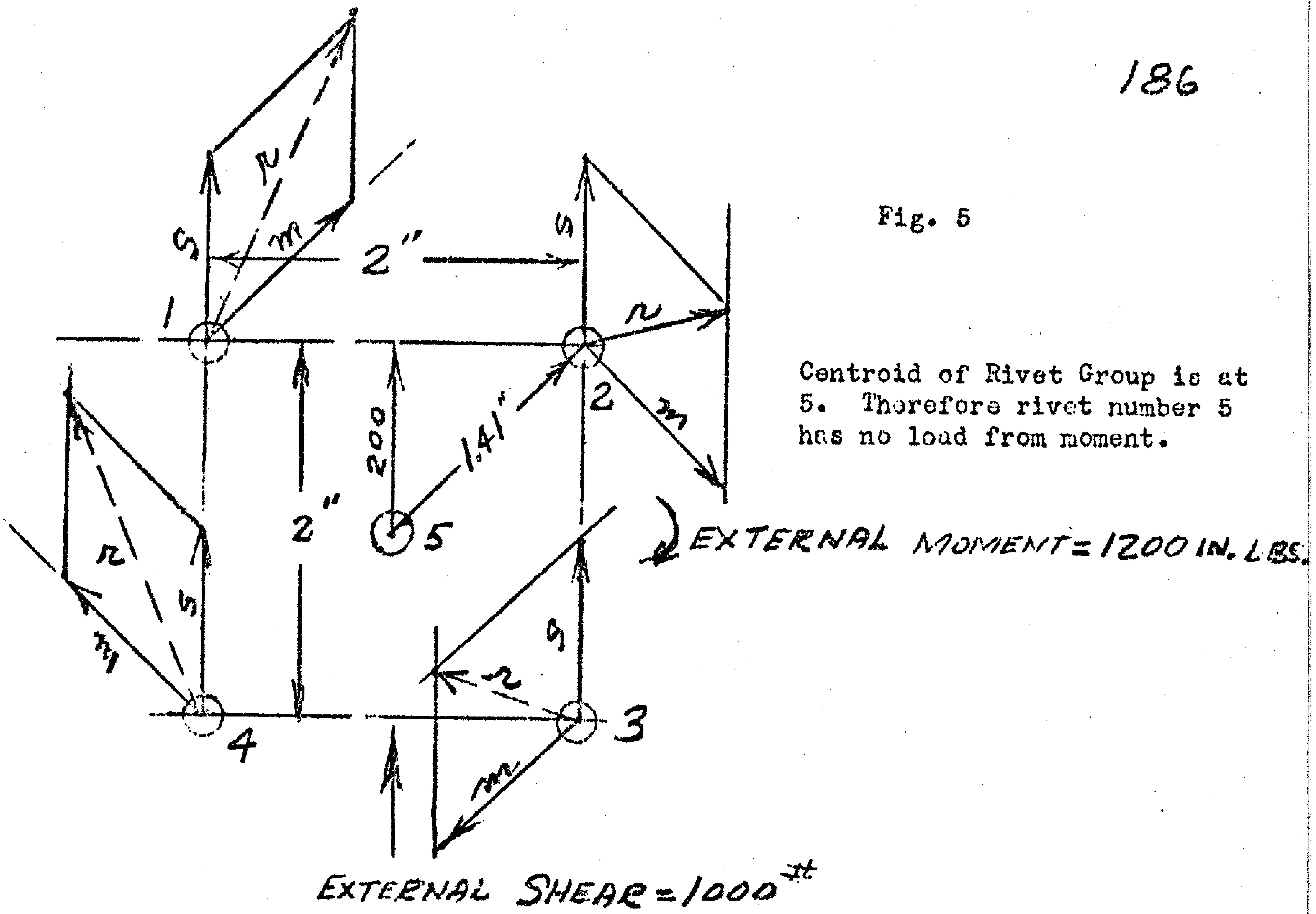


Fig. 5

Centroid of Rivet Group is at 5. Therefore rivet number 5 has no load from moment.

Per rivet shear loads are $\frac{1000}{5}$ = 200 lbs. each = s

Moment loads are $\frac{1200}{(4)(1.41)}$ = 212 lbs. each = m

The final loads on the respective rivets are the resultants of the shear and moment loads, r, shown by dashed lines on Fig. 5.

****

If the solution had been made using the "Moment of Inertia" of the rivet group the work would be as follows:

Centroid of rivet group is at 5. Radial distance to each rivet effective in bending = 1.41 inches.

$$I = \Sigma d^2 = (4)(1.41)^2 = 8$$

Then the load on any rivet, $m = \frac{Md}{I}$ where M is the external moment on the rivet group. In this case

$$m = \frac{(1200)(1.41)}{8} = 212 \text{ lbs.}$$ which checks our initial result.

The solution of forces for a group of rivets in line subjected to bending is rather simple if we continue the beam analogy that the force on each rivet is proportional to its distance from the centroid. See Fig. 6. This is true of course only if the rivets are all of the same size. Where rivets have different sizes an additional factor k is introduced. To illustrate the case of rivets of different strengths let us assume that rivets 1 and 5 of the cluster shown on Fig. 6 are 3/16 diameter and rivets 2, 3, and 4 are 1/8 diameter and that the rivets are critical in shear; that is that the bearing strength is ample. Then if we say that the strength of rivets 2, 3, and 4 is unity, (1.0) that of rivets 1 and 5 is $\left(\frac{3/16}{1/8}\right)^2 = 2.25$. This value we term k.

The moment of inertia for this rivet group which has a centroid at rivet 3 would be $\sum k(d)^2$. Putting the solution into tabular form

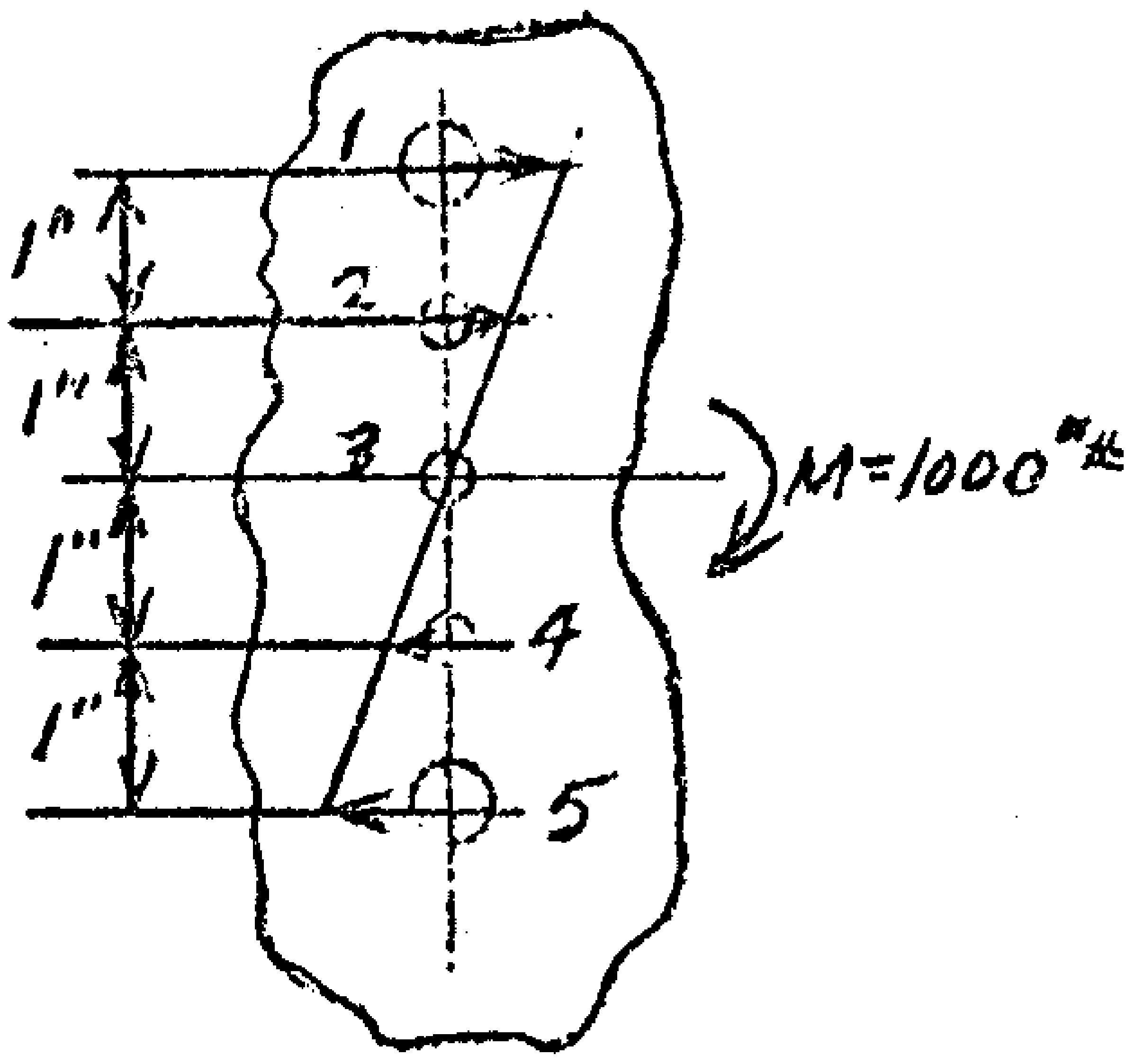


Fig. 6

Rivet No.	d	d^2	k	kd^2
1	2	4	2.25	9.0
2	1	1	1.0	1.0
3	0	0	0	0
4	1	1	1.0	1.0
5	2	4	2.25	9.0
			$I = \sum =$	20.0

Note that the above solution is made as if we had replaced each 3/16 diameter rivet with 2.25 1/8 diameter rivets. Then the actual load on any rivet is equal to $\frac{(k)(M)d}{I}$

The load on rivets 1 and 5 is $\frac{(2.25)(2)(1000)}{20} = 225$ lbs. and the load on rivets 2 and 4 is $\frac{(1)(1)(1000)}{20} = 50$ lbs.

As a check consider the total resisting moment which is equal to the summation of Force times distance.

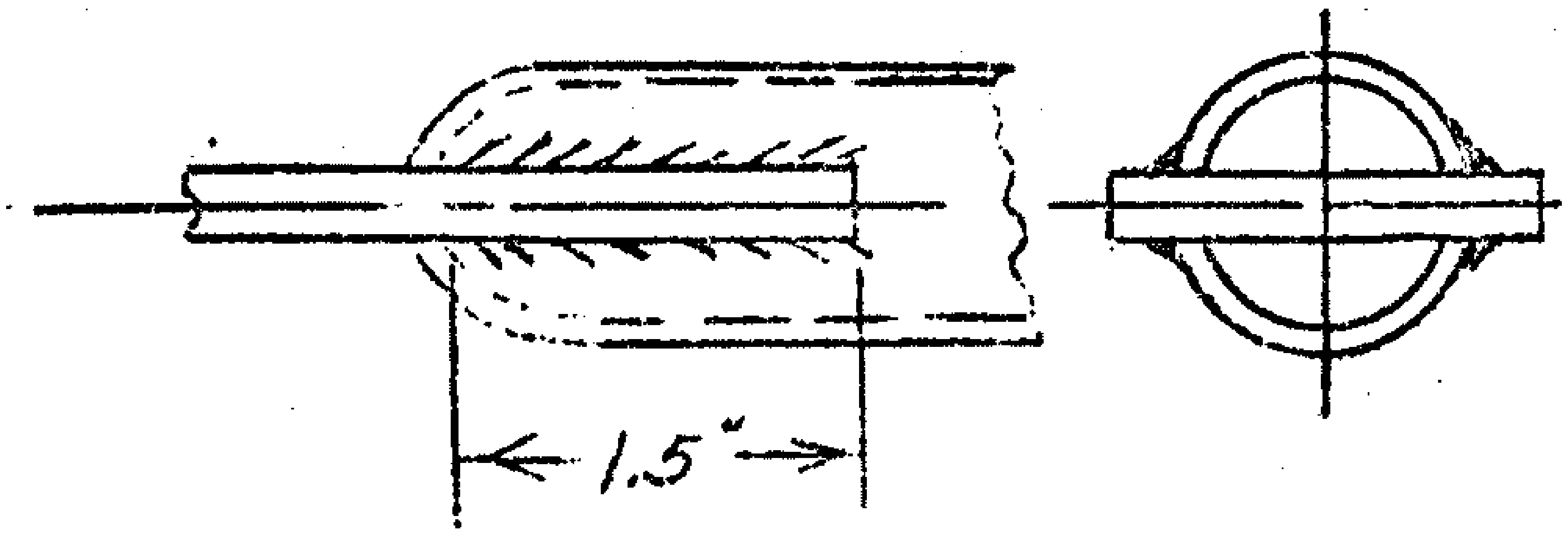
Resisting moment = (2)(225)(2) + (2)(50)(1) = 900 + 100 = 1000 which equals the external moment.

A similar solution can be extended to cover any combination of rivets or bolts in a joint. Such a solution is good only when the rivets or bolts are used in metal construction, and would not apply to a joint in wood, where the direction of the load with respect to the grain would determine the bearing value of the rivet or bolt.

Problem Assignment.

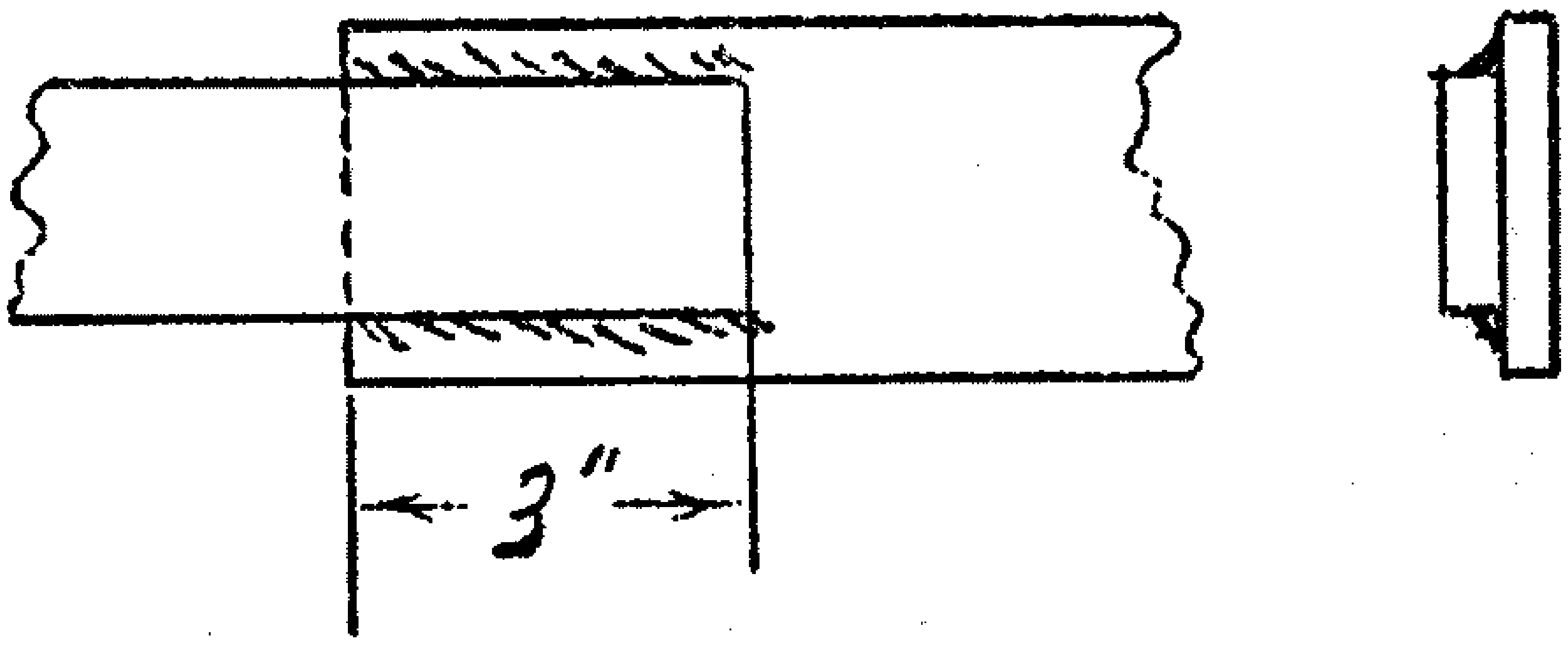
19:1 Calculate the strengths of the following welded joints:

(a)



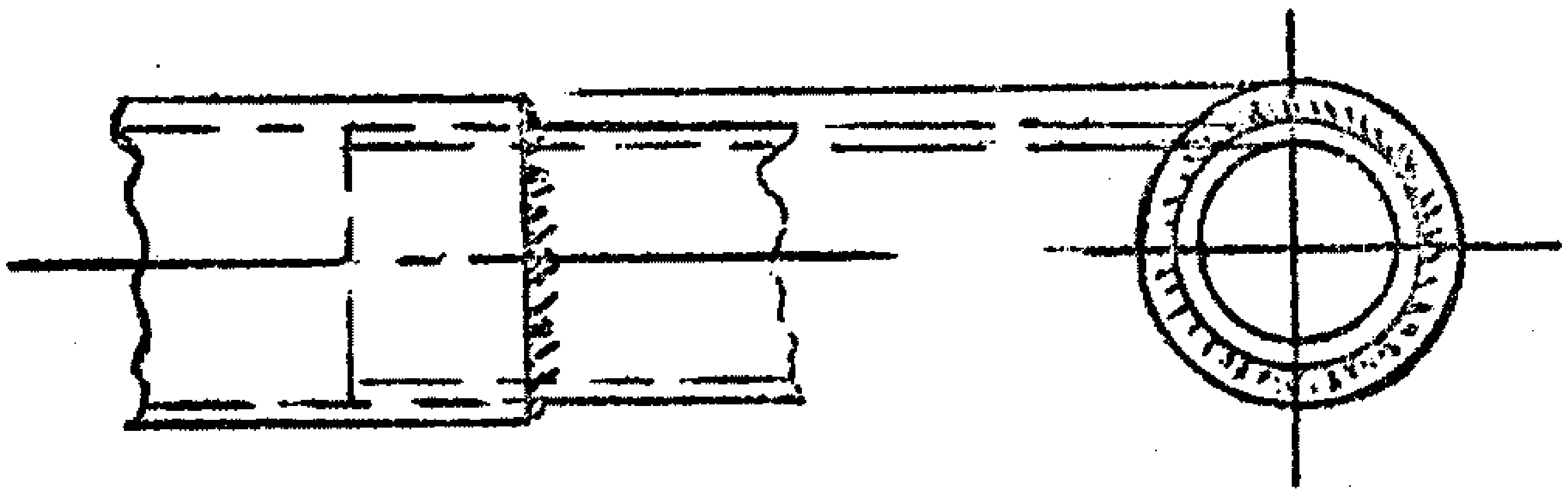
Tube is .065 wall
Gusset is .125 thickness
Heat treatment 125,000
psi.

(b)



Plates are .093 C.M.
Steel. Heat treatment
is 180,000 psi.

(c)



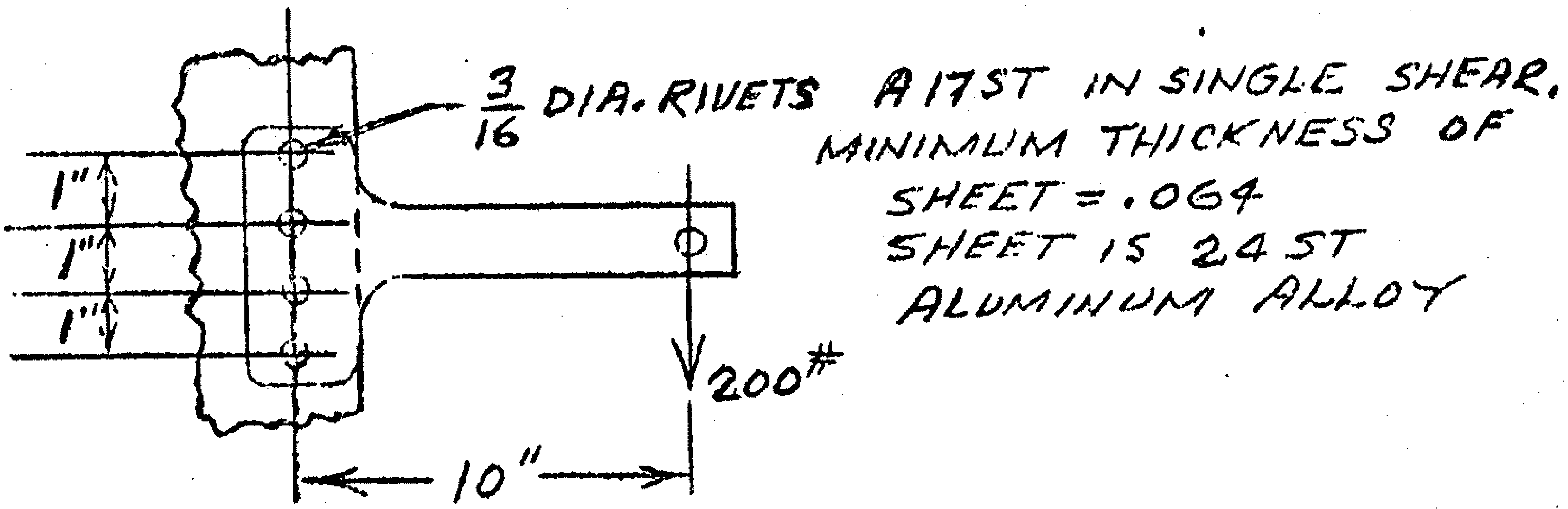
Outside Tube is $1\frac{1}{2}$ - .065

Inside Tube is 1 3/8 - .058

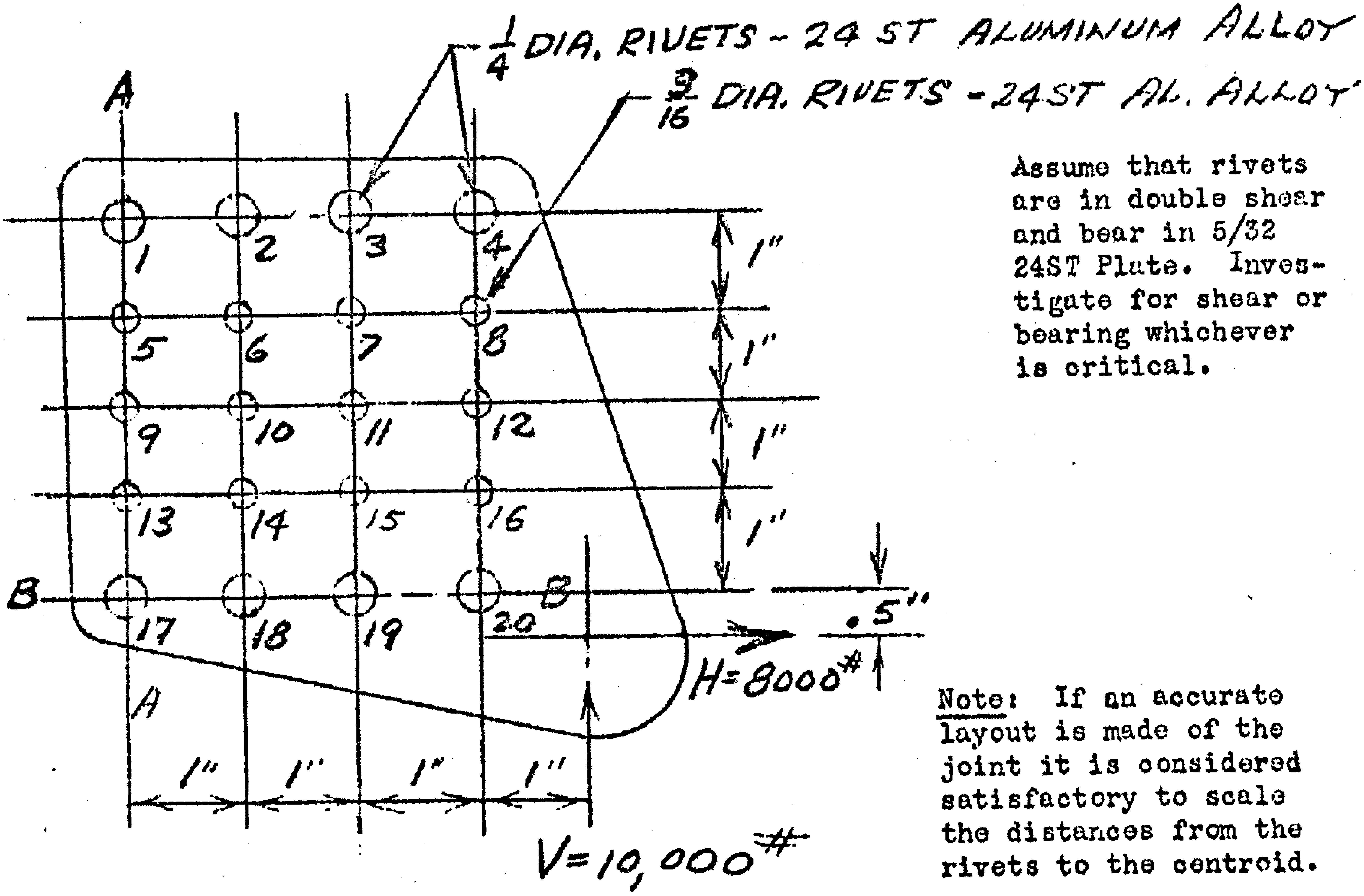
Tubes are C.M. steel
normalized.

19:2 Calculate the maximum loads on the rivets in the joints shown and compare them with the allowable loads for rivets shown in ANC-5.

(a)


3/16 DIA. RIVETS A17ST IN SINGLE SHEAR.
MINIMUM THICKNESS OF SHEET = .064
SHEET IS 24ST ALUMINUM ALLOY
1"
1"
1"
200#
10"

(b)


1/4 DIA. RIVETS - 24 ST ALUMINUM ALLOY
3/16 DIA. RIVETS - 24ST AL. ALLOY
A
A
B
B
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
1"
1"
1"
1"
.5"
H=8000#
1"
1"
1"
1"
V=10,000#

Assume that rivets are in double shear and bear in 5/32 24ST Plate. Investigate for shear or bearing whichever is critical.

Note: If an accurate layout is made of the joint it is considered satisfactory to scale the distances from the rivets to the centroid.

BASIC STRESS ANALYSIS

ASSIGNMENT TWENTY

Assignment 20

FITTINGS AND JOINTS

(ANALYSIS OF LUGS AND THREADED CONNECTIONS)

Lugs.

Lugs are present in nearly every type of fitting and their analysis presents a general outline applying to all types of bolted connections. Lugs are investigated for their shear strength, tensile strength and bearing strength. End connections for standard steel tie rods have been reduced to standard sizes and are listed in ANC-5, page 4-26. Since it is often desired to design a lug which is decidedly non-standard a brief discussion of the process is given here.

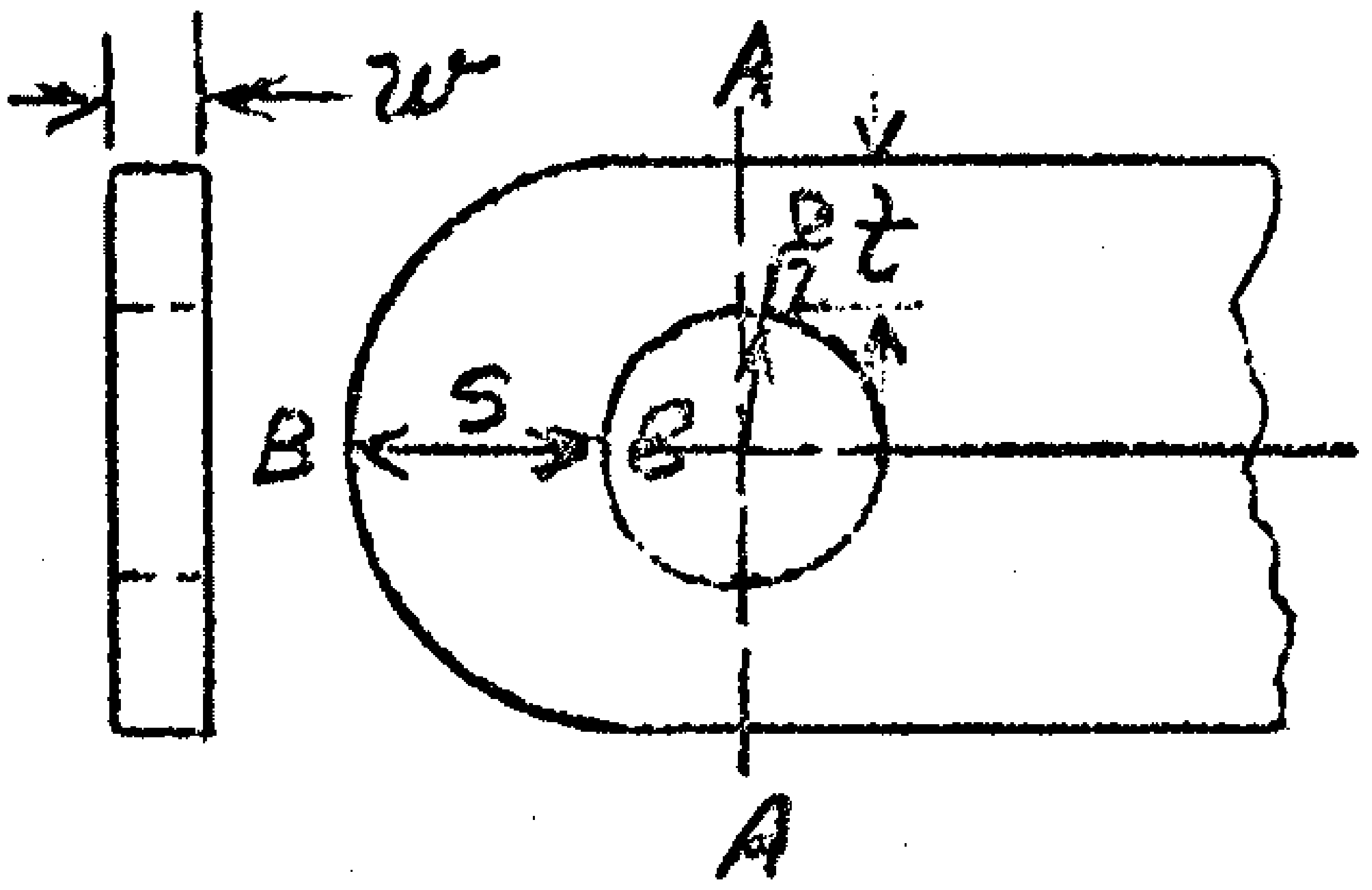


Fig. 1

Let us assume for example that we are designing the lug shown in Fig. 1, from 24 ST. - Al. Alloy plate.

This plate has the following physical properties:

Ultimate tensile	62,000 psi.
Yield in tension	40,000 psi.
Shear ultimate	37,000 psi.
Bearing ultimate	90,000 psi.

Assume we are transmitting a load in tension of 15,000 lbs. and that for clearance with the mating parts it is advisable to use a 1/2 inch diameter nickel steel bolt, heat treated to 125,000 psi. If the part were on a commercial airplane a margin of safety for all fittings of 20% would be required and the bolt would be considered a part of the fitting. The design would proceed as follows:

1/2 diameter bolt in double shear, shear value of bolt = 2 x 14,700 = 29,400 lbs.

$$M.S. = \frac{29,400}{15,000} - 1 = \underline{.96} \qquad \text{o.k.} \qquad .20 \text{ required}$$

Required bearing width in lug, "w" is found next.

$$w \times \tfrac{1}{2} \times 90,000 = 15,000 \times 1.20$$

$$w = \frac{15,000 \times 1.20}{45,000} = .40 \text{ inches.}$$

A width of 7/16 inches will be used.

Actual bearing value = 7/16 x 1/2 x 90,000 = 19,700 lbs.

$$M.S. = \frac{19,700}{15,000} - 1 = \underline{.31}$$

The use of the full 90,000 psi. in bearing would be permissable only where there was no shock load or reversal of stress. Where shock loads were carried a bearing value of only 45,000 psi. would be allowed, and if the lug were subject to loads reversing from tension to compression only 60,000 psi. would be used. Bearing between metals is of course always computed on the basis of the softest material. In this case it would have been possible to increase the bearing value of the lug by inserting a steel bushing, the bearing value increasing in direct ratio to the outside diameter of the bushing.

The shear value of the lug is computed by taking the shearing area as twice that of section B-B. Dimension "S" is then computed as follows:

$$2 \times S \times 7/16 \times 37{,}000 = 15{,}000 \times 1.20$$

$$S = \frac{15{,}000 \times 1.20}{2 \times 7/16 \times 37{,}000} = .555 \text{ inches, use } 5/8 \text{ inches}$$

$$\text{Shear value} = 2 \times \frac{5}{8} \times \frac{7}{16} \times 37{,}000 = 20{,}200 \text{ lbs.}$$

$$\text{M.S.} = \frac{20{,}200}{15{,}000} - 1 = \underline{.34}$$

It is actually impossible for the bolt to shear through the lug at section B-B but use of this section as the shear plane has been standardized in ANC-5.

The tension width, "t", is computed:

$$2 \times t \times \frac{7}{16} \times (40{,}000 \times 1.5) = 15{,}000 \times 1.20$$

$$t = \frac{15{,}000 \times 1.20}{2 \times 7/16 \times 60{,}000} = .343 \text{ inches, use } 3/8 \text{ inches}$$

$$\text{Tensile value} = 2 \times \frac{3}{8} \times \frac{7}{16} \times 60{,}000 = 19{,}700 \text{ lbs.}$$

$$\text{M.S.} = \frac{19{,}700}{15{,}000} - 1 = \underline{.31}$$

Note particularly that instead of using the ultimate tensile value of 62,000 psi, we have used 1.5 times the yield point value of 40,000 psi. This reduction is made wherever the ultimate tensile value exceeds 1.5 times the yield point. Its purpose is to insure that the part does not yield at the expected (or yield point) load.

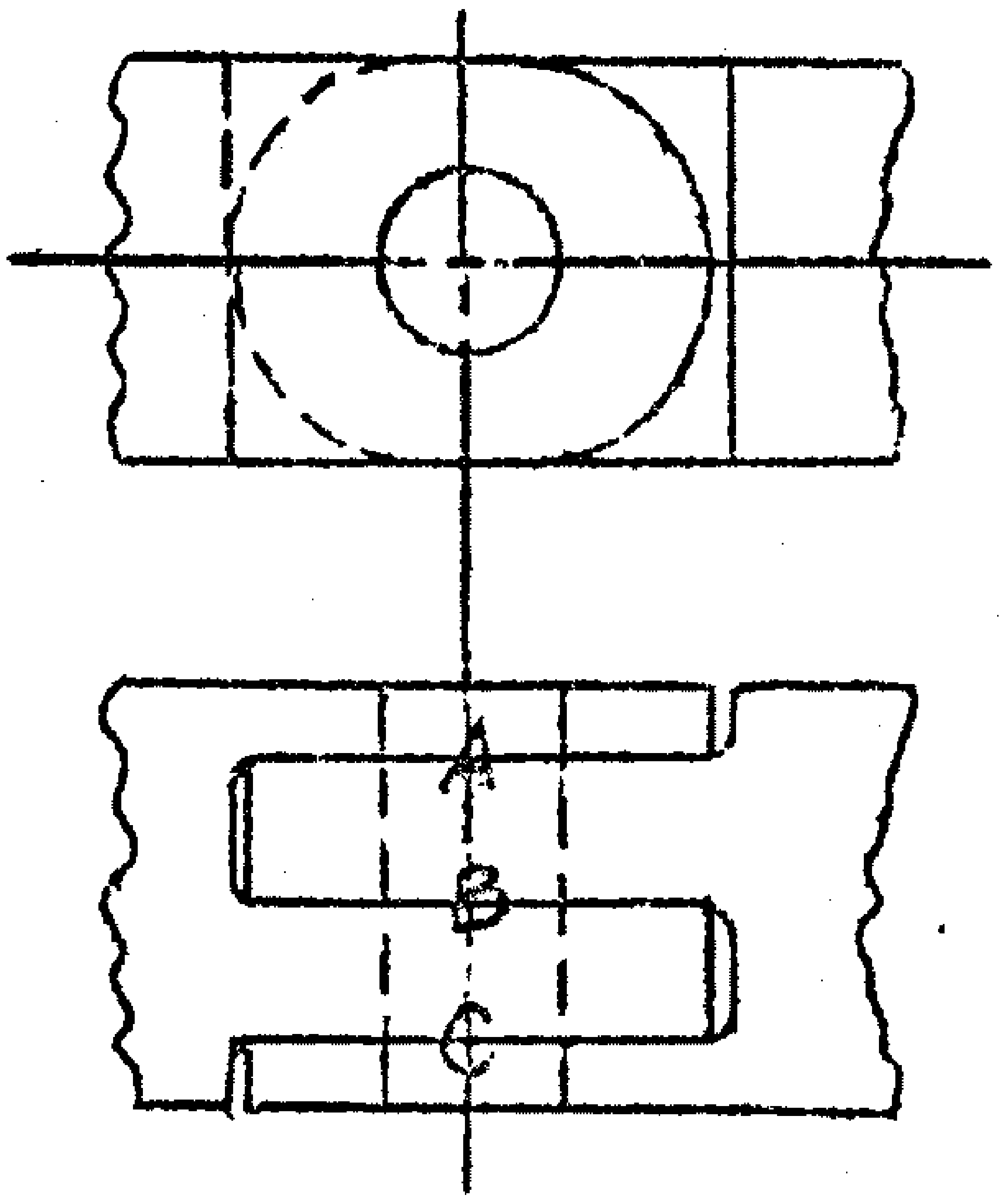


Fig. 2

It is sometimes possible to introduce more than two shear planes on a bolt connecting a pair of lugs and thus keep down the bolt size. A typical example is shown in Fig. 2.

Assume the parts connected are of steel heat-treated to 125,000 psi. Then the bearing value is 175,000 psi and the shear value 75,000 psi. Let us transmit a load of 75,000 lbs., and assume that we may have reversal of stress. This means that we may use only $\frac{175,000}{1.5}$ or 116,500 as our ultimate value in bearing. We have three shear planes on the bolt, A, B, and C. It is therefore possible to use a 3/4 inch diameter nickel steel bolt, heat treated to 125,000 psi.

Shear value of bolt = 3 x 33,135 = 99,400 lbs.

$$\text{M.S.} = \frac{99,400}{75,000} - 1 = \underline{.32} \qquad \text{Required} = \underline{.20}$$

The required thickness of lugs is computed as follows:

Lugs above A and below C must each carry the load from one shear plane of the bolt while lugs between A and B, and between B and C carry the loads from two shear planes. The thickness "w" of the outer lugs is calculated:

$$w \times \frac{3}{4} \times 116,500 = \frac{1}{3} \times 75,000 \times 1.20$$

$$w = \frac{25,000 \times 1.20}{.75 \times 116,500} = .344 \text{ inches, use } 3/8 \text{ inches}$$

Lugs A-B and B-C would each have a width of 3/4 inch. The shear and tension allowables for each lug would be computed the same as for the example shown in Fig. 1.

Bolts:

Aluminum Alloy

It is not good practice to use aluminum alloy bolts and nuts in sizes less than 1/4 inch diameter because of the danger of stripping the threads in tightening down the nut. Aluminum alloy nuts should not be used with steel bolts, even when the bolts are in shear, for airplanes which are to be used around salt water. This combination should never be used where bolts are in tension.

Special Steel Bolts

It is often possible to obtain a more economical joint by using a bolt heat treated to higher values than the 125,000 psi. standard. Nickel steel bolts are used up to 150,000 psi. minimum value, but above this value special Nickel Chrome-Molybdenum steels are advisable. In choosing the steel to be used it is necessary to consider the ultimate tensile, yield point, elongation, the fatigue resistance, and susceptibility to impact loads or to notch effect. A balanced value of ultimate tensile where the other properties are also good is desirable. The ability to machine the steel must also be considered.

It is sometimes advantageous to use hollow pins. Where the D/t of the pin used exceeds (4) reduction in the shear value must be made to account for distortion of the pin by crushing. This reduction varies from (1/3) at D/t = (8) to no reduction at D/t = (4). The variation in allowable shearing stress is assumed to be linear and is based on results of tests.

Threaded Joints.

Threaded joints in axially loaded members must be handled with caution since they represent one type of fitting in which the load carrying capacity decreases sharply as the component parts deflect under load. In addition they are subject to stress concentrations from the notch effects at the bottom of the threads. Consider one tubular member threaded into another as shown in Fig. 3. Consider this joint to be carrying an axial tension load in the members. The component parts are subjected to

1. Tension loads on the net area of each tube critical either in the solid section or at the root of the threads. Where possible the net area at the root of the threads should considerably exceed the area of the tubes connected to allow for concentration of stress at the bottom of the threads.

2. Shear loads across the threads at Section A-A for inner member and across Section B-B for outer member.

3. Radial tensile loads in outer member due to "wedge" action of the threads.

4. Radial compressive loads in inner member due to wedge action of the threads.

The following standard nomenclature is used for the threaded joint:

T = Design Axial Tension on joint.
O.D. = Body diameter for threading.
I.D. = Internal diameter for threading.
R.D. = Root diameter or minimum diameter of internal member.
P.D. = Pitch diameter of thread.
L = Length of engagement of thread.

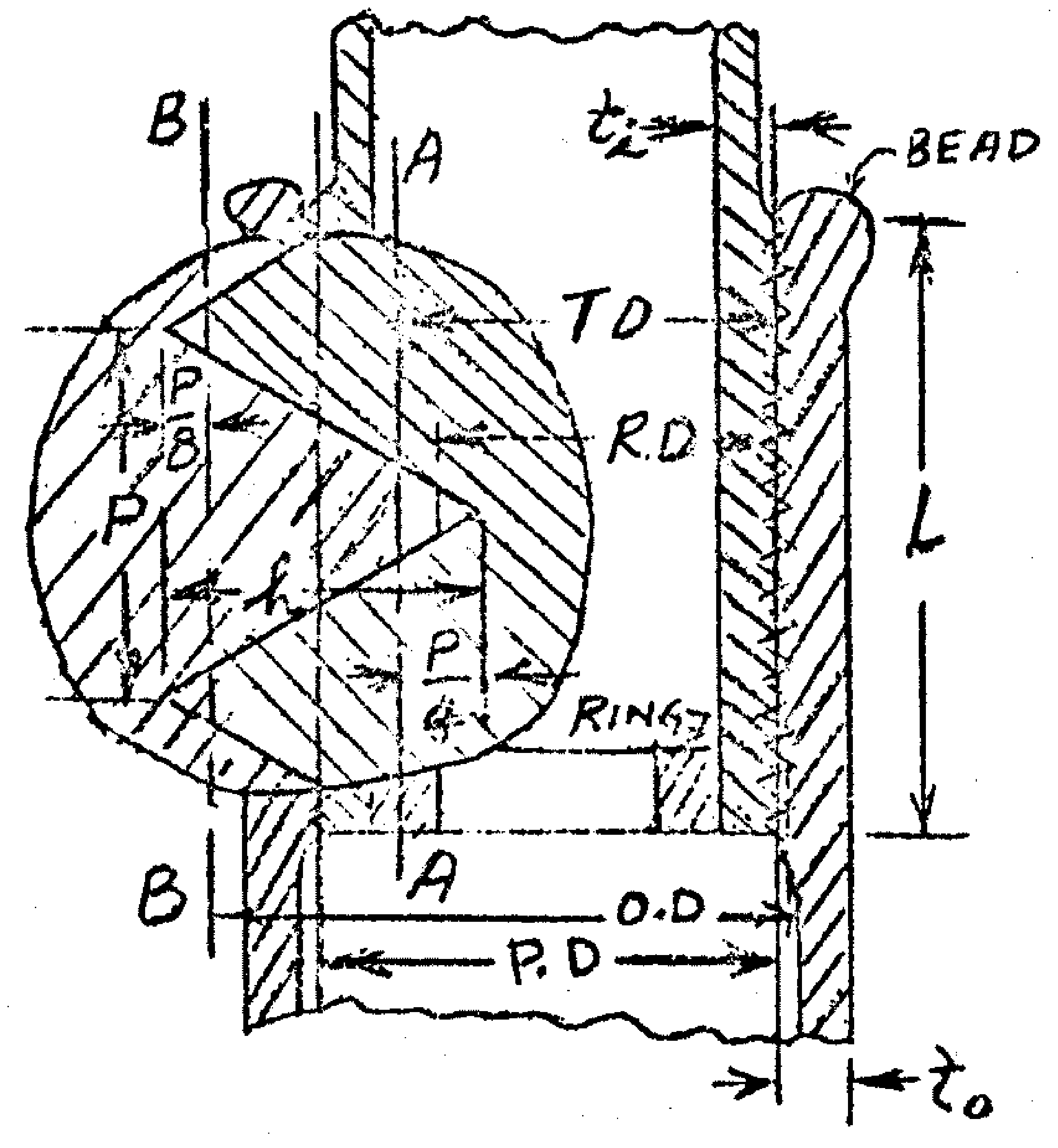


Fig. 3

$$p = \text{pitch of thread} = \frac{1}{\text{no. of threads per inch.}}$$

h = height of sharp thread = .866 p

t_o = Wall thickness of outer member, measured from pitch line.

t_i = Wall thickness of inner member, measured from pitch line.

E = Modulus of elasticity.

****

The analysis of the joint is made as follows:

1. The minimum area of each connecting member is computed. This will be either in the unthreaded section, at the thread relief or at the root

of the threads for the inner tube; and in the unthreaded section, the thread relief or at the bottom of the thread on the outer section. With no allowance for stress concentration, the average unit stress is of course $\frac{T}{A\text{ min.}}$

Stress concentration factors may be taken from Timoshenko, "Strength of Materials" or from Product Engineering, April and May, 1938 - Roark on "Factors of Stress Concentration".

It is sufficient on materials which have good elongation, such as 17ST Al. Alloy and the Chromo-Molybdenum and Nickel steels to use the yield point stress in tension as the average stress for design purposes.

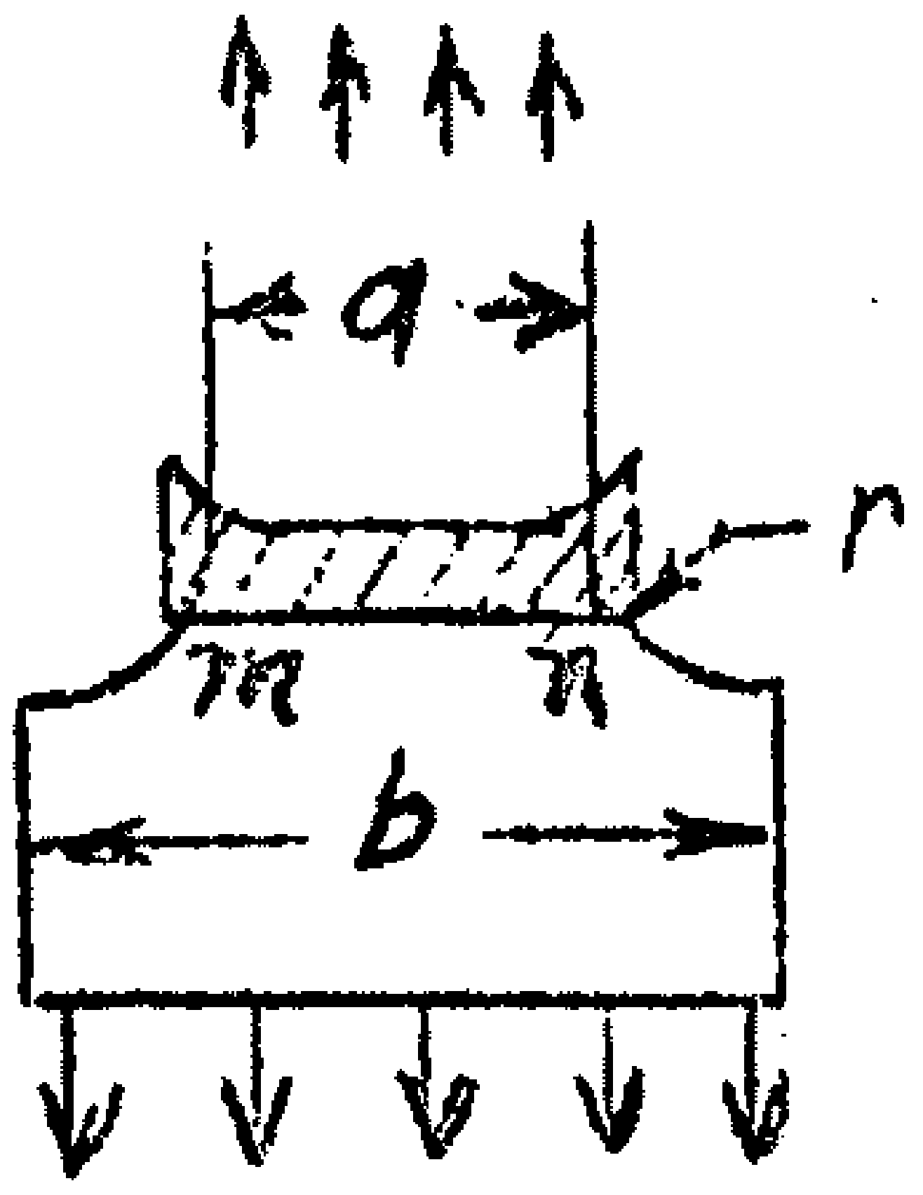


Fig. 4

2. For standard threads the radial unit tensile stress in the outer member is computed from the formula

$$f_t = .0919 \times \frac{T}{Lt_o}.$$

Good practice demands that this value not exceed the yield point for steel or 75% of the yield point for Aluminum Alloy or bronze.

3. The radial unit compressive stress in the inner member

$$f_c = .0919 \times \frac{T}{Lt_i} \qquad (3)$$

The allowable stress for f_c is the same as for f_t.

4. For axial load along section A-A $f_s = \frac{T}{\text{Area along A-A}}$ and maximum unit combined shear $= \left(f_s^{\,2} + \left(\frac{f_c}{2}\right)^2\right)^{\frac{1}{2}}$

The effective section along A-A is dependent upon the following variables:

(a) Deflection due to radial stress in both the inner and outer members.

$$d_i = \frac{f_c \times \text{P.D.}}{E}, \qquad d_o = \frac{f_t \times \text{P.D.}}{E}$$

(b) Amount cut off sharp thread for flat generally .25h.

(c) Pitch diameter tolerance of both inner and outer members.

(d) Internal diameter tolerance on tube.

The shear area along A-A

$$= \frac{.75h - \frac{d_i + d_o}{2} - \text{P.D. tolerance} - \frac{\text{I.D. Tolerance}}{2}}{h}$$

times L(I.D. + I.D. Tol.) x π

f_c is either that from formula (3) above or $f_c = \frac{.577T}{\text{Area along A-A}}$, whichever is the greater. The unit stress for this condition should not exceed 50% of the ultimate shear value of the material used. If the ratio of net shear length to length of thread is less than .45 a coarser pitch thread should be used.

5. For axial load along section B-B $f_s = \frac{T}{\text{Area along B-B}}$ and maximum combined

$$f_s\text{max.} = \left[f_s^2 + \left(\frac{f_t + f_c^1}{2}\right)^2\right]^{\frac{1}{2}}$$

$$\text{Shear area along B-B} = \frac{.875h - \frac{d_i + d_o}{2} - \text{P.D. tol.} - \frac{\text{O.D. tol.}}{2}}{h}$$

times L(O.D. - O.D. tol.)π

f_t is taken from the formula of item 2

$$f_c^1 = \frac{.577T}{\text{Area along B-B}}$$

It is desirable to reinforce the free end of a threaded member with a ring or a bead to prevent excessive deflection from eccentric loads or from secondary bending in the members connected.

Materials.

Machined fittings are made from bar or plate stock or from forgings or castings. Choice of the material will be governed by the number of parts to be used, the space limitations, the strength weight ratios and the special limitations set up by the procurement or licensing agency. In general, parts machined from bar or plate stock or welded up steel sheet stock fittings would be used on an experimental airplane, to be replaced by forgings or castings on a production airplane. Choice of the material will also depend upon whether or not a shock load is to be carried.

Machined Fittings.

The direction of the grain is important in most fittings since there is generally a choice. The major axial stresses whether due to axial load or to bending should parallel the grain. Sudden changes in section or sharp corners are to be avoided since these produce stress concentrations which may cause failure in the part. Stress concentration is more important in a brittle material than in one which is ductile, since a ductile material if overloaded locally will yield and transfer load into the remaining material. Stress concentrations can not be entirely eliminated but they may be held to a minimum by careful design. As an example let us consider the part shown in Fig. 4, taken from Timoshenko "Strength of Materials", Vol. II. The maximum stresses occur at points "m" and "n" and may be represented by the equation $f_{max.} = K \times f_o$ where f_o is the uniform tensile stress at the end of the narrow portion and K is a factor depending on the ratios b/a and r/a. Typical values are given in the table below taken from the above reference, page 618.

b/a	r/a =				
	1/8	3/16	1/4	3/8	1/2
6	2.7	2.3	2.0	1.8	1.6
3	2.7	2.3	2.0	1.8	1.6
2½	2.6	2.2	1.9	1.7	1.5
2	2.3	2.0	1.7	1.5	1.4
1½	2.0	1.8	1.6	1.4	1.3

Stress concentrations are also present when parts are notched or have holes drilled out of solid sections. The use of even small radii as relief reduce the stress concentrations considerably.

Forgings.

Forgings have a decided advantage over parts machined from bar stock in that the direction of grain may be more flexibly controlled. In general the same considerations apply to parts made from forgings as to those machined from plate or bar stock. Certain of the Aluminum Alloy forging materials must be handled with care, in particular 14ST. This material has a wide difference in the elongations and fatigue resistances with and across the grain and should be used with extreme caution where any appreciable loads will be carried across the grain.

Castings.

Because of the difference between physical properties of actual castings and parts cast from the same melts in the form of test bars it is necessary to reduce the physical properties of castings by one-half for use in design. This allowance of 100% margin is also depended upon to cover blowholes, shrinkage, etc. in a small degree. Castings should be used with caution where shock loads are expected because they are susceptible to fracture.

Eccentricities.

Eccentricities should be avoided wherever possible, but where they do exist should be accounted for in design. In general this consists merely of designing certain component parts for end load plus a bending load.

Fatigue.

The normal working load on any part (the load to which it will be subjected steadily over long periods of time) should never produce a unit stress in excess of the fatigue limit of the material. Fatigue limits are expressed in terms of the unit stress at which fatigue failure will occur when 300,000,000 cycles of completely reversed stress are applied in bending or 20,000,000 cycles of completely reversed stress are applied in torsion.

AIRPLANE
STRESS
ANALYSIS

PART EIGHT

ADV. STRESS 1

ADV. STRESS 2

ADV. STRESS 3

ADV. STRESS 4

ADV. STRESS 5

ADVANCED STRESS ANALYSIS

ASSIGNMENT 1

ASSIGNMENT 1

ADVANCED AIRPLANE STRESS ANALYSIS

Introduction.

You have now completed the basic stress analysis study material. Beginning with this lesson, we shall deal with problems intended to prepare the student for responsible analysis work in the industry. So that we may thoroughly be in accord in later discussions, we shall first carefully review the fundamentals required.

The methods presented in the Advanced Analysis Course will be based upon the requirements and accepted practices of the Civil Aeronautics Authority, the controlling regulatory body for commercial aircraft. It will be to the student's advantage to supply himself with copies of Bulletins 6-CFR-04 and ACM-04 of the Civil Aeronautics Authority. These may be obtained upon request to the United States Government Printing Office, Washington, D.C.

The examples used will be hypothetical but will closely parallel practical applications on modern commercial airplanes. No reference will be made to Army and Navy work, since the analysis practices of these agencies are held confidential. The student, however, may be assured that the fundamental methods presented here will ground him thoroughly in the work, so that, as the opportunity presents itself to him to work on Army or Navy models, he will have no trouble in adapting himself to the special requirements involved. On certain points the Army, Navy, and Civil Aeronautics Authority are already in exact agreement. For example, Army, Navy Commerce Bulletin 1 (ANC-1) giving span distribution of air loads on wings, and ANC-5 the strengths of materials handbook are equally applicable to all types of airplanes. These Bulletins may be procured from ~~the Superintendent of Documents, Washington, D. C. Prices are:~~ JOHN ROBY, SAN DIEGO CALIF,

ANC-1 ....
ANC-5 ....

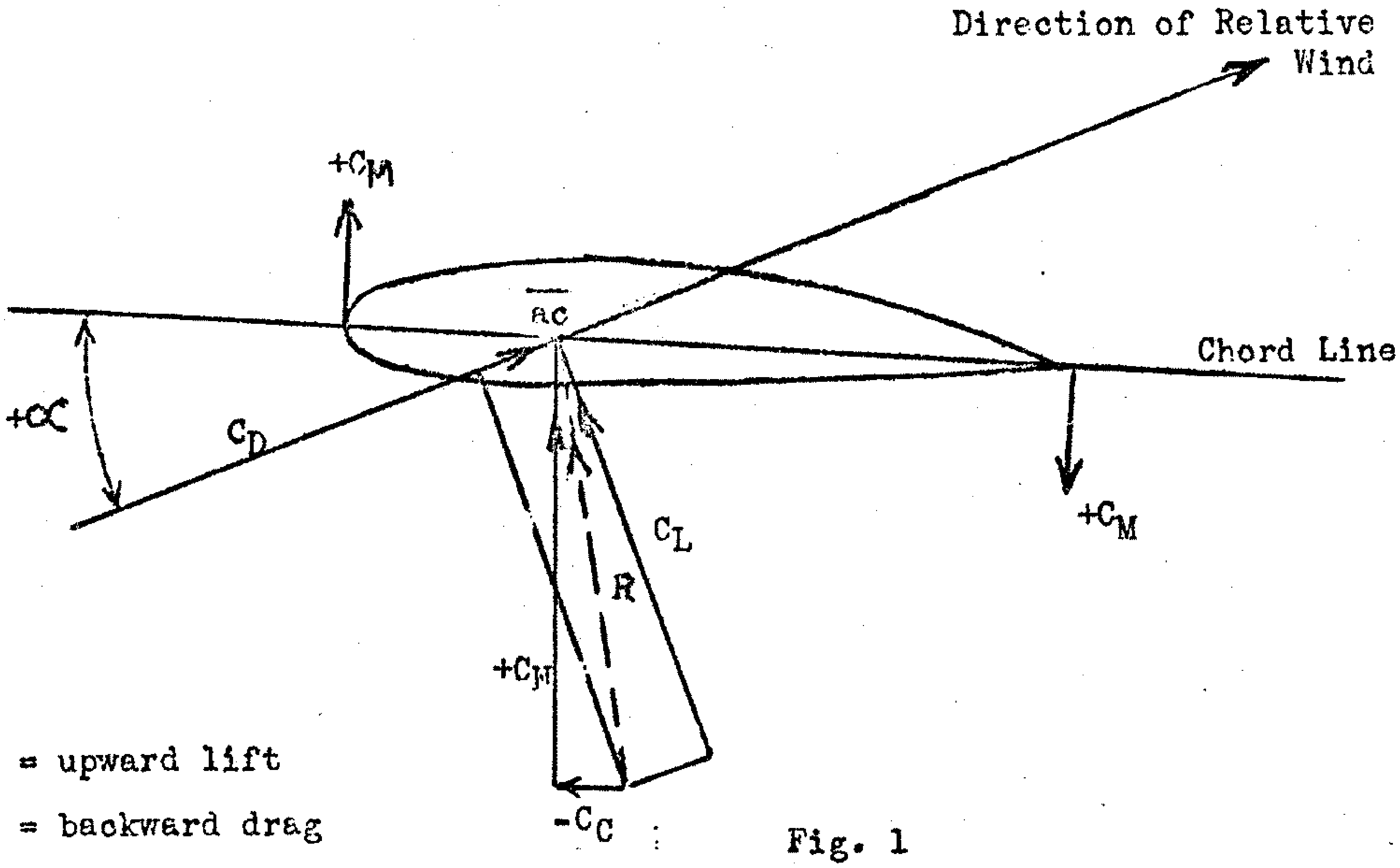
The list of recommended reference bulletins is as follows:

6CFR-04	ANC-1
ACM-04	ANC-5

It is advised that the student who has not already done so, procure the Bulletins as soon as possible, in order to obtain the greatest value from the course.

Basic Aerodynamic Theory.

Before we may proceed to the calculation of loads on an airplane structure we must know something about aerodynamic loads. A brief resume of the basic aerodynamic theory will be given here. Forces on an airplane in flight are generated by the relative movement between the air and the airplane, by the tractive forces set up by the propeller and by inertia forces due to gravity and to accelerations and decelerations set up by the air loads and the propeller.



+ C_L = upward lift

+ C_D = backward drag

+ C_M = a moment tending to increase the angle of attack.

Fig. 1

Aerodynamic Forces on an Airfoil

The aerodynamic theory is based on the premise that air is a low density fluid, and follows the laws of fluids, for all practical purposes.

Bernoulli's Theorem.

Bernoulli's Theorem states that the total energy of a fluid particle is constant at all points on its path in a steady flow. The equation is

$$\frac{P}{gM} + \frac{1}{2} MV^2 + h = K \text{ (a constant), where}$$

$\frac{P}{gM}$ is the pressure head

$\frac{1}{2} MV^2$ is the velocity head

h is the potential head, due to position

In its original form the equation was intended to apply only to steady flow in an incompressible fluid. It may be applied to solution of aerodynamic problems also if each term is given its proper relative significance.

Pressure Head.

In general the airplane is moving through the medium in such a fashion that the pressure in the undisturbed air ahead of it is the same for the portion which passes under the wings as for the portion which passes over them. The pressure head therefore is of minor importance in providing a lifting force on the airplane.

Potential Head.

The same argument may be advanced as for the pressure head to show that this item is of no great importance.

Velocity Head or Dynamic Pressure.

This item constitutes practically all of the energy available in the stream of air moving past the wings and from it must come the forces which counteract the effect of gravity in flight. The dynamic pressure is designated as "q".

$$q = \frac{1}{2} MV^2$$

where M = mass of air
V = velocity in fps.

For standard air at sea level this formula may be reduced as follows:

Wt. per cubic foot of air at sea level = $\frac{1}{13}$ lb.

$M = \text{Mass} = \frac{w}{g} = \frac{1/13}{32.2} = .00238$

Then $q = \frac{1}{2} \times .00238 \times V^2 = .00119V^2$, ($V$ in fps.)

1 Mile per hour = 1.468 ft. per sec.

When V is given in miles per hour

$$q = (.00119)(1.468V)^2 = .00256V^2 \quad (V \text{ in mph})$$

The dynamic pressure at any other altitude is given by

$$q = .00119V^2\sigma \text{ , where } \sigma = \frac{P \text{ altitude}}{P \text{ sea level}} \quad (V \text{ in fps, } P = \text{density})$$

The purpose in using an "airfoil section" for a wing is to provide a lifting surface that will provide a maximum of lift with a minimum of drag. The drag forces are those which slow up the airplane. For purposes of comparing airfoils and for use in load and performance data the properties of an airfoil are reduced to the following, expressed as coefficients:

Lift = a measure of the supporting force available
Drag = a measure of the resistance to forward motion
Moment = a measure of the twisting force on the section

All of the values are obtained with respect to the Angle of Attack, α. See Fig. 1.

Definitions.

The Chord Line of a wing section is an arbitrary reference line chosen for convenience in locating the airfoil attitude. See Fig. 1. It is adopted for convenience in reference and not because it has any particular significance with regard to the airfoil characteristics.

The Angle of Attack, α, of the airfoil section is the angle between the Chord Line and the direction of the relative wind. See Fig. 1.

C_L = the Lift Coefficient measured perpendicular to the relative wind.

C_D = the Drag Coefficient measured parallel to the relative wind.

Forces on wing sections are determined in Wind Tunnel tests. The values so obtained are reduced to terms of lift and drag per sq. ft. of wing at unit speed. The lift per square foot of wing at any speed and angle of attack is $C_L q$ and the drag is $C_D q$.

(Note: An efficient airfoil might appear to obtain more lift per sq. ft. of surface than the formula $\frac{1}{2}MV^2$ shows to be available. This could only be considered true, however, if only the first cubic foot of air adjacent to each square foot of wing were considered effective. Actually a greater amount of air is affected; according to Prandtl the volume of air affected when a wing section moves through it is a cylinder whose diameter is equal to the wing span. The air is also deflected downward, thus giving up potential energy.)

The Moment Coefficient, C_M, is a convenient term used in locating the center of pressure (centroid of air loads) in a chordwise (fore and aft) direction along a section of wing. It is given with reference to some particular station along the chord and is the measure of the moment of the air forces about that station, in terms of C_L and C_D, divided by the wing chord. It may be thought of as a couple acting at the leading and trailing edges and balancing the moments in a chordwise direction along a section of the wing.

The standard airfoil characteristics may be obtained for most airfoils from Technical Reports and Technical Notes published by the National Advisory Committee for Aircraft. When a new airfoil is developed its characteristics are obtained from wind tunnel tests. The standard test section is one for which the chord is held constant and the aspect ratio is 6.0.

$$\text{The Aspect Ratio} = \frac{(\text{span})^2}{\text{Area}}$$

The usual modern airplane wing is tapered both in plan form and in thickness ratio, a scheme which gives a more efficient lifting surface both for performance and for structure. Also the Aspect Ratio is generally greater than 6.0 on a monoplane design. The standard wind tunnel data must then be corrected by the method outlined in Air Commerce Manual 04.1-11. Given the airfoil characteristics for an Aspect Ratio of 6.0 (R is used to denote actual aspect ratio existing)

K = Correction Factor = $\frac{1}{R} - 0.1667$

$$\alpha = \alpha_6 + 18.24KC_L$$

where α_6 = angle of attack in degrees for a given C_L when aspect ratio = 6.0

α = angle of attack in degrees for the same C_L when the Aspect Ratio is R.

$$C_D = C_{D_6} + 0.318KC_L^2$$

where C_{D_6} = C_D for given C_L when aspect ratio = 6

C_D = C_D for given C_L when aspect ratio = R

m = slope of the lift curve = Increment of C_L per radian

$$m = m_6\left[\frac{4}{3 + \frac{6}{R}}\right]$$ where

m_6 = slope of the lift curve when Aspect Ratio = 6

m = slope of the lift curve when Aspect Ratio = R

****

For our type problem we will assume a wing tapered from a chord of 180 inches at the root to 90 inches at the tip and taper the thickness ratio from 18% at the root to 9% at the tip.

The thickness ratio is the ratio of the maximum depth of a wing section to its chord. We will choose an airfoil of the NACA 2200 series which has good all around characteristics. The characteristics of this airfoil for an Aspect Ratio of 6.0 may be taken from NACA Technical Note No. 487. The plan-form of the wing we are to use is shown on Fig. 2. We must correct the NACA data for our actual aspect ratio.

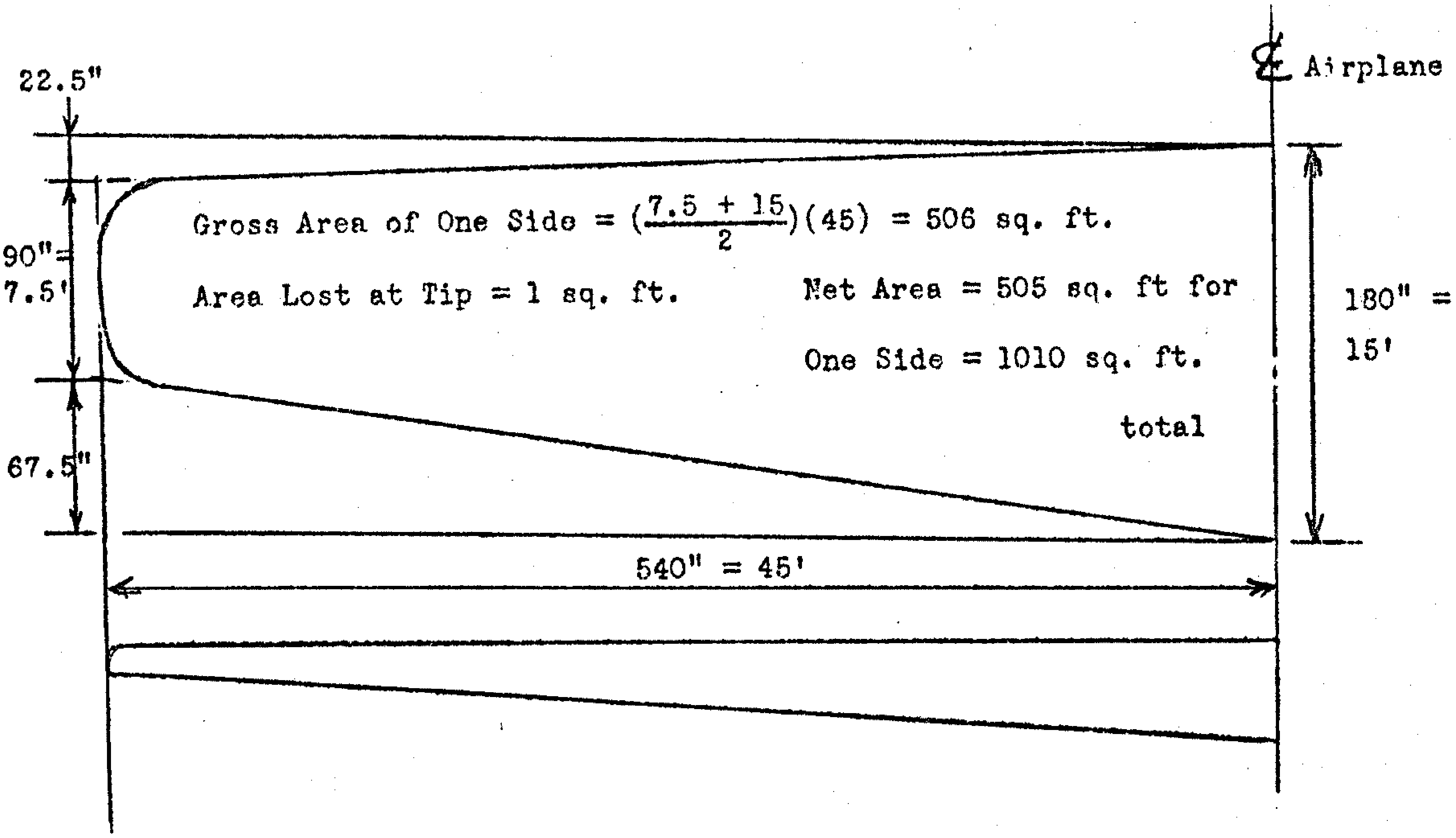

22.5"
℄ Airplane
90" = 7.5'
67.5"
180" = 15'
540" = 45'
Gross Area of One Side = $(\frac{7.5 + 15}{2})(45)$ = 506 sq. ft.
Area Lost at Tip = 1 sq. ft.
Net Area = 505 sq. ft for One Side = 1010 sq. ft. total

Fig. 2

Monoplane Wing - Semi-Span

$$\text{Aspect Ratio} = \frac{(90)^2}{1010} = 8.0 \quad \longleftarrow \text{R}$$

The airfoil characteristics for aspect ratio of 6.0 and the taper ratios shown here are plotted on NACA Technical Note 487, Fig. 4. The data given there will be reduced to tabular form for use in Table I, page .1-12 of ACM04. Full directions for obtaining the correct values for the Aspect Ratio of 8 are given on the table. The student who wishes to go further into application of the aerodynamic theory is referred to Diehl, "Engineering Aerodynamics," The Ronald Press Company, New York City.

TABLE I

COMPUTATION OF AIRFOIL CHARACTERISTICS

1	C_L	-1.0	-.8	-.6	-.4	-.2	0	.2	.4	.6	.8	1.0	1.2	1.4	1.6
2	α_6	-16.2	-13.4	-10.6	-7.9	-5.1	-2.3	.4	3.2	6.0	8.8	11.5	14.3	17.0	19.8
3	$\Delta\alpha$ 18.24 KC_L														
4	$\alpha = (2) + (3)$														
5	C_{D_6}	.076	.050	.032	.020	.013	.010	.012	.018	.027	.045	.068	.094	.126	.158
6	$\Delta C_{D_i} = .318KC_L^2$														
7	$C_D = (5) + (6)$														
8	$\cos \alpha$														
9	$\sin \alpha$														
10	$C_L \cos \alpha$														
11	$C_D \sin \alpha$														
12	$C_N = (10) + (11)$														
13	$C_L \sin \alpha$														
14	$C_D \cos \alpha$														
15	$C_C = (14) - (13)$														
16	C_M abt .25 chd.	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029
17	$\overline{CP} = .25 - \frac{C_M}{C_N}$														

$R = 8.0$ $\quad K = 1/8 - 1/6 = -.0417$ $\quad C.P. = .25 - \frac{C_M}{C_N}$

Problem Assignment.

1:1 Complete Table I to give the complete airfoil data for the wing of Fig. 2 with Aspect Ratio = 8.0.

1:2 Plot the corrected characteristics against the value C_L as shown on Fig. 11, page 1-10 of ACM-04. C_N, C_C, α , C_M and $\overline{CP}$ curves should be plotted.

1:3 Plot values of C_L, and C_D corrected against corrected values of α for reference in future assignments. Values will be taken from the solution of problem 1:1 on Table I.

ADVANCED STRESS ANALYSIS

ASSIGNMENT 2

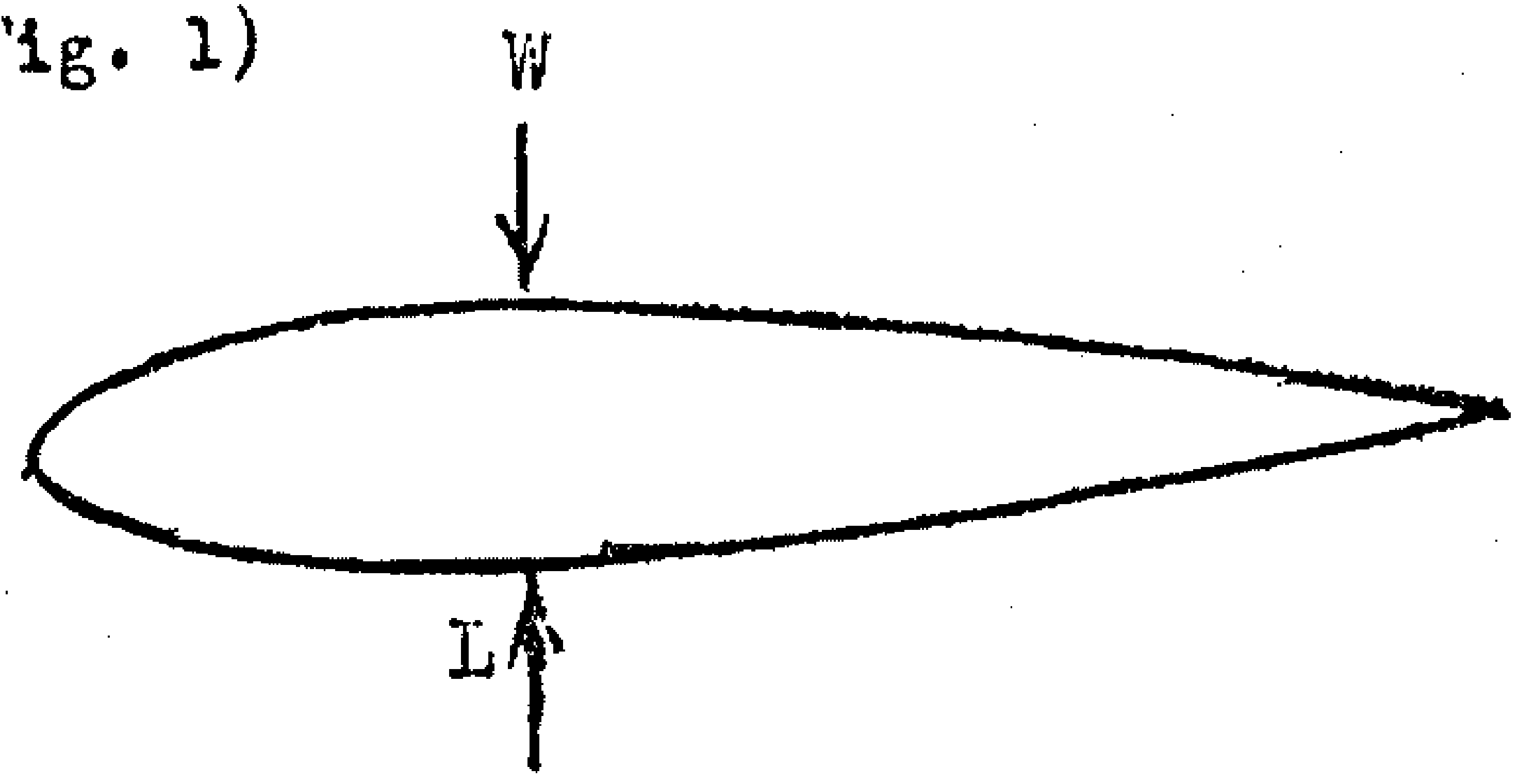
Assignment 2

FLIGHT LOAD FACTORS ON WING WITH WING FLAPS RETRACTED

When we speak of a flight load factor for commercial airplanes we are referring to a wing load factor as differentiated from an inertia or dead weight load factor. The load factor is that multiplying factor by which the steady flight forces are multiplied to obtain the equivalent static effect of dynamic forces acting during acceleration of the airplane.

Dynamic Forces.

Consider the loads on the wing sustaining an airplane in unaccelerated flight and neglecting for the moment the effect of load on the tail surfaces. (See Fig. 1)



L = Wing Lift

W = Gross Weight of Plane

$L = W$

Fig. 1

Now suppose the lift force is increased on the plane to some value nL. The plane will tend to accelerate upward. The inertia effect of the weight, W, will be added to its actual weight. If "a" equals the acceleration produced, the additional force $F = Ma$, and since $M = \frac{W}{g}$, $F = \frac{Wa}{g}$. The total effect of the weight sustained is therefore $W + \frac{Wa}{g}$ or $W\left(1 + \frac{a}{g}\right)$

$$nL = W\left(1 + \frac{a}{g}\right) \quad \text{or} \quad n = \left(1 + \frac{a}{g}\right)$$

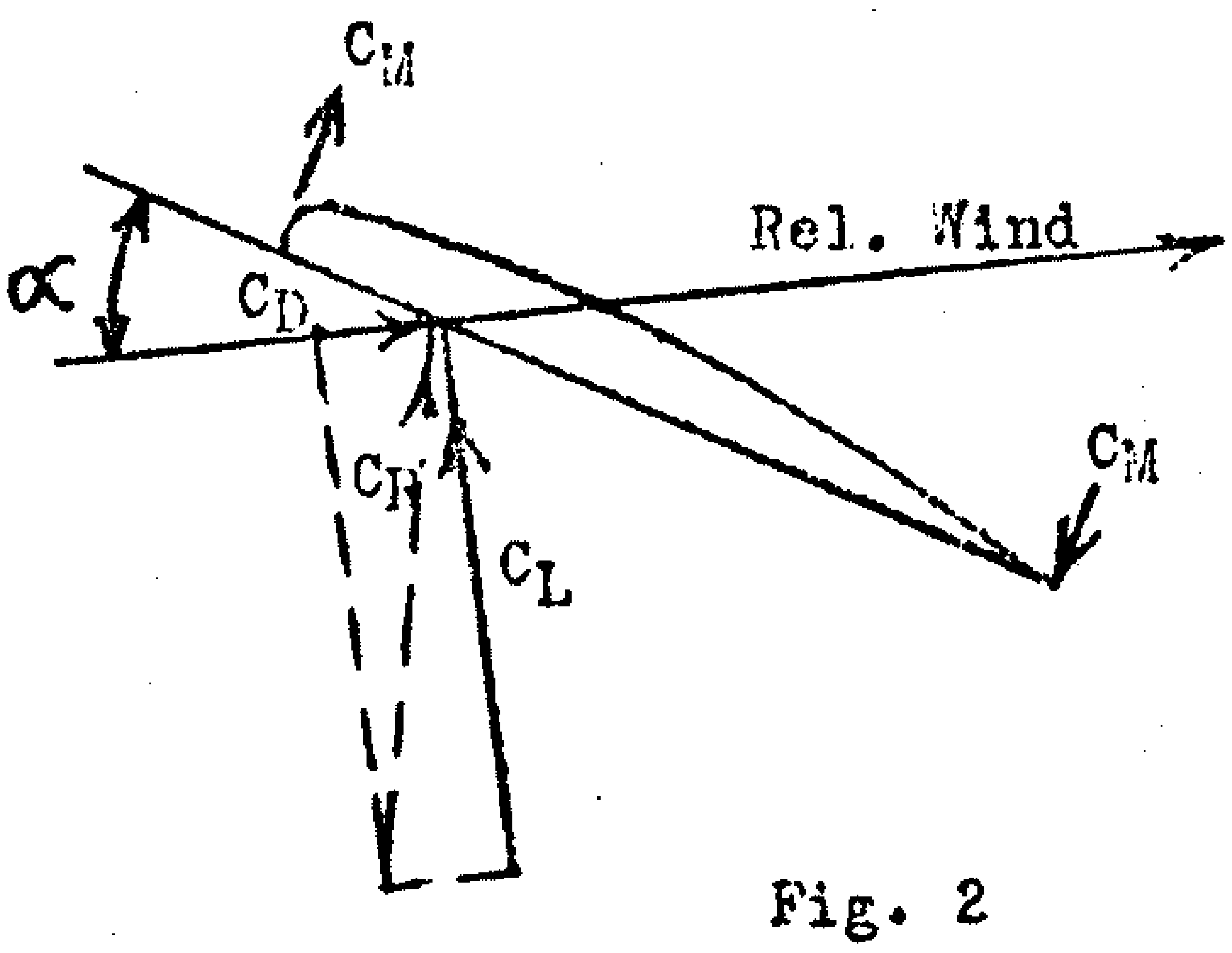
n is the applied load factor for the wing and is defined as the maximum load factor which is to be expected in any normal operation. An airplane should therefore be designed so that the applied loads do not exceed the yield point of the structure.

The factor of safety is introduced to give a material safety factor and to provide some factor of ignorance.

$$n' = \text{design factor} = \text{F.S.} \times n$$

Airplanes are designed to their ultimate strength for the load factor, n'. The usual factor of safety is 1.5.

Load factors are introduced into airplanes either from the effect of maneuvering (controlled by the pilot), or from the effects of gusts in rough air. Basically the results produced are the same, namely to change the attitude of the plane suddenly without decreasing its speed to correspond to the changed attitude. Let us consider the forces acting on an airplane wing in flight. (Fig. 2)



Resultant force vector, $R = C_R S q$

C_R = Coefficient

S = Wing Area

q = Dynamic pressure = $.00119V^2$

$(\sigma = \frac{P\ \text{altitude}}{P\ \text{sea level}})$, V = fps

σ = 1.0 at sea level

Fig. 2

Resolving these forces parallel and perpendicular to the relative wind:

C_L = Lift coefficient + = upward

C_D = Drag coefficient + = backward

$$C_R = (C_L{}^2 + C_D{}^2)^{\frac{1}{2}}$$

If the beams and drag truss are not parallel and perpendicular to the wing chord (Fig. 3) the resultant load, C_R, may be resolved parallel to the beams and the drag truss respectively giving $C_{N'}$ and $C_{C'}$ which are not necessarily perpendicular to each other.

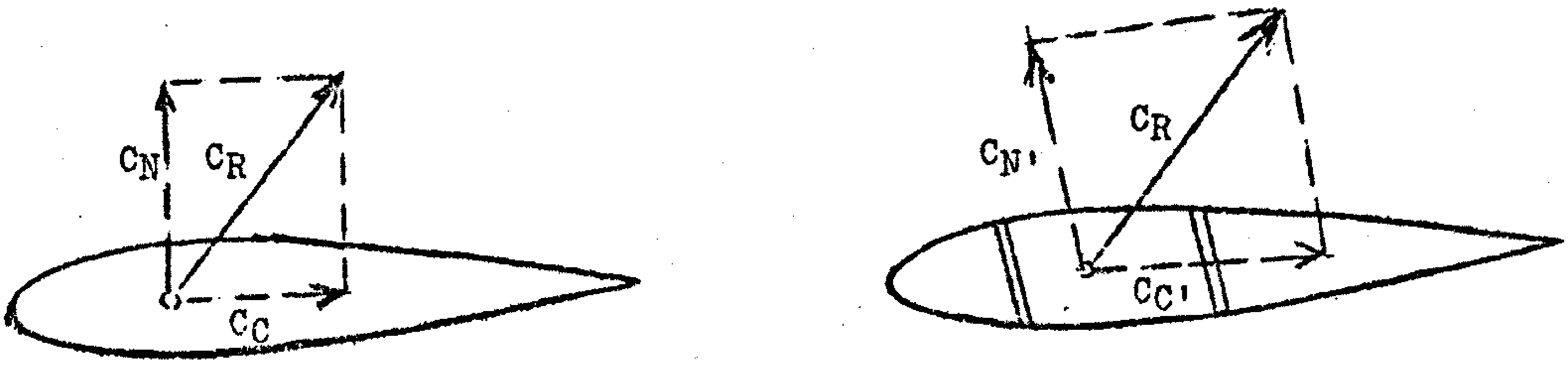


Fig. 3

For design purposes it is assumed satisfactory to use $C_L = C_N = C'_N$.

Position of Lift Force Along Wing Chord.

The total lift on the wing is $C_L Sq$ (or $C_N Sq$ or $C'_N Sq$).

The total twisting moment of the lift forces on a wing $M = C_M Sq \times Ch$ where Ch = wing chord. The moment coefficient, C_M, must be given about some definite location point on the wing chord. Usually it is given about the aerodynamic center. The aerodynamic center (AC) may be defined as that location on the wing chord about which the moment of the air forces is substantially constant;

that is the product of C_N x distance to aerodynamic center in percentage of wing chord = A constant. For any point, "X", along the wing chord then

$$C_{M_X} = C_N\,(X - \overline{CP}) \text{ where } \overline{CP} = \text{center of pressure in percent of chord.}$$

Assuming that X = distance from leading edge to aerodynamic center = .25 then

$$\overline{CP} = .25 - \frac{C_M}{C_N} \text{ where } C_M = \text{mom. coefficient about the .25 of chord point.}$$

(+ C_M shows a moment tending to increase the angle of attack)

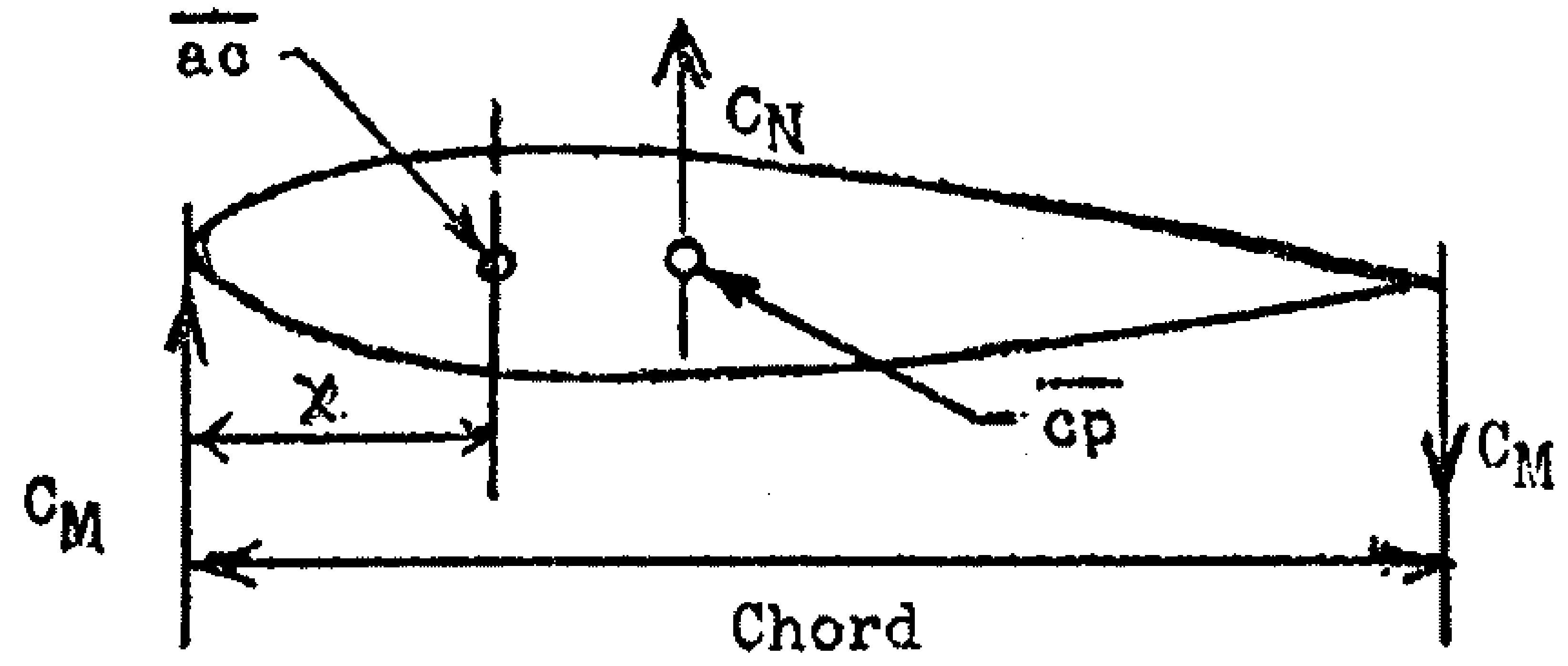


Fig. 4

C_M may also be thought of as the value of a couple at the leading and trailing edges of the wing, tending to cause rotation. It represents the moment introduced when C_N is moved from the $\overline{CP}$ to the $\overline{AC}$.

The method used to determine load factors and the design conditions used will correspond to those given in 6CFR04 and ACM04. The load factors are dependent upon the ratio of weight to wing area, wing loading, ratio of weight to power, power loading, and the limiting speed of the airplane. They also depend upon the aerodynamic coefficients of the airfoil, especially upon the maximum lift coefficient possible. In this respect it should be noted that it is possible to momentarily attain lift coefficients greater than those shown in the wind tunnel since there must be a time lag occurring before the airfoil will burble. For this reason if in calculations of load factors the value of C_N exceeds the maximum lift coefficient for the airfoil the calculated value shall be used.

The following data are required for calculation of the wing and inertia load factors in flight:

1. Design weight of the airplane.
2. Effective wing area and the airfoil section.
3. Rated horsepower of the engines.
4. Design level flight high speed = V_L.
5. Estimated terminal velocity = V_M.
6. Lift, drag and moment curves for the airfoil.

7. Location and length of the mean aerodynamic chord.
8. The center of gravity location for the airplane or the limits of center of gravity travel--fore and aft.
9. The tail length. (Distance from the center of gravity of the airplane to the center of pressure of balancing loads on the tail. The center of pressure of tail loads is assumed at 20% of the mean chord of the entire tail surface)

The above items will be discussed in sequence.

Design Weight.

The design gross weight of the airplane is the maximum weight at which it is licensed to fly. Usually this weight condition will be critical in loading most of the airplane but this is not necessarily the case as will be shown later. Certain portions of the airplane for which the inertia loads remain substantially constant while the airplane as a whole becomes lighter from using up fuel and oil; such as engine mounts, nacelles, and the forward portion of the fuselage may be designed by the effects of gusty air on the airplane when it is flying at a minimum weight. The Minimum Design Weight is defined as the weight empty with standard equipment, plus minimum crew, plus fuel of .25 lb. per maximum (except takeoff) horsepower, plus a full load of oil.

Effective Wing Area.

Based upon wind tunnel tests on complete models the Design Effective Wing Area is taken as that of the complete wing without any deduction for that portion, blanketed by fuselages or nacelles. For these parts the wing outline is taken as though the wing ran straight through.

Rated Horsepower.

The engines are rated in three ways:

1. A maximum rating for continuous operation.
2. A takeoff rating for one minute operation only.
3. An emergency rating for continuous operation when one or more engines has failed. (Applies, of course, to multiengined ships only.)

The wing load factors are calculated on the basis of the maximum rating for continuous operation.

The Mean Aerodynamic Chord is an imaginary chord on which all the wing area may be considered concentrated without changing the lift or moment of the air forces on the airplane.

The Center of Gravity of an airplane is a variable which may travel within the stability limits as determined by actual flight test. It is necessary in design of the airplane to use limiting locations determined from the wind tunnel tests. Generally these limits are arbitrarily extended in both directions to insure that the full range determined from the flight tests will be covered by the analysis.

Calculation of Design Speeds.

V_L = high speed of the airplane in level flight corresponding to the maximum rated power available for continuous operation reduced to equivalent "indicated" air speed at sea level in standard air (fps). Note: Indicated and actual speeds are the same at sea level. The use of indicated speeds allows the use of higher speeds relative to the ground when flying at altitude.

V_G = design gliding speed with power off ("indicated" - fps). V_G shall not be less than $V_L + K_g\ (V_M - V_L)$, except that it need not be greater than either V_L + 100 mph or 1.5 V_L whichever is the lower.

$$K_g = 0.03 + \frac{1350}{W + 3000}$$ except that K_g shall not be less than 0.15.

V_M, the terminal velocity with power off is estimated from the formula

$$V_M = 29\ (d)^{\frac{1}{2}} \text{ where}$$

$$d = \frac{W}{A_D} \text{ and } A_D = \text{estimated total drag area in sq. ft.}$$

BASIC ACCELERATED FLIGHT CONDITIONS

(With normal wing - Flaps Up.)

The following symbols are used:

s = effective wing loading $= \frac{W}{S} = \frac{\text{Design Weight}}{\text{Wing Area}}$ = lbs. per sq. ft.

P = power loading $= \frac{W}{P}$ = lbs. per H.P.

R = aspect ratio $= \frac{(\text{Wing span})^2}{\text{Area}}$

q = dynamic pressure $= .00119V^2$ when V in fps

$$= \frac{V^2}{391} \text{ when } V = \text{MPH}$$

m = slope of lift curve, C_L per radian, corrected for aspect ratio.

The flight load factors which follow are wing load factors. Dead weight or inertia load factors must be arrived at by considering the effect of balancing loads on the tail. All load factors calculated are applied or limit load factors. That is, the structure must not yield when these factors are applied.

The limit maneuvering factors are derived largely from experience with conventional types of airplanes and are considered minimum allowable values.

The limit gust load factors are considered the results of the airplane going into sharp edged up or down air currents acting normal to the flight path. To more fully appreciate these effects it is necessary to consider the effect of changes in angle of attack on the lift coefficient, C_L. In the design range of angles of attack C_L versus the angle of attack, α is a straight line variation (Fig. 5). The effect of a sudden change in angle of attack without any change in speed (at least momentarily) is to increase the total air load in the wing by the ratio of the respective values of C_L. (In this case $\frac{C_L \text{ at } B}{C_L \text{ at } A}$ in Fig. 5.) C_L at A represents the lift coefficient necessary to maintain level flight at the speed V. C_L at B represents the forces on the wing after a sharp edged gust, KU has suddenly changed, the angle of attack by $\Delta\alpha$, without any slackening of speed. Since for small angles, the angle is equal to the tangent

$$\Delta\alpha = \tan^{-1}\frac{KU}{V}$$

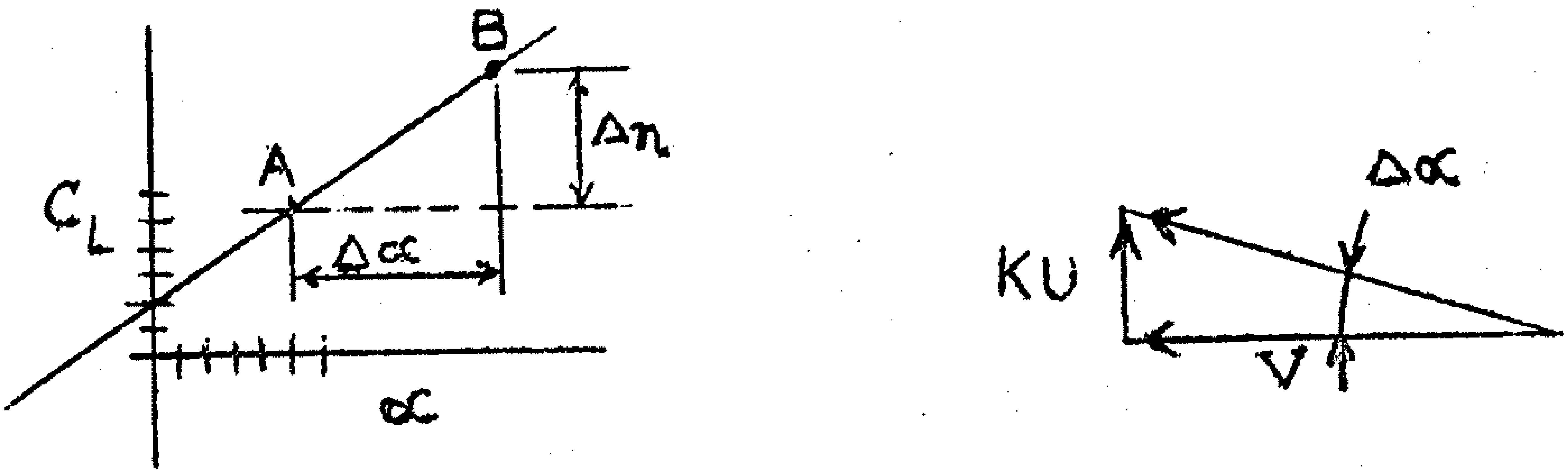


Fig. 5

Δn in Fig. 5 represents the increment of load due to the change, $\Delta\alpha$, in angle of attack. From 6CFR04

$$\Delta n = \frac{KUV}{575}\frac{m}{s} \quad \text{where}$$

Δn = limit (or applied) load factor increment.

K = gust reduction factor = $\frac{1}{2}(s)^{1/4}$ but need not exceed 1.0. (Note: above s = 16, K = 1.0)

U = gust velocity in feet per second.

V = indicated airspeed in miles per hour.

FLIGHT CONDITIONS

Condition I - Positive High Angle of Attack.

1. $n_1 = 1 + \Delta n_1$ (shall not be less than 2.50) where Δn_1 is the higher value determined from

   (a) $\Delta n_{1_a} = \dfrac{KU\,V_L}{575\,s} \times (m)$, where U = 30 fps up gust.

   "Power loading need not be less than 12"

   (b) $\Delta n_{1_b} = \left[0.77 + \dfrac{32{,}000}{W + 9200}\right]\left[\dfrac{3.25}{\rho^{.435}}\right]$ (ρ = power loading)

2. $C_{N_1} = \dfrac{n_1 s}{q_L}$ $\left(C_{N_1} = \dfrac{n_1 W}{Sq}\right.$ but $\dfrac{W}{S} = s$ Therefore $\left.C_{N_1} = \dfrac{n_1 s}{q_L}\right)$

   q_L = dynamic pressure corresponding to speed V_L.

3. C_c' = value of drag coefficient corresponding to C_{N_1} or $-0.20\ C_N$, whichever has the greater negative value.

4. $\overline{C.P.}'$ = most forward position of the center of pressure between $C_L = C_{N_1}$ and C_L max.; when C_{N_1} exceeds C_L max. the $\overline{CP}$ curve shall be extended accordingly.

5. C_M' = moment coefficient necessary to give the required $\overline{CP}$ in conjunction with C_{N_1}.

Condition I_1 - Positive High Angle of Attack - Modified.

The smaller of the two values of C_c specified in Condition I, and the most rearward $\overline{CP}$ position between $C_L = C_{N_1}$ and C_L max. shall also be investigated when Condition I is critical for the rear spar (or its equivalent) or if any portion of the front spar (or its equivalent) is likely to be critical in tension. Only the wings and wing bracing need be investigated for this condition. (Balancing loads on the tail are not required and their effect on the fuselage is not considered.)

Condition II - Negative High Angle of Attack (30 fps down gust)

1. $n_2 = 1 - \Delta n_{1_a}$ where Δn_{1_a} is taken from Condition I.

2. $C_{N_2} = \dfrac{n_2 s}{q_L}$

3. C_c = actual value corresponding to C_{N_2}. Note: When C_c is positive or has a negative value smaller than 0.02, it may be assumed zero.

4. C_M = actual value corresponding to C_{N_2}.

Condition III - Positive Low Angle of Attack

1. $n_3 = 1 + \Delta n_3$ (shall not be less than 2.0) where Δn_3 is the higher value obtained from the following equations:

   (a) $\Delta n_{3_a} = \dfrac{KUV_G}{575\,s} \times (m)$

   U = 15 fps upward

   V_G = glide speed used in design

   (b) $\Delta n_{3_b} = 0.60\,\Delta n_{1_b}$ as determined in Condition I.

2. $C_{N_3} = \dfrac{n_3 s}{q_G}$; q_G = dynamic pressure corresponding to flight speed V_G.

3. C_c = actual value corresponding to C_{n_3}. (When C_c is positive or has a negative value smaller than 0.02 it may be assumed zero.)

4. C_M = the actual value corresponding to C_{N_3}.

Condition III_1 - Positive Low Angle of Attack - Modified

If the moment coefficient of the airfoil section at zero lift has a positive value or a negative value smaller than 0.06, the effects of displaced ailerons on the moment coefficient shall be accounted for in Condition III for that portion of the span incorporating ailerons. To cover this point it will be satisfactory to combine 75 per cent of the loads acting in Condition III with the loads due to a moment coefficient of $-0.08 - C_{M_{III}}$ acting over that portion only of the span incorporating ailerons. The design dynamic pressure for the additional moment forces shall be equal to $0.75q_G$. Only the wings and wing bracing need be investigated for this condition.

Condition IV - Negative Low Angle of Attack

1. $n_4 = 1 -\quad n_{3_a}$ where $\quad n_{3_a}$ is taken from Condition III.
2. $C_{N_4} = \dfrac{n_4 s}{q_G}$
3. C_c = actual value corresponding to C_{N_4}.
4. When C_c is positive or has a negative value smaller than 0.02 it may be assumed to be zero.
5. C_M = the actual value corresponding to C_{N_4}.

The foregoing conditions are the principal ones which must be investigated for the airplane, when flying with flaps up. Other special conditions and the effect of flaps down will be treated in the next assignment.

Problem Assignment.

2:1 Using the airfoil data from the curves of Problem 1:2 in Assignment 1 and the following additional data:

1. Gross weight of airplane = 20,000 lbs.
2. Wing area = 1010 sq. ft.
3. Normal rated power at sea level = 2000 (2-engines of twin-motored airplane)
4. Level flight speed at sea level = 220 mph.
5. Estimated terminal velocity = V_M = 600 mph.
6. Value of m, the slope of the lift curve to be taken from the curves of Problem 1:2.

$$m = \frac{C_N}{\qquad}, \text{ where } \quad \text{ is in radians.}$$

Note: "m" in the design range should be 4.4 approximately.
1 radian = 57.3 degrees.

Calculate the values for V_G, q_L, q_G and n, C_N, C_c or C'_c, $\overline{CP}$ and C_M or C'_M for Conditions I, I_1, II, III, III_1, and IV.

ADVANCED STRESS ANALYSIS

ASSIGNMENT 3

Assignment 3

FLIGHT LOAD FACTORS

There are certain special conditions of balanced flight with flaps up, which are important only on special types of airplanes. These will be given briefly for reference.

Condition V - Inverted Flight

This condition applies only to the rear (or single) lift truss system of externally braced wing structures; i.e. those braced by external struts or wires.

1. $N_5 = -1.0 - \Delta n_5$ (Min. value = -1.50)

   Where Δn_5 is the higher of the two values

   $\Delta n_{5_a} = 0.50\ \Delta n_{1_a}$ (From Condition I)

   $\Delta n_{5_b} = 0.25\ \Delta n_{1_b}$ (From Condition I)

2. $C_{N_5} = \frac{n_5 s}{q_L}$ (Speed = V_L)

3. $C'_c = 0$

4. $\overline{CP} = 25\%$

The purpose of this requirement is to insure a rigid external bracing.

Condition VI - (Gliding - At limited diving speed, V_g)

1. C_{N_6} = Value corresponding to maximum + C_c
2. C'_c = Maximum positive C_c + 0.01
3. C_M = the actual value corresponding to C_{N_6}
4. The drag of the nacelles and other items attached to the wings must be conservatively estimated and included in the investigation of this condition.
5. Only the wings and wing bracing need be investigated for this condition.

All of the foregoing conditions, and those in Assignment 2, have been assumed to occur without wing flaps or other auxiliary lift devices in operation.

Symmetrical Flight Conditions - With Flaps Down

When flaps or similar high lift devices are used on wings, the design conditions must be modified to account for their use in flight. These modifications are based on the intended use of the flaps and the aerodynamic characteristics of the wing. The following conditions are considered the minimum required to cover a suitable range of symmetrical flight loadings in cases where the wing flaps are used only at relatively low air speeds, that is at speeds reduced for landing.

Condition VII - Positive Gust With Flaps Down

1. The design speed for this condition is V_f which is the restricted speed with flaps down. V_f may not be less than 2 V_{sf}, where V_{sf} is the stalling speed of the airplane with flaps down.

$$V_{s_f} = \left[\frac{391W}{C_{L_{Max.}} S}\right]^{\frac{1}{2}}$$

where W = gross weight of airplane
S = total wing area
$C_{L_{Max.}}$ = Max. effective lift coefficient with flaps down.
V_{s_f} in this formula is in miles per hour.

2. Gust velocity, $U = 15$ fps.

3. $n_7 = 1 + \Delta n_7$ (Min. value = 2.0)

$$\Delta n_7 = \frac{KUV_f m}{575 \frac{W}{S}}$$

4. $$C_{N_7} = \frac{n_7 (W/S)}{qV_f}$$

5. C_c = actual value corresponding to C_{n_7}

6. C_M = actual value corresponding to C_{n_7}

Note: The most critical deflection of the flap must be investigated, generally maximum throw.

Values given are for the entire wing and must be redistributed to flapped and unflapped portions.

Condition VIII - Negative Gust With Flaps Down

1. Speed $= V_f$ Gust speed = -15 fps.
2. $n_8 = 1 - \quad n_7$ (With no lower limiting value)
3. $C_{N_8} = \dfrac{n_8\ (W/S)}{q_{V_f}}$
4. C_c and C_m correspond to the values for C_{N_8}

Condition IX - Dive With Flaps Down

This condition is a check for the maximum rearward chord loads on the wing structure. The maximum deflection of the flap is used, and the load factor and the magnitude and distribution of normal, chord and moment forces over the wing correspond to the angle of attack at which the greatest rearward chord loads are produced.

1. Speed $= V_f$ No gust factor.
2. C_{c_9} = maximum rearward chord coefficient with flaps down.
3. C_{N_9} = value of C_N corresponding to C_{c_9}
4. $n_9 = \dfrac{C_{N_9}\ q_{V_f}}{W/S}$
5. C_{M_9} = value corresponding to C_{N_9}

Only the wings and wing bracing need be investigated for this condition. That is, balancing loads on the tail and their effect on the fuselage need not be computed.

Unsymmetrical Flight Conditions

In the unsymmetrical flight conditions the unbalanced rolling moment of the airplane is to be assumed to be resisting by angular inertia of the entire airplane. (The angular inertia of the airplane is a measure of its resistance to "wing over" rotation. A given rotating force will accelerate a compact airplane with most of the weight in the fuselage, faster than one which has most of its weight in the wings.) If it is not desired to make complete calculations for the angular inertia of the complete airplane it is acceptable and conservative to consider that only major items in the wing such as nacelles, landing gear or fuel in tanks are effective in resisting rotating acceleration, and that the remaining twist is resisted by the fuselage. The unsymmetrical air forces which produce rotation are assumed as follows:

Condition Iu - Unsymmetrical - High Angle Gust

Condition I of Assignment 2 is modified to assume 100% of the air load to be acting on one side and 40% on the other. For airplanes over 1000 lbs. gross weight the latter factor may be increased linearly with gross weight up to 80% at 25,000 lbs.

Condition IIIu - Unsymmetrical - Low Angle Gust

Condition III of Assignment 2 is modified as described for Condition Iu.

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Up to this point all the design conditions have been based on the gross weight of the airplane. It is also necessary to investigate certain portions of the structure, which carry fixed weights, for the effect of gusts when the airplane is flying light; since under light load conditions the effect of gusts is more severe. (Note: the increment of load factor due to gust is inversely proportional to the wing loading.) The light load conditions in particular design engine mounts and the front portions of fuselages. On large airplanes where most of the fuel is carried in the wings it is generally necessary to investigate the fuselage for some intermediate weight conditions in which the fuselage is carrying its maximum load, while the airplane as a whole is not up to gross weight because fuel has been used up.

The minimum design weight (which gives the highest possible gust load factors) is defined as the weight empty of the airplane with standard equipment, plus crew, plus fuel of .25 lbs. per normal rated horsepower, plus a full load of oil.

Wing Tanks Empty

If fuel tanks are supported by the wing structure such structure and its bracing must be investigated for Conditions I, II, III, and IV with wing tanks empty. The design weight may be reduced by 0.9 lb. per normal rated horsepower. This means that the load factors originally determined for Conditions I, II, III, and IV at gross weight must be recomputed at a reduced weight. The purpose of this requirement is rather vague unless one investigates the regulations covering the requirements for minimum fuel load. In 6CFR 04.740 we find that an airplane which is not designed for airlines service may receive a rating at a gross weight which contains full payload and at least .15 gal. (.90 lb.) of fuel per maximum (except takeoff) rated horsepower. The weight of gasoline is 6 lbs. per gallon. The purpose of this condition is then to investigate the loads on the airplane when it is flying with full payload but with wing tanks practically empty. It fits an airplane which just meets the minimum requirement for fuel. In all cases where the amount of fuel which may be carried with full payload exceeds the requirement of .90 lbs. per horsepower the reduction in weight may be taken equal to the weight of the fuel which may be carried simultaneously with full payload. The importance of the location of the load which makes up the gross weight will be shown by later examples.

The Distinction Between Wing Load Factor and the Dead Weight Factor

All of the lift load factors previously determined are wing load factors. They represent the ratio between the forces actually necessary to sustain the airplane in steady flight, and the forces built up on the wings when the angle of attack is changed without a corresponding change in speed. If the resultant air forces on the airplane passed through the center of gravity of the ship there would be no tendency toward rotation, except that induced by unbalanced moment from thrust and drag. Actually there is nearly always a tendency for the airplane to rotate about the center of gravity which must be balanced by loads on the horizontal tail. To keep the summation of vertical forces equal to zero the force at the center of gravity must balance the effect of wing load plus tail load. If all these values are expressed in terms of the airplane weight then the balancing force at the center of gravity is the dead weight or inertia load factor.

Since the moment about the center of gravity determines the vertical load on the horizontal tail, and thus the dead weight load factor, the location of the center of gravity is important. On a transport airplane the travel of the center of gravity may be anywhere within the stability range as determined by flight tests. It is generally sufficient to investigate the extremes of forward and aft center of gravity travel. For analysis purposes one must rely on wind tunnel data if available, or make a conservative estimate of the center of gravity limits. Even when wind tunnel results are to be had it is general practice to allow for limits two or three per cent in either direction outside the indicated stability range. The importance of the location of the center of gravity becomes clearer when all of the forces acting on the airplane are taken into account. This point will be considered further in the computations for balancing loads on the airplane. Assignment 5.

Wing Flaps

The purpose of trailing edge wing flaps is to raise the lift coefficient by changing the effective camber of all of the wing area forward of the flaps. The lift and drag, and the pitching moment (a moment tending to put the nose of the airfoil down) are all increased by lowering the wing flaps. The effect of the flaps varies with their angular throw, being greatest when they are deflected to their maximum. Wing flaps which are used primarily for landing accomplish two effects:

(1) They lower the landing speed so that less field is required to stop in after landing
(2) They steepen the angle of glide so that it is easier to get into a landing field surrounded by obstacles.

The aerodynamic effect of the flaps may be taken from wind tunnel tests. Wind tunnel models are made with the wing flaps to scale so that the entire data may be taken both for flaps up and for flaps down. This type of data is very satisfactory for calculation of the performance with flaps down, but can be used directly for calculation of the loads on the wing only when the flaps extend across the entire trailing edge of the wing. Since the outer portion of the wing trailing edge is nearly always reserved for ailerons it is necessary to separate the aerodynamic forces on the part of the wing ahead of the flaps from those on the part of the wing not affected by flaps.

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Calculation of Loads on Wing with Flaps Down.

METHOD 1. To be used where Wind Tunnel data are available for the full wing with flaps down.

Assume that we have the airfoil characteristics for an entire wing such as that shown on our basic data furnished with this assignment, (Fig. 2), taken with wing flaps up and with wing flaps down.

Two sets of wing characteristics will be plotted--one for flaps up and the other for flaps down, but both with reference to the same angles of attack, α , referred to the attitude of the normal unflapped wing.

The data for the entire wing with flaps down may be used directly to compute load factors for the airplane as a whole, and balancing loads to maintain equilibrium (loads on the tail, etc.), even though the difference in effect is all furnished by that portion of the wing which lies ahead of the flaps. To design the wing and properly apportion its loads we must separate out the forces on the portion not affected by flaps from that which lies ahead of the flaps. This is relatively simple since for that portion of the wing not affected by flaps we have the data from the wind tunnel tests with flaps up. Our data then divides into two parts.

(1) Over the portion of wing not affected by wing flaps the values of C_L, C_D and C_M are the same as for the wing with flaps up for any given angle of attack.

(2) Over the portion of the wing ahead of the flaps the values are $C_L + \Delta C_L$, $C_D + \Delta C_D$, and $C_M + \Delta C_M$. The values ΔC_L, ΔC_D, and ΔC_M for the flapped portion at any given angle of attack are found by dividing the difference between the characteristics (flaps up and flaps down) for the entire wing at any given angle of attack by the ratio $\left[\frac{\text{area ahead of flap}}{\text{total wing area}}\right]$ expressed as a decimal.

Suppose that for the entire wing of the airplane we are investigating we find the following characteristics at a given angle of attack,

With flaps up $C_L = .58$, $C_D = .028$, $C_M = -.029$

With flaps down $C_L = 1.17$, $C_D = .130$, $C_M = -.240$

The differences due to flaps are +.59, +.102, -.211

.585 times the area lies ahead of the flaps.

Then over the portion ahead of the flaps

$$\Delta C_L = \frac{+.59}{.585} = +1.00, \qquad \Delta C_D = \frac{+.102}{.585} = +.174,$$

$$\Delta C_M = \frac{-.211}{.585} = -.361,$$

while over the area not affected by flaps the characteristics remain the same as for flaps up.

We may compute the overall effect of the entire wing with flaps up as a check on the increments calculated for the flapped portion.

Area affected by flaps = (.585)(total)
Area not affected by flaps = (.415)(total)

Over flapped portion
$C_L = .58 + 1.00 = 1.58$
$C_D = .028 + .174 = .202$
$C_M = -.029 - .361 = -.390$

Over unflapped portion
$C_L = .58$
$C_D = .028$
$C_M = -.029$

For entire wing $$C_L = \frac{(.585)(1.58) + (.415)(.58)}{1.0} = 1.17$$

$$C_D = \frac{(.585)(.202) + (.415)(.028)}{1.0} = .130$$

$$C_M = \frac{(.585)(-.390) + (.415)(-.029)}{1.0} = -.240$$

A complete plot of ΔC_L, ΔC_D and ΔC_M for all angles of attack in the design range can be computed by the foregoing method.

METHOD 2. Wind Tunnel Data for the entire wing with flaps down not available.

If we do not have wind tunnel data for a certain model the increments of C_L, C_D and C_M over the flapped portion may be estimated by the methods given in Diehl's "Engineering Aerodynamics". For the airfoil of our example with a flap covering 20 per cent of the chord we may take the following increments for the characteristics over the part affected by the flaps, assuming the flaps go down 45 degrees.

α =	-12°	-8°	-4°	0°	4°	8°	12°	16°
ΔC_L =	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.98
ΔC_D =	.075	.105	.130	.150	.175	.205	.250	.325
ΔC_M =	-.230	-.265	-.290	-.320	-.350	-.373	-.390	-.400

These values should be plotted as curves so that they may be readily determined for any value of angle of attack, α. Having the above values we already have the correct breakdown between flapped and unflapped portions of the wing. To obtain the overall characteristics for the wing with flaps down, we use the method used for checking in the Method 1. Thus

C_L for entire wing with flaps down

$$= (.585)(C_L + \Delta C_L) + (.415)(C_L)$$

$$C_D = (.585)(C_D + \Delta C_D) + (.415)(C_D) \; .... \text{etc.}$$

The airfoil characteristics for the entire wing are then plotted for future reference.

The average values are used as follows in obtaining design load factors for flaps down conditions.

(a) For the calculation of the maximum effective value of C_L with flaps down for the entire wing use $C_{L\ max.}$ for the unflapped portion equal to 1.41 at $\alpha = 16^\circ$.

Then the maximum C_L on the flapped portion is $C_L + \Delta C_L = 1.41 + .98 = 2.39$.

The average maximum value for the entire wing is

$$(.585)(2.39) + (.415)(1.41) = 1.98$$

The minimum flying speed with flaps down

$$V_{s_f} = \left[\frac{(391)(\frac{W}{S})}{C_{L\ max.}}\right]^{\frac{1}{2}} = 19.75 \left[\frac{\frac{W}{S}}{C_{L\ max.}}\right]^{\frac{1}{2}}$$ where V_{s_f}

is in miles per hour. In our case for normal weight of 20,000 lbs.

$$\frac{W}{S} = \frac{20,000}{1010} = 19.8 \text{ lbs. per sq. ft.}$$

$$V_{s_f} = (19.75) \left[\frac{19.8}{1.98}\right]^{\frac{1}{2}} = 62.5 \text{ miles per hour}$$

(b) The design V_f is taken as $2V_{s_f} = 125$ mph.

(c) Since the increase in the lift coefficient for the flapped condition is a constant value ($\Delta C_L = 1.0$) in the design range, the slope of the lift curve, $m = \frac{\Delta C_L}{\Delta \alpha}$, is the same for flaps down as for flaps up.

($m = 4.4$ where $\Delta\alpha$ is in radians.)

(d) Compute Δn_7 for the entire wing. See page 2 for method.

(e) Compute $n_7 = 1 + \Delta n_7$ (but cannot be taken less than 2.0. If the value comes out less than 2.0, use 2.0 for n_7 in the succeeding work.)

$$\text{Then } C_{N_7} = \frac{(n_7)(\frac{W}{S})}{q_{V_f}}$$

$q_{V_f} = .00256\ V_f^2$, where V_f is miles per hour.

From our plot of airfoil data with flaps up in Assignment 1 we have seen that in the design range (approximately -8° to $+12^\circ$) we may take C_N equal to C_L.

Using $C_N = C_L$, if x is taken as C_L over the portion of the wing not affected by flaps, C_L over the portion affected by flaps is $x + \Delta C_L = x + 1.0$. Then $(.585)(x + 1.0) + (.415)(x) = C_{N_7}$.

The solution for x gives the value for C_L on the unflapped portion of the wing. This may be referred back to the plot of C_L versus α in Assignment 1 to obtain the angle of attack corresponding to speed V_f and from it values of C_D and C_M for the unflapped portion. The values of C_D and C_M for the flapped portions may then be obtained by adding the increments for flaps corresponding to the angle of attack at speed V_f. This method gives data for the solution of the flapped conditions without plotting the complete data for flaps down. Note that ΔC_L and ΔC_D for the flapped portions can not be added to C_N and C_c for the unflapped portions in obtaining characteristics for the part of the wing affected by flaps until they have been resolved into coordinates perpendicular and parallel to the wing chord as was done in Assignment 1.

It is advisable with either method to plot complete aerodynamic data

(a) For the entire wing with flaps up (neutral).
(b) For the entire wing with flaps down to their maximum throw.
(c) The increments of the flapped portion ΔC_L, ΔC_D and ΔC_M.

Problem Assignment.

3:1 Plot the curves of ΔC_L, ΔC_D, and ΔC_M versus the angle of attack, α, from the data given on page 3-8.

3:2 Calculate the values C_L, C_D and C_M for the entire wing with flaps down and plot curves of these values versus α.

3:3 Compute the values of q, C_N, C_c, and C_M for conditions VII, VIII, and IX which are outlined in this assignment.

Assignment 3 - APPENDIX

BASIC DATA FOR A TYPE AIRPLANE

Complete analysis of this airplane in all essential parts is to be completed in this course. Not all design conditions will be completely investigated, but each type of loading which may be critical for any part of an airplane will be considered and the general outline will be comprehensive enough to serve as a guide for a most complete stress analysis. The airplane which we will take as our example will have the following characteristics:

Type - Twin engined all metal commercial transport monoplane.
Gross Weight - 20,000 lbs.
Empty Weight - 12,000 lbs.
Wing Area - 1010 sq. ft., Span = 90 ft.
Normal Horsepower - 2000 (for two engines) at 2400 RPM
Gear Ratio 2:1 (Propellers turn 1200 RPM when engines turn 2400 RPM)
Take off Horsepower - 2400 (for two engines) at 2600 RPM
Estimated High Speed at Sea Level = 220 mph.
Maximum Gasoline Capacity - 750 gals.
Maximum Oil Capacity - 60 gals.
Total Cargo and Baggage Capacity - 3000 lbs.
19 - Place 16 passengers and crew of 3

It is necessary that we know the loadings for various conditions in order to choose those critical for the structure. The following conditions will be taken as covering those likely to put worst loads in the various parts of the structure.

A. Normal Loading

Empty weight (ready for loading) = 12,000 lbs.
Crew (Pilot and Co-pilot at 170 lbs. each, Stewardess at 130 lbs.) = 470
Full passenger load = (16)(170) = 2,720
Baggage at 35 lbs. ea. = (16)(35) = 560
Cargo, express, etc. = 1,600
Fuel = 400 gallons at 6 lbs. = 2,400
Equivalent oil = 33 gal. at 7.5 = 250
Total = 20,000 lbs.

****

B. Minimum Flying Weight (For Light Load Gust Conditions)

Empty weight = 12,000 lbs.
Crew (pilot and co-pilot) = 340
Minimum fuel = (.25)(2000) = 500
Full oil = (60)(7.5) = 450
Total = 13,290 lbs.

C. Maximum Load in Fuselage with Full Gross Weight

Weight empty = 12,000 lbs.
Crew = 470
Passengers (16)(170) = 2,720
Baggage (16)(35) = 560
Maximum cargo = 2,440
Fuel (277 gals.) = 1,660
Oil (20 gals.) = 150
Total = 20,000 lbs.

Approximate range at 75% power = 500 miles.

****

D. Maximum Load in Fuselage with Fuel Tanks Empty (End of trip condition)

Same as Condition C except with fuel out gross weight is
20,000 - 1,660 = 18,340 lbs.
This condition is likely to design the fuselage.

****

E. Loading for Maximum Range at Full Gross Weight

Weight empty = 12,000 lbs.
Fuel = 750 gals. = 4,500
Oil = 60 gals. = 450
Crew (three) = 470
Passengers = (12)(170) = 2,040
Baggage = (12)(35) = 420
Cargo = 120
Total = 20,000 lbs.

Approximate range at 75% power = 1350 miles.

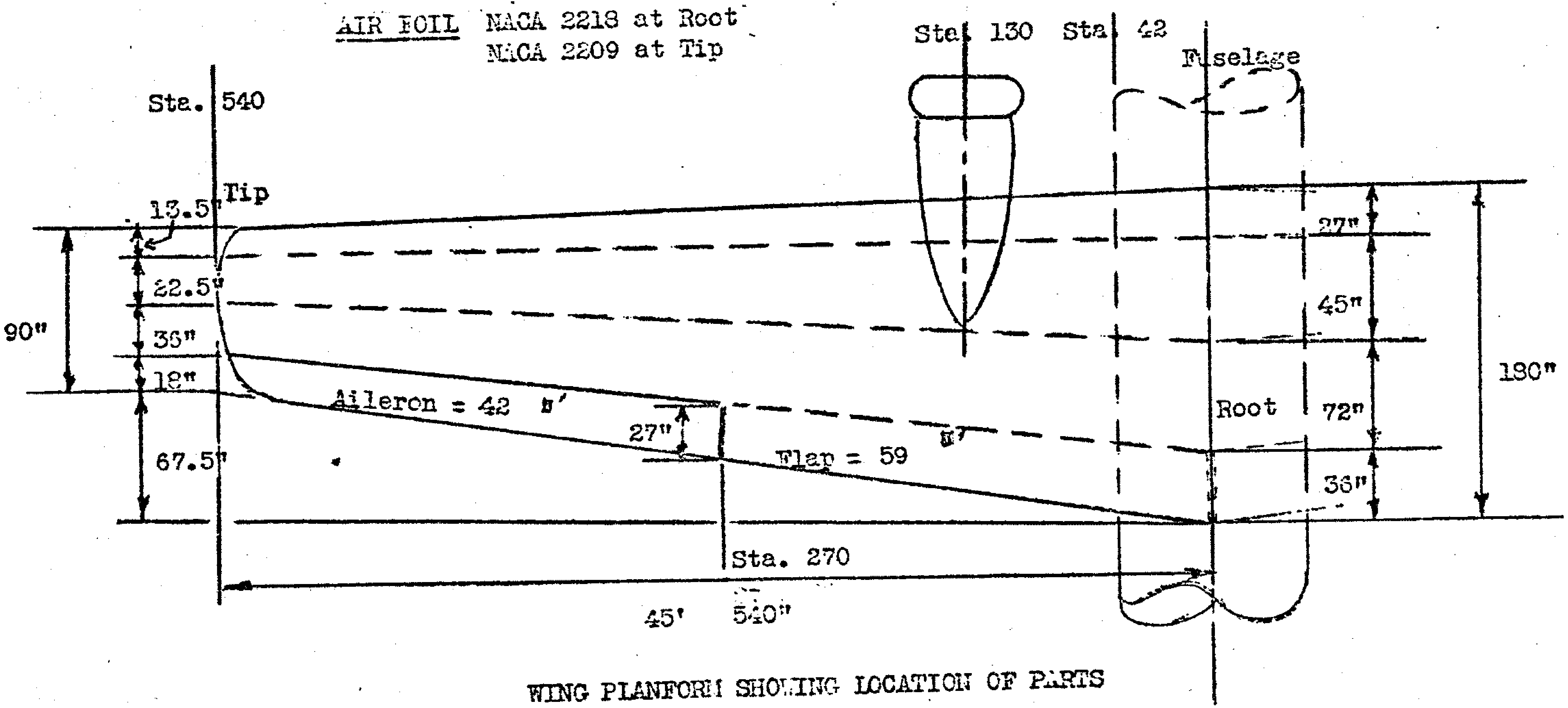
Wing Area 1 - Side = $\left[\frac{90 \quad 180}{(2)(12)}\right]$ (45) = 506 gross = 505 sq. ft. net after deducting lost area at tip.

Total for 2 sides = (2)(505) = 1010 sq. ft.

This includes all area blanketed by nacelles and the fuselage.

Aileron area = 42 sq. ft. per side. Flap area = 59 sq. ft. per side.
Wing area affected by flaps = 590 sq. ft. = 58.5%; not affected = 41.5%.

AIR FOIL NACA 2218 at Root
NACA 2209 at Tip



WING PLANFORM SHOWING LOCATION OF PARTS

Fig. 2

ADVANCED STRESS ANALYSIS

ASSIGNMENT 4

Assignment 4

SPANWISE DISTRIBUTION OF LIFT
CHORDWISE PRESSURE DISTRIBUTION
CALCULATION OF THE MEAN AERODYNAMIC CHORD

At the beginning of this Assignment it is well to define the terms used.

Definitions:

Spanwise Distribution of Lift refers to the variation in the value C_L as one travels along the span from the root chord, at the center-line of the airplane, to the tip chord. C_L or C_N, as we have used them in previous assignments refer to an average value for each square foot of wing area. Actually each square foot of wing never lifts the same amount as every other square foot of wing area because there is a variation in the efficiency in both the span and chord directions. This variation is much smaller in the spanwise direction than in the chordwise direction and in some cases the spanwise variation may be disregarded.

Chordwise Pressure Distribution refers to the variation of the lifting effort at sections along the chord from the nose to the tail. This difference is great and varies with the angle of attack. In previous computations this variation has been allowed for by the use of the moment coefficient, C_M, which we have utilized in determining the chordwise centroid of the air loading. See Fig. 1. The total effect of the suction on the top surface

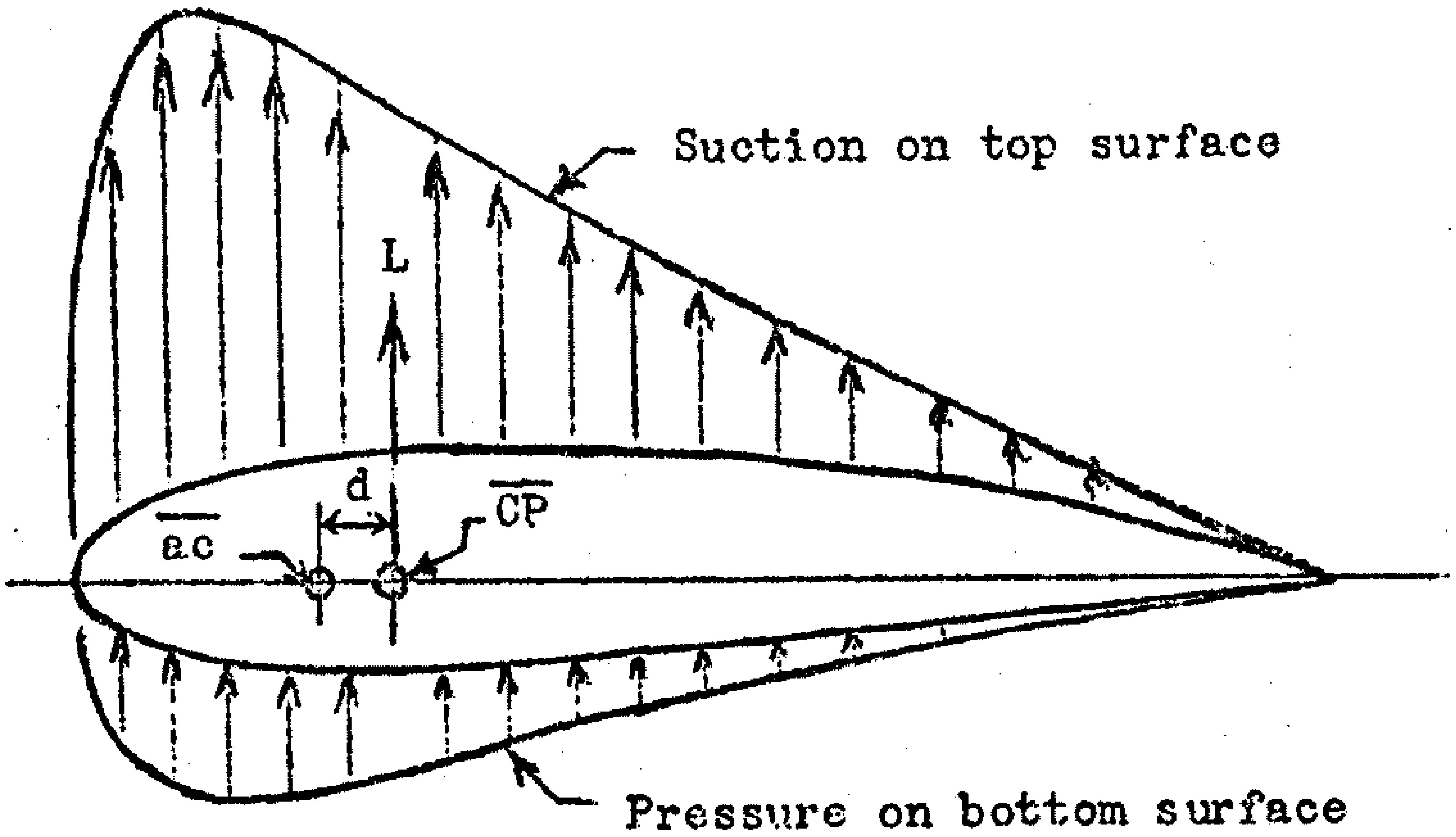


Fig. 1

and the pressure on the bottom surface is represented by the single force L acting at the center of pressure. The moment of this force about the aerodynamic center is represented by

$$\frac{-Ld}{\text{Chord in Ft.}} . \qquad C_M = \frac{-Ld}{\text{Chord in Ft.}}$$

C_M has the minus sign because the moment of the air force about the aerodynamic center tends to decrease the angle of attack. This tendency is sometimes expressed as providing a "pitching moment;" that is, tending to put the nose of the airfoil down.

The Mean Aerodynamic Chord is that chord of the airplane at which all the air forces could be considered to be concentrated without changing the total air forces or the moments of these forces about any of the three coordinate axes. The mean aerodynamic chord for symmetrical wings then lies along the longitudinal axis of the fuselage. We may also have a mean aerodynamic chord for the wing surface on one side of the airplane. It will have the same length and fore and aft location as that at the ℄ of the airplane but the forces considered acting upon it would be for one side only. A method of computing the Mean Aerodynamic Chord is given in detail in Bulletin ACM04. This method is more cumbersome than is necessary for many simple wing plan forms but it will be discussed in detail here because its application is universal, and it covers the principles involved very thoroughly.

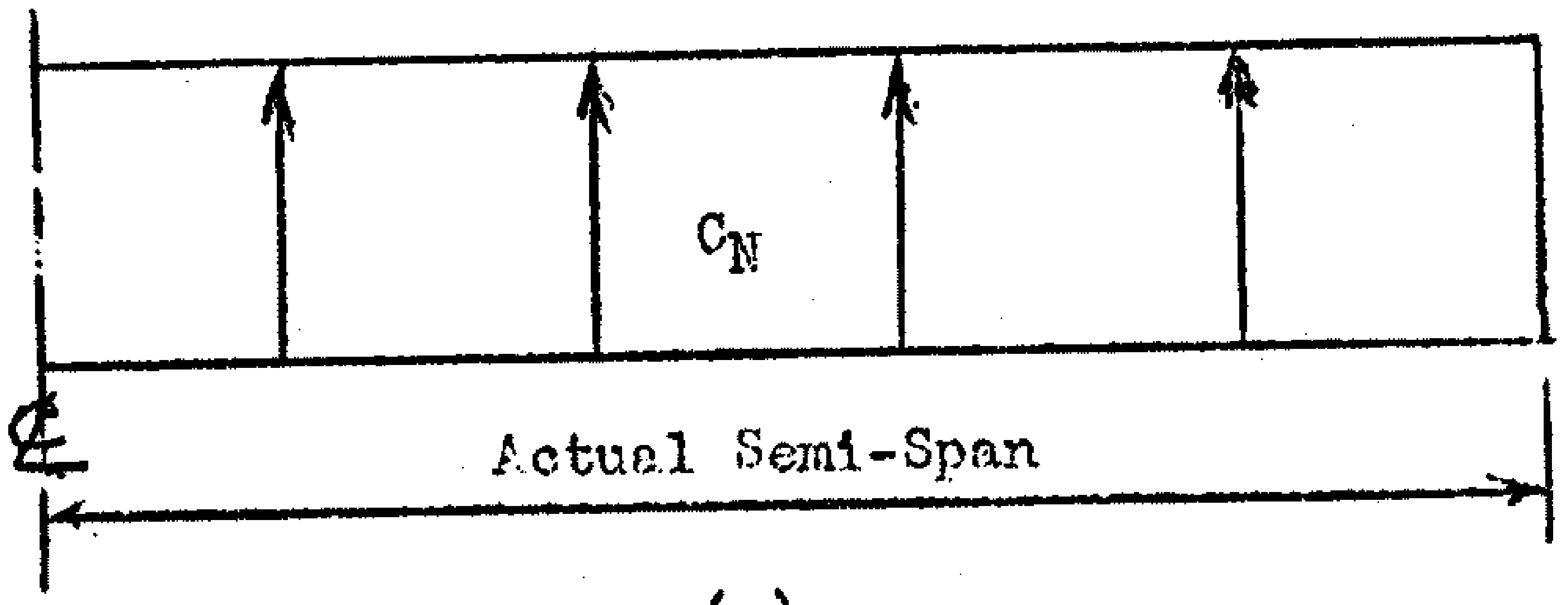
****

Spanwise Distribution of Lift.

A method for obtaining the spanwise lift distribution, (variation of lift force in lbs. per sq. ft. along the span) is given in great detail in ANC-1, "Spanwise Air-Load Distribution", issued by the Army-Navy-Commerce Committee on Aircraft Requirements. This method involves a large amount of tabulations, and for many cases increases the accuracy only one or two per cent over that obtained by using a uniform span load distribution. Equally usable results may be obtained by using simplifying assumptions, or by making certain corrections for the bending moment increase to be expected at certain sections of a wing. The method given in ACM04 section .217 covers a great many cases. According to ACM04 if the wing has a mean taper ratio of 0.5 or more and does not have aerodynamic twist the variation in the value of C_N may be assumed according to Fig. 2 (a) or (b).

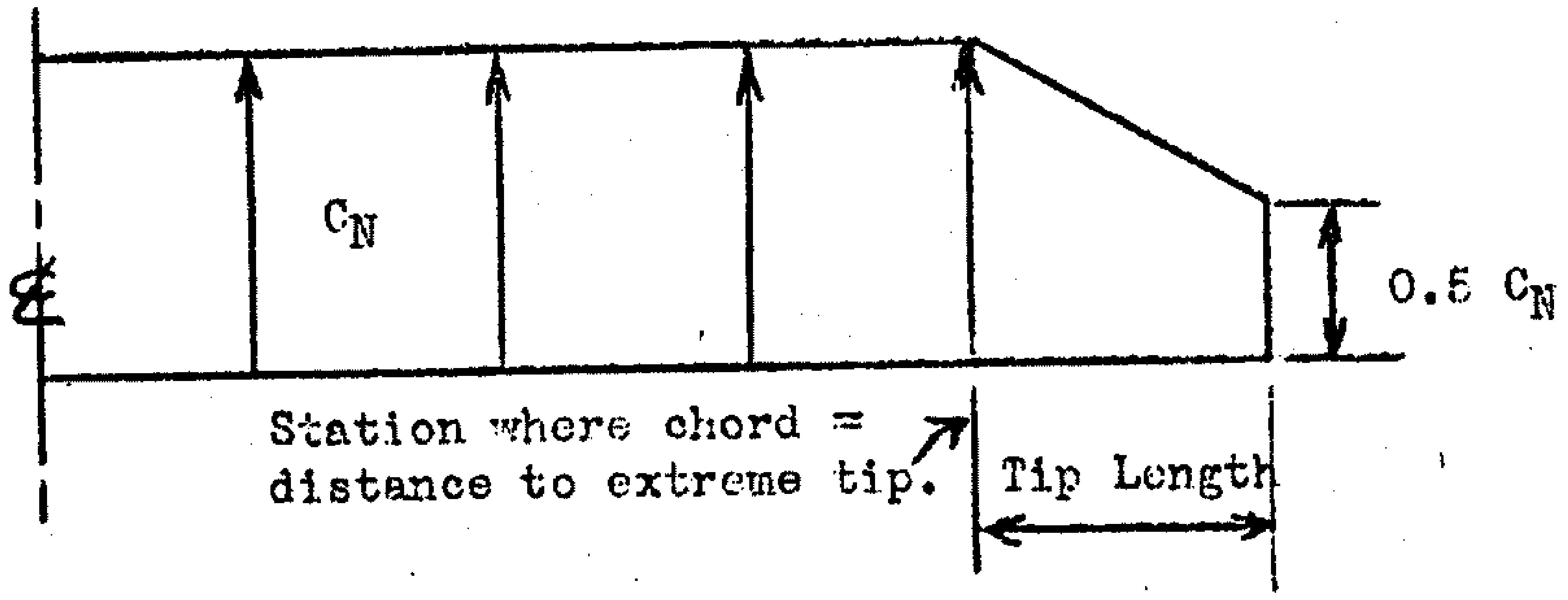
The mean taper ratio may be defined as the ratio of the tip chord (in our case 90 inches) to the root chord (in our case 180 inches). The taper ratio for our wing is therefore 0.5 and it just meets the condition for which complete computations of distribution are not required.

Aerodynamic twist exists when the reference planes of the chord sections at different stations along the wing span are not parallel. That is when the angles of attack are different for different portions of the wing. Aerodynamic twist is sometimes used to prevent tip stall. We have seen that the lift curve drops off sharply at an angle of attack of about 16°. This is the stalling angle for the airfoil, where the flow over it breaks down into turbulent eddies. For maintaining lateral control it is desirable that the stall if it comes should occur first on the center portion of the wing: the pilot is then warned by a loss of altitude before the tips stall and he loses lateral control. It is possible to attain this result if the tip section of the wing is twisted down so that it is at an angle of attack of say 14° when the inner portion of the wing is at 16°.



(a)

Span Distribution with no Tip Loss



(b)

Span Distribution with Tip Loss

Fig. 2

Fig. 2 (a) shows the distribution which would be critical for bending on a monoplane wing. The distribution of Fig. 2 (b) would give the highest wing loading inboard of the tip length and therefore highest loads on the ribs. It would also be critical for a braced wing, where the higher loading on the tip would reduce the moments inboard of the tip joint. See Fig. 3.

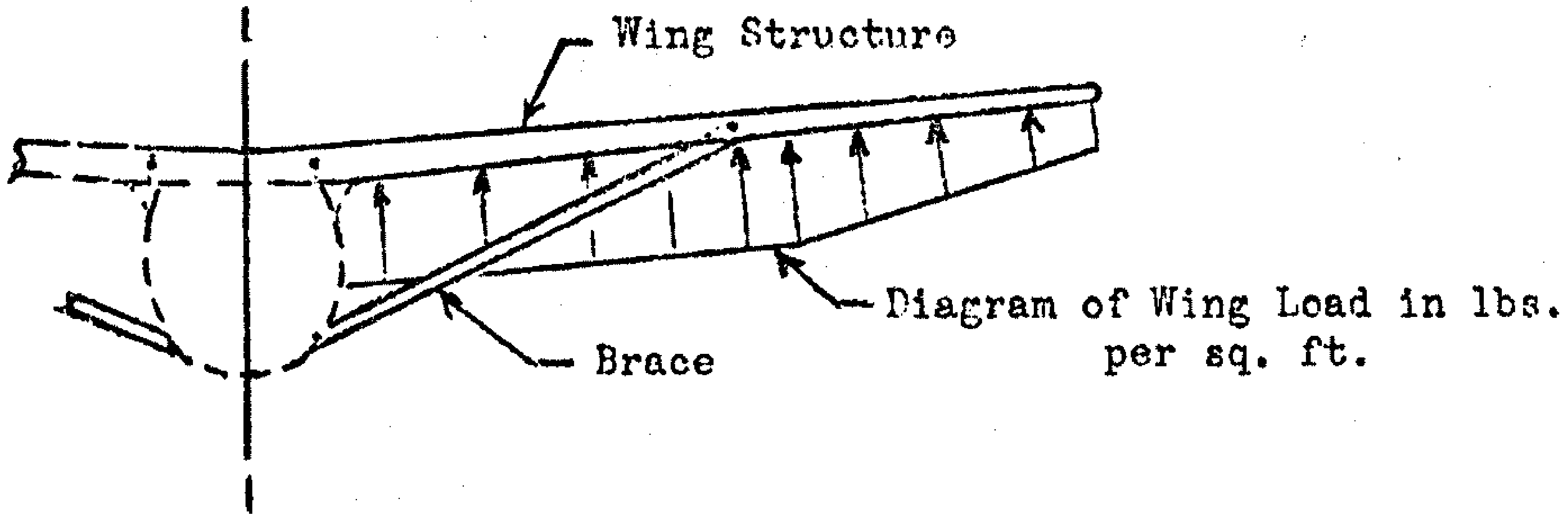


Fig. 3 Showing how reduced load on the tip and increased load inboard can be critical for bending on a braced wing.

With any lift distribution it is of course necessary that the (Average value of C_N)(total area) = $\sum$ (corrected values of C_N)(areas to which they apply)

The simplest method of obtaining C_N for Fig. 2 (b) is to compute an "effective semi-span" for which the total loading is (C_N inboard)(area of effective semi-span). This can be done by "shrinking" the chords at several stations along the tip lengths by multiplying the actual chord by the ratio.

$$\frac{C_N \text{ at station}}{C_{N \text{ max.}}}.$$

The effective tip area can then be calculated or taken by planimeter. This method will be illustrated for the wing we are using although the loading of Fig. 2 (b) could only be critical in our case for rib design.

For the distribution of the lift coefficient by the methods of Fig. 2, we may make the following simplifying assumptions:

1. The effect of nacelles may be neglected. This assumption is supported by wind tunnel data.
2. The effect of trailing edge cutouts which remove less than 50 per cent of the chord may be neglected.
3. The spanwise distribution of the chord force may be assumed constant. Tip loss is not assumed to affect the chord coefficient.

****

For further data on spanwise distribution the student is referred to NACA, "Technical Report No. 585," and NACA "Technical Note No. 606."

Chordwise Distribution of Pressure.

We have said that the chordwise distribution of pressure varies with the angle of attack. It also is affected by the use of wing flaps or ailerons, but full use of these control surfaces occurs at speeds so reduced that the distribution is important only on the structure supporting the ailerons and flaps, i.e. the hinge bracket ribs.

For pressure distribution on the wing of our example, we use the data for the NACA 2212 series airfoil as taken from NACA Technical Report No. 465. Pressure distribution is plotted for percentage of chord with the ordinates values of p/q. A family of curves are plotted for different values of C_L. To determine the actual pressure it is necessary to know the speed (to obtain q) and the angle of attack or C_L. Where values of C_L fall between those given, the pressure curve may be interpolated. The chordwise pressure distribution is important for rib design and for obtaining the twist on the wing. Chordwise pressure diagrams are shown on Fig. 4.

CHORDWISE PRESSURE DISTRIBUTION
NACA 2212 AIRFOIL TR-465

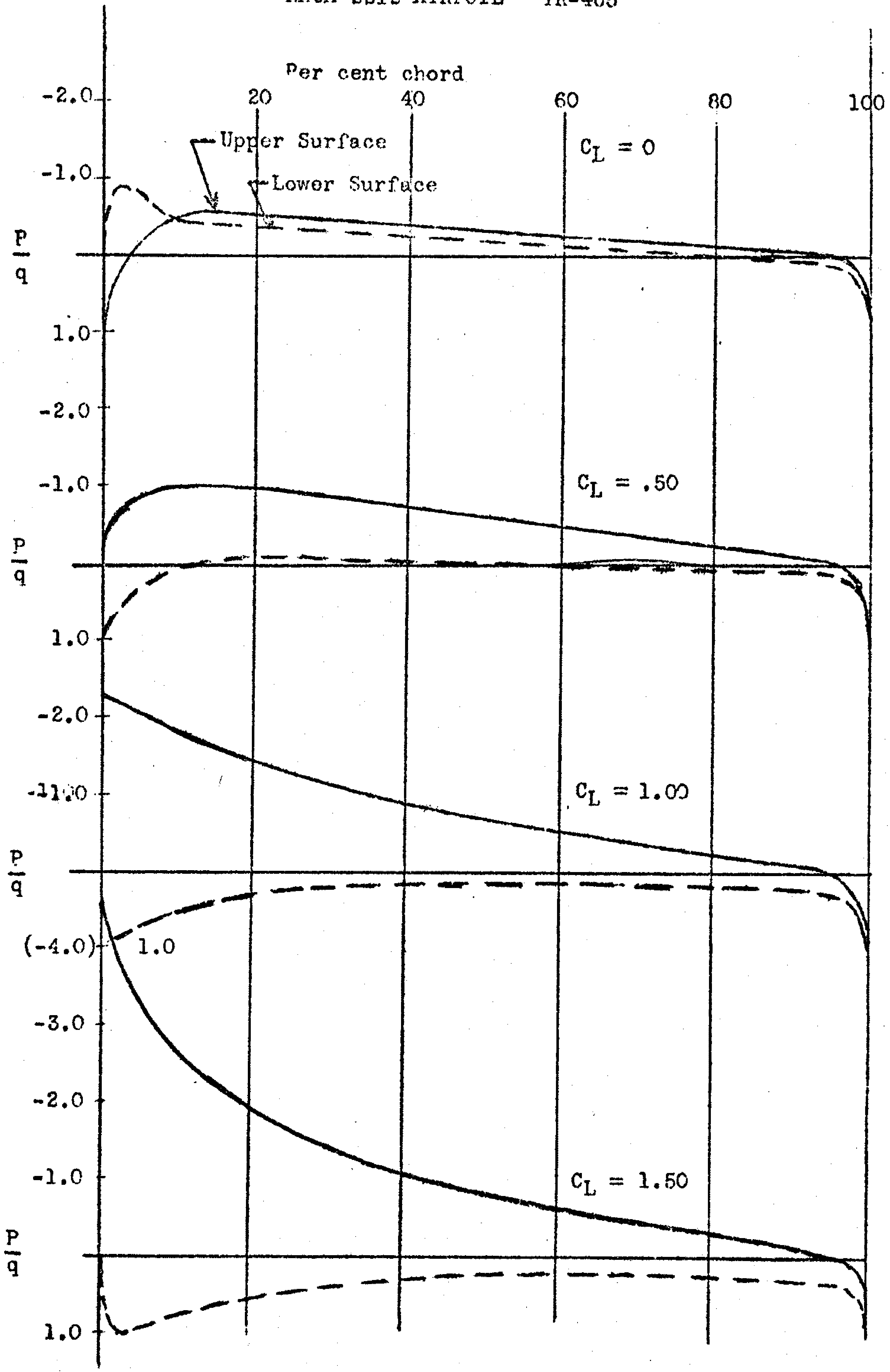


Fig. 4

Determination of the Resultant of all Air Forces on a Wing and Location of their Application Points. The Mean Aerodynamic Chord.

For the case we are considering we may use the load distribution along the wing from Fig. 2 (a) and 2 (b). Since we are dealing with a monoplane wing it is obvious that case (a) is critical for every purpose except rib design. We will therefore base our entire analysis on this case, except that the loads for rib design inboard of the tip section will be computed for case (b). The balancing forces on the airplane, and the mean aerodynamic chord will be determined for case (a). A general method will be illustrated for determining the mean effective value of the normal force coefficient, the average moment coefficient, location of the mean aerodynamic center and the value of the mean aerodynamic chord. The most general case will be given, although we can omit certain steps because we are dealing with a monoplane wing.

In general the summation of all the forces acting on the wing may be expressed as a single resultant force acting at a certain point and a moment of air forces about this point. If the point is so chosen that the moment of the air forces does not change with changes in angle of attack at a given speed the point is called the mean aerodynamic center of the wing. In effect it is the average of the aerodynamic centers for the different chordwise strips into which the wing may be considered divided. The resultant force may be resolved into the beam and chord components and represented by the average coefficients C_N and C_C, while the moment is represented by the average moment coefficient, C_M, times a distance which is considered to be the mean aerodynamic chord.

The values of the above forces and the location of the M.A.C. depend on the wing planform and the type of span distribution curve assumed.

The method of the Civil Air Authority for locating the mean aerodynamic center and determining the mean aerodynamic chord is illustrated for our wing in the curves of Figs. A to G. Fig. C illustrates the method of determining the normal force coefficient. It is simplified in our case (a) by the fact that C_N is constant over the entire wing. That is, each square foot of area in the wing span is assumed to lift the same weight. Briefly the method is as follows:

Fig. A - Plot the wing planform to scale (one side only is required).

Fig. B - Plot the variation of wing chord with span to scale. The area should be accurately determined. The total should be one-half the total effective wing area. Units are inches and square inches. For this purpose the actual tip will be used as a trapezoid neglecting the small loss in area from the actual shape.

Fig. C - Plot the variation of the lift coefficient, C_N versus span. In our case C_N is constant so that the ratio

$$R_b = {C_{N_b}}/{C_{N_o}} = 1.0$$

No plot is actually required.

Fig. D - In the general case where R_b varies over the span, Fig. D is a plot of $R_b \times C$. Actually in our case where $R_b = 1$ over the entire span this curve is the same as the plot of chord. (Fig. B). The difference is illustrated later for the loads in case (b).

Fig. E - Fig. E is drawn to determine the location of the mean aerodynamic center along the span. The ordinates are obtained by multiplying the ordinates of Fig. D by their distance along the span. The area under curve E divided by the area under curve D gives the distance from the wing root to the chord on which the mean aerodynamic center of the wing panel is located. This distance is indicated on Fig. A by the distance $\overline{b}$.

Note: The locus of the aerodynamic centers of the individual wing chords are plotted as the dotted line on Fig. A. Line A-B. In Table I the distances x from the baseline O-E to line A-B are entered under item (6).

Fig. F - Fig. F is plotted using as ordinates the values R_bCx from item (7) of Table I. The area under curve F divided by the area under curve D, gives the distance of the mean aerodynamic center from the base line O-E of Fig. A. This distance is shown as $\overline{x}$ on that figure.

Fig. G - Fig. G is a plot of the values C^2 taken from item (8) of Table I. The area under curve G divided by the area under curve D gives the value of the mean aerodynamic chord.

Note: According to the method given in ACM-04, for any lift distribution the mean aerodynamic chord would be A_G/A_B.

This amounts to assuming a constant value of C_N in determining the MAC. It is not as exact as the method we have given here, which puts the MAC definitely under the center of lift on a wing panel, when C_N varies along the span.

TABLE I

Determination of Resultant Air Forces and Mean Aerodynamic Chord

No.	Item	Station Root	42	130	270	405	540
(1)	span = b	0	42	130	270	405	540
(2)	chord = C	180	173	158.4	135	112.5	90
(3)	R_b	1.0	1.0	1.0	1.0	1.0	1.0
(4)	$(R_b)(C)$	180	173	158.4	135	112.5	90
(5)	$(R_b)(C)(b)$	0	7,270	20,600	36,400	45,600	48,600
(6)	x	45	45	45	45	45	45
(7)	$(R_b)(c)(x)$	8,100	7,780	7,125	6,075	5,060	4,050
(8)	C^2	32,400	29,930	25,100	18,225	12,650	8,100

$\bar{b} = 239.8''$

$\bar{x} = 45''$

MAC = 140.2"

It is readily apparent that the mean aerodynamic chord determination for the example wing is simplified by the planform chosen. The general method however is given in order that the student may have it available for other cases. The procedure we have followed is to calculate a few points on a curve bounding an area, establish the curve by these points and take the area under it. In the calculation of $\bar{b}$ for example:

If we had taken increments of chord of only one inch width all along the semi-span, the area of each strip in square inches would have been the length of chord. Then if each square inch lifts the same load and we wish to find the spanwise location of the center of lift, we multiply each strip by its arm to the center line, sum up the values and divide by the total area of all the strips. The average arm from the center line, $\bar{b}$, results. What we have actually done instead of choosing a point every inch along the span, is to choose a few points on the curve and determine the rest by plotting the curve, as illustrated in Fig. 5.

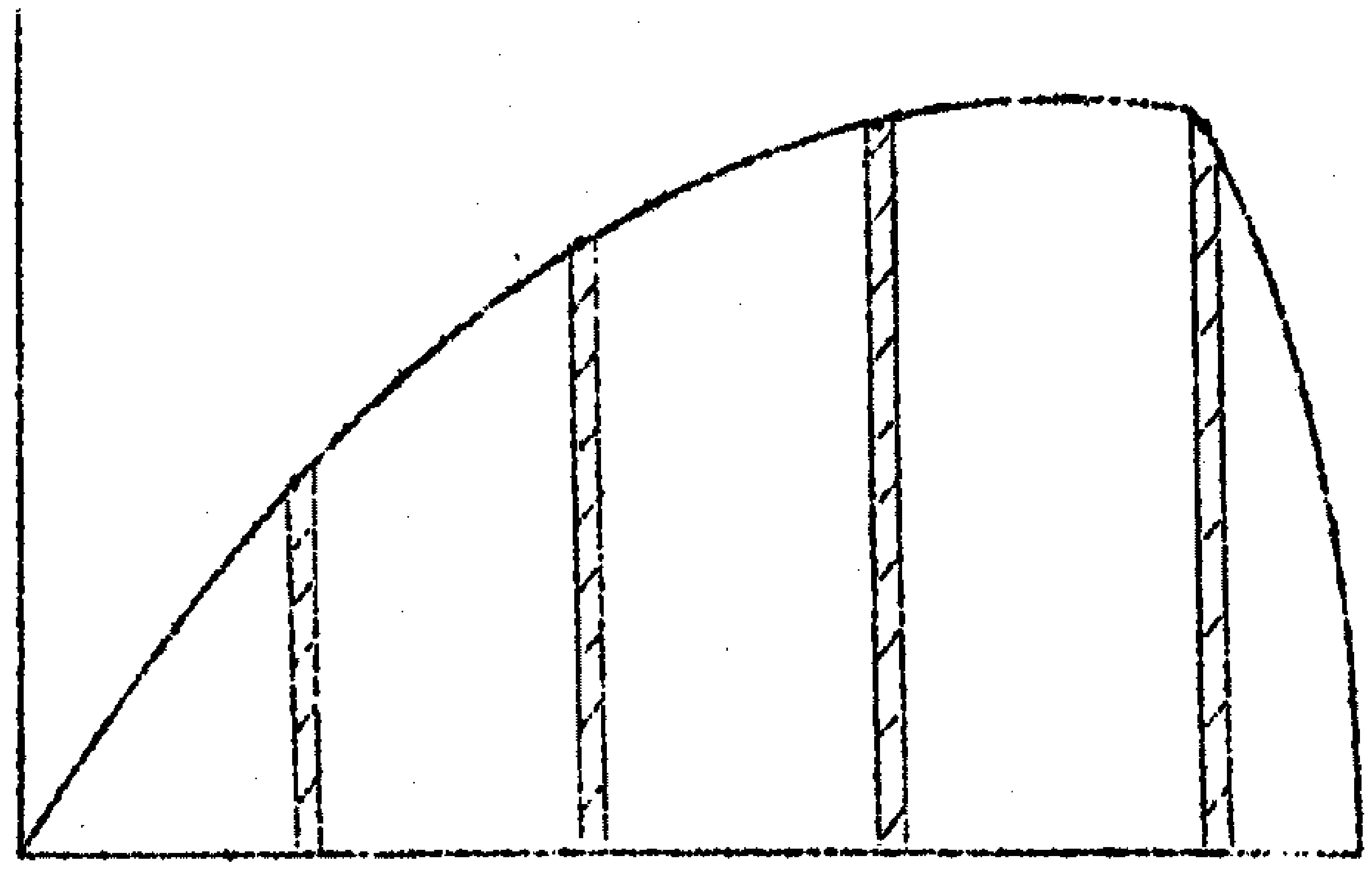


Fig. 5

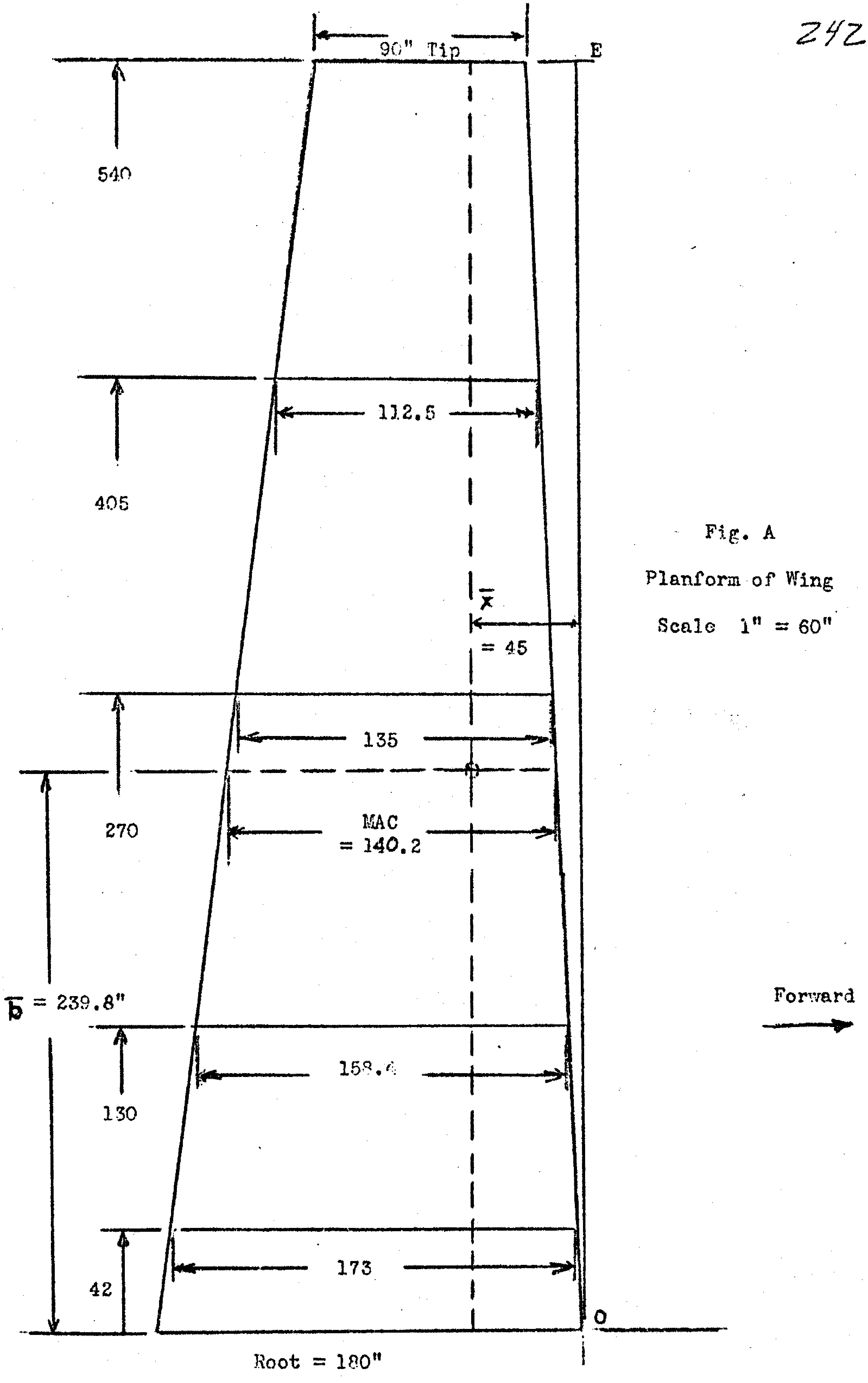


Fig. A

Planform of Wing

Scale 1" = 60"

Ordinates are chord, C in inches.
Area under curve = area of semi-span.

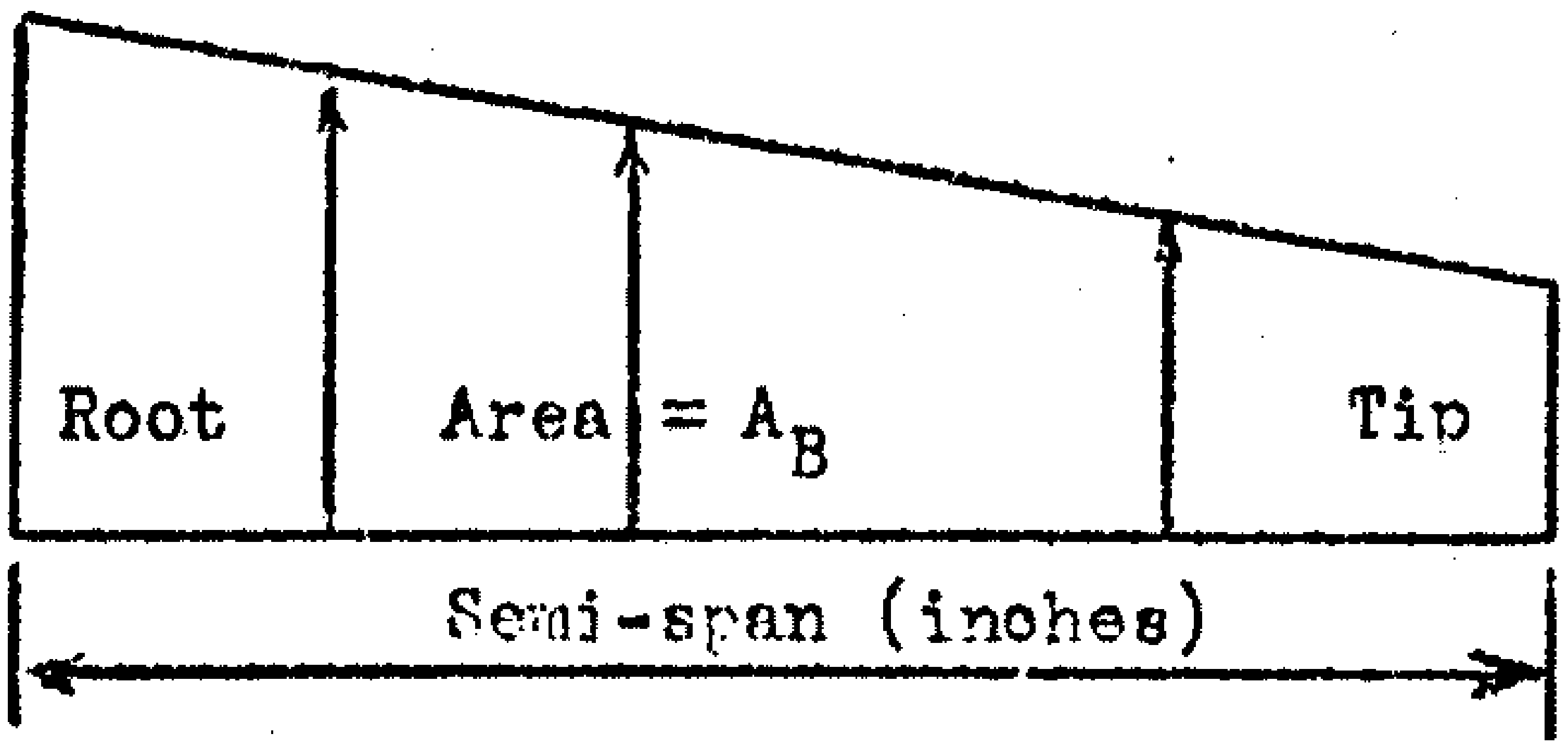


Fig. B

Ordinates = $R_b = \frac{C_{N_b}}{C_{N_o}}$

Our case $R_b = 1.0$ throughout

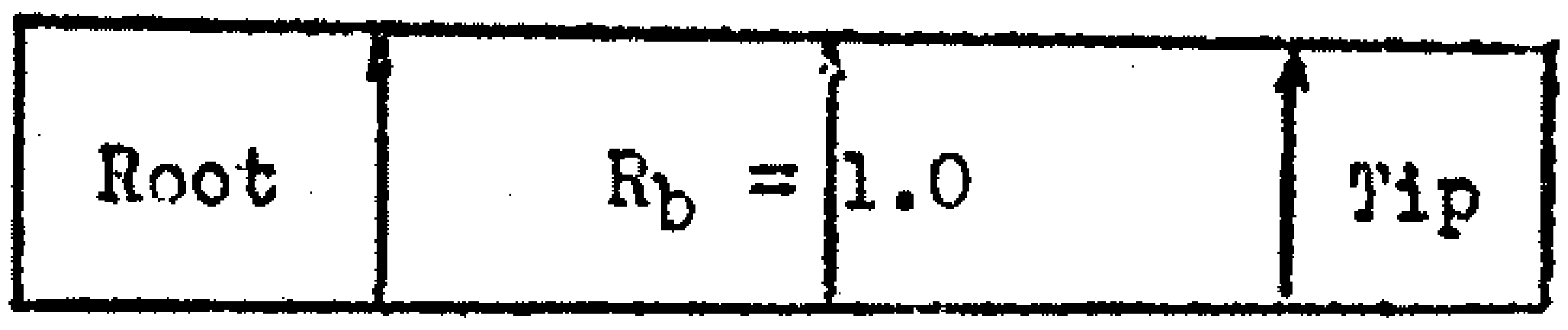


Fig. C

Ordinates = $(R_b)(c)$

Since in our case $R_b = 1.0$ Fig. D is same as Fig. B

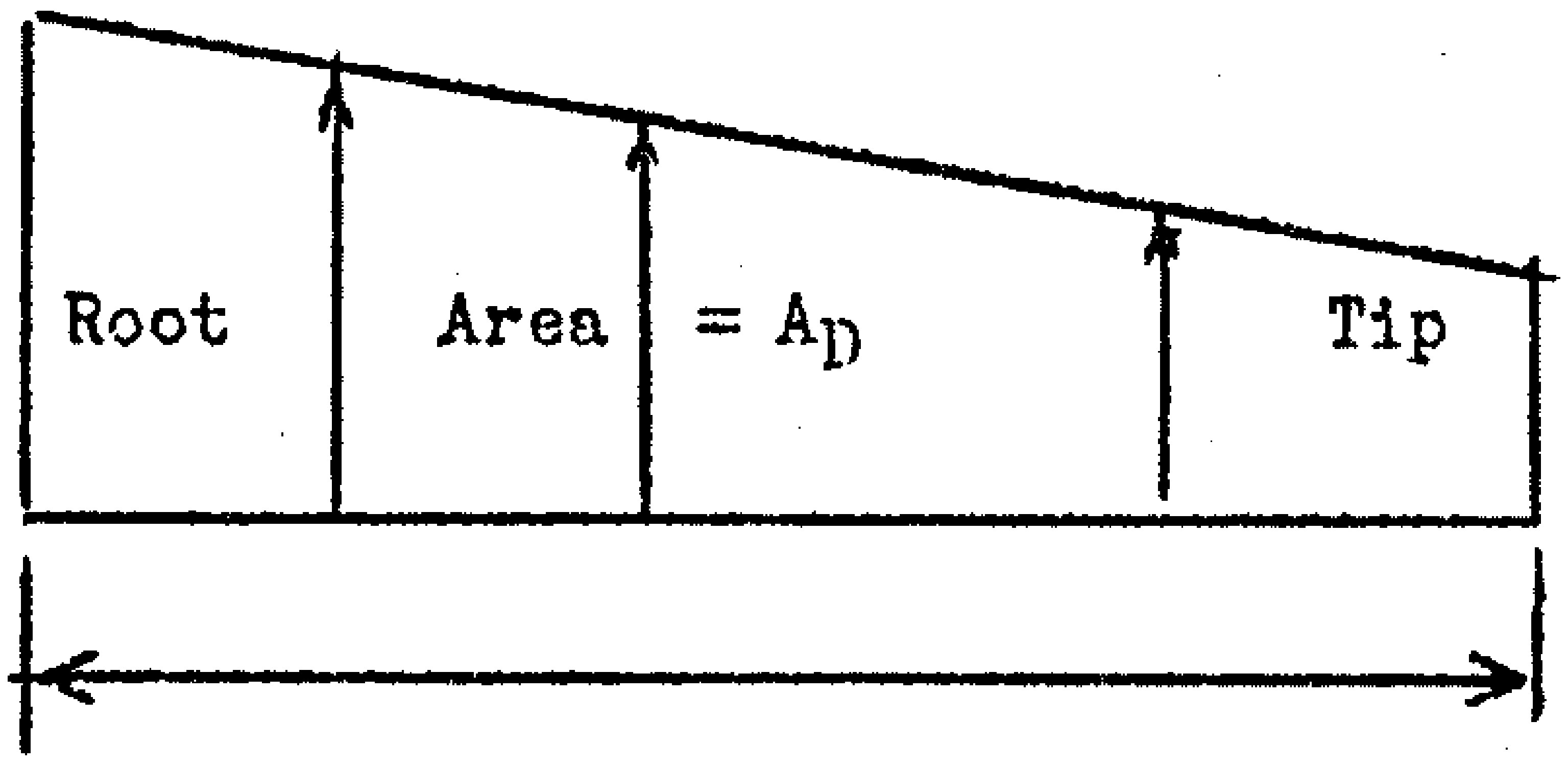


Fig. D

Ordinates = $(R_b)(C)(b)$

$\overline{b} = \frac{A_E}{A_D}$

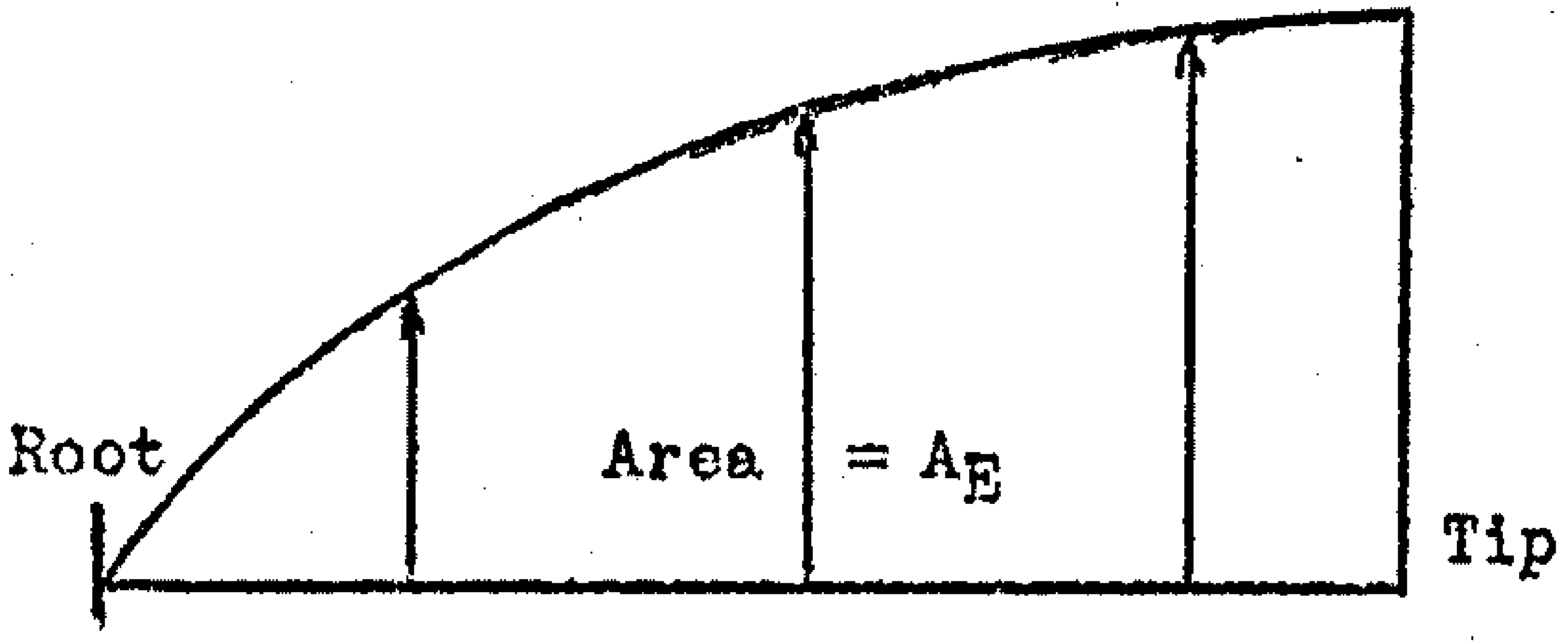


Fig. E

Ordinates = $(R_b)(c)(x)$

x is from Col. (6), Table I.

$\overline{x} = \frac{A_F}{A_D}$

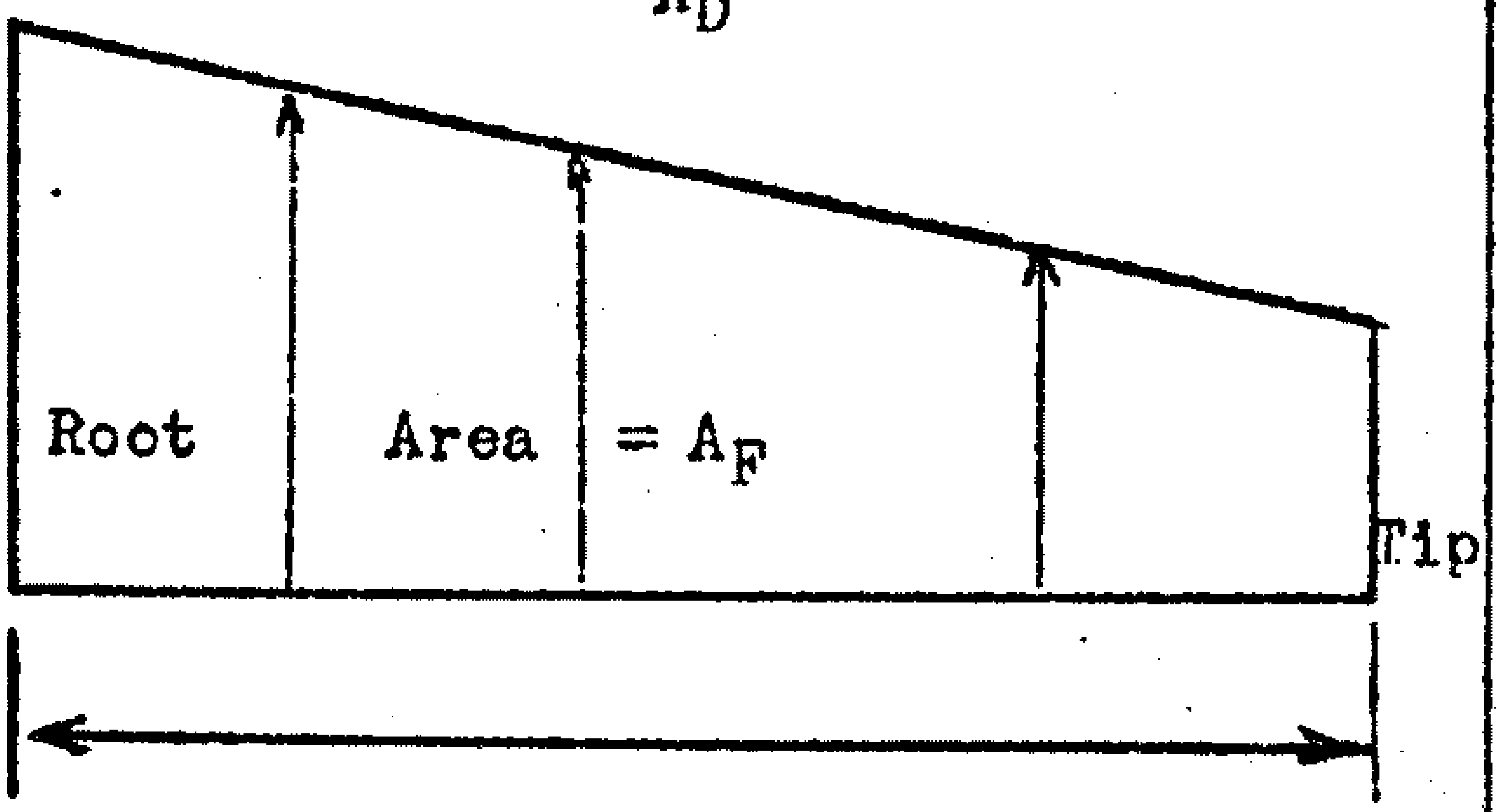


Fig. F

Ordinates = C^2 from Col. (8), Table I.

MAC = $\frac{A_G}{A_D}$

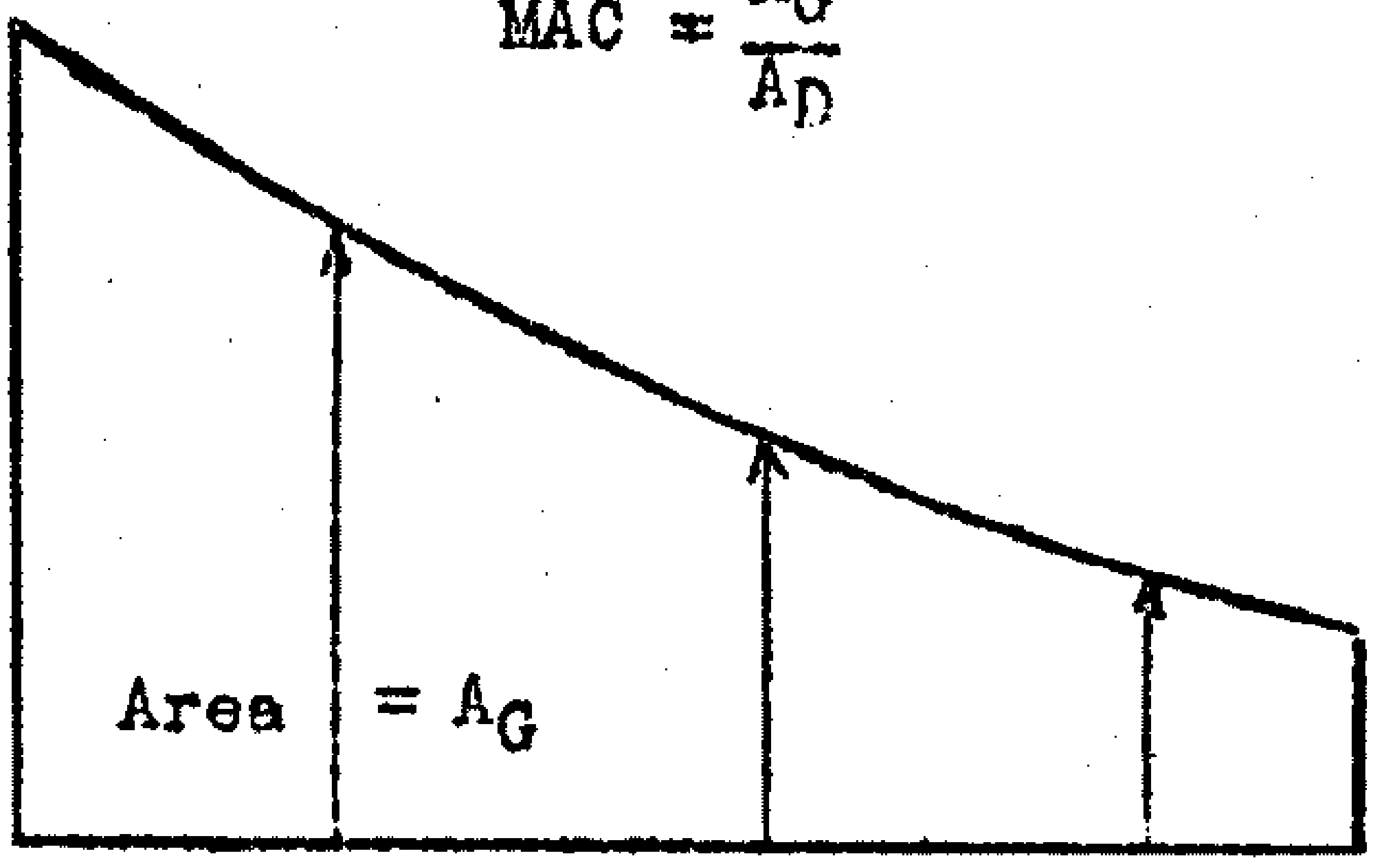
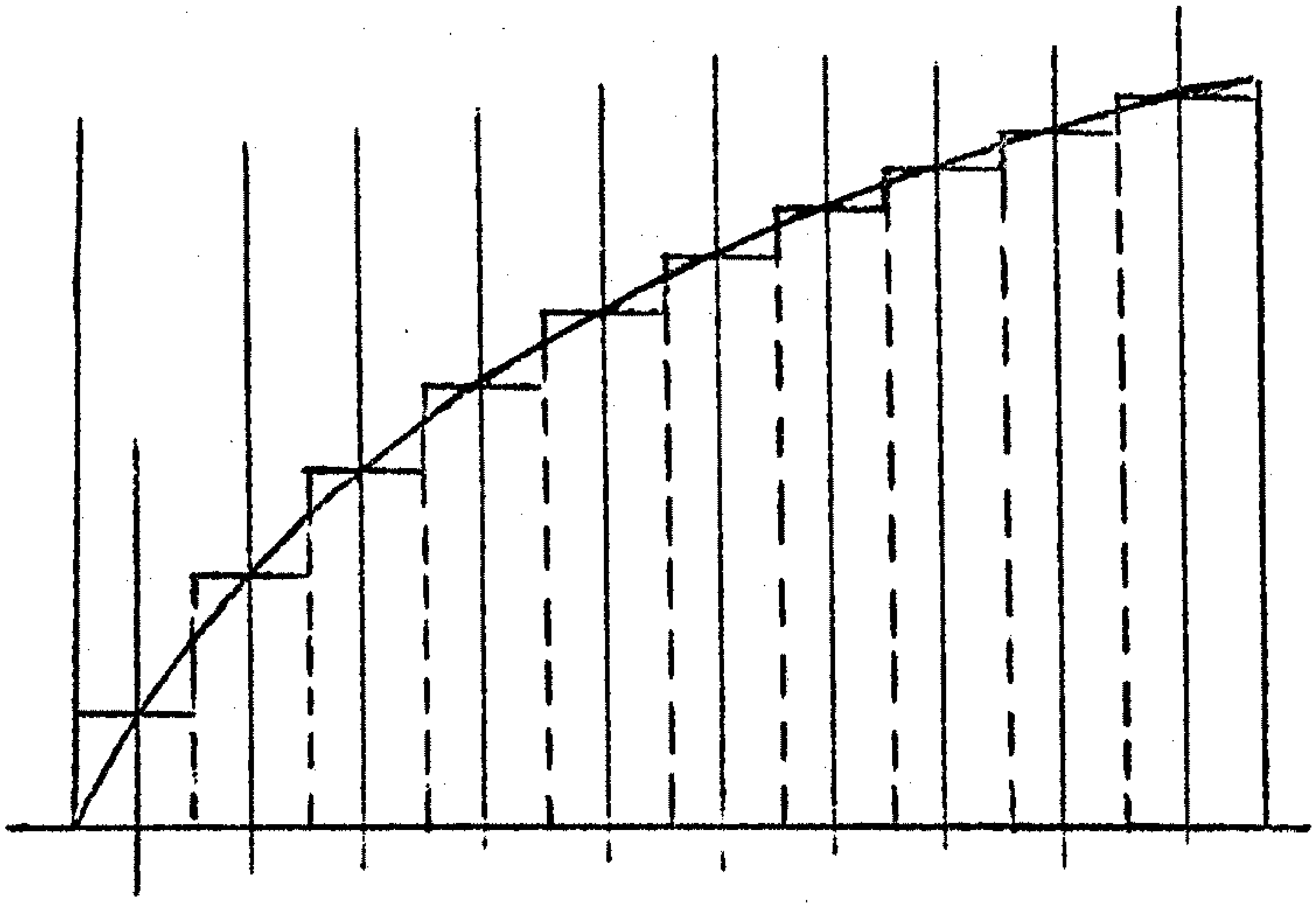


Fig. G

Problem Assignment.

4:1 Plot the actual curves indicated for Figs. B, C, D, E, F, and G and check the values for $\bar{b}$, $\bar{x}$, and the MAC.

Note: In actual practice the curves are plotted to a fairly large scale and areas are taken by planimeter. If a planimeter is not available they should be plotted on cross-section paper and areas taken by considering the curves to be a series of rectangles, thus:



ADVANCED STRESS ANALYSIS

ASSIGNMENT 5

Assignment 5

BALANCING LOADS ON THE AIRPLANE

The basic design conditions must be converted into conditions representing the external loads applied to the airplane, before a complete stress analysis can be made. This process is known as balancing the airplane and represents a complete static condition in which the fundamental equations $\Sigma V = 0$, $\Sigma H = 0$, $\Sigma S = 0$, and $\Sigma M = 0$ must be satisfied. It is a momentary condition of equilibrium. Actually there are both angular and linear accelerations acting to change the velocity and attitude of the airplane, but it is customary to represent a dynamic condition, for analysis purposes, as a static condition by assigning to each item of mass in the airplane the increased "inertia force" with which it resists acceleration. The acceleration force $= \frac{wa}{g}$ where "w" is the weight of the item, "a" its linear acceleration and "g" the acceleration of gravity.

If the direction of the resultant air forces on the airplane is not through the center of gravity there will be an angular acceleration tending to rotate the plane. An exact analysis would require the computation of this angular acceleration and its application to each item of mass in the airplane. Instead of this laborious method it is customary to eliminate the angular inertia forces by providing the necessary moment with a load on the tail surfaces. This is convenient, since the balancing load can be thought of either as an aerodynamic force on the tail surfaces or as a summation of the required balancing inertia forces. In a gust condition it is probable that angular inertia loads resist most of the unbalanced couple added by the gust, while in a steady pullup condition the tail load is mostly a balancing air load from the tail surfaces.

The following general assumptions are made in balancing the airplane:

(1) Full "power on" is assumed for conditions at V_L (I and II). For conditions at V_G (III and IV) the propeller thrust is assumed to be zero.

(2) It is assumed that the applied load factors specified for the basic flying conditions are wing load factors. A solution is therefore made for the net load factor acting on the entire airplane. The net load factor is then used in connection with each item or group of items in the analysis of the fuselage. For balancing purposes the net load factor can be assumed to act at the center of gravity of the airplane. When the center of gravity is variable, as on a transport airplane, the fore and aft limits must be determined and the load factors for the limiting conditions must be computed.

(3) In balancing the airplane we are concerned primarily with tail loads. Assuming that it is possible for the stabilizer and elevator loads to be acting in opposite directions, the civil air authority recommends that the center of pressure of the loads on the horizontal tail be placed at 20% of the mean chord of the entire tail surface. This arbitrary location is also considered as the point of application of inertia forces resulting from angular acceleration, thus simplifying the balancing process.

(4) The simplifying assumptions are made that the external forces act at four points on the airplane only. The forces consist of

- (a) Wing lift, wing drag and wing moment acting at the mean aerodynamic center. The mean aerodynamic center was computed in assignment 4.
- (b) Weight of the airplane and parasite drag of the airplane acting at the center of gravity.
- (c) Thrust of the propeller acting at the center line of thrust. (If the line of propeller thrust is within 3 degrees of the same angle as the reference line of the fuselage the difference is neglected.)
- (d) The tail force acting up or down at 20% of the mean tail chord.

For our problem Fig. 1 shows the locations and lever arms for these forces. Note that lengths are reduced to terms of the mean aerodynamic chord, and forces are expressed in terms of the gross weight of the airplane.

A tabular form is used to simplify the computation of the balancing loads for the various flight conditions. Both the forward and aft extreme positions of the center of gravity must be considered.

TABLE I

Center of Gravity at 15% MAC

		V_L = 220 mph.		V_G = 281 mph.	
No.	Item	Cond. I	II	III	IV
(1)	W = gross wt. lbs.	20,000			
(2)	$q = \frac{V^2}{391}$ (V in mph.)	123.7			
(3)	s = W/wing area	19.9			
(4)	$\frac{q}{s} = \frac{(2)}{(3)}$	6.21			
(5)	n_1 = applied wing load factor	3.54			
(6)	$C_N = \frac{(5)}{(4)}$	.57			

TABLE I (Continued)

Center of Gravity at 15% MAC

No.	Item	V_L = 220 mph. Cond. I	II	V_G = 281 mph. III	IV
(7)	α corresponding to C_N	6.3°			
(8)	C_c	-.114			
(9)	$n_{x_1} = (8) \times (4)$	-.708			
(10)	$n_{x_4} = F_{pr}/_W$	-.1365			
(11)	C'_m = design moment coefficient	-.0114			
(12)	$m_1 = (11) \times (4)$	-.0708			
(13)	n_3 = tail load factor	-.11			
(14)	$n_2 = - (5) - (13)$ = net load factor	-3.43			
(15)	$n_{x_2} = - (9) - (10)$ = chord load factor	.8445			
(16)	$T = (13) \times W$ = tail load	-2200			

Consistent signs are as follows:

+ = upward.
+ = aft.
+ = moment clockwise about the mean aerodynamic center.

It is advisable to make a sketch for each analysis condition putting the forces on and indicating the directions in which they act. This practice will avoid confusion as to the proper directions and signs to be used when the tail loads are computed. In calculations for the tail load factor, moments must be taken about the center of gravity of the airplane since both the tail coefficient and the net load factor through the center of gravity are unknown.

The equations may be set up with all forces positive as shown on Fig. 1. If a force is known to act in a direction to give a negative sign the negative sign should be prefixed before the force is entered in the equation.

A sample computation is made in Table I to show the value of the tail load for Condition I with the center of gravity in the forward position.

c.g. forward is at 15 per cent of the mean aerodynamic chord.
c.g. aft is at 30 per cent of the mean aerodynamic chord.

The value F_{pr}, the propeller thrust, is calculated from the formula

$$F_{pr} = \frac{(375)(HP)(\eta)}{V}$$

where F_{pr} is thrust in pounds.

η is the propeller efficiency (may be assumed .80).

V is miles per hour.

The propeller thrust is used for the conditions at level flight speed, V_L. (I and II)

At the gliding speed, V_G, the propeller thrust may be assumed to be zero. (III and IV)

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Sample computations are given below for Condition I.

$n_1 = 3.54$

$n_{x_1} = -.708$ (acts forward)

$m_1 = -.0708$ (diving moment ↶)

Horse power = 2000 (2 engines)

$$F_{pr} = \frac{(375)(2000)(.80)}{220} = 2730 \text{ lbs.}$$

$$n_{x_4} = \frac{F_{pr}}{W} = \frac{-2730}{20,000} = -.1365$$ (minus because it acts forward)

$$n_{x_2} = -(n_{x_1}) - (n_{x_4}) = +.708 + .1365 = .8445$$

Moments about c.g. forward = 0 ↶+

Forces are placed in the diagram acting in their correct directions.

$$(n_3)(x_3 + x_{2_F}) + m_1 + (n_1)(x_{2_F}) - (n_{x_1})(h_2) - (n_{x_4})(h_2 + h_4) = 0$$

$$(n_3)(3.10) + .0708 + (3.54)(.10) - (.708)(.10) - (.1365)(.15) = 0$$

$$3.10\ n_3 = -.0708 - .354 + .0708 + .0205 = -.3335$$

$$n_3 = -\frac{.3335}{3.10} = -.1076 \quad \text{use } -.11$$

The tail load was assumed to act upward.
The minus sign indicates that it acts downward.

$$n_2 = -n_1 - n_3$$

$$= -3.54 + .11 = -3.43 \qquad \text{acting downward}$$

$$T = (-.11)(20{,}000) = -2200 \text{ lbs.} \qquad \text{acting downward}$$

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Problem Assignment.

5:1 Check the solution for Condition I and complete the tables for Conditions II, III, and IV with forward center of gravity.

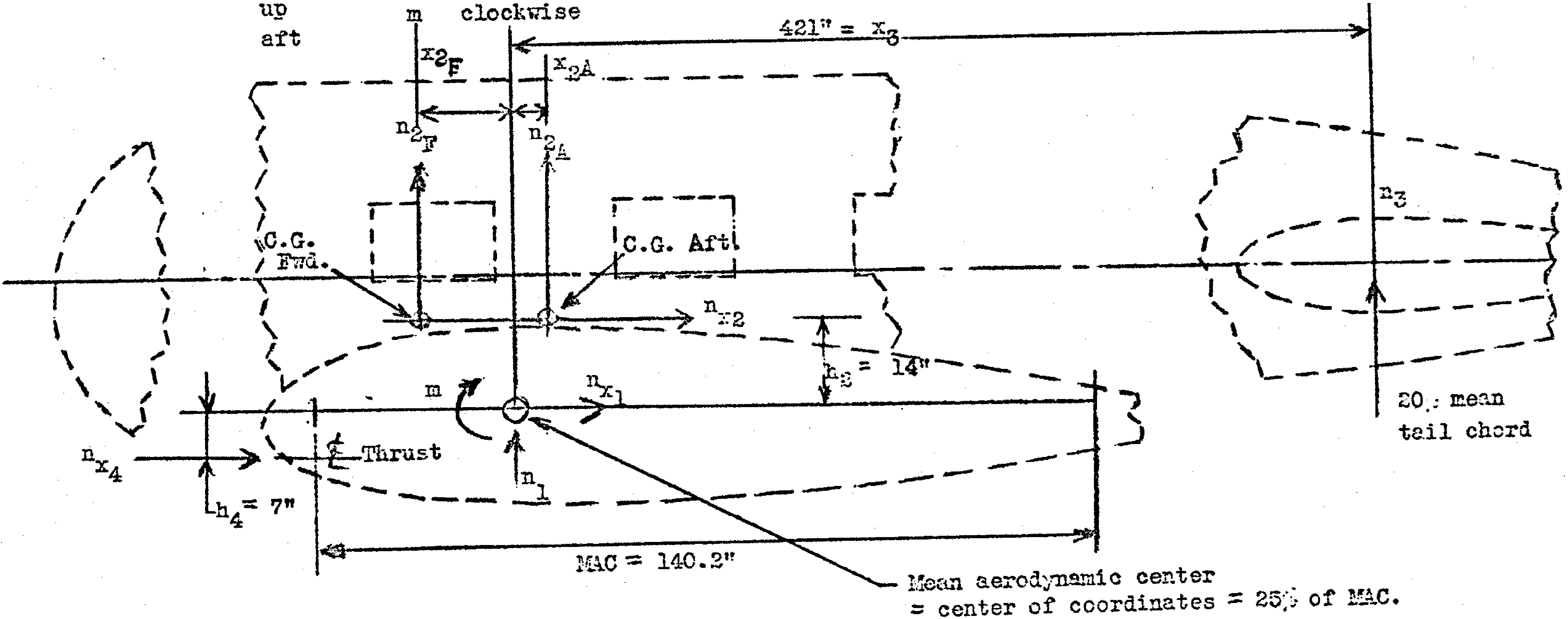
5:2 Complete the tables for Conditions I, II, III, and IV for aft center of gravity.

BALANCE DIAGRAM

n_1 = lift load factor
n_3 = tail load factor
m = moment factor

n_2 = dead weight factor
n_{x_1} = drag load factor on wing
n_{x_2} = parasite drag factor

n_{2_F} = C.G. fwd. = 15%
n_{2_A} = C.G. aft. = 30%
n_{x_4} = propeller thrust factor



Mean aerodynamic center = center of coordinates = 25% of MAC.

In terms of MAC Length:

$x_{2_F} = -(.25 - .15) = -.10$
$x2_A = .30 - .25 = .05$
$x_3 = 421/140.2 = 3.0$

$h_2 = 14/140.2 = .10$
$h_4 = -7/140.2 = -.05$

Fig. 1

AIRPLANE
STRESS
ANALYSIS

PART NINE

ADV. STRESS 6

ADV. STRESS 6A

ADV. STRESS 7

ADVANCED STRESS ANALYSIS

ASSIGNMENT 6

Assignment 6

LOAD, SHEAR, MOMENT, AND TWIST ON A CANTILEVER WING

Loads.

The loads on a wing consist of the air loads, (which we have previously resolved into the components in the chord and beam directions), the thrust of the propellers, and the inertia or dead weight loads due to the weight of the wing structure itself, and the weights of fuel tanks and any fuel they may contain, and of engines, engine mounts and nacelles, landing gear, skis or floats which are carried in or attached to the wing. The factors to be used for the dead weights are resolved into components in the beam and chord directions in Assignment 5. (These factors are listed as N_2 and N_{x_2} in the tables at the end of Assignment 5.) The magnitude and the distribution of the dead weight of items in or attached to the wing are taken from a weight estimate.

Dead Weight Distribution--Spanwise.

An accurate determination of the dead weight distribution involves estimation of the weight per inch of the structure at various stations along the span. Usually it is accurate enough to estimate the wing weight in lbs. per sq. ft. of area, basing the estimate for the total on the actual weight for previous wings of the same type. The wing weight is then distributed as being proportional to the chord lengths and to the thickness. When there is a straight line variation in both the chord and the thickness, the plot of wing structure weight per inch will be a slight curve. It is considered permissible in such a case to replace the curve by a straight line which approximates it. It is then possible to write a linear equation for the distribution of structure weight, from which the dead weight shear and moment at any station along the wing may be easily calculated.

The effect of concentrated items of dead weight such as fuel tanks and nacelles must also be included. The weights of tanks and fuel may be uniformly distributed over that portion of the span in which they are located. Nacelles, landing gear, etc., are considered to be concentrated loads at the stations at which they attach to the wing.

It should be noted that wing dead weight loads always act opposite to the air loads, and that an underestimate on structure weight, which nearly always occurs, is conservative for design.

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Dead Weight Distribution--Chordwise.

The chordwise distribution of the weight of the wing structure must be estimated by calculation of the centroids of weight at various stations along the wing. It will generally be found accurate enough to assume the centroid of dead weight to be at 40 per cent of the wing chord behind the leading edge. The chordwise distributions of other dead weight loads in the wing are readily calculated from the actual locations of the centers of gravity of the items involved.

Spanwise and Chordwise distribution of the air loads over the wing were discussed in Assignment 4.

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Distribution of Loads to the Wing Structure.

The distribution of the loads to a wing structure is dependent upon the type of bending structure used, particularly with regard to its resistance to twisting, termed its "torsional rigidity". A fabric covered wing is torsionally flexible, while a metal covered wing is torsionally rigid. The distribution of the load to a torsionally flexible wing will be considered first. Such a wing is analyzed as follows in the general case with any variation of chord length and loading along the span:

1. The wing is divided along its span into chordwise strips of equal width.

2. The magnitude and centroid of the air load on each strip is determined from the wind tunnel data for the particular angle of attack being designed for.

3. The air load is distributed to the spars inversely proportional to their distances from the center of pressure. This assumes that the ribs act as simple beams supported at the spars. The dead weight loads are distributed in the same manner.

4. The drag loads are carried by a "drag truss" in the chordwise plane, for which the front and rear spars act as chord members.

Assuming first a torsionally flexible wing, the forces acting upon it will be as shown in Fig. 1, which represents the loading condition for any strip along the span.

S = area of strip of wing considered.

$N = C_N S_q$

$C = C_c S_q$

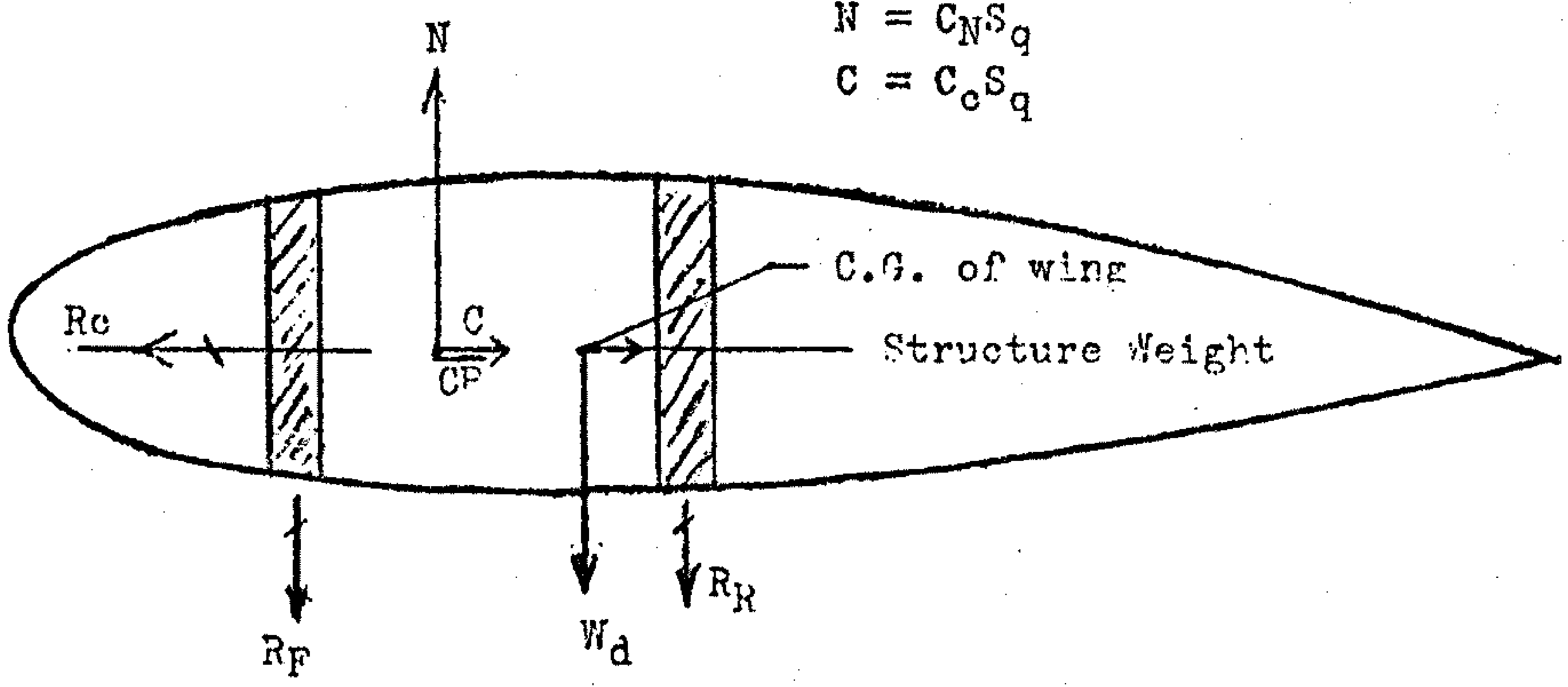


Fig. 1

Balance of Forces on a Typical Strip of Wing

R_F = vertical reaction at front spar.

R_R = vertical reaction at rear spar.

R_c = chordwise reaction on drag truss.

Since we have seen that the air load factors and the dead weight load factors are not the same (Assignment 5) it is generally simpler to compute the air load shears and moments, and the dead weight shears and moments separately and combine the results in tables to obtain the true shears and moments, accounting for both air loads and dead weight loads.

The simplest and most accurate method of determining the air loads, shears and moments for a number of flight conditions is to compute these values for a unit loading over the wing of 1 lb. per sq. ft., and to obtain the actual values for the flight conditions investigated by multiplying the unit load, (1 lb. per sq. ft.), values by the average design load values for a particular loading condition being investigated.

For the dead weight loads the actual weight of the wing structure, etc. is taken for the computation of unit load shears and moments from dead weight. This corresponds to weights under the acceleration of gravity.

Then the true values in a given condition are found by multiplying the values for the unit dead weight by the dead weight factor for the condition being investigated. The methods referred to will be illustrated for the wing of our type problem.

Unit Air Loads, Shears and Moments.

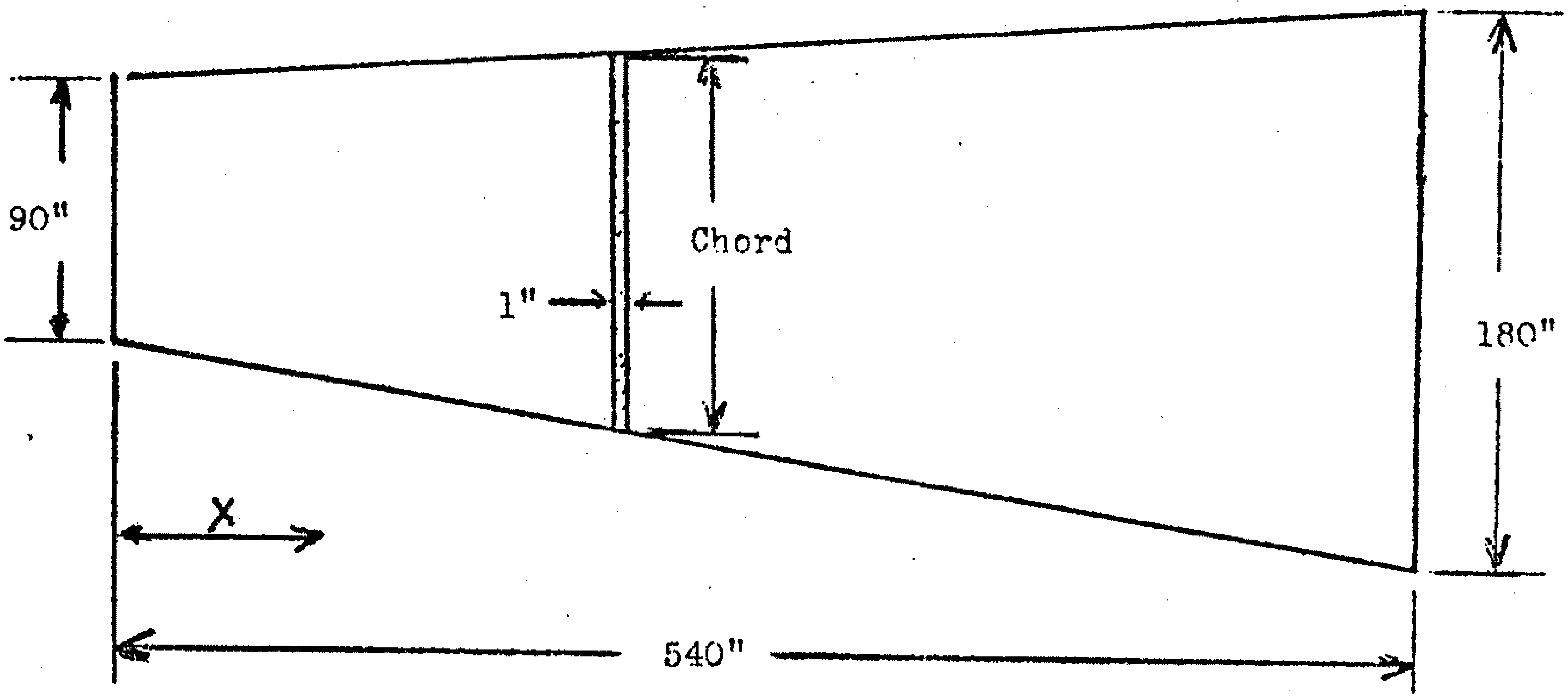


Fig. 2

For an air load of 1 lb. per sq. ft. on the wing the running load per inch on the wing will be $\frac{(1)(\text{chord in inches})(1\text{ inch})}{144}$, since the area of a strip 1 inch wide times the length of the chord is $\frac{(C)(1)}{144}$ sq. ft.

Then the unit air load at the tip is $\frac{90}{144} = .625$ lbs/inch.

and at the root chord is $\frac{180}{144} = 1.25$ lbs/inch.

The difference between root and tip chords is .625 lbs. per inch. Then the equation of the total load on the wing beginning at the tip is

$w = .625 + \frac{.625}{540}x$ and the variation of the load per inch is a linear function of x, the distance along the spar.

We will first develop the load, shear and moment values for the entire wing and then apportion them to wing beams.

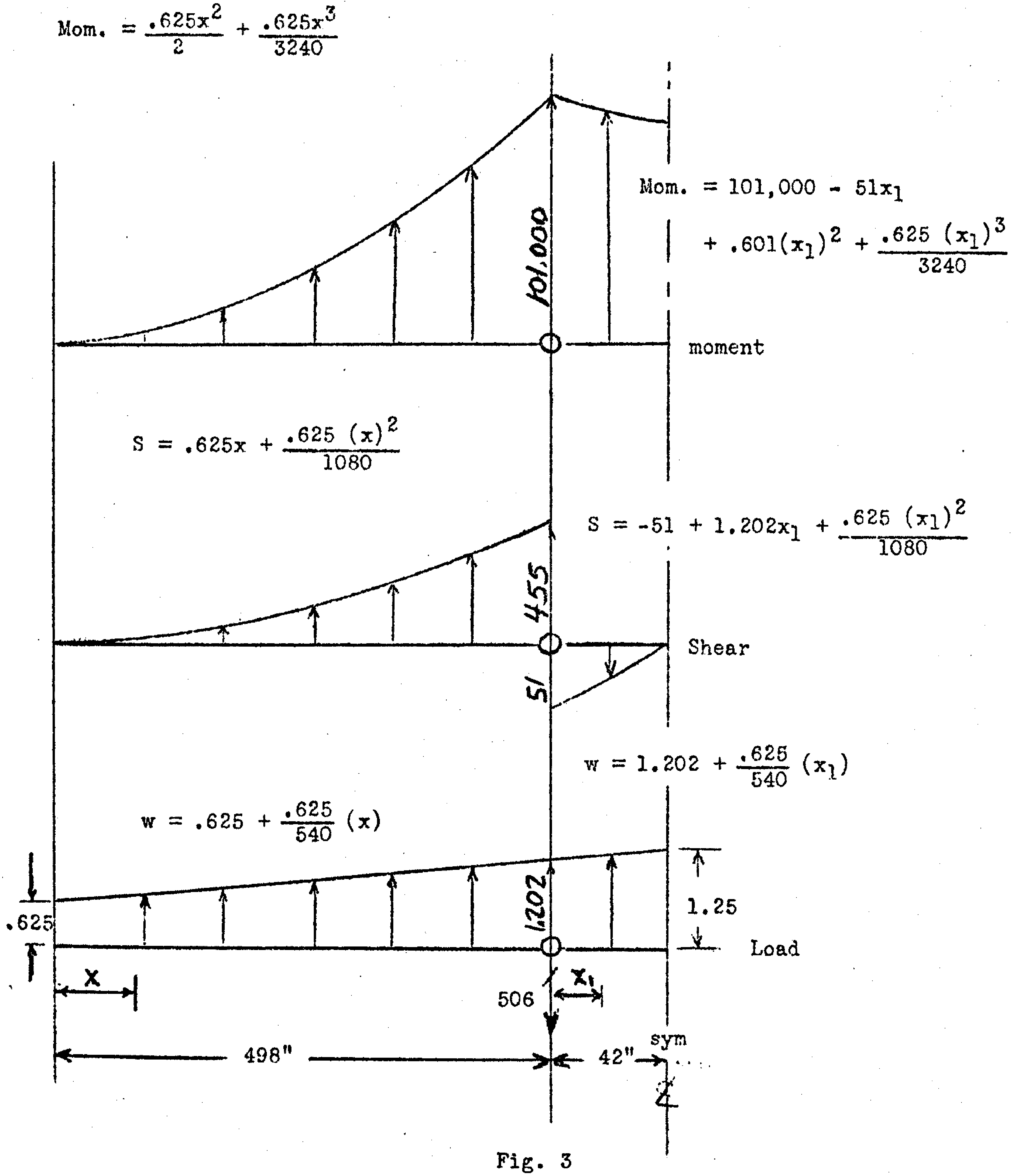

Mom. $= \frac{.625x^2}{2} + \frac{.625x^3}{3240}$
Mom. $= 101,000 - 51x_1 + .601(x_1)^2 + \frac{.625\,(x_1)^3}{3240}$
101,000
moment
$S = .625x + \frac{.625\,(x)^2}{1080}$
$S = -51 + 1.202x_1 + \frac{.625\,(x_1)^2}{1080}$
455
51
Shear
$w = 1.202 + \frac{.625}{540}\,(x_1)$
$w = .625 + \frac{.625}{540}\,(x)$
.625
1.202
1.25
Load
x
x_1
506
498"
42"
sym
℄

Fig. 3

Unit Air Loads, Shears and Moments on Wing
(Load = 1 lb. per sq. ft.)

A unit torque from air load on the wing is defined as the torque from the displacement of the unit load on the strip by one per cent of chord. This is a convenient unit since for air loads the center of pressure is at the same percentage of chord at any section along the span. Where both the equation of running load and the equation of one per cent of the chord may be written it is possible to secure the equation for unit torsion by multiplying one equation by the other. For our case the equation of unit load from Fig. 3 is

$$w = .625 + \frac{.625}{540}(x)$$

The equation of the chord at any section, taking the origin for x at the tip, is

$$\text{Chord} = 90 + \frac{90}{540}(x)$$

Then the equation for 1 per cent of the chord at any station is

$$90 + \frac{90}{540}x \quad (.01)$$

The unit torque load is $(.01C)(w) = tq$

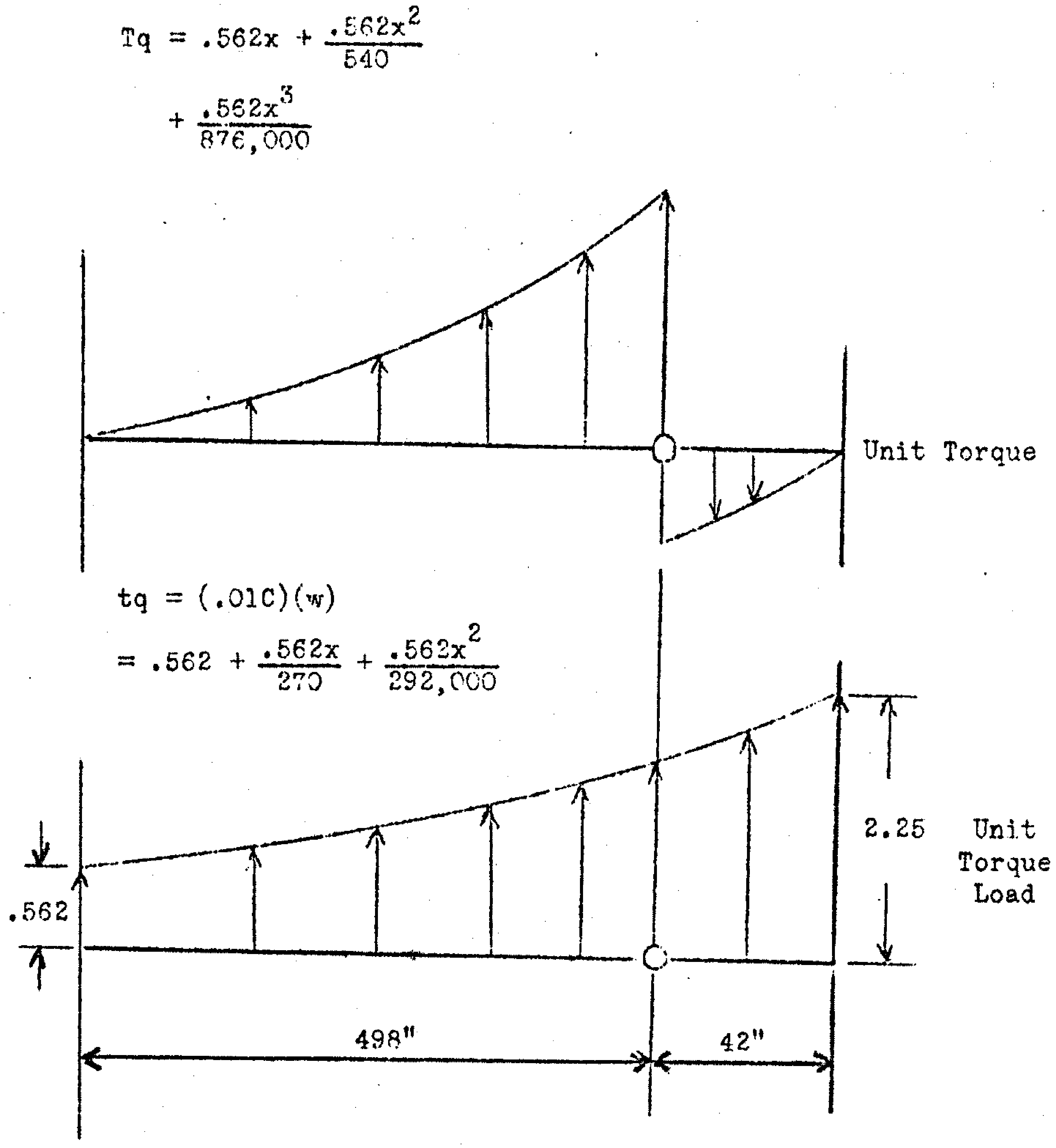
$$tq = (90 + \frac{90}{540}x)(.01)(.625 + \frac{.625}{540}x)$$

$$= (90)(1 + \frac{x}{540})(.01)(.625)(1 + \frac{x}{540})$$

$$= (.9)(.625)(1 + \frac{x}{540})(1 + \frac{x}{540})$$

$$= (.562)(1 + \frac{x}{540})^2 = .562 + \frac{.562x}{270} + \frac{.562x^2}{292,000}$$

The unit torque load equation gives the value of the torque for any strip along the wing. To obtain the summation of the torque (the sum of the torque outboard of a given station) it is only necessary to integrate the equation for the torque load curve. The results are shown on the curves of Fig. 4.



$$Tq = .562x + \frac{.562x^2}{540} + \frac{.562x^3}{876,000}$$

$$tq = (.010)(w) = .562 + \frac{.562x}{270} + \frac{.562x^2}{292,000}$$

Fig. 4

When the equations for shears, moments, and torques can be written it is convenient to solve them in tabular form. Table 1 gives the solution for our example for a unit loading of 1 lb. per sq. ft. Values given are for the air loads acting in the beam direction.

Table 1

Unit Lift Shear, Moment and Torque

Sta.	42	115	145	270	405
			SHEAR		
x	498	425	395	270	135
x^2	247,000	180,600	156,000	73,000	18,250
$.625x$	312	266	246	169	84.2
$\frac{.625x^2}{1080}$	143	104	90	42	10.6
Unit Shear	455	370	336	211	94.8
			MOMENT		
x^3	123,000,000	76,800,000	61,500,000	19,700,000	2,470,000
$\frac{.625x^2}{2}$	77,300	56,500	48,800	22,800	5,700
$\frac{.625x^3}{3240}$	23,700	14,800	11,900	3,800	470
Unit Moment	101,000	71,300	60,700	26,600	6,170
			TORQUE		
$.562x$	280	239	222	152	76
$\frac{.562x^2}{540}$	258	188	162	76	19
$\frac{.562x^3}{876,000}$	79	49	40	13	2
Unit Torque	617	476	424	241	97

The use of the unit torque is illustrated when we design a box wing.

Distribution of Dead Weight.

The total dead weight carried by the wings consists of the following items:

Wing structure (distributed load)........		= 2590 lbs.
Fuel System in wing (distributed load)...		= 300 lbs.
Landing gear (concentrated at nacelles)..		= 900 lbs.
Nacelle Weights (2 nacelles)............		= 4560 lbs.
Engines and accessories...........	2600	
Propellers........................	800	
Oil systems.......................	60	
Oil...............................	150	
Engine Mount and Nacelle Structure	950	
Total	= 4560 lbs.	

Total dead weight on wings--both sides = 8350 lbs. with fuel tanks empty. Since the fuel carried will vary with the condition fuel is considered as a separate item.

The nacelle weights are taken into the wing equally at stations 115 and 145. The chordwise locations of the weight items which are concentrated at the nacelle are shown on Fig. 5. Values are divided by 2 to show the loads for one nacelle. Values given above were for the entire airplane.

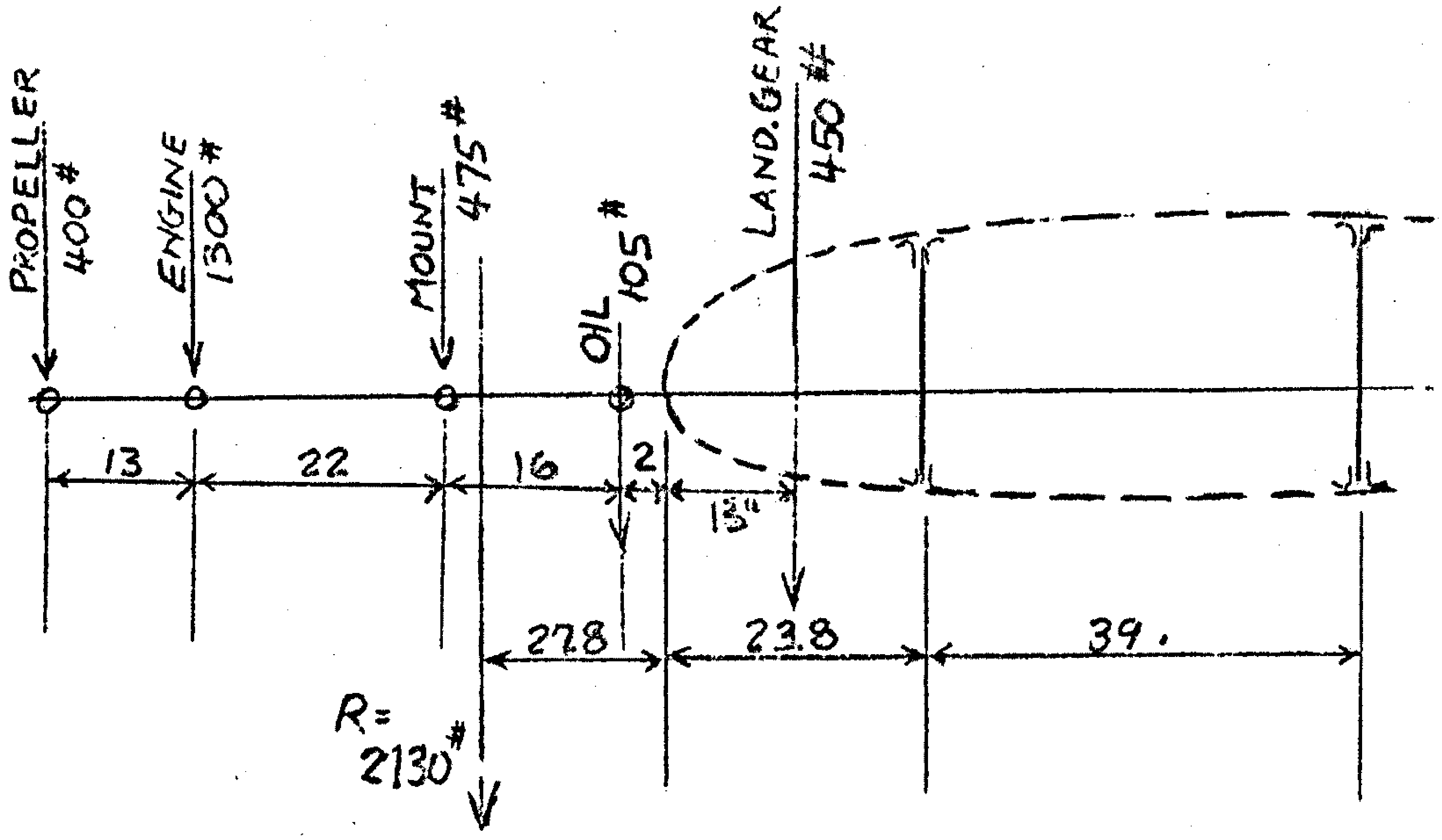


Fig. 5

Chordwise Distribution of Dead Weight Load at a Nacelle

For one Nacelle - Reference at Leading Edge of Wing.

Item	Weight	Horizontal Arm	Horizontal Moment
Propeller.........	400	-53	-21,200
Engine............	1300	-40	-52,000
Mount.............	475	-18	- 8,520
Oil and Tank......	105	- 2	- 210
Landing gear......	450	+13	+ 5,850
Total..........	2730 lbs.	-27.8 in.	-76,080

From preliminary analysis and calculation of section areas the dead weight of the wing structure is found to vary as shown on Fig. 6. The equations of load, shear, and moment are developed for the wing structure separately. Fig. 7. Then in the tables (see Table 2) allowance is made for the effects of the dead weights concentrated at the nacelles, and the weights in the fuel system, and fuel. In the equations the dead weight is assumed plus (acting upward). The sign convention for dead weight load factors has already accounted for the correct direction based on this assumption. See Assignment 5.

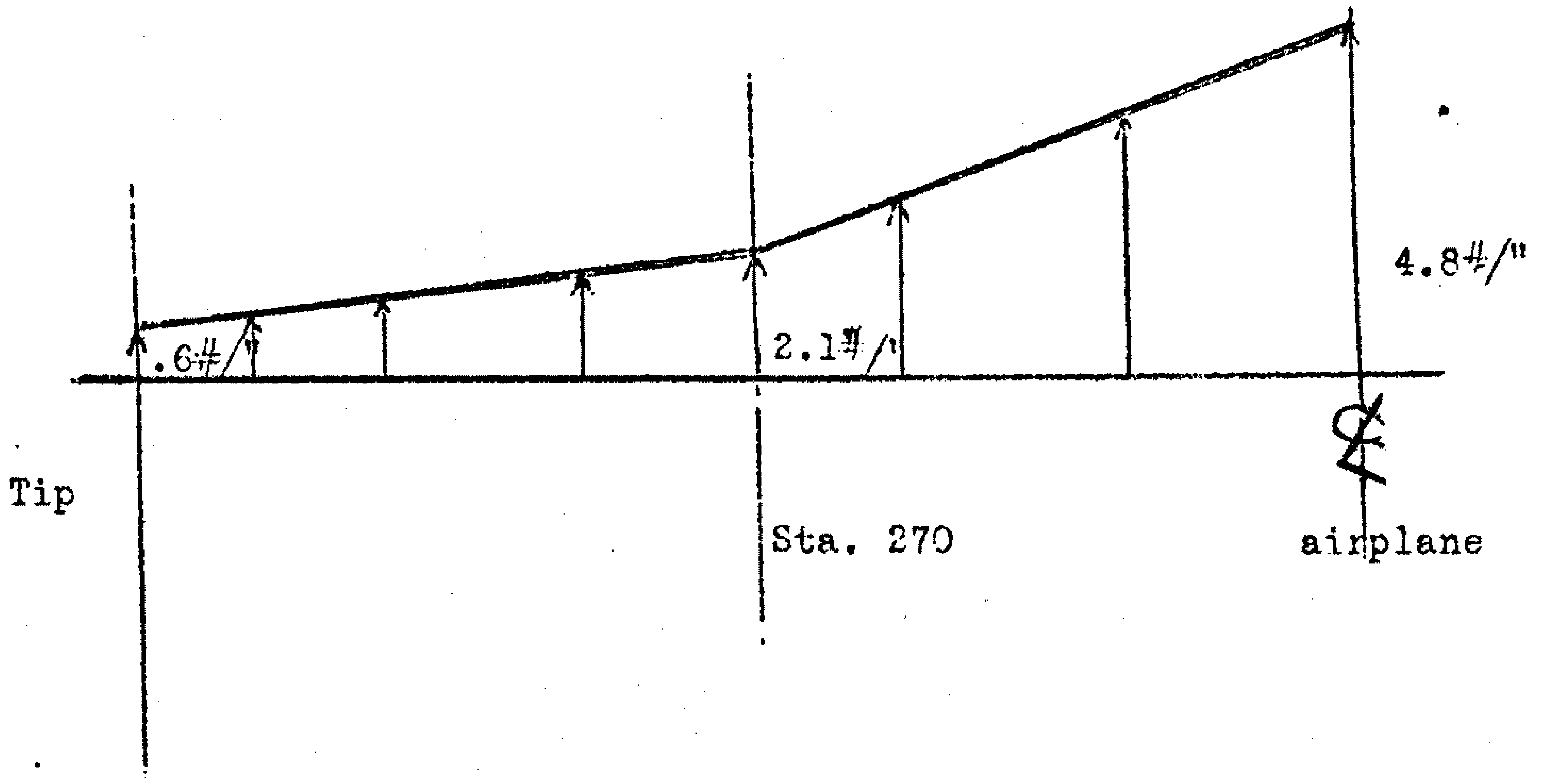


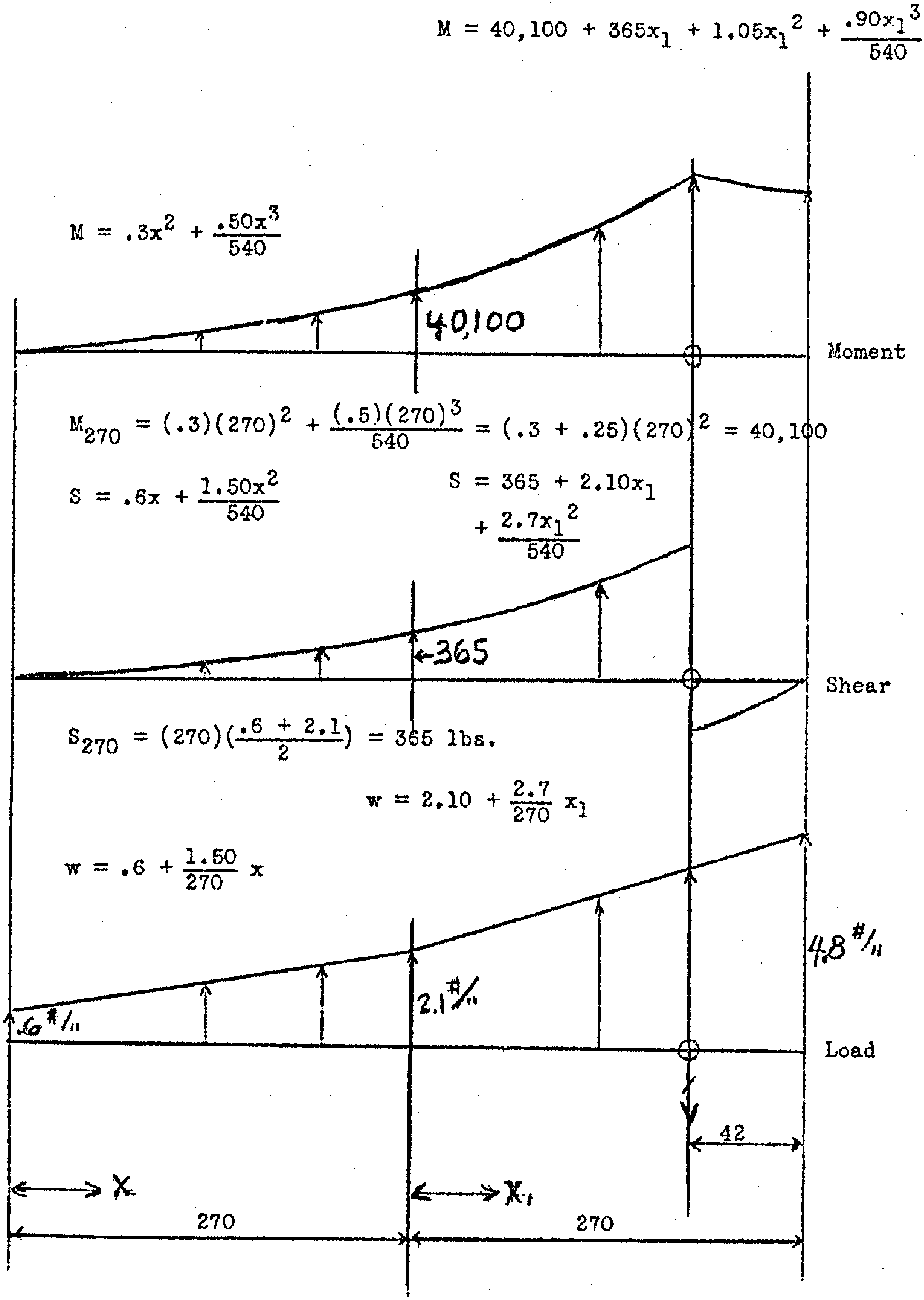
Fig. 6

Dead Weight Variation of Wing Structure

Structure Weight on one side

$$= \left(\frac{.6 + 2.1}{2}\right)(270) + \left(\frac{2.1 + 4.8}{2}\right)(270) = 1295 \text{ lbs.}$$

Wing structure for total airplane = (2)(1295) = 2590 lbs.



$$M = 40,100 + 365x_1 + 1.05{x_1}^2 + \frac{.90{x_1}^3}{540}$$

$$M = .3x^2 + \frac{.50x^3}{540}$$

$$M_{270} = (.3)(270)^2 + \frac{(.5)(270)^3}{540} = (.3 + .25)(270)^2 = 40,100$$

$$S = .6x + \frac{1.50x^2}{540}$$

$$S = 365 + 2.10x_1 + \frac{2.7{x_1}^2}{540}$$

$$S_{270} = (270)\left(\frac{.6 + 2.1}{2}\right) = 365 \text{ lbs.}$$

$$w = 2.10 + \frac{2.7}{270}x_1$$

$$w = .6 + \frac{1.50}{270}x$$

Fig. 7

Load, Shear and Moment Curves for Dead Weight of Wing Structure

Effect of Dead Weight Loads at Nacelles on Load, Shear and Moments on Spars.

For the case of a flexible wing (one in which the wing beams are assumed to act independently in bending) the loads may be apportioned directly to each beam; and the shears and moments for each beam, from the effect of nacelle loads only may be computed directly from the load. All dead weight values will be kept in terms of one load factor, for convenience in combining the effects of wing structure dead weight, nacelles dead weight and weight of fuel and fuel tankage.

For computing the effect of weights at the nacelles we need only to consider the resultant of loads, R = 2730 lbs., from Fig. 5. The nacelle trusses and the bulkheads between the spars, which occur at stations 115 and 145 act as beams loaded at the center of gravity of the nacelle weights and supported on the spars. See Fig. 8. The computations will be made for the total nacelle load and the reactions on the spars will then be divided by 2 to give the loads acting at stations 115 and 145 respectively.

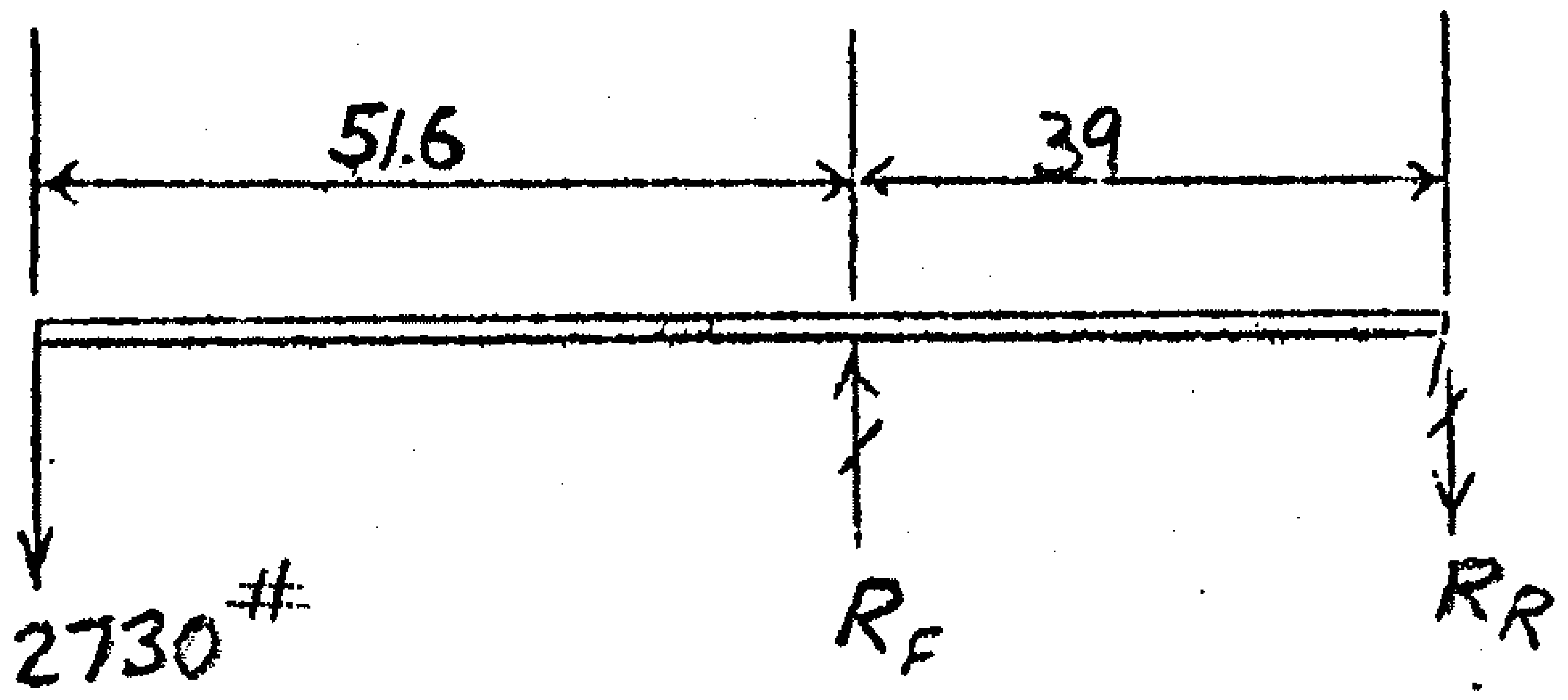


Fig. 8

Balance of Loads from Nacelle Dead Weights

Taking moments about the front spar:

$$M_{R_F} = 0 \qquad +\circlearrowright$$

$$(R_R)(39) - (2730)(51.6) = 0$$

$$39R_R = 141{,}000$$

$$R_R = 3{,}610 \text{ lbs.} \qquad R_F = 6{,}340 \text{ lbs.}$$

Check by taking moments about R_R

$(90.6)(2730) = 248,000$ in. lbs.

$(39)(6340) = 248,000$ in. lbs.   (Checks)

The loads on the wing beams are equal and opposite to the reactions to the wing bulkheads. Therefore the load of 6,340 lbs. acts downward on the front spar and the load of 3610 lbs. acts upward on the rear spar. The total values divided between stations 115 and 145 are as follows:

Front Spar: 3170 lbs. down at Stations 115 and 145
Rear Spar: 1805 lbs. up at Stations 115 and 145

To determine the shear and moment curves on the spars for the nacelle dead weight:

For the Front Spar, the loads will subtract from the air loads.

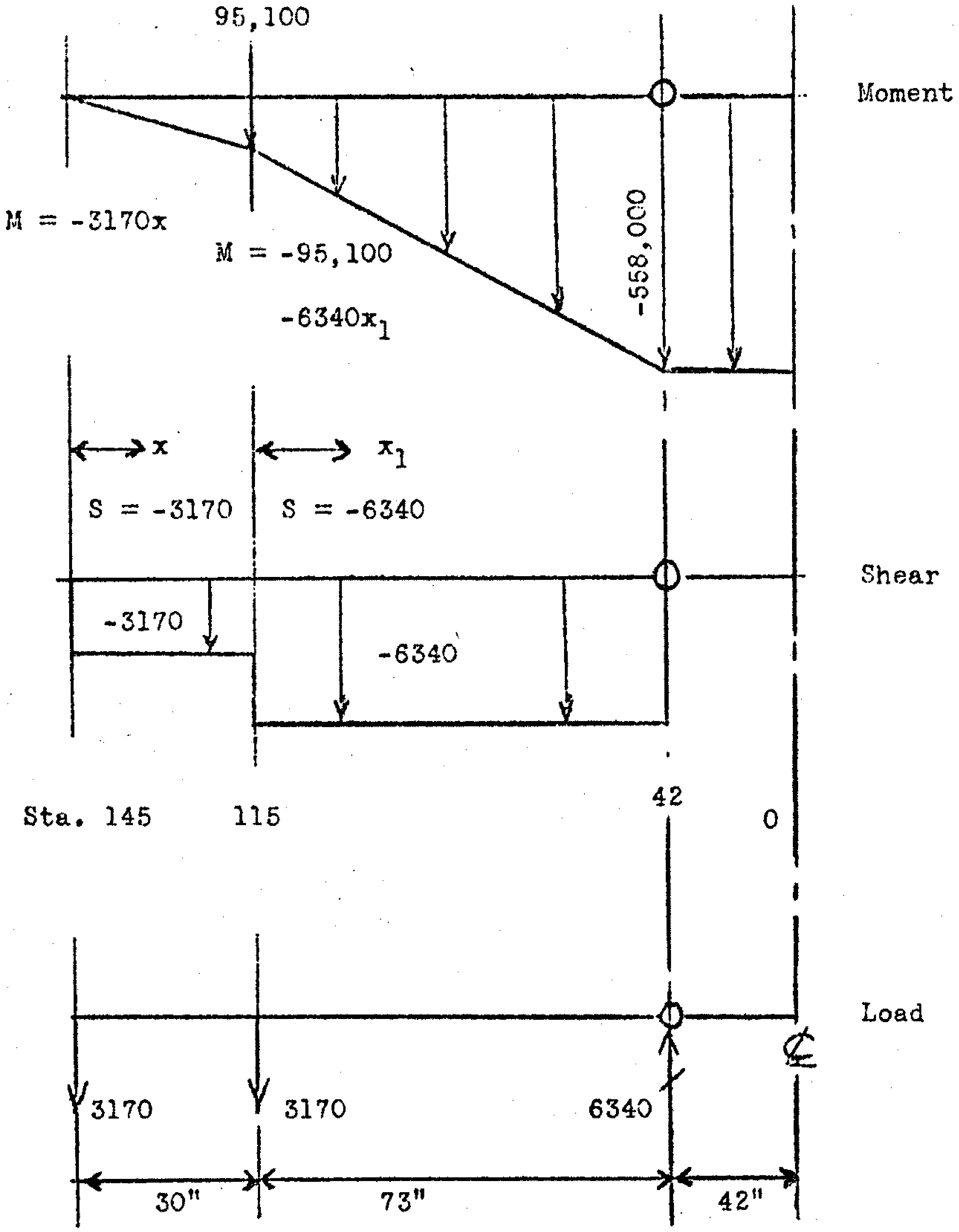


Fig. 9

Effect of Nacelle Dead Weight on Front Spar
Load Factor = -1.0

For the Rear Spar, the loads will add to the air loads.

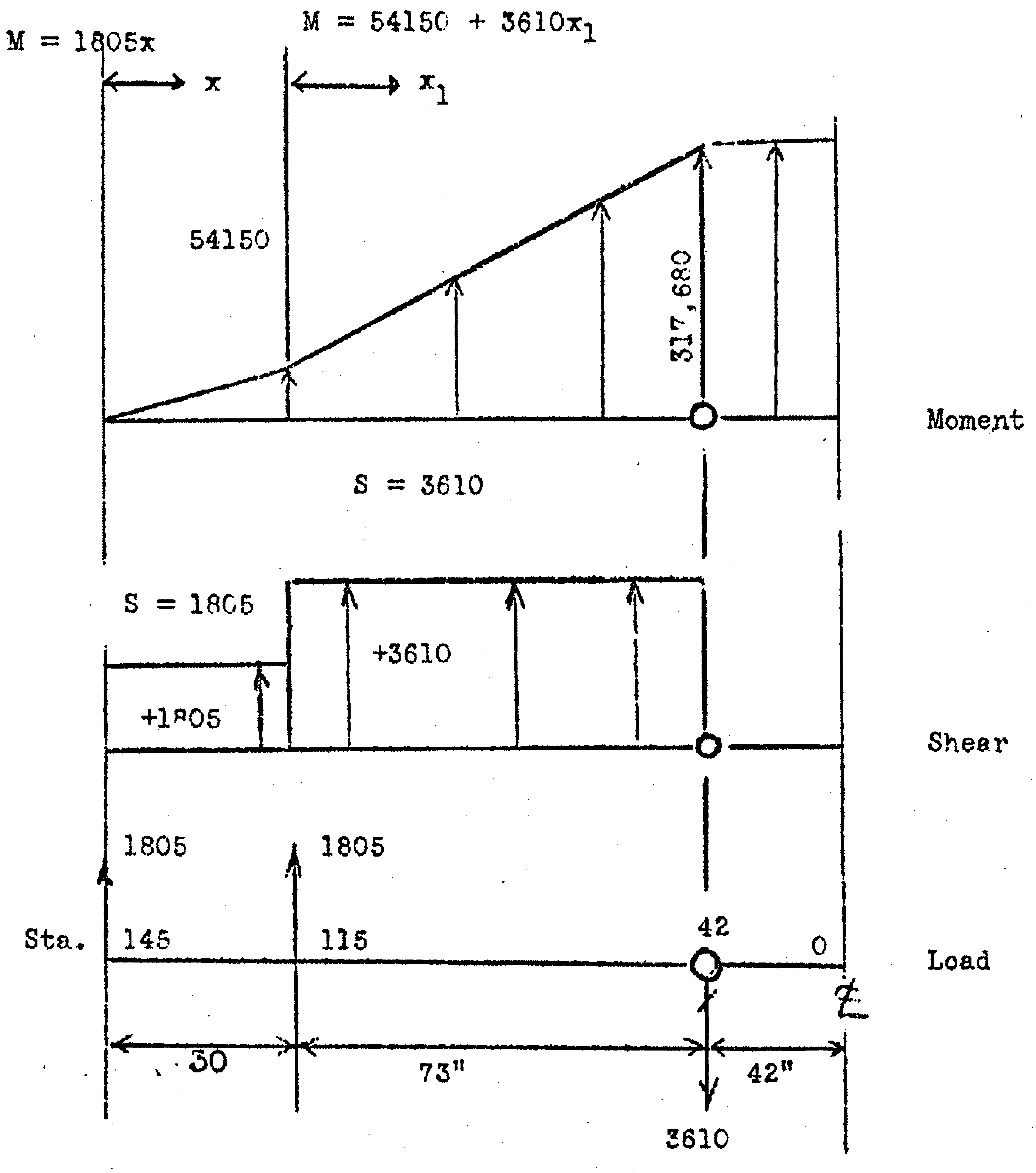


Fig. 10

Effect of Nacelle Dead Weight on Rear Spar
Load Factor = -1.0

Effect of Dead Weight of Tankage and Fuel.

The fuel system is estimated to weigh a total of 300 lbs. or 150 lbs. per side. This load is located spanwise uniformly between the center-line of the fuselage and wing station 115 and is located chordwise so that its center of gravity is at 27 per cent of the wing chord. The location of fuel carried will be assumed to coincide with that of the fuel system.

For our investigation we will take a loading condition for maximum load in the fuselage with full gross weight. For this condition the fuel load is 277 gals. (= 1660 lbs.). See Assignment 3, page 3-19.

The total load of fuel plus tankage for this condition is 300 + 1660 = 1960 lbs. or 980 lbs. per side.

The load per inch of wing spanwise would then be $\frac{980}{115} = 8.52$ lbs. for fuel plus tankage at a load factor of 1. The load, shear and moment curves resulting from this loading are shown on Fig. 11.

Except for the Weights at the Nacelles the dead weight loads, shears and moments have been solved for the entire wing. For a division of load to the spars we must examine the chordwise centroids of loading. For the wing structure the chordwise center of weight is at the 40 per cent point on the chord. It therefore coincides with the rear spar and it may be assumed that the dead weight of the structure is carried on the rear spar. The fuel and tankage is at 27 per cent of the wing chord. Since the front spar is at 15 per cent and the rear spar at 40 per cent the tankage load will go

$\frac{12}{25} = .48$ to the rear spar and

$\frac{13}{25} = .52$ to the front spar.

The above distribution assumes that the tank supports act as pin ended beams supported at the wing spars.

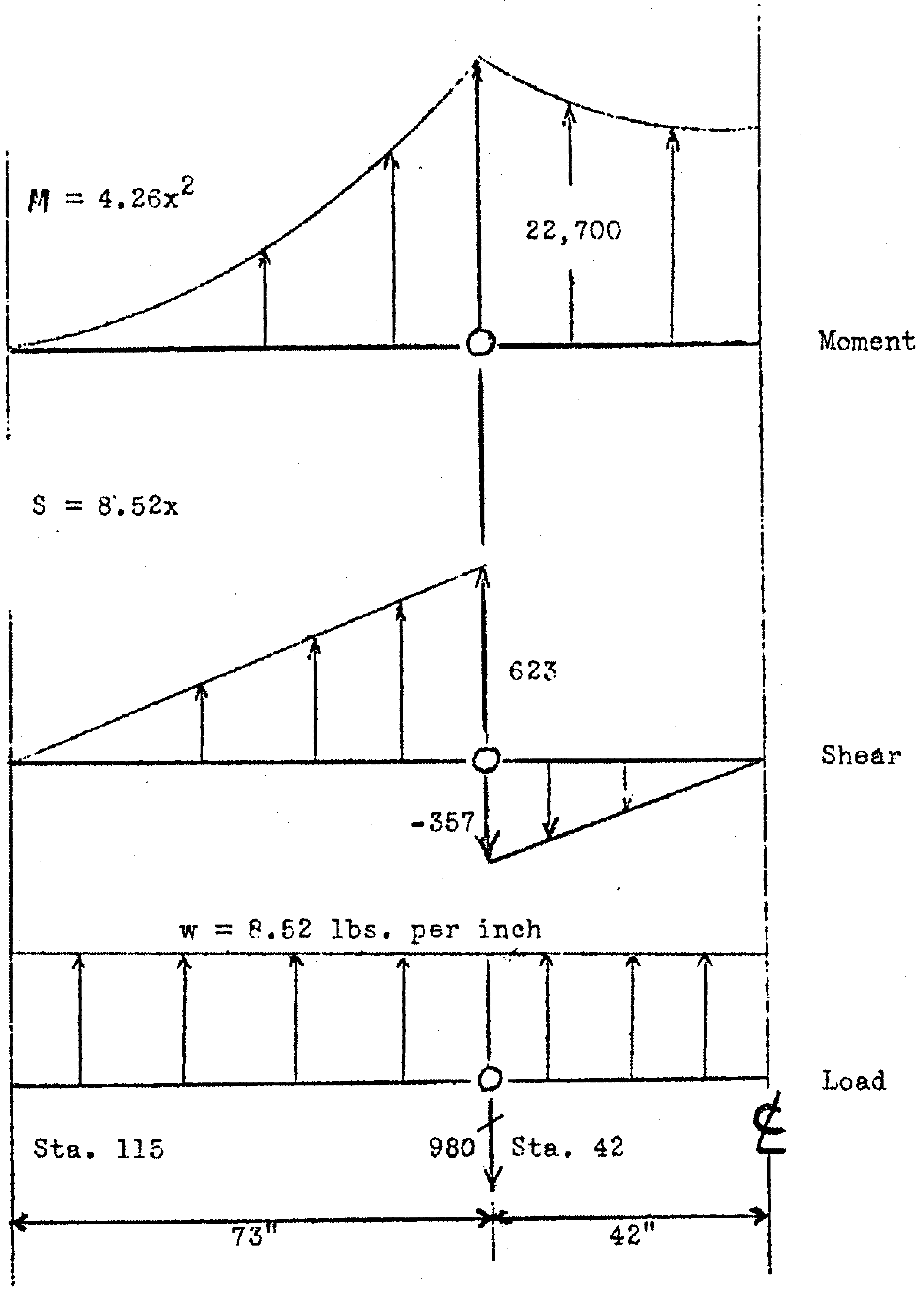


Fig. 11

Dead Weight Load, Shear and Moment from Fuel and Fuel Tankage

Load Factor = +1.0

Total for Wing Shown

Assignment:

Using the dead weight data developed in the curves of Figs. 7, 9, 10, and 11 and the ratios given above compute the total dead weight shears and moments on each spar for a dead weight factor of -1.0, that is for a downward acting dead weight. Note that on Figs. 9 and 10, concerning the effects of nacelles, a downward load on the nacelles has been assumed, while the general equations of Figs. 7 and 11 have been written for an upward acting dead weight.

Calculate the values for each spar for stations 42, 115, 145, 270 and 405. The values for each station may be obtained for the structure, nacelles, and (fuel + fuel system) separately and then added to obtain the final results.

ADVANCED STRESS ANALYSIS

ASSIGNMENT 6 A

Assignment 6A

DESIGN SHEARS AND MOMENTS ON A TWO SPAR WING

The work of this assignment is a continuation of that in Assignment 6. In Assignment 6 we calculated the values of the dead weight shears and moments at various stations along the wing. In the first part of this assignment we will distribute the air loads to the spars and determine our final shears and moments, which are the resultant of all dead weight and air load effects.

Distribution of the Lift Air Loads to Spars.

In the general case the load to each spar, from air loads on a given segment of a wing may be obtained by calculating the required reaction at the spar to hold the segment in equilibrium. See Fig. 1, page 6-3. Since the reaction from the spar and the load to the spar must be equal and opposite, if the required reaction acts downward the load on the spar acts upward. For a perfectly general case where the spars are not at a constant percentage of the wing chord the percentage of load carried by each spar will vary along the span. In this case, if an equation representing the variation can not be written, it is possible to consider the wing in small segments spanwise, determine the spar reactions, and load the spars with a series of concentrated loads of equal value and acting opposite to the reactions.

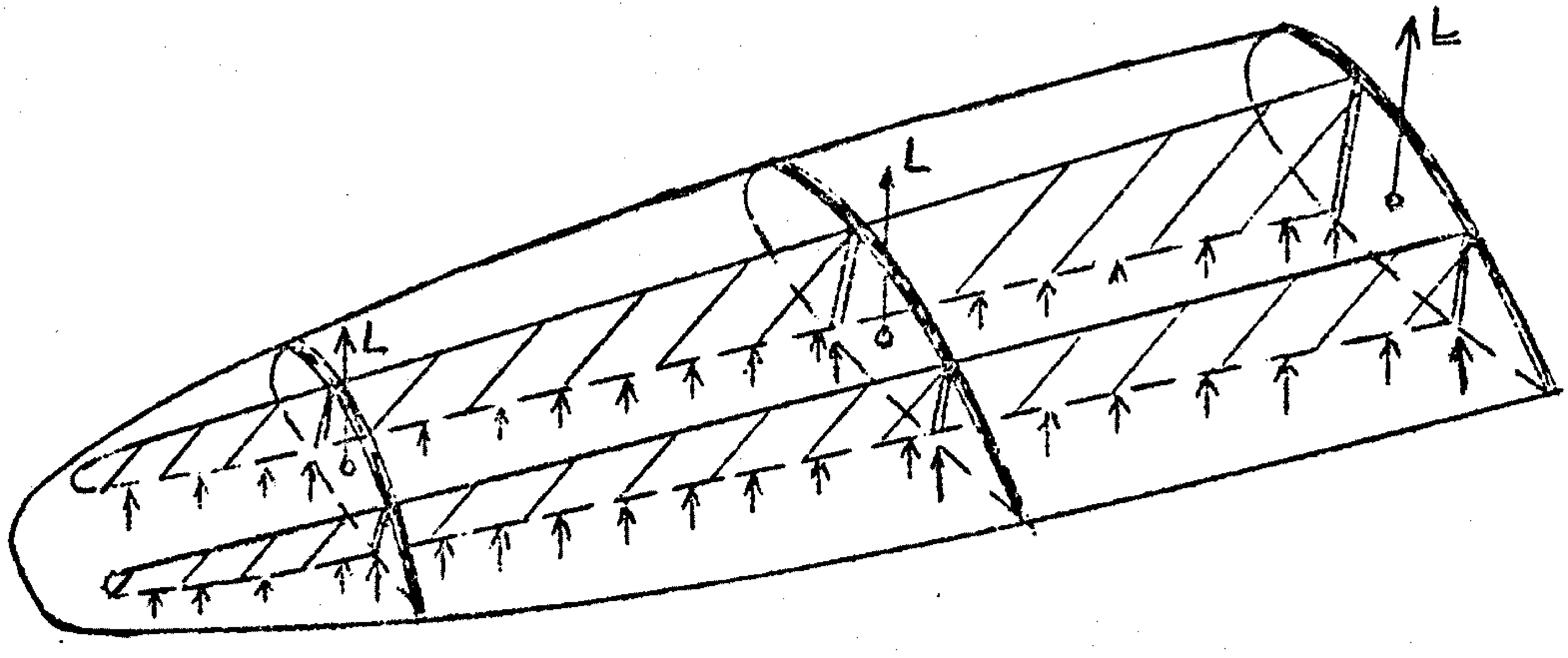


Fig. 1

Lift Loads on Spars by Spanwise Segments

Where the relation between the loads on the front and rear spars can be expressed as an equation for all points along the span it is possible to determine the loads on each algebraically from the total loads on the wing. This was already done in Assignment 6 for dead weight. For the air loads, since the center of pressure is different for different angles of attack, the percentage going to each spar will vary with the condition being investigated, but the percentage can always be evaluated since the spars are at a constant percentage of chord.

For Condition I for example:
Front Spar is at 15% of chord.
Rear Spar is at 40% of chord.
Center of Pressure is at 27% of chord (see page 2-9).

Then for any chordwise section along the span the load will divide between spars as shown by Fig. 2. Distances are in percentages of chord.

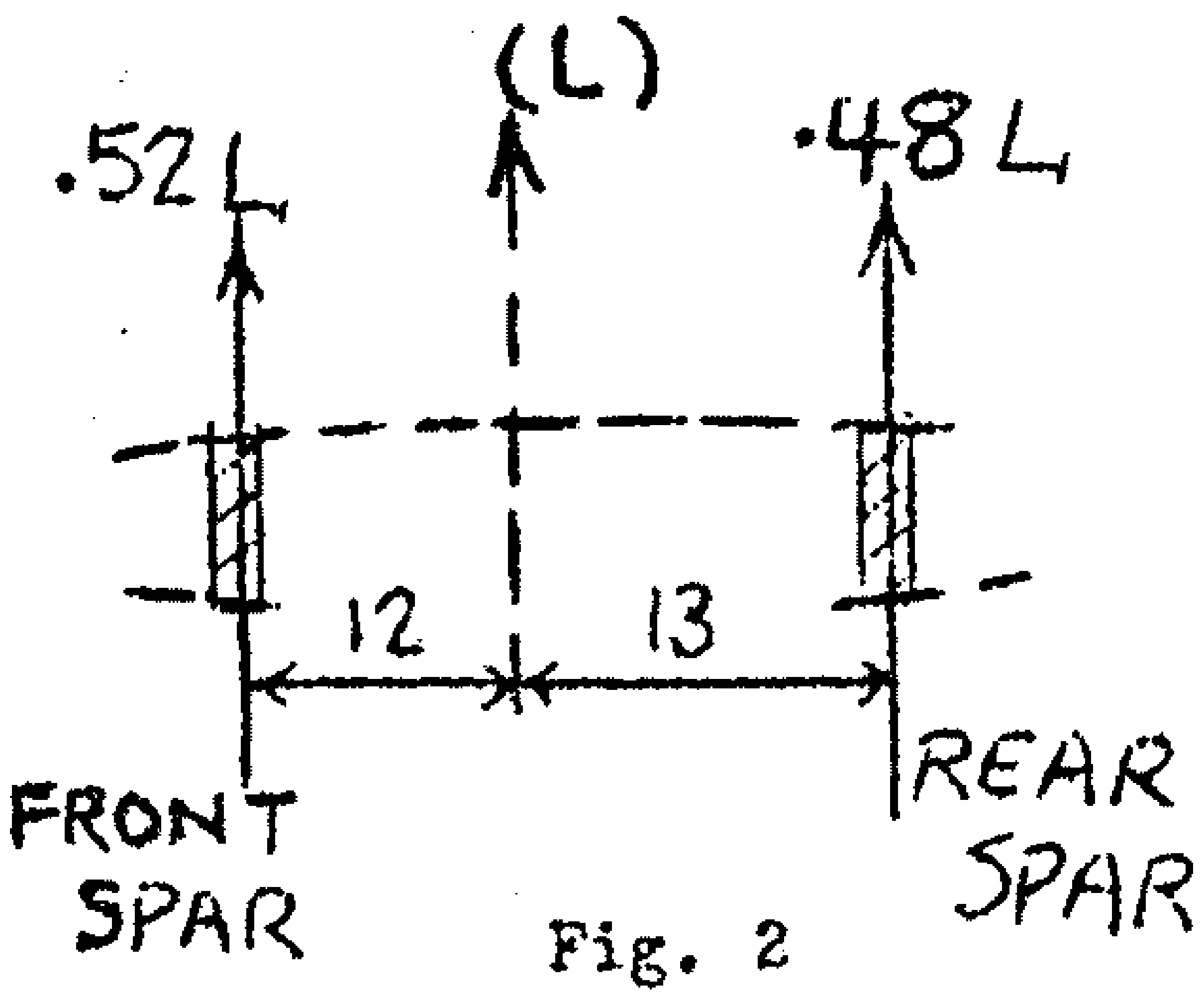


Fig. 2

Load to Front Spar $= \frac{13}{25}$ L = .52L

to Rear Spar $= \frac{12}{25}$ L = .48L

Then the lift load shears and moments may be taken from the values shown on Table I of page 6-8 by proportion.

*****

For a unit lift load of one lb. per sq. ft. with $\overline{CP}$ for Condition I.

Table I - Unit Lift Shear and Moment on Spars

STA.	405	270	145	115	42
Shear-Wing	94.8	211	336	370	455
S-F. Spar	49.3	110	175	192	237
S-R. Spar	45.5	101	161	178	218
Mom.-Wing	6,170	26,600	60,700	71,300	101,000
M.-F. Spar	3,210	13,840	31,580	37,100	52,500
M.-R. Spar	2,960	12,760	29,120	34,200	48,500

Shears and moments on Front Spar are (.52)(Total for Wing)
Shears and moments on Rear Spar are (.48)(Total for Wing)

Dead weight shears and moments for a Load Factor of -1.0 are also taken from Assignment 6 where they have already been distributed to the front and rear spars.

*****

Table II - Dead Weight

For dead weight - Load Factor = -1.0

Station	405	270	145	115	42
. Fr. Spar	0	0	-3170	-6340	-6664
. Rr. Spar	-132	-365	+1100	+2800	+2206
M. Fr. Spar	0	0	0	-95,100	-569,800
M. Rr. Spar	-7750	-40,100	-105,500	-74,050	+109,000

*****

In order to compute the actual shears and moments on the spars we must compute the multiplying factors. These factors are the ratios of the design loads on the wing to those used in computing the unit loads. For example our unit lift load from air forces was taken as 1 lb. per sq. ft. Shears and moments computed on this basis must be multiplied by the ratio

$$r = \frac{\text{actual load per sq. ft.}}{\text{unit load per sq. ft.}}$$

For Condition I the limit or applied load factor, n, is 3.54. The design load factor is (1.5)(3.54) = 5.31.

For a load factor of 1 the wing loading is $\frac{20,000}{1,010}$ = 19.9 lbs. per sq. ft.

The multiplying factor for the lift loads for Condition I is therefore (5.31)(19.9) = 105.7

This figure represents the design average loading on the wing for Condition I, in lbs. per sq. ft. The ratio may also be checked by computing from the basic formula. Load per sq. ft. = $C_N q$, $C_N = .570$, $q = 123.7$ from Assignment 2, page 9.

Applied load = $C_N q$ = (.570)(123.7) = 70.5

Design load = (1.5)(70.5) = 105.7

*****

The limit dead weight factor is -3.43 for Condition I, from Table I, page 3 of Assignment 5. The design dead weight factor is (-3.43)(1.5) = -5.14

Since the dead weight shears and moments are based on a load factor of -1.0 the multiplying factor for these values is

$$r = \frac{-5.14}{-1.0} = \underline{+5.14}$$

Note that the dead weight factor for c.g. forward, which gives the lower value is used. This is conservative.

Table III shows a method for combining the values to obtain the final values of design shears and moments on the front spar.

*****

Table III - Design Shears and Moments - Condition I. Front Spar

(1) Station	405	270	145	115	42
(2) Unit Lift Shear	49.3	110	175	192	237
(3) Des. Lift Shr.	5,220	11,640	18,500	20,300	25,000
(4) Unit Dd. Wt. Shr.	0	0	- 3,170	- 6,340	- 6,664
(5) Des. Dd. Wt. Shr.	0	0	-16,300	-32,600	-35,300
(6) Des. Shear	5,220	11,640	2,200	-12,300	-10,300
(7) Unit Lift Mom.	3,210	13,840	31,580	37,100	52,500
(8) Des. Lift Mom.	339,500	1,463,000	3,340,000	3,920,000	5,550,000
(9) Unit Dd. Wt. Mom.	0	0	0	-95,100	-569,800
(10) Des. Dd. Wt. Mom.	0	0	0	-489,000	-2,930,000
(11) Design Mom.	339,500	1,463,000	3,340,000	3,431,000	2,620,000

Notes: Column (3) = (col. 2)(105.7) Col. (5) = (col. 4)(5.14)
Column (8) = (col. 7)(105.7) Col. (10) = (col. 9)(5.14)

Drag Loads from Air Forces and Dead Weight Chord Loads.

The effect of these loads may be taken by proportion from tables I, II, and III, and from Assignment 6, except for the nacelles, where the drag of the nacelles, the thrust loads from the propellers and the nacelle dead weight forces in a chordwise direction must be computed separately. From ACM 04.217-C it would be permissible to neglect the effect of thrust in designing the wing for chordwise loads. However, for a complete analysis it should be included. The estimated drag of each nacelle is obtained as follows:

Drag Area (estimated) = 2.0 = S_D

Drag Force, $F_D = S_D q$ $\qquad q_I = 123.7$

Then F_D for Condition I = (2)(123.7) = 247.4 lbs.

Applying the usual factor of safety of 1.5 for the ultimate $F_D' = 370$ lbs. per nacelle. This force is taken equally at stations 115 and 145.

*****

The applied thrust per nacelle for Condition I is $\frac{-2730}{2} = -1365$ lbs.

Design thrust is -(1365)(1.5) = -2050 lbs. per nacelle (acts forward) (see Assignment 5, page 5-4).

The applied chord load factor for dead weight loads in Condition I is +.8445. The chord loads act aft. The design load factor is (1.5)(.8445) = 1.27.

For the purpose of illustration we will assume that the nacelle chord forces act along the chord plane of the wing at stations 115 and 145. Where this is not the case the effect of the moment of the chord forces about the airfoil section will be to introduce a vertical couple on the spars which modifies their shear and bending loads in the vertical plane. For example if the resultant chord forces on a nacelle act backward above the wing they would introduce a twisting moment on the wing which would put an up load on the front spar and a down load on the rear spar. See Fig. 3.

In our case for one nacelle the dead load is 2730 lbs. The dead weight chord force is therefore (2730)(1.27) = 3460 lbs. acting backward.

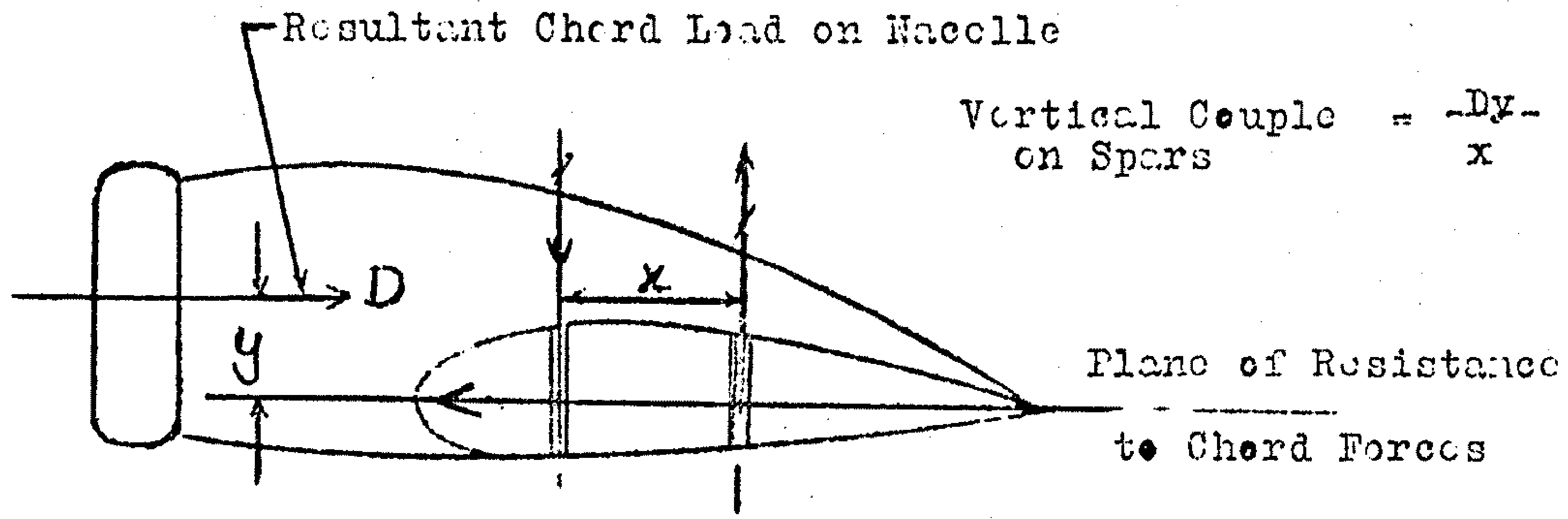

Resultant Chord Load on Nacelle
Vertical Couple on Spars = $\frac{Dy}{x}$
D
x
y
Plane of Resistance to Chord Forces

Fig. 3

Summing the total chord force per nacelle

From propeller thrust	-2050 lbs.	(fwd.)
From nacelle drag	370 lbs.	(back)
From dead weight	3460 lbs.	(back)
Total per nacelle	1780 lbs.	(back)

Loads at stations 115 and 145 are $\frac{1780}{2}$ = 890 lbs. at each.

*****

The chord loads are carried by an effective chordwise truss in the wing for which the front and rear spars act as the chord members. The force in each spar due to chordwise bending may be found by dividing the chordwise bending moment at any section by the distance between spars.

Chordwise shears and moments will be summed up for the entire wing. The ratio of chord air loads to beam air loads for Condition I is $\frac{-.114}{.570} = -.20$ from Assignment 5. This indicates forward chord loadings from the air forces. The shears and moments are taken by proportion from the lift forces for the entire wing. Table I. The multiplying factor for the design chord shears and moments from air load is $(-.20)(105.7) = -21.1$

The effect of the running dead weight loads in the wing (excepting those in the nacelle) may also be taken by proportion from the values in the beam direction. The ratio of factors here is $\frac{1.27}{-1.0} = -1.27$

The original values were for a load factor of -1.0 so that the signs are automatically corrected. The dead weight chord loads act aft in this condition and therefore tend to put tension on the front spar.

Table IV - Chord Shears on Wing

(1)	Station	405	270	145	115	42
(2)	Unit Lift Shear, Wg	94.8	211	336	370	455
(3)	Des. Chord Shear = (col. 2)(-21.1)	-2000	-4450	-7100	-7800	-9600
(4)	Unit Dd. Wt. Shear-Wing	- 132	- 365	- 705	- 810	-1105
(5)	Des. Dd. Wt. Shear-Wing = (col. 4)(-1.27)	168	463	895	1030	1410
(6)	Unit Dd. Wt. Shear Fuel	0	0	0	0	- 625
(7)	Des. Dd. Wt. Shear Fuel = (col. 6)(-1.27)	0	0	0	0	794
(8)	Des. Dd. Wt. Shear. Nac.	0	0	890	1780	1780
(9)	Total Des. Chord Shear	-1832	-3987	-5315	-4990	-5616

Col. (9) = Cols. (3) + (5) + (7) + (8)

In the above table minus shears denote that the portion toward tip would tend to move forward; that is for a cantilever wing minus shears show the summation of chord loads outboard acts forward.

Table V - Chord Moments on Wing

(1)	Station	405	270	145	115	42
(2)	Unit Lift Wing-Mom.	6,170	26,600	60,700	71,300	101,000
(3)	Des. Chord Mom. = (col. 2)(-21.1)	-130,200	-561,000	-1,280,000	-1,505,000	-2,130,000
(4)	Unit Dd. Wt. Mom.-Wing	- 7,750	- 40,100	- 105,600	- 128,200	-198,000
(5)	Des. Dd. Wt. Mom.-Wing = (col. 4)(-1.27)	+ 9,850	50,900	134,000	163,000	251,000
(6)	Unit Dd. Wt. Mom.-Fuel	0	0	0	0	- 23,000
(7)	Des. Dd. Wt. Mom.-Fuel = (col. 6)(-1.27)	0	0	0	0	29,200
(8)	Des. Dd. Wt. Mom. Nac.	0	0	0	26,700	156,640
(9)	Total Des. Mom.	-120,350	-510,000	-1,146,000	-1,315,300	-1,693,160

Col. (9) = Cols. (3) + (7) + (8) +(5)

For col. (8). Effect of Nacelles:

Chord load from nacelles = 890 lbs. aft at each of stations 145 and 115.

Mom. Station 115 = (890)(30) = 26,700

Mom. Station 42 = (890)(103) + (890)(73) = 156,640

Given the design chord moments on the wing the loads in the front and rear spars are found as follows: (See Fig. 4)

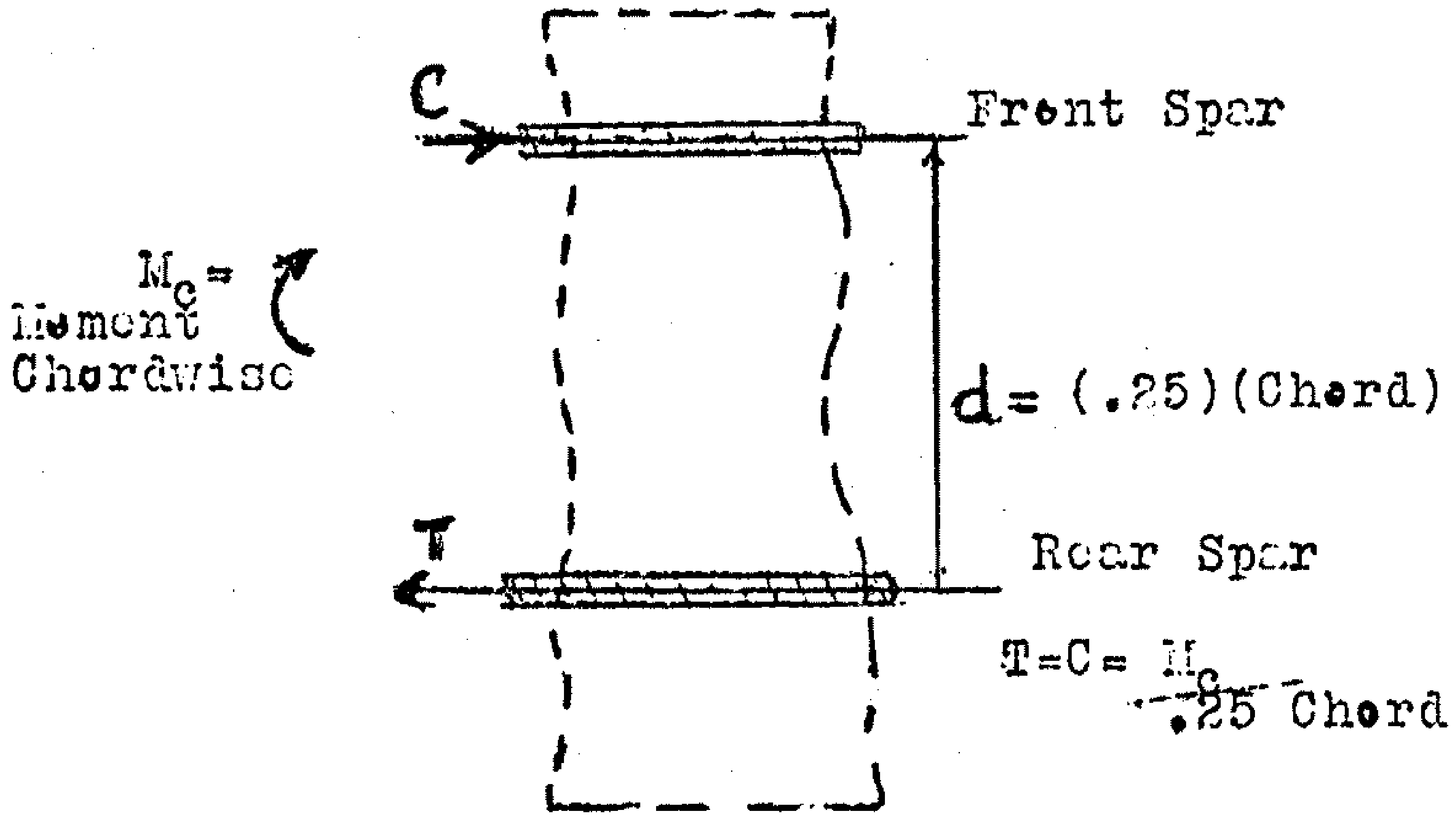


FIG.4

Table VI

End Loads in Spars from Chordwise Moment - Condition I

(1)	Station	405	270	145	115	42
(2)	Chordwise Moment	-120,350	-510,000	-1,146,000	-1,315,300	-1,693,160
(3)	Chord Length	112.5	135	155.8	160.8	173
(4)	(.25)(Chord Length)	28.1	33.7	38.9	40.2	43.3
(5)	Compression Load Front Spar	- 4280	- 15,100	- 29,400	- 33,000	- 39,000
(6)	Tension Load Rear Spar	+ 4280	+ 15,100	+ 29,400	+ 33,000	+ 39,300

Note: In calculation of the axial loads in the above table the small component of the spars in the chordwise direction has been neglected.

Problem Assignment:

(1) Check through the computations given for the front spar for Condition I until you thoroughly understand all steps.

(2) Make similar computations for the rear spar for Condition III, taking required values from assignments 5, 6, and 6A. In some cases shears and moments may be taken from Condition I by using ratios of loads or load factors. Care must be used however because of the difference in loads from the nacelle. When in doubt work the loading up from the beginning.

Values required are shears, moments, and axial loads due to chordwise bending.

ADVANCED STRESS ANALYSIS

ASSIGNMENT 7

Assignment 7

DESIGN OF MULTIPLE SPAR OR BOX WING

We have determined shears and moments in the beam direction, shears and moments in the chord direction and a torque about the elastic centrum of the wing. The calculation of section properties at any station along the wing will be referred to the same axes about which the shears and moments have been calculated. To arrive at the worst possible conditions of load distribution it is also necessary to check the sections for bending about their principal axes.

The principal axes of any section are the axes about which its resistance to bending are maximum and minimum; that is the axes of maximum and minimum moments of inertia. Before we go into the calculation of the principal axes it is well to review the basic assumptions on which the beam theory is based.

Important: The design of a box wing is based on the theory that the bending material in the wing follows the same laws as the fibers in a beam; that is that planes before bending remain planes after bending, and unit stress (and deformation) are proportional to the distance from the neutral axis.

If we take any irregular body and consider it made up of small particles, dA, Fig. 1, the following conditions exist:

I about any axis:

$$I_{x_o} = \int y^2 dA$$

$$I_{x_1} = \int (y \pm a)^2 \, dA$$

$$= \int y^2 dA + a^2 \int dA \pm 2a \int y dA$$

$$= I_{x_o} + (A)(a)^2 \pm 2a \int y dA$$

If $x_o - x_o$ passes through c.g.,

$$\int y dA = 0, \text{ since } y = o$$

Therefore $I_{x_1} = I_{x_o} + Aa^2$

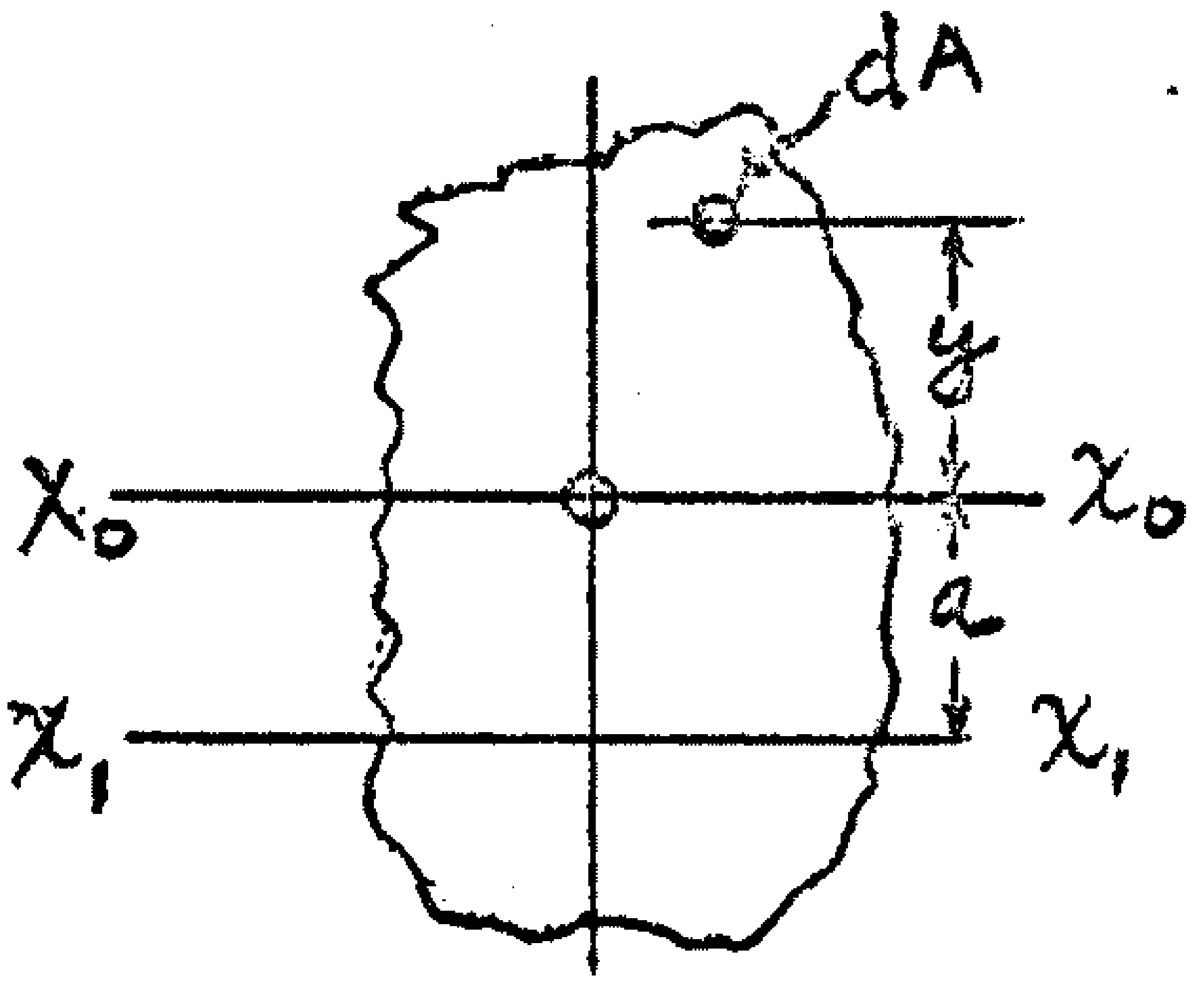


Fig. 1

(axis x_1 may lie either above or below axis x_o)

Product of Inertia: "K"

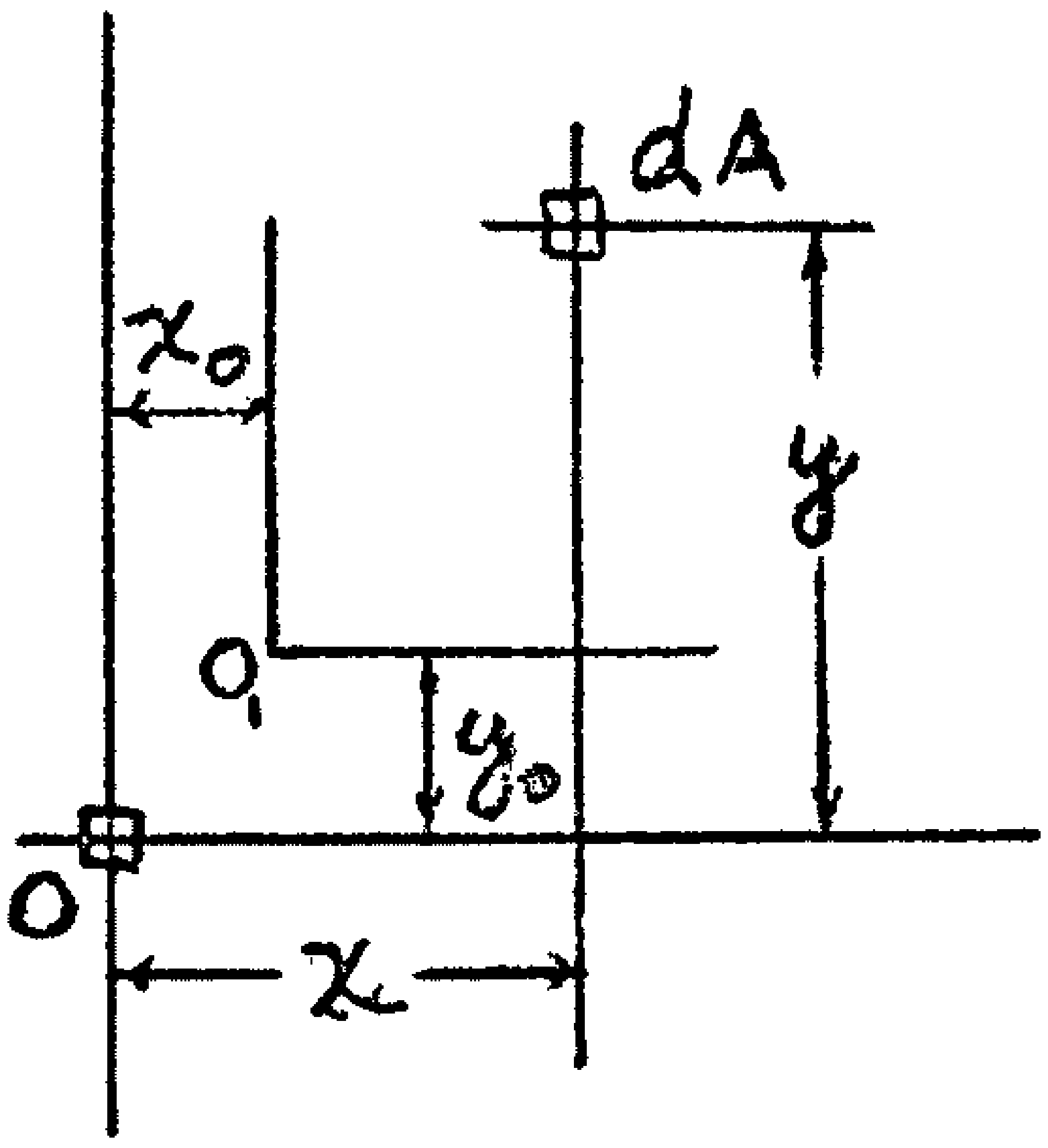


Fig. 2

$$K = \int xydA = \iint xydxdy \quad (dA = dxdy)$$

$$K_o = \int xydA \quad \text{origin at "0"}$$

$$K_1 = \int (y - y_o)(x - x_o)dA$$

$$= \int xydA + x_oy_o \int dA - y_o \int xdA - x_o \int ydA$$

If "0" is c.g. of section $\int ydA = 0$,

$$\int xdA = 0$$

Also $\int xydA = K_o$

Therefore $K_1 = K_o + x_oy_oA$

Product of Inertia about any set of axes parallel to axes through c.g. = Product of inertia about gravity axes plus product of coordinates of new axis x area.

Our case: 0 at c.g. of part considered $K_o = \int xydA$ about own c.g.

0 at c.g. total section $K_1 = \sum (K_o + x_oy_oA)$

Polar Moment of Inertia.

$$I_p = \int r^2dA, \quad r = (x^2 + y^2)^{\frac{1}{2}}$$

$$r^2 = x^2 + y^2$$

$$I_p = \int (x^2 + y^2)dA = \int x^2dA + \int y^2dA = Iy + Ix$$

The polar moment of inertia of an area about any point in its plane equals the sum of the moments of inertia about any two rectangular axes in its plane passing through the point.

$$Ix + Iy = Ix_1 + Iy_1$$

Use as check on calculations.

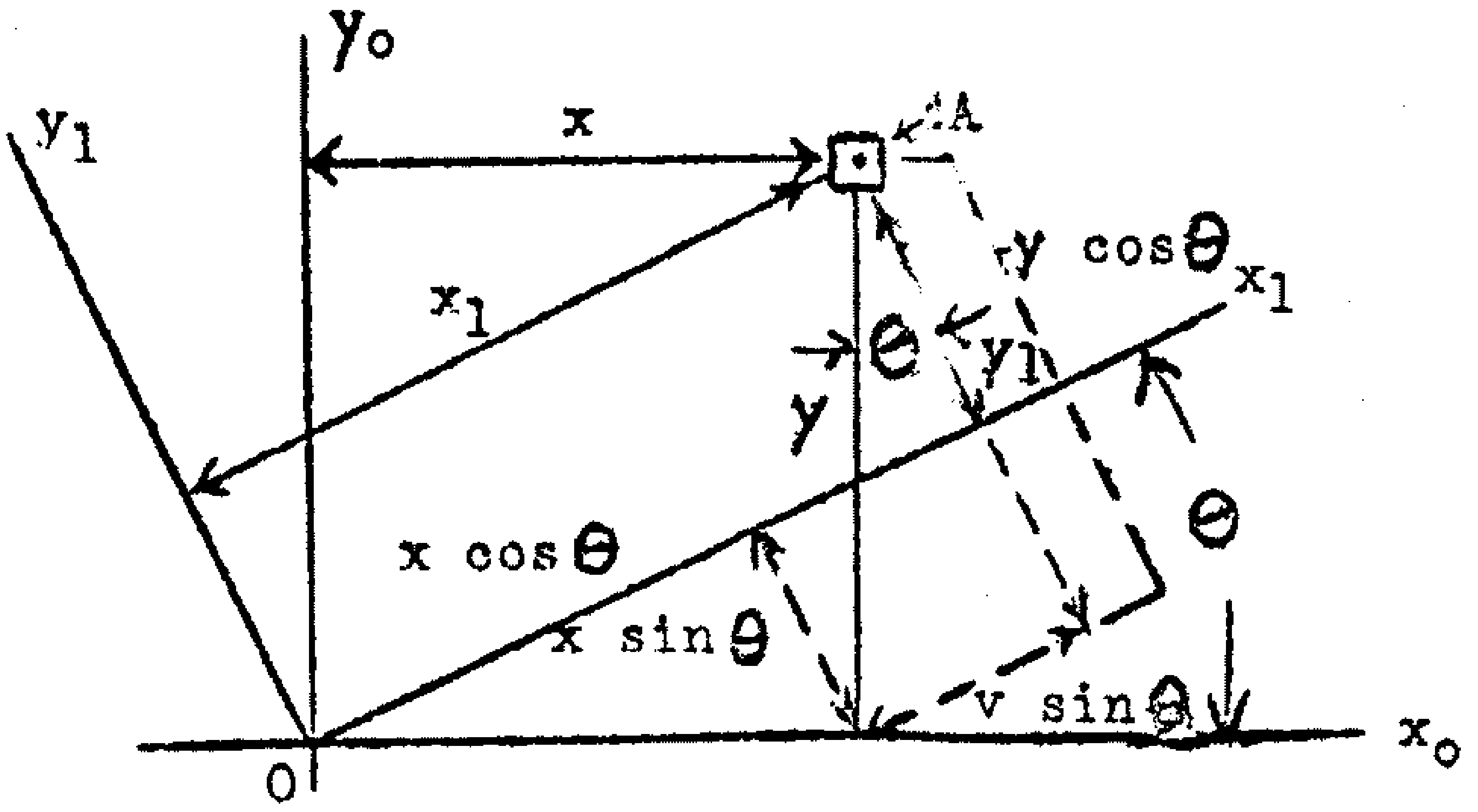


Fig. 3

$$x_1 = y \sin\theta + x \cos\theta$$

$$y_1 = y \cos\theta - x \sin\theta$$

$$I_{x_1} = \int (y \cos\theta - x \sin\theta)^2 \, dA$$

$$= \cos^2\theta \int y^2 dA - 2 \sin\theta \cos\theta \int xy dA + \sin^2\theta \int x^2 dA$$

$$= I_x \cos^2\theta - 2 \sin\theta \cos\theta \int xy dA + Iy \sin^2\theta$$

Similarly:

$$Iy_1 = \int {x_1}^2 dA = \int (y \sin\theta + x \cos\theta)^2 \, dA$$

$$= \sin^2\theta \int y^2 dA + 2 \sin\theta \cos\theta \int xy dA + \cos^2\theta \int x^2 dA$$

$$= I_x \sin^2\theta + 2 \sin\theta \cos\theta \int xy dA + Iy \cos^2\theta$$

Check: $Ix_1 + Iy_1 = Ix \cos^2\theta - K \sin 2\theta + Iy \sin^2\theta$

$$+ Ix \sin^2\theta + K \sin 2\theta + Iy \cos^2\theta$$

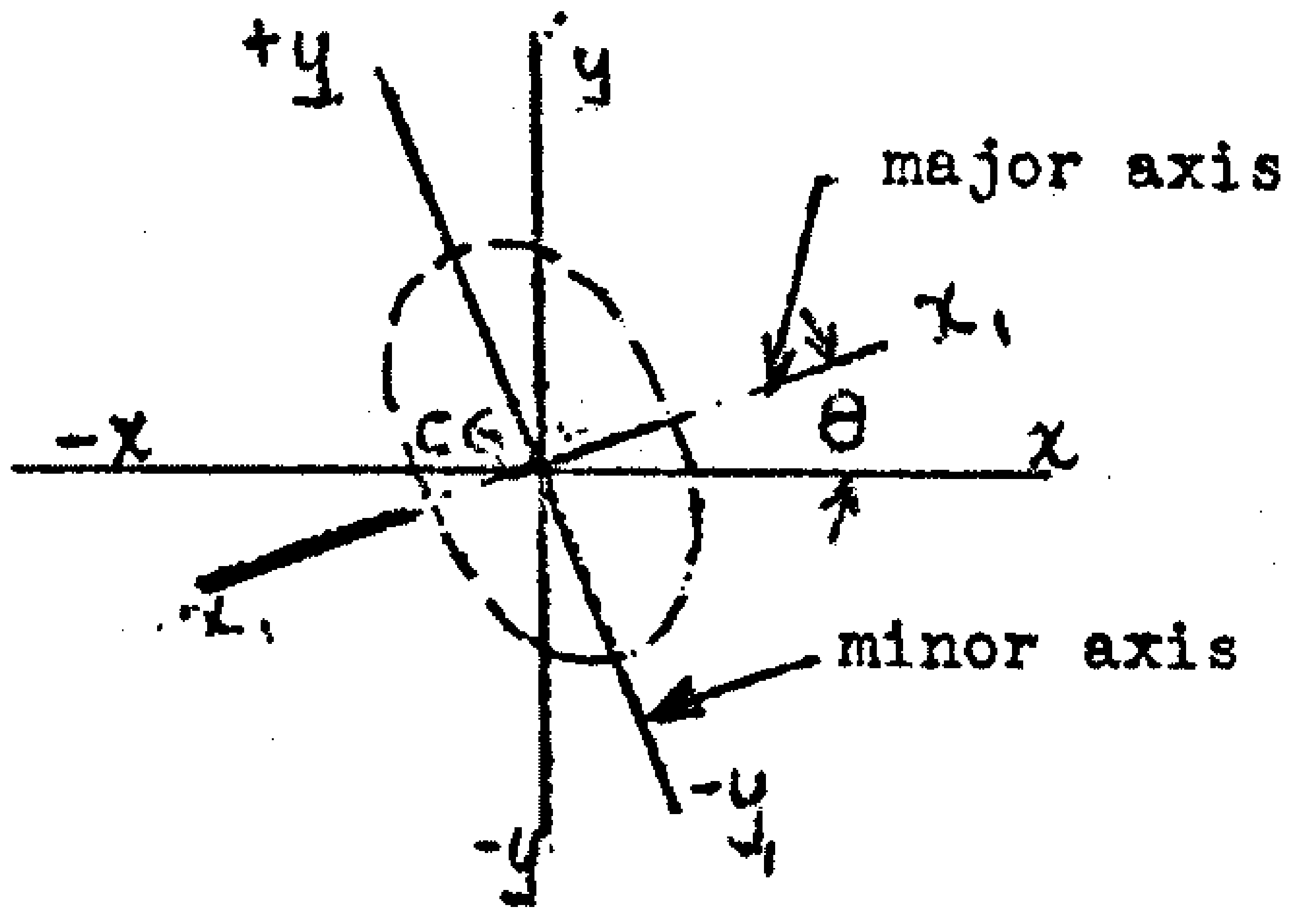
$$= Ix (\sin^2\theta + \cos^2\theta) + Iy (\sin^2\theta + \cos^2\theta)$$

$$\sin^2 + \cos^2 = 1.0$$

$Ix_1 + Iy_1 = Ix + Iy$ (Check)

Ellipse of Inertia: Physical Significance.

If the variation of I_x with θ is plotted as a polar the locus will be an ellipse the axes of which are the principal axes of the section, axes of greatest and least resistance to bending.



A difference of up to 5% on maximum stresses on a section of wing, may be obtained by neglecting the principal axes.

Fig. 4

Variation of Ix with θ is zero at the principal axis.

$$I_{x_1} = I_x \cos^2\theta - 2\sin\theta\cos\theta\int xydA + Iy\sin^2\theta$$

$$I_{x_1} = I_x\left(\tfrac{1}{2} + \frac{\cos 2\theta}{2}\right) - 2\left(\frac{\sin 2\theta}{2}\right)\int xydA + Iy\left(\tfrac{1}{2} - \frac{\cos 2\theta}{2}\right)$$

$$0 = \frac{\delta I_{x_1}}{\delta\theta} = -Ix\sin 2\theta - 2\cos 2\theta\int xydA + Iy\sin 2\theta$$

(dividing by $\cos 2\theta$)

$$-Ix\tan 2\theta - 2\int xydA + Iy\tan 2\theta = 0$$

$$\tan 2\theta\,(Iy - Ix) = 2\int xydA$$

$$\tan 2\theta = \frac{2\int xydA}{Iy - Ix}$$

$$\cos^2\theta = \left(\tfrac{1}{2} + \frac{\cos 2\theta}{2}\right)$$

$$\sin\theta\cos\theta = \frac{\sin 2\theta}{2}$$

$$\sin^2\theta = \left(\tfrac{1}{2} - \frac{\cos 2\theta}{2}\right)$$

$$\sin^2\theta + \cos^2\theta = 1$$

--- Trigonometric functions for a check.

We will now apply the foregoing theory to the design of a wing section. The moments of inertia of the wing section are first obtained about x - x and y - y axes through the c.g. of the section || and ⊥ to the most convenient reference line, generally the wing chord. The problem of obtaining the principal axes and moments of inertia about them is somewhat complex for a box or multispar wing and will be considered here in some detail.

Section Properties.

1. Choose any convenient set of perpendicular reference axes. We will use the beam and chord directions used for computing wing loads.

2. Divide the section into numbered component parts, such as sections of skin, stiffeners, shear webs, etc. On the tension side use all the skin area, on the compression side use 30t of skin at each longitudinal stiffener.

3. Compute the area of each part.

4. Tabulate as follows: (numbers in ( ) refer to columns in table.)

   (1) Number of part
   (2) Area of part = A (this is actually a "finite" dA)
   (3) y coordinate distance to reference axis } Scaled from layout.
   (4) x coordinate distance to reference axis }
   (5) Ay
   (6) Ax

5. Determine position of c.g. of section. Locate to scale.

$$\frac{\sum Ay}{\sum A} = y \text{ coordinate of c.g.}$$

$$\frac{\sum Ax}{\sum A} = x \text{ coordinate of c.g.}$$

6. Choose new axes ∥ to first ones but through c.g.

7. Tabulate

   (7) y_o coordinate distance to new axis } computed from original x and y locations.
   (8) x_o coordinate distance to new axis }
   (9) Ay_o^2
   (10) Ax_o^2
   (11) I_{x_o} = moment of inertia of part about its own c.g.
   (12) I_{y_o} = moment of inertia of part about its own c.g.

   (Some parts will have negligible values of I_{x_o} and I_{y_o})

   (Trial computations will show which may be neglected.)

8. Determine the moments of inertia of the section about the new axes.

$$\text{Total } I_{x_o} = \sum Ay_o^2 + \sum I_{x_o} \text{ of parts}$$

$$\text{Total } I_{y_o} = \sum Ax_o^2 + \sum I_{y_o} \text{ of parts}$$

9. Tabulate

(13) Ax_oy_o for each part (taking due account of sign -- see Fig. 4)
(14) K_o = product of inertia of part about its own c.g. (will be negligible for the stiffeners)

10. Determine the angle between the new axes and the principal axes.

$$\tan 2\theta = \frac{2\sum (Ax_oy_o + K_o)}{I_{y_o} - I_{x_o}}$$

11. Determine principal moments of inertia.

$$I_{x_1} = I_{x_o} \cos^2\theta - 2 \sin\theta \cos\theta \sum (Ax_oy_o + K_o) + I_{y_o} \sin^2$$

$$I_{y_1} = I_{y_o} \cos^2\theta + 2 \sin\theta \cos\theta \sum (Ax_oy_o + K_o) + I_{x_o} \sin^2$$

(for derivation of formulae see preceding pages)

The formulae for I_o and K_o for the skin portion are listed. It is customary to use the full value of the skin on the tension side and only 30 x skin thickness at each stiffener on the compression side, for each row of rivets attaching skin to stiffener. This is in agreement with the effective widths from ANC-5, in the range covered by present allowable unit stresses.

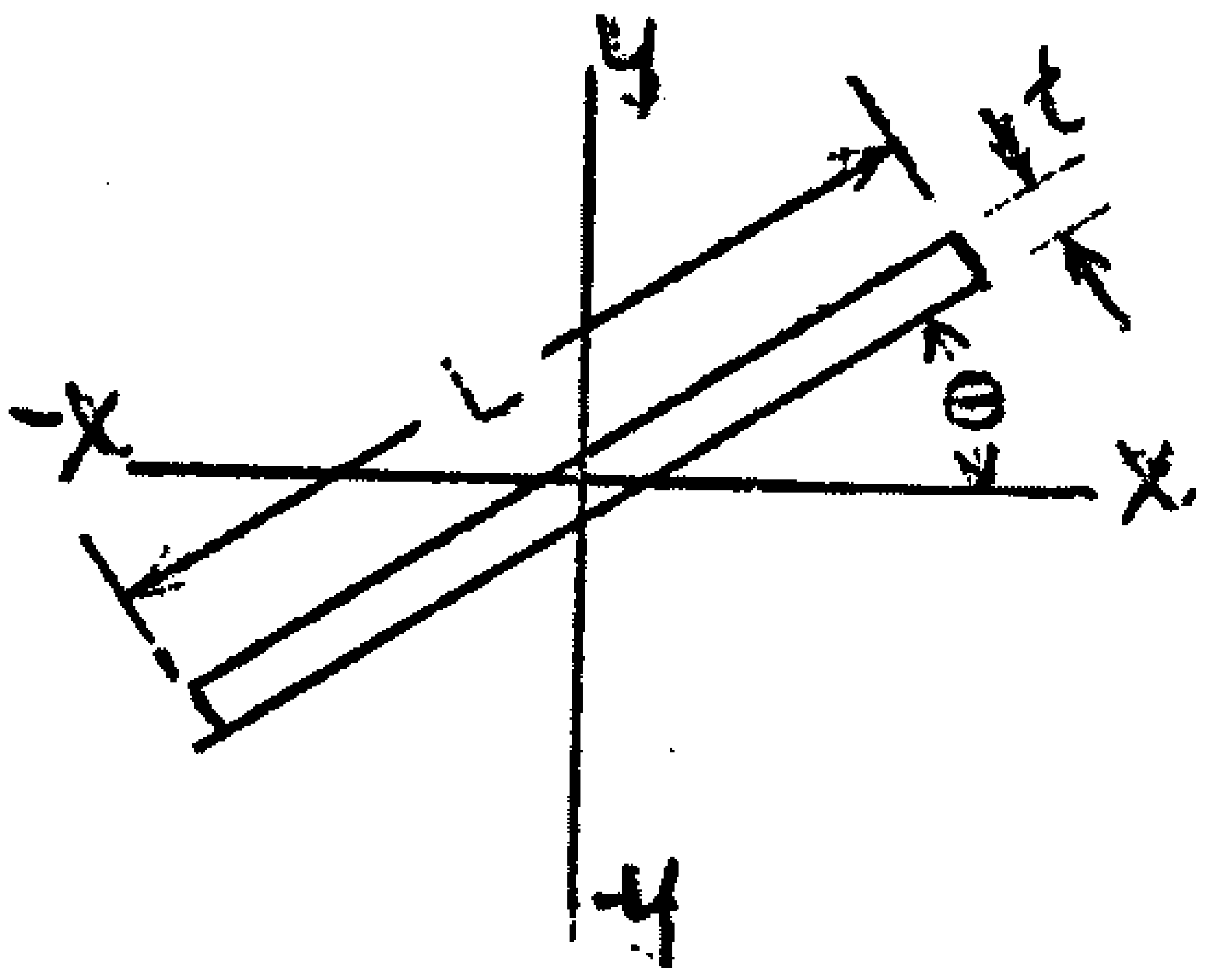


Fig. 5

$$I_x = \frac{L^2A}{12} \sin^2\theta = \frac{L^3t}{12} \sin^2\theta$$

$$K_o = \frac{L^2A}{12} \sin\theta \cos\theta = \frac{L^3t}{12} \sin\theta \cos\theta$$

$(A = Lt)$

θ as used here is the angle between the long axis of any element and the coordinate axes and is scaled from the layout.

Use of Width of 30 Times skin thickness as effective width with stiffener on compression side of structure:

From ANC-5 the effective width of sheet acting with a stiffener at the same unit stress as the stiffener is

$w = Ct\ (E/Fc)^{\frac{1}{2}}$ where

c = a constant, t = sheet thickness

E = Modulus of Elasticity = 10,000,000 for dural.
Fc = Unit compression stress

Fc for sheet stiffener combinations near failing load lies between 25,000 and 40,000 psi.

C for one row rivet connection = 1.70

Then for $Fc = 25{,}000$

$$w = 1.70t\ \left(\frac{10{,}000{,}000}{25{,}000}\right)^{\frac{1}{2}} = 34t$$

For $Fc = 40{,}000$

$$w = 1.70t\ \left(\frac{10{,}000{,}000}{40{,}000}\right)^{\frac{1}{2}} = 27t$$

30t is used as an average value in the range of allowable ultimate loads.

Assignment.

Make a complete check of the theory and equations developed in the text in preparation for calculation of wing section properties in Assignment 8.

FINAL CHECK TABLE

No. of Part (1)	A (2)	y (3)	x (4)	Ay (5)	Ax (6)	y_o (7)	x_o (8)	Ay_o^2 (9)	Ax_o^2 (10)	Ix_o (11)	Iy_o (12)	Ax_oy_o (13)	K_o (14)
	Area of individual parts	y-ordinate to reference axis	x-ordinate to reference axis	Area of part x its y-ordinate	Area of part x its x-ordinate	y_o-distance to part from x_o- axis through (c.g.)	x_o-distance to part from y_o-axis through (c.g.)	(Aea. part)(y_o of part)^2	(Aea. part)(x_o of part)^2	Mom. of inertia of part about its own c.g. parallel to x_ox_o axis	Mom. of inertia of part about its own c.g. parallel to y_oy_o axis	Product of each area by its x_oy_o coord. = product of inertia	Product of inertia of part about its own c.g.
	ΣA			ΣAy	ΣAx			ΣAy_o	ΣAx_o	ΣIx_o	ΣIy_o	ΣAx_oy_o	ΣK_o

In computing product of inertia special attention to proper sign.

$$K_1 = \Sigma Ax_oy_o + \Sigma K_o$$

$$\text{y ord. to c.g.} = \frac{\Sigma Ay}{\Sigma A}$$

$$\text{x ord. to c.g.} = \frac{\Sigma Ax}{\Sigma A}$$

$$\text{Total } Ix_o = \Sigma Ay^2 + \Sigma Ix_o \text{ of parts}$$

$$Iy_o = \Sigma Ax_o^2 + \Sigma Iy_o \text{ of parts}$$

$$Ix^1 = Ix_o \cos^2\theta - 2 \sin\theta \cos\theta \,\Sigma Ax_oy_o + Iy_o \sin^2\theta$$

$$Iy^1 = Iy_o \cos^2\theta + 2 \sin\theta \cos\theta \,\Sigma Ax_oy_o + Ix_o \sin^2\theta$$

Check: $Ix_o + Iy_o = Ix^1 + Iy^1$

AIRPLANE
STRESS
ANALYSIS

PART TEN

ADV. STRESS 8

ADV. STRESS 9

ADV. STRESS 9A

ADV. STRESS 10

ADVANCED STRESS ANALYSIS

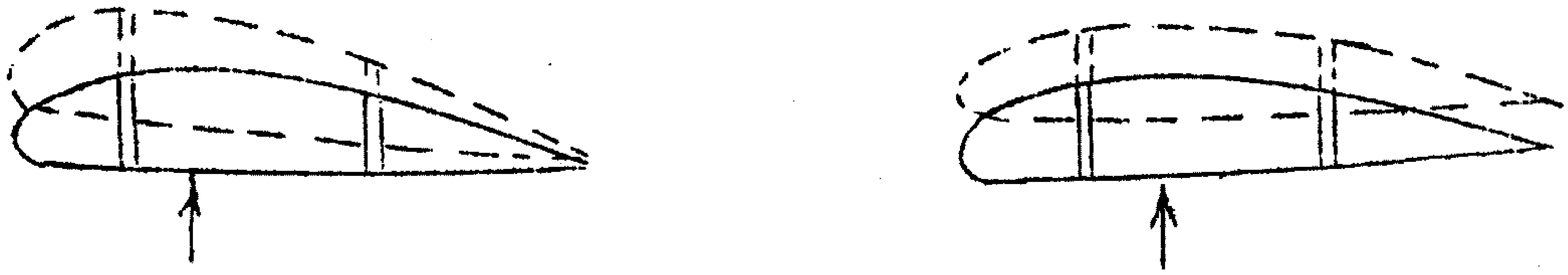
ASSIGNMENT 8

Assignment 8

DESIGN OF SEMI-MONOCOQUE WING

In Assignments 6 and 6-A we covered the methods used in securing loads, shears and bending moments on the beams of a torsionally flexible wing. We will now present the methods used in designing a torsionally rigid wing. A wing which is torsionally rigid is one in which the bending members (beams) are restrained from deflecting independently. This means in effect that a beam which is loaded more highly than its neighbor is restrained from deflecting away from it, but must transfer some of its excess loads to its more lightly loaded neighbors.

If, for example, in a torsionally rigid wing the front and rear shear webs had equal bending rigidity and the center of pressure of the load from air forces were at the one third point between them, a simple beam distribution as in Assignments 6 and 6-A would place two-thirds of the load on one spar and one-third on the other. This distribution would give bending deflections on the heavier loaded spar twice those on the lighter loaded spar. But in a torsionally rigid wing this cannot occur because differential deflection of the spars would assume that the wing section could twist without restraint. The result is that a part of the load is transferred from the heavily loaded spar to that which is more lightly loaded. See Fig. 1.



(a)

Deflection of a Torsionally
Flexible Wing

(b)

Deflection of a Torsionally
Rigid Wing

Fig. 1

*****

The method for computing conversion factors for unit loads was given in Assignment 6-A, for Conditions I and III. Conversion factors for all conditions are conveniently calculated in tabular form, when one does a complete wing analysis. The method is illustrated in Table I which gives the factors for five design conditions. The unit torque factors for lift forces on the wing were developed in Table I of Assignment 6. The torque due to dead weight will be considered in this assignment, in its application to the design of the box wing.

TABLE I

Conversion Factors for Unit Air Loads,
Shears, Moments and Torques

Condition	I	II	III	IV	Sect. Inbd. of Aileron III_1	Aileron Section
n_1	3.54	-1.54	2.62	-.62	1.96	1.96
$1.5n_1$	5.31	-2.31	3.93	-.93	2.94	2.94
n_{x_1}	-.708	0	0	0	0	0
$1.5n_{x_1}$	-1.06	0	0	0	0	0
Beam factor $= \frac{W}{S}(1.5n_1)$	105.7	-46.0	78.1	-18.5	58.5	58.5
Chord factor $= \frac{W}{S}(1.5n_{x_1})$	-21.1	0	0	0	0	0
n_2	-3.43	+1.54	-2.44	+ 70	-1.83	-1.83
$1.5n_2$ *	-5.14	+2.31	-3.66	+1.05	-2.75	-2.75
n_{x_2}	+.8445	+.1365	0	0	0	0
$1.5n_{x_2}$	+1.27	+.205	0	0	0	0
$\overline{C.P.}$	27.0	13.3	36.2		36.2	81.0
e - E.C. at 20%	-7.0	+6.7	-16.2		-16.2	-61.0
Tq Factor - E.C. at 20%	-742	-306	-1260		-950	-3570 (incr. = -2620)
e - E.C. at 30%	+3.0	+16.7	-6.2		-6.2	-51.0
Tq Factor - E.C. at 30%	+318	-768	-485		-360	-2980 (incr. = -2620)

---

* Conversion factors for dead weights in this table are based on upward acting dead loads.

Assume Elastic Centrum Limits - 20% to 30%.

(The Elastic Centrum is defined as that chordwise location on a section at which all lift load could be concentrated, and the wing would deflect vertically without rotation. It is therefore the centroid of vertical moment of inertia of all the bending material in a wing section.)

*****

For the two-spar, torsionally flexible wing treated in Assignment 6-A the dead weight shears, moments and torques were treated separately. For a torsionally rigid wing it is advantageous to treat them for the wing as a whole. This requires a variation of the table given on page 21 of Assignment 6, where one spar was treated separately. For the entire wing as a unit:

TABLE II

Unit Dead Weight Shear, Moment and Torque

<table>
<tr><th colspan="8">SHEAR</th></tr>
<tr><th></th><th></th><th></th><th colspan="2">145</th><th colspan="2">115</th><th>42</th></tr>
<tr><th>Station</th><th>405</th><th>270</th><th>Inbd.</th><th>Outbd.</th><th>Inbd.</th><th>Outbd.</th><th></th></tr>
<tr><td>S - Wing</td><td>132</td><td>365</td><td colspan="2">705</td><td colspan="2">810</td><td>1105</td></tr>
<tr><td>S - Nac.</td><td>0</td><td>0</td><td>1365</td><td>0</td><td>2730</td><td>1365</td><td>2730</td></tr>
<tr><td>S - Fuel and System</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>623</td></tr>
<tr><td>S - Total</td><td>132</td><td>365</td><td>2070</td><td>705</td><td>3540</td><td>2175</td><td>4458</td></tr>
<tr><th colspan="8">MOMENT</th></tr>
<tr><td>M - Wing</td><td>7,750</td><td>40,100</td><td colspan="2">105,500</td><td colspan="2">128,200</td><td>198,000</td></tr>
<tr><td>M - Nac.</td><td>0</td><td>0</td><td colspan="2">0</td><td colspan="2">41,000</td><td>240,000</td></tr>
<tr><td>M - Fuel and System</td><td>0</td><td>0</td><td colspan="2">0</td><td colspan="2">0</td><td>23,000</td></tr>
<tr><td>M - Total</td><td>7,750</td><td>40,100</td><td colspan="2">105,500</td><td colspan="2">169,200</td><td>461,000</td></tr>
</table>

Table continued

TABLE II

Torque with Elastic Centrum assumed at 20 per cent of the Chord

Station	405	270	Station	145 In.	145 Out.	115 In.	115 Out	42
-10.8x	-1460	-2920	$-56.7x_1$	- 7080		- 8,780		-12,870
$\frac{-32.4x^2}{540}$	-1090	-4370	$\frac{-110.7x_1^2}{1080}$	- 1600		- 2,460		- 5,330
$\frac{-x^3}{16,200}$	- 150	-1210	$-x^3/1800$	- 1,080		- 2,060		- 6,600
			Tq - 270	- 8,500		- 8,500		- 8,500
Tq. Wing	-2700	-8500		-18,260		-21,800		-33,300
Tq. Nac.	0	0		81,100	0	162,200	81,100	162,200
Tq. Fuel Syst.	0	0		0	0	0	0	- 7,300
Total - Tq.	-2700	-8500		62,840	-18,260	140,400	59,300	121,600
With Elastic Centrum at 30 per cent of the Chord								
Tq. Wing	-1350	-4250		- 9,130		-10,900		-16,650
Tq. Nac.	0	0		103,000	0	206,000	103,000	206,000
Tq. Fuel Syst.	0	0		0	0	0	0	3,130
Total - Tq.	-1350	-4250		93,870	-9130	195,100	92,100	192,480

Notes:

1. Center of gravity for wing structure weight is at 40 per cent of chord.
2. Center of gravity of fuel and fuel system is at 27 per cent of chord.
3. Torque from the nacelles is equally distributed to stations 115 and 145.
4. Values of torque constants for the Elastic Centrum at 20 per cent of chord are from Fig. 2.

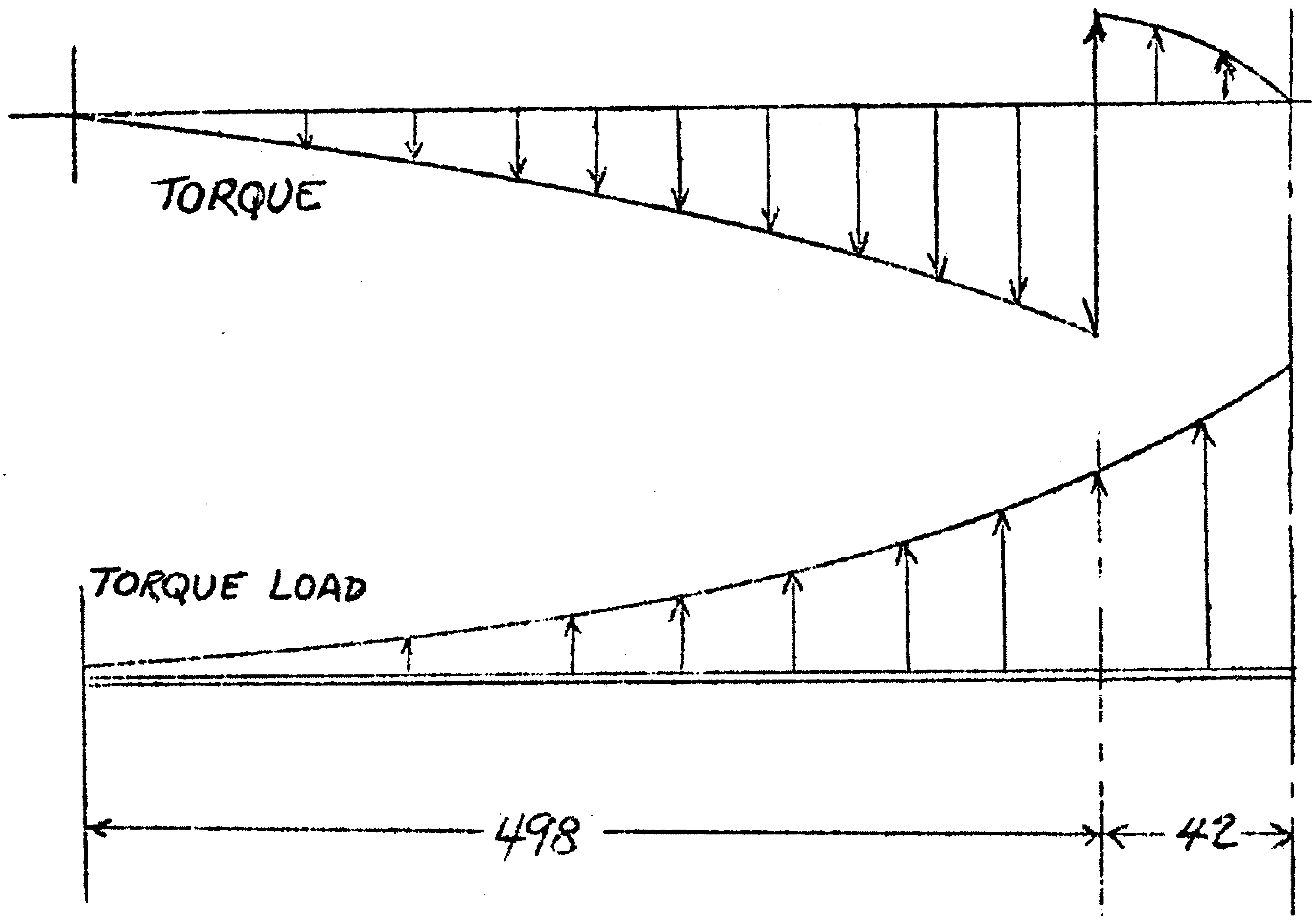


Fig. 2

Dead Weight Torque

With wing dead weight at 40% of chord and Elastic Centrum assumed at 20%:

Wing chord in inches = $90 + \frac{90x}{540}$

Wing weight = $(.6 + \frac{1.50}{270})x$ for outboard of station 270.

For upward acting dead weight loads a diving moment (minus) is produced.

Unit torque loading of the wing structure equals

$$(.20 - .40)(\text{chord})(\text{unit weight loading})$$

$$= (-.20)(90 + \frac{90x}{540})(.6 + \frac{1.50x}{270})$$

$$= -10.8 - (\frac{10.8}{540} + \frac{27}{270})x - \frac{27x^2}{(540)(270)}$$

$$= -10.8 - \frac{64.8x}{540} - \frac{x^2}{5400}$$

This equation represents the torque loading between the tip and station 270. The actual torque is the summation of the torque loading.

$$(-10.8 - \frac{64.8x}{540} - \frac{x^2}{5400})dx = -10.8x - \frac{32.4x^2}{540} - \frac{x^3}{16,200} + C$$

C is zero since the torque at the extreme tip is zero.

This equation represents the dead weight torque for a factor of 1.0 at any station "x" between the tip and station 270.

*****

At station 270 the torque loading is $-10.8 - 32.4 - 13.5 = -56.7$

Inboard of station 270 the equation of the wing structure dead weight load is

$$w = 2.10 + \frac{2.70x_1}{270}$$

The equation of the chord lengths from station 270 inboard is

$$c = 135 + \frac{90x_1}{540}$$

Then the equation of the unit torque load inboard of station 270 is

$$\text{Tq. Load} = -56.7 - (.20)(135 + \frac{90x_1}{540})(2.10 + \frac{2.70x_1}{270})$$

$$= -56.7 - (\frac{37.8}{540} + \frac{72.9}{540})x_1 - (\frac{1}{6})(\frac{1}{100})x_1^2$$

$$= -56.7 - \frac{110.7x_1}{540} - \frac{x_1^2}{600}$$

$$(-56.7 - \frac{110.7x_1}{540} - \frac{x_1^2}{600})dx = -56.7x_1 - \frac{110.7x_1^2}{1080} - \frac{x_1^3}{1800} + C$$

In this case C = torque at station 270 = -8500

$$\text{Tq. inboard of Station 270} = -8500 - 56.7x_1 - \frac{110.7x_1^2}{1080} - \frac{x_1^3}{1800}$$

We will next tabulate final values of shears, bending moments and torques for the entire wing.

TABLE III

Final Shears

Station	Factor	42 Outbd.	115 Inbd.	115 Outbd.	145 Inbd.	145 Outbd.	270	405
UNIT LOADS								
Wing Lift	1.0	455	370	370	336	336	211	94.8
Dead Wt.	1.0	4460	3540	2175	2070	705	365	132
CONDITION I								
Wing Lift	105.7	48,200	39,200	39,200	35,600	35,600	22,400	10,030
Dead Wt.	-5.14	-23,000	-18,250	-11,220	-10,700	-3,630	-1,880	-680
Total		25,200	20,950	27,980	24,900	31,970	20,520	9,350
CONDITION II								
Wing Lift	-46.0	-20,900	-17,020	-17,020	-15,470	-15,470	-9,700	-4,360
Dead Wt.	+2.31	10,300	8,190	5,020	4,800	1,630	840	300
Total		-10,600	-8,830	-12,000	-10,670	-13,840	-8,860	-4,060
CONDITION III								
Wing Lift	78.1	35,600	28,900	28,900	26,300	26,300	16,500	7,400
Dead Wt.	-3.66	-16,380	-12,980	-8,000	-7,600	--2,590	-1,330	-485
Total		19,220	15,920	20,900	18,700	23,710	15,170	6,915
CONDITION I								
Chord Load	-21.1	-9,620	-7,820	-7,820	-7,100	-7,100	-4,470	-2,000
Dead Wt.	+1.27	5,680	4,500	2,765	2,630	890	460	170
Thrust	-2,730	-2,730	-2,730	-1,365	-1,365	0		
Total		-6,670	-6,050	-6,420	-5,835	-6,210	-4,010	-1,830
CONDITION II								
Dead Wt.	+.205	+915	+725	+445	+425	+144	+75	+27
Thrust	-2,730	-2,730	-2,730	-1,365	-1,365	0		
Total		-1,815	-2,005	-920	-940	+144	+75	+27

Thrust = (-.1365)(20,000) = -2,730 lbs.

TABLE IV

Final Bending Moments

Station	Factor	42 Outbd.	115 Inhd.	145 Outbd.	270	405
UNIT LOADS						
Wing Lift	1.0	101,000	71,300	60,700	26,600	6,170
Dead Wt.	1.0	461,000	169,200	105,500	40,100	7,750
CONDITION I						
Wing Lift	105.7	10,700,000	7,570,000	6,420,000	2,820,000	652,000
Dead Wt.		-2,380,000	-870,000	-543,000	-206,000	-40,000
Total		8,320,000	6,700,000	5,877,000	2,614,000	612,000
CONDITION II						
Wing Lift	-46.0	-4,650,000	-3,280,000	-2,790,000	-1,225,000	-284,000
Dead Wt.	+2.31	+1,065,000	+390,000	+244,000	+93,000	+18,000
Total		-3,585,000	-2,890,000	-2,546,000	-1,132,000	-266,000
CONDITION III						
Wing Lift	78.1	7,900,000	5,580,000	4,740,000	2,080,000	481,000
Dead Wt.	-3.66	-1,690,000	-616,000	-384,000	-147,500	-28,300
Total		6,210,000	4,964,000	4,356,000	1,932,500	452,700
CONDITION I						
Chord Load	-21.1	-2,130,000	-1,505,000	-1,280,000	-561,000	-130,000
Dead Wt.	+1.27	588,000	215,000	134,000	51,000	+9,800
Thrust		-240,000	-41,000			
Total		-1,782,000	-1,331,000	-1,146,000	-510,000	-120,200
CONDITION II						
Dead Wt.	+.205	+95,000	34,700	21,600	8,200	1,600
Thrust		-240,000	-41,000			
Total		-145,000	-6,300	21,600	8,200	1,600

Thrust Moment --
Station 115 - $M = (-2,730)(15) = -41,000$
Station 42 - $M = (-2,730)(88) = -240,000$

TABLE 1 - FINAL TORQUES

	Station	Factor	42 Outbd.	115 Inbd.	115 Outbd.	145 Inbd.	145 Outbd.	270	405
	UNIT LOADS								
	Wing Lift	1.0	617	476	476	424	424	421	97
	Dd.Wt.--E.C. at 20%	1.0	121,600	140,400	59,300	62,840	-18,260	-8,500	-2,700
	Dd.Wt.--E.C. at 30%	1.0	192,480	195,100	92,100	93,870	-9,130	-4,250	-1,350
E. C. at 20% C	CONDITION I								
	Lift	-742	-417,000	-353,000	-353,000	-315,000	-315,000	-179,000	-72,000
	Dead Weight	-5.14	-626,000	-724,000	-306,000	-323,000	94,200	43,800	13,900
	Total		-1,043,000	-1,077,000	-659,000	-648,000	-220,800	-135,200	-58,100
	CONDITION II								
	Lift	-306	-188,500	-146,000	-146,000	-130,000	-130,000	-77,000	-29,700
	Dead Weight	2.31	280,500	325,000	137,300	145,000	-42,300	-19,600	-6,200
	Total		92,000	179,000	-8,700	15,000	-172,300	-94,600	-35,900
	CONDITION III_1								
	Lift Without Aileron	-950	-585,000	-453,000	-453,000	-402,000	-402,000	-229,000	-92,000
	Lift - Aileron Increment	-2620	-631,000	-631,000	-631,000	-631,000	-631,000	-631,000	-254,000
	Dead Weight	-2.75	-334,000	-387,000	-163,000	-172,000	50,200	23,400	7,440
	Total		-1,550,000	-1,471,000	-1,247,000	-1,205,000	-982,800	-836,600	-338,560
E. C. at 30% C	CONDITION I								
	Lift	318	196,000	152,000	152,000	135,000	135,000	76,800	30,800
	Dead Weight	-5.14	-993,000	-1,005,000	-475,000	-483,000	-47,100	-21,900	-7,000
	Total		-797,000	-853,000	-323,000	-348,000	87,900	54,900	25,800
	CONDITION II								
	Lift	-768	-473,000	-366,000	-366,000	-325,000	-325,000	-185,000	-74,400
	Dead Weight	2.31	445,000	451,000	213,000	216.000	21,100	9,800	3,100
	Total		-28,000	85,000	-153,000	-109,000	-303,900	-175,200	-71,300
	CONDITION III_1								
	Lift Without Aileron	-360	-222,000	-172,000	-172,000	-153,000	-153,000	-86,800	-35,000
	Lift - Aileron Increment	-2620	-631,000	-631,000	-631,000	-631,000	-631,000	-631,000	-254,000
	Dead Weight	-2.75	-530,000	-538,000	-254,000	-258,000	25,200	11,700	3,700
	Total		-1,383,000	-1,341,000	-1,057,000	-1,042,000	-758,800	-706,100	-285,300

For the design of bending material in the wing we will assume the following conditions:

Rib spacing = 15 in.

Corrugation material--24 SRT Aluminum Alloy

Pitch = 3.5   Depth = 1 1/4 $\frac{P}{D} = 2.8$

$\frac{R}{D} = .73$ $R = (.73)(1.25) = .912$

$\frac{P}{D} = .357$ $= (.357)(1.25) = .446$ $\frac{L}{\ } = \frac{15}{.446} = 33.6$

$K_w = 1.33$ D.W. per pitch = $(1.33)(3.5) = 4.65$

Allowable Stresses:

.064 - $\frac{R}{t} = 14.2$ - 42,100 lbs. per sq. in.

.051 - $\frac{R}{t} = 17.9$ - 42,100 lbs. per sq. in.

.036 - $\frac{R}{t} = 25.4$ - 38,000 lbs. per sq. in.

.025 - $\frac{R}{t} = 36.5$ - 30,800 lbs. per sq. in.

See ANC-5

Stations:

Fuselage 42

Nac. 115
145

Ail-Flap 270
405

Tip 540

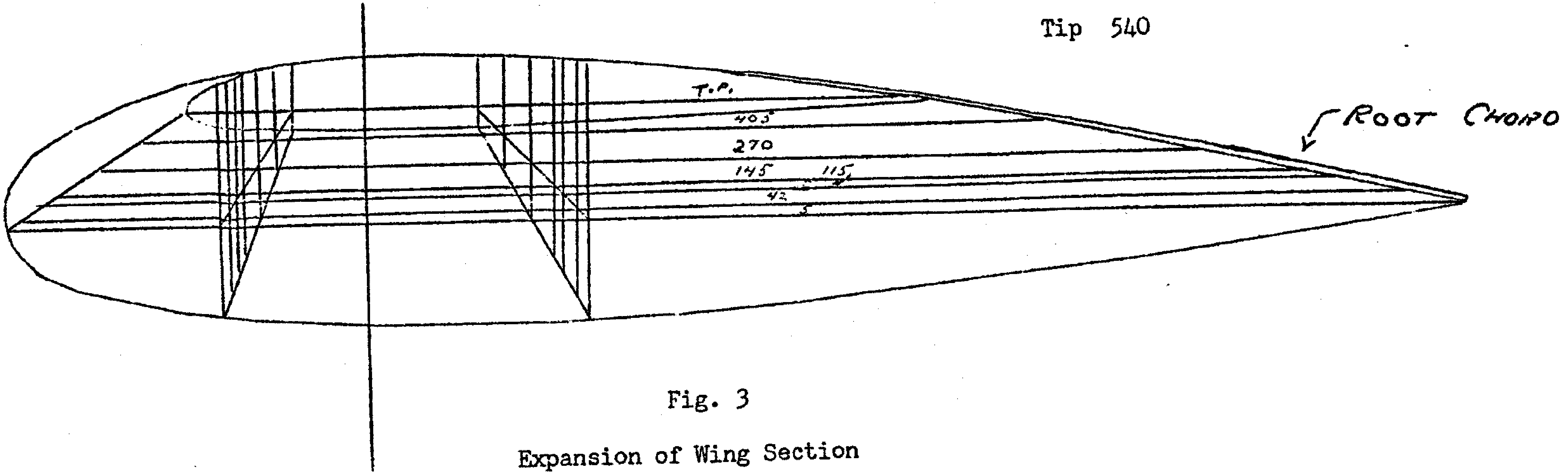


Fig. 3

Expansion of Wing Section

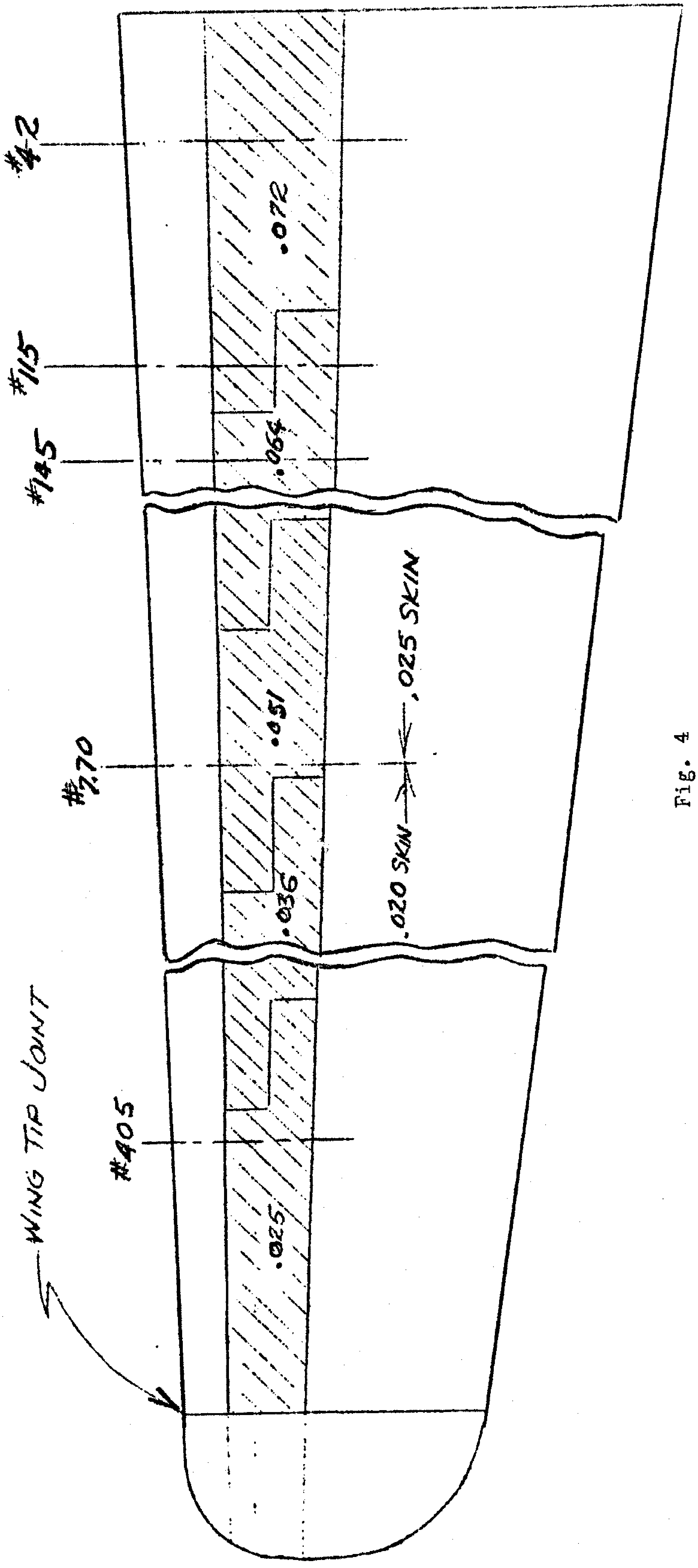


Fig. 4

Distribution of Material along Wing Span

The dimensions of the airfoil section at any particular station are scaled from the expansion shown on Fig. 3. In practice the dimensions are taken either from large scale layouts or from "lofted" dimensions laid out full scale. For the purpose of illustration here the approximate dimensions from the small scale layout are satisfactory.

Calculation of the section properties at Station 405 will be given as an example. Fig. 5 shows the dimensions of this section and the distribution of material.

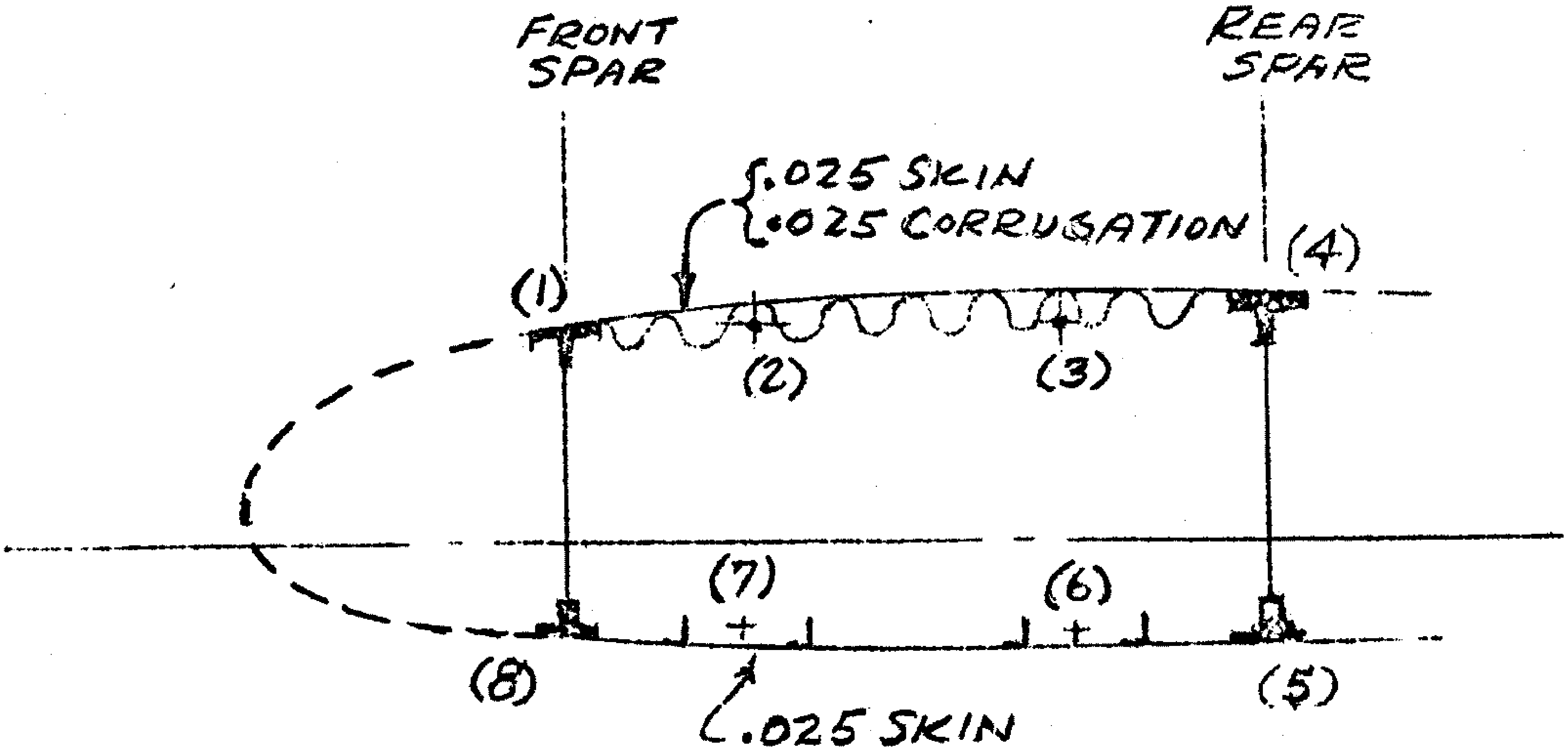


Fig. 5

Cross Section at Station 405

Calculation of Section Properties

Station 405

No.	A	y	Ay	x	Ax	y_o	x_o	Ay_o^2	Ax_o^2	Ix_o	Iy_o	Ix_o	Ax_oy_o
1	.52	-8.1	-4.21	0	0	-6.6	+14.3	22.6	106				-49.2
2	.42	-8.8	-3.70	8	3.4	-7.3	+6.3	22.4	17		6		-19.3
3	.42	-9.0	-3.78	21	8.8	-7.5	-6.7	23.6	19		6		+21.1
4	.52	-8.9	-4.63	28.5	14.8	-7.4	-14.2	28.5	105				+54.7
5	.50	+4.4	2.20	28.5	14.2	5.9	-14.2	17.4	102				-41.9
6	.65	+4.4	2.86	20.5	13.3	5.9	-6.2	22.6	25		9		-23.8
7	.65	+4.2	2.73	8	5.2	5.7	6.3	21.1	26		9		+23.3
8	.50	+3.8	2.20	0	0	5.3	14.3	14.0	102				+37.9
	4.18	-1.5	-6.33	14.3	59.7			172.2	502		30		+ 2.8

(1) (4) A = .45 + 1.5 (.020 + .025) = .517

(2) (3) A = $(3\frac{1}{2})(.025)(4.65) + 3\,(20)(.020)(.020) = .418$

(5) (8) A = .45 + 1.5 (.025) = .497

(6) (7) A = (14)(.025) + (2)(.15) = .65

$I_{x-x} = 172.2$

$I_{y-y} = 502 + 30 = 532$

Figs. 6, 7, 8, and 9 show the cross sections at Stations 270, 145, 115, and 42. The pertinent dimensions are shown in the tables following.

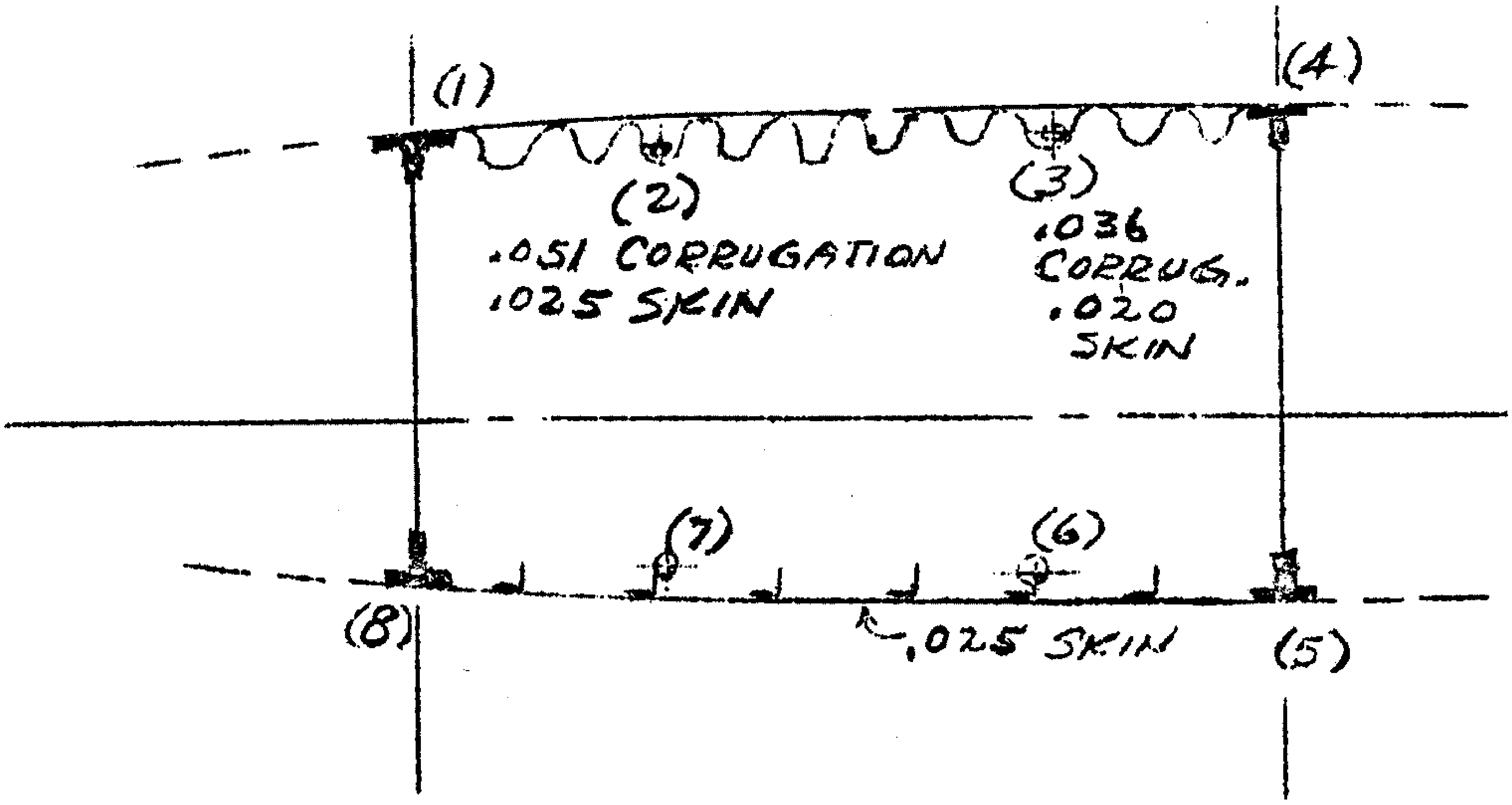


FIG 6 - STATION 270

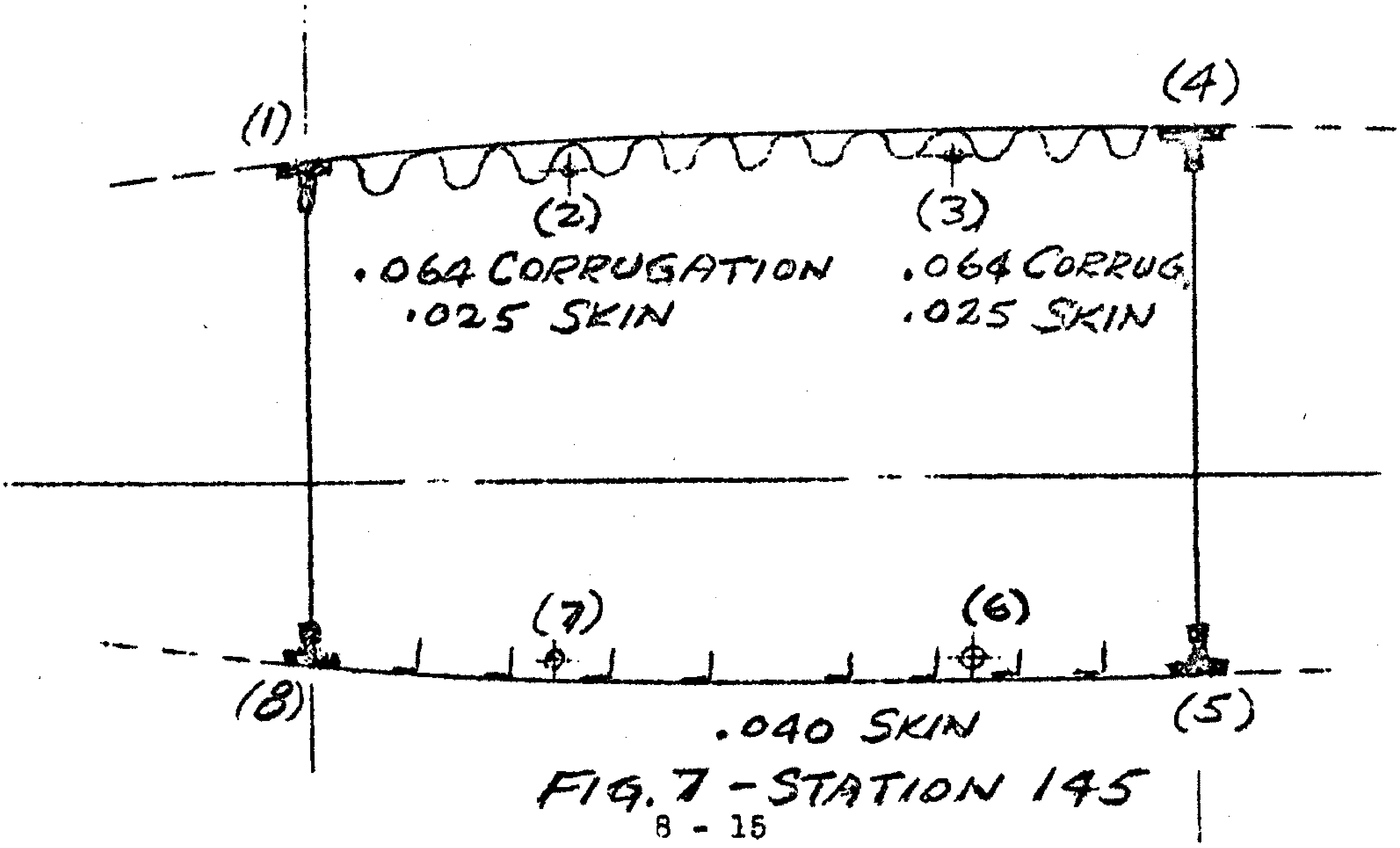


FIG. 7 - STATION 145

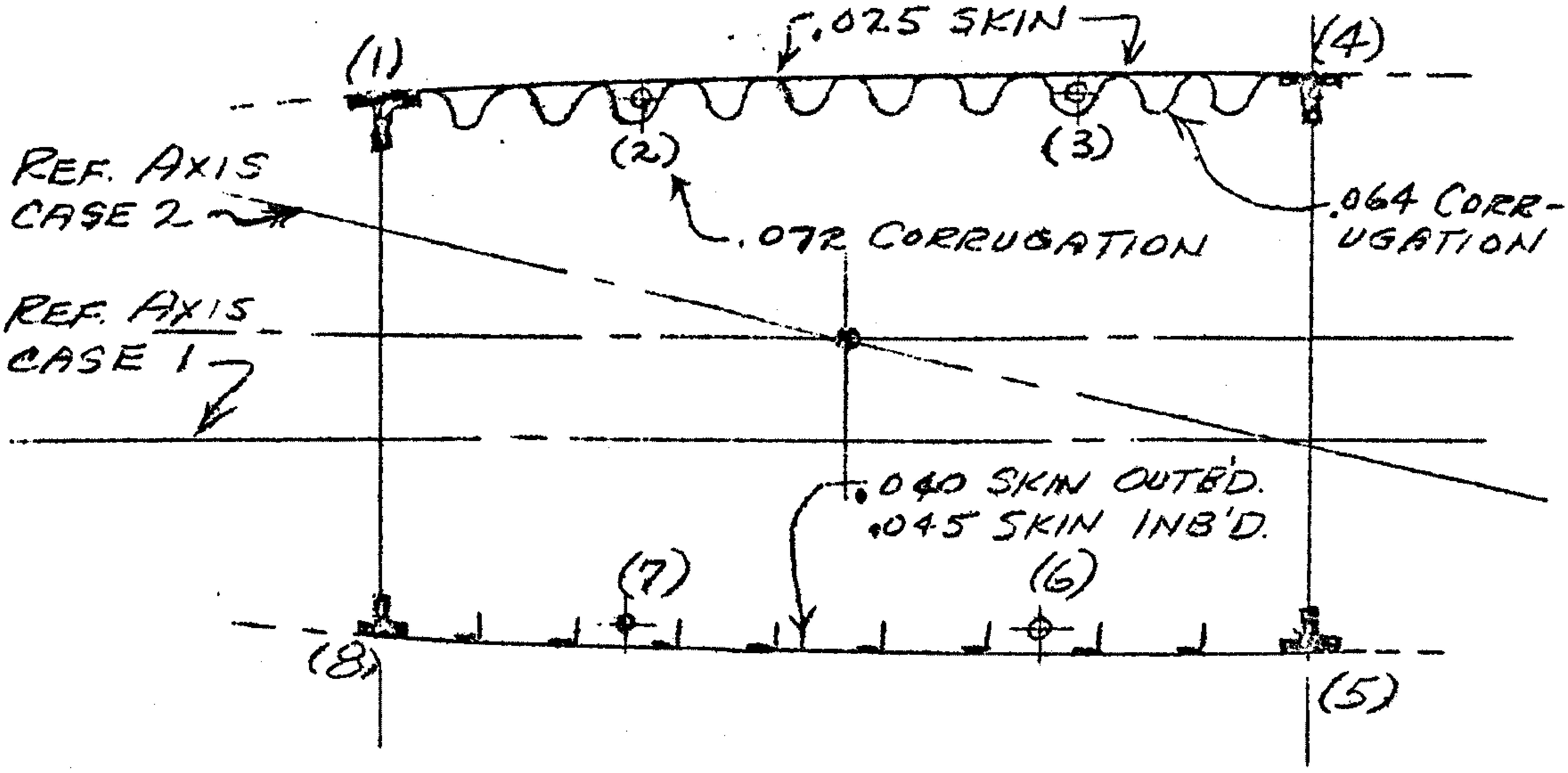


FIG. 8 - STATION 115

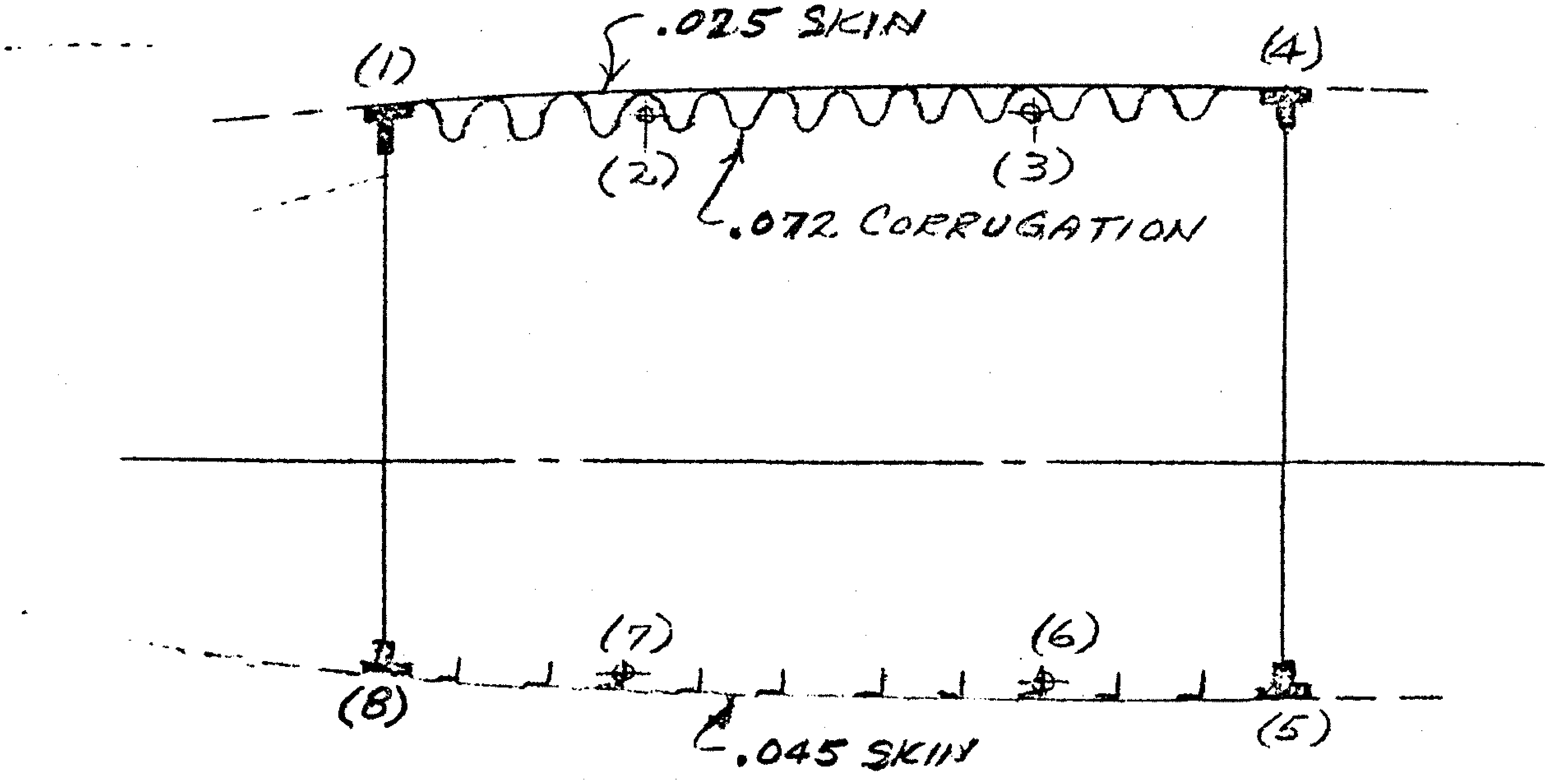


FIG. 9 - STATION 42

Problem Assignment:

Given the data of Figs. 7 and 8 and the dimensional data corresponding, as shown in the following tables:

Calculate the Section Properties for Compression on the top surface for stations 145 and 115. Use tables similar to those given in the text for tabulating the data.

Station 145

No.	A	y	x
1	2.07	-14.0	0
2	1.54	-15.0	10.0
3	1.54	-15.2	28.2
4	2.07	-15.1	38.6
5	1.92	8.5	38.6
6	1.32	8.8	28.2
7	1.32	8.5	10.0
8	1.92	7.8	0

Station 115

No.	A	y	x
1	2.27	-14.6	.0
2	1.90	-16.0	12
3	1.55	-16.1	29.8
4	2.27	-15.9	40.0
5	2.12	9.1	40.0
6	1.34	9.3	29.3
7	1.34	9.1	11.0
8	2.12	8.2	0

Note in the above tables that the reference axis for x is taken at the front spar and for y at the chord line of the section. The sign convention for y (-y = up) corresponds to the sign of the bending stresses (compression on the top fibers).

The areas given in the above tables are calculated as follows:

Station 145

(1) and (4)

$$A = 1.8 + (3.0)(.064 + .025) = 2.07 \text{ sq. in.}$$

(2) and (3)

$$A = (5)(.064)(4.65) + (4)(20)(.025)(.025) = 1.54$$

(5) and (8)

$$A = 1.8 + (3)(.040) = 1.92$$

(6) and (7)

$$A = (18)(.040) + (4)(.15) = 1.32$$

*****

Station 115

(1) and (4)

$A = 2.0 + (3)(.064 + .025) = 2.27$ sq. in.

(2) $A = (5.5)(4.65)(.072) + (5)(20)(.025)(.025) = 1.90$

(3) $A = (5)(4.65)(.064) + (5)(20)(.025)(.025) = 1.55$

(5) and (8)

$A = 2.0 + (3)(.040) = 2.12$

(6) and (7)

$A = (18.5)(.040) + (4)(.15) = 1.34$

ADVANCED STRESS ANALYSIS

ASSIGNMENT 9

Assignment 9

THE DESIGN OF STRESSED SKIN WINGS

General.

There are a number of types of internally braced cantilever wing designs each having certain advantages or disadvantages when compared to another type. The development of construction types is bound up with shop manufacturing developments and has followed economic as well as technical lines. Because of the fact that, for monocoque or semi-monocoque wings the analysis and design must proceed together, it is well at this time to sum up the design types used and point out their major features.

Let us review the minimum objectives which must be attained before we can consider our wing structure satisfactory.

1. It must not show excessive deflections or objectionable wrinkling of the outer cover under normal working loads; that is when in level flight under load factors of from 1 to 1.25 times the acceleration of gravity. Formulas are available in Timoshenko's "Strength of Materials" for computing the loads at which sheet covering will first buckle in compression, or go into the wave state in shear. A guide is also available in the curve of Fig. 34 in ACM-04 which gives allowable panel sizes for different thicknesses of sheet at various radii of curvature. If doubt exists it is advisable to build a test panel and note its appearance when subjected to unit stresses corresponding to the normal working loads on the airplane.

2. The structure must not yield in any part when subjected to the maximum "expected" or limit loads.

3. The structure must not fail before it carries loads equal to 1.5 times the limit loads.

In addition to the above considerations care must be taken that parts which are not a part of the structure, such as fairings or fillets, are free to deflect out of the way when the strength-carrying structure deflects. Otherwise a part which is not considered a load carrier may cripple in following the deflection of the structure.

In placing material in a wing certain fundamental principles must be observed.

1. The bending material should be as far from the neutral axis as possible to carry loads in the beam direction.

2. Some bending material must be placed so that bending loads may be carried in the chord direction.

3. The major portion of the bending material should be near the thickest portion of the airfoil cross-section to maintain efficiency. (See also (1) above.)

4. Distribution of the shear webs in the wing should be proportioned so that the shear carried by each web is proportional to the bending strength of the beam material adjacent to that web.

5. Bending material in the top and bottom of the wing should be concentrated near the major shear webs wherever possible.

6. Sudden changes in areas of wing sections should be avoided, and changes in guages of wing covering should be made in small increments to avoid concentrations of wrinkles along the joints.

7. Eccentricities at joints should be avoided where possible.

8. Allowance should be made for the effects of the air loads on the local wing structure such as the ribs and the stringers between the ribs.

Types of Stressed Skin Wings.

1. Two spar wing with plate girder type of Wagner type shear web and metal covering beaded in the fore and aft direction or flat.

2. Two spar wing with trussed beams and metal covering beaded in the fore and aft direction or flat.

3. Semi-monocoque or box wing with one or more beams with the skin between them stiffened by corrugations or extruded shapes.

Since there are any number of variations of the foregoing types which allow one type to incorporate features of the others a comparison will be made by considering the major components of the wing separately.

*****

Bending Resisting Material.

In type 1 the bending resisting material is nearly all in the spar caps, which are usually extruded shapes riveted to the shear webs. The advantages of using this type of section to resist bending are as follows:

1. By careful design of the cap to avoid free edges and sharp transition in section high allowable bending stresses may be obtained. For aluminum alloy extrusions the ultimate values may be as high as 45,000 to 50,000 psi.

2. By machining off the portions of the cap not needed in the outer part of the wing, the material may be used up to its full strength all along the span, with a resultant weight saving.

3. The Elastic Axis of the wing may be kept near the center of pressure of the load in nearly all conditions of flight, thus avoiding a heavy twisting moment on the wing.

4. Shear web or plate girder beams may be made economically as separate assemblies and are lower in cost than other types.

The obvious disadvantages are:

1. The panels of metal covering which form the wing surface must be stiffened in some way to enable the sheet to carry the shear loads, and to avoid drumming or excessive wrinkling. This requires either very close rib spacing or the presence of spanwise stiffeners on the sheet, which are susceptible to crippling when the spars deflect under load.

2. In a two-spar wing it is impossible to obtain torsional rigidity and still utilize the maximum depth of the wing for more than one spar. A true monospar wing could accomplish the purpose of putting the bending material at the section of greatest depth but it would be too flexible in twisting, and rigidity in this respect (torsional rigidity) has been found to be more important than bending rigidity in preventing wing flutter. (Note: The wing constructions generally referred to as monospar actually contain three shear webs, viz. the center or main spar, the curved leading edge of the wing, and the "false" spar ahead of the aileron and wing flaps. See Fig. 1)

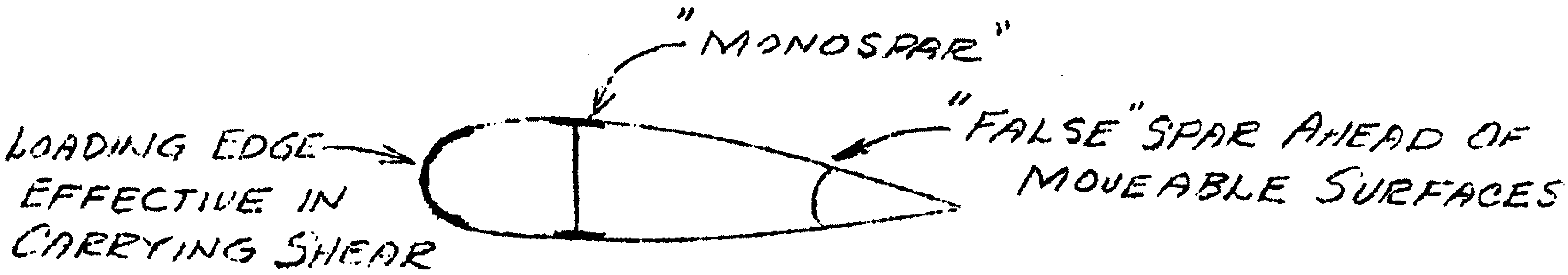

"MONOSPAR"
LOADING EDGE EFFECTIVE IN CARRYING SHEAR
"FALSE" SPAR AHEAD OF MOVEABLE SURFACES

FIG. 1 - "MONOSPAR" ARRANGEMENT

3. The machine operations on beam flange extrusions are very expensive unless production is in large quantity, where elaborate tooling costs can be spread over a large number of units.

In the type 2 wing steel or aluminum alloy tubes are used to form trussed spars. The proponents of this type of structure claim the following advantages for it over type 1:

1. By using square steel tubes on the highest loaded sections inboard and heat treating to 160,000 to 180,000 psi a favorable strength weight ratio is secured.

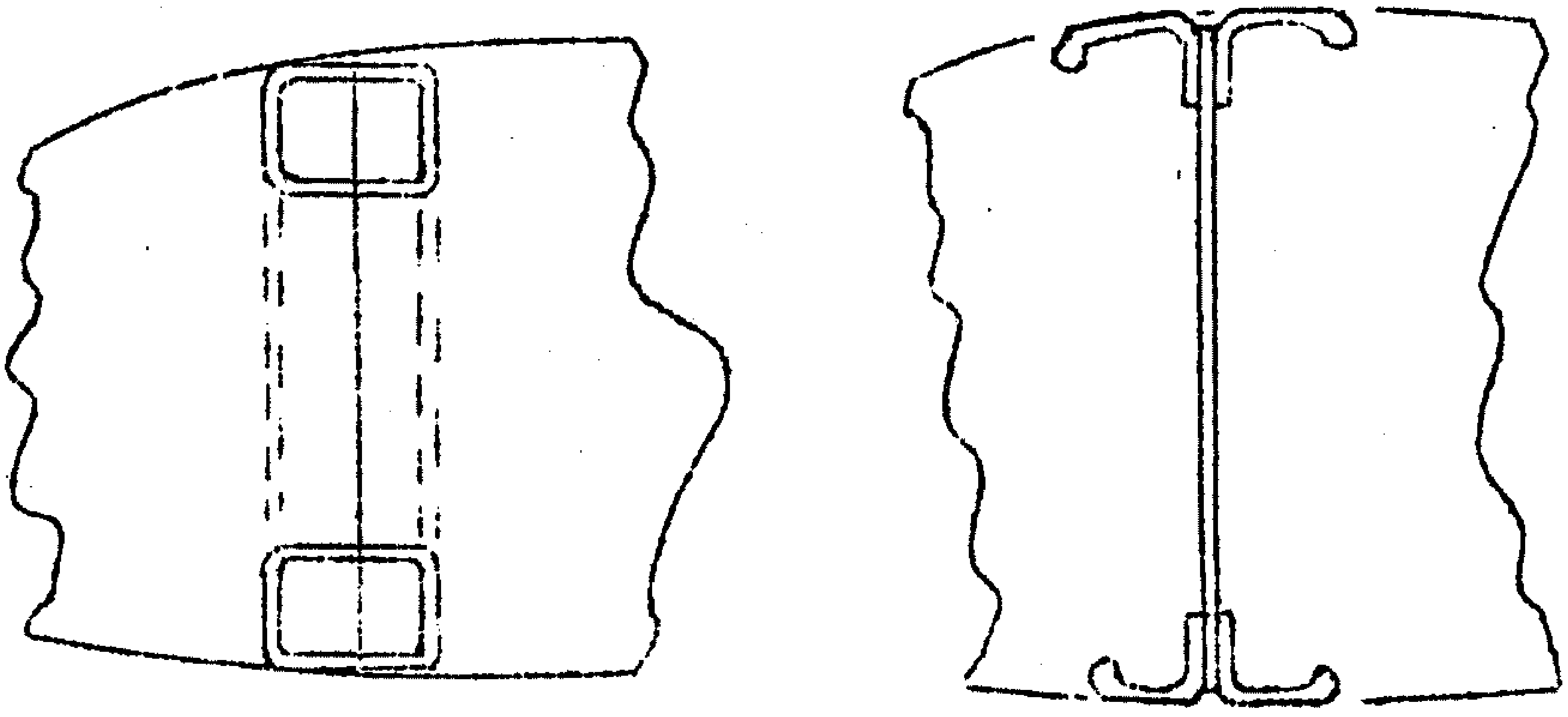
2. On multi-engined ships where the engines are carried in the wings, the use of steel tubing from the power plant section to the center line of the ship promotes safety in case of fire in the engine section. (This advantage is nullified on ships using aluminum alloy by shutting off the power plant section with a stainless steel firewall.)

3. The use of the shear truss permits a definite load analysis and the use of higher working stresses in the material in the web members.

The evident disadvantages of the trussed type spars are:

1. It is difficult to design them so that the tubes utilize the full depth of the wing contour, and nearly always less than the full depth must be used. See Fig. 2.

2. The area of the cap may be changed only by splicing in a different size tube, or by using supplementary material between spars; that is by introducing other bending material into the wing in the form of spanwise stiffeners or longitudinals attached to the skin.



CROSS-SECTION OF TRUSSED SPAR

PLATE GIRDER SPAR

FIG. 2

3. Secondary stresses are present in the chord members because they must bend with the wing as a whole. (The latter disadvantage is also present in a slightly different guise in the Wagner type beam because of the pull of the wrinkled sheet on the cap strips. This condition is covered by assignment 10.)

4. Assembly costs are very high because of the large number of gusset fittings required at the joints. It is also necessary to use a large amount of inside riveting in the final assembly. Special equipment must be developed for both the riveting operation and inspection, and inspection of the inside riveting on the spars is impossible in service.

The wing of type 3 usually incorporates plate girder or "shear web" beams with flanges of extruded material. See Fig. 2. Between these beams the wing covering is stiffened either by a number of extruded or formed shapes or by continuous spanwise corrugations formed from sheet stock. Advantages of this type of construction are:

1. A large portion of the bending resisting material may be located at the maximum depth of the wing.

2. The skin stiffeners, which are nearly always found necessary to prevent the covering from wrinkling excessively, are put to work in bending. (This is not always an advantage since the stiffeners will generally fail at lower unit stresses than the beam caps will and partial failure may occur before the actual ultimate strength is reached.)

3. The area of bending material is tapered easily and efficiently by discontinuing certain stiffeners as one progresses toward the wing tip.

4. Partial failure may occur without complete wing failure, since buckling of the stiffeners at the maximum depth on the wing will give warning before the strength of the wing is reached. That is, the intermediate stiffeners will fail before the main shear web caps are crippled.

Disadvantages are:

1. Cutouts in the skin for access and inspection become troublesome. The area removed must be replaced by gussets and additional stiffeners or corrugation reinforcements.

2. Wing joints must be continuous over a major portion of the wing chord, which is costly, heavy and complicates assembly and disassembly in service.

Since wings of type 3 are in extensive use on modern airplanes we will go further into the quantitative limitations which affect them. These wings normally incorporate bending material between the shear webs made either of 24SRT aluminum alloy corrugations or 24ST extruded shapes, such as Z's, T-sections, or angles. A comparison of these stiffener types follows:

	Corrugations	Stiffeners
1. Maximum compression stress in bending........................	46,000 - 50,000	35,000
2. Taper in area....................	Fairly good	Very good
3. Minimum practical area per inch of chord.......................	.032 (.020 corrugation)	.010 (.060 stiffeners at 6" spacing)
4. Contributes to torsional strength of wing thus reducing skin thickness required...................	yes	no
5. Cost of manufacture..............	High	Low
6. Cost of assembly.................	High	Low

*****

Features to bear in mind when designing a wing of this type are:

1. The depth of the corrugation or stiffener should be proportional to the depth of the wing.

$$\frac{\text{Maximum depth of wing}}{\text{Depth of Bending-Resisting Material}} = \frac{25}{1} \text{ to } \frac{40}{1}$$

In corrugations the $\frac{R}{t}$ ratio should be $\frac{15}{1}$ to $\frac{25}{1}$ (see curves in ANC-5)

2. Ribs should be spaced to give and L/ρ of aluminum alloy corrugations or approximately 40 to 60. This keeps the material in the local crippling range and minimizes the importance of the exact end fixity.

3. Put as much bending material at the maximum depth of the wing as possible.

4. Use enough spars to prevent excessive shear lag, or group the greater area of stiffeners close to the spars.

5. Use heavy enough skin to prevent buckling on the compression side at one load factor. This buckling action is a function of both skin thickness and rivet spacing.

*****

Shear-Resisting Material.

The most common shear resisting medium is a shear web of the "Wagner" type. The advantages of this are:

1. It is cheap.

2. It is efficient.

3. In special cases it facilitates the watertighting of certain compartments of the wing.

The disadvantages are:

1. Minimum gages may be reached if too many spars are used, which reduces the efficiency.

2. They tend to divide the wing into separate compartments making access for service inspection and repairs difficult.

3. Shear deflections are high.

A second type is the truss for which the advantages are:

1. It can be tapered efficiently by change in tube sizes in adjacent bays.
2. Combined with truss-type ribs a very accessible wing is obtained.

The disadvantages are:

1. Relatively poor strength-weight ratio when compared to a well designed Wagner beam.
2. Cost of assembly.
3. Failure of one member of truss cripples entire wing.
4. Deflections are greater than for the web type.

Ribs.

The principal types of ribs are hydro-press, truss-type and web-and-cap type. Of the three the hydro-press ribs are probably the cheapest in production and can be made fairly efficient. The truss type is efficient, particularly in large wings, and is not excessively expensive. The web and cap ribs are well adapted to high loads but are apt to be heavy in the normal case. Hence they are used for bulkheads or where the rib spacing is unusually high. For ribs the designer should be careful to make assumptions of reactions which are consistent with the type of wing construction used. In other words the effect of shear in the wing covering and of the inward loads from the bending resisting material should be allowed for. Otherwise an inefficient and possibly weak design may result.

Special Considerations.

Various other features must be borne in mind in the design of the wing. These include the effects of concentrated loads from landing gear, nacelles, fuel or oil tanks, aileron, flap and control systems. These will probably control certain features of the wing such as the number and spacing of the wing spars. All that can be said about the detail design of the wing in these regions is that, insofar as possible, the structure for these items should be made an integral part of the wing to prevent duplication and, what may be more serious, relative distortion of the various parts of the structure.

Shear Stresses in Semi-Monocoque Construction

The word monocoque is used rather loosely in referring to any type of all metal construction where the covering forms a part of the structure. Actually the airplane structures with which we are familiar are stiffened or semi-monocoque. In a pure monocoque construction *the outer covering would be the structure*, and all bending and shear loads would be carried through it. It should be understood that reference to monocoque construction throughout this text is made with reference to semi-monocoque entirely, which coincides with the popular usage of the term.

****

The shear stresses in monocoque wings are due to torsion (twisting) and to lift and drag forces. The effect of each will be considered separately.

Torsional Shear

A typical monocoque wing section under pure torsion is shown on Fig. 4 (a). Torsional shear acts uniformly over the outer skin. The actual values of torsional shear are found as follows: The actual cross-sectional area enclosed by the skin is taken with a planimeter, or calculated. This area will be termed A. Then

$$\text{Load per inch from torsion} = \frac{\text{Torsional moment}}{2A}$$

$$\text{Then } f_s/\text{inch torsion} = \frac{(90{,}000)}{(2)\ (300)} = 150 \text{ lbs. per running inch}$$

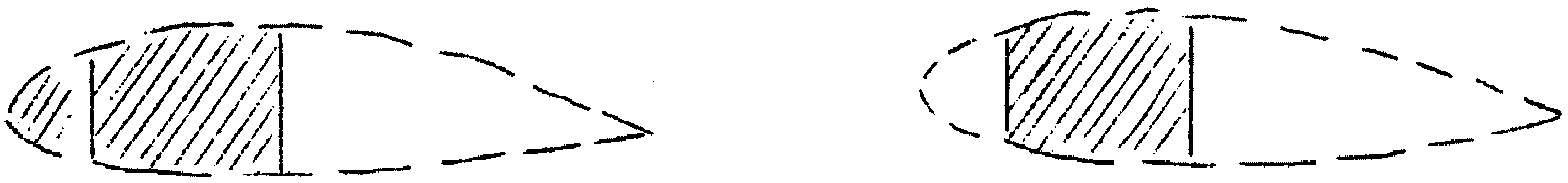
For .040, shear = 150/.040 = 3750 lbs. per sq. in.

For .030, shear = 150/.030 = 5000 lbs. per sq. in.

For .020, shear = 150/.020 = 7500 lbs. per sq. in.

The above method is somewhat approximate in that it neglects the effect of the beams in resisting torsion.

If all of the wing covering is not effective in resisting torsion a reduced torsional box or tube must be used, in which case the enclosed area must also be reduced. For example, if as in our type airplane the section behind the rear spar is fabric covered it cannot aid in carrying wing twist the effective section will then be as shown in Fig. 3 (a) with the effective area shown shaded. If for example a cutout for a landing light is placed in the leading edge, the section becomes as shown on Fig. 3 (b).



(a) Rear Section Fabric Covered

(b) Nose Section also Removed

Fig. 3
Effective Torque Boxes

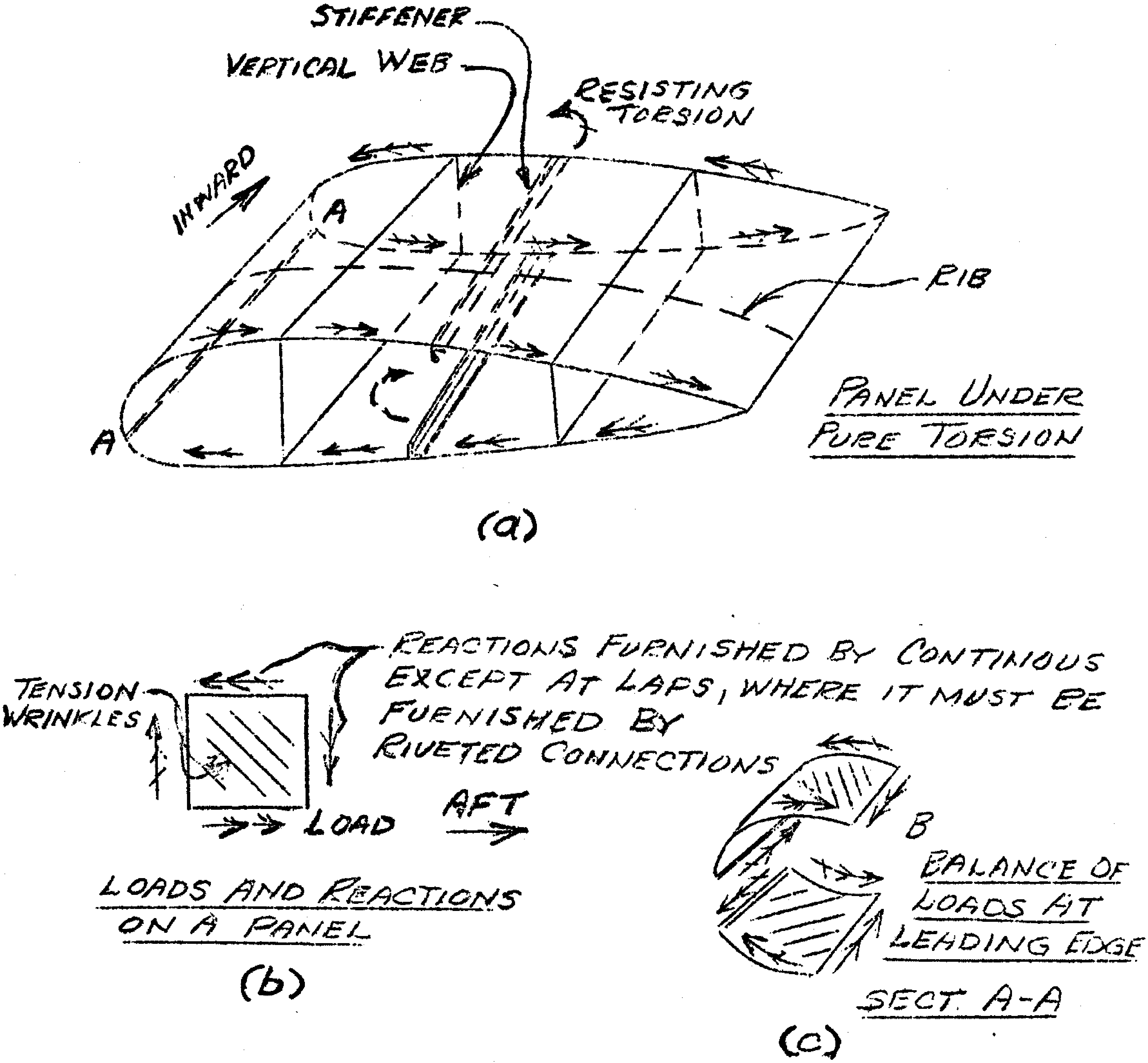
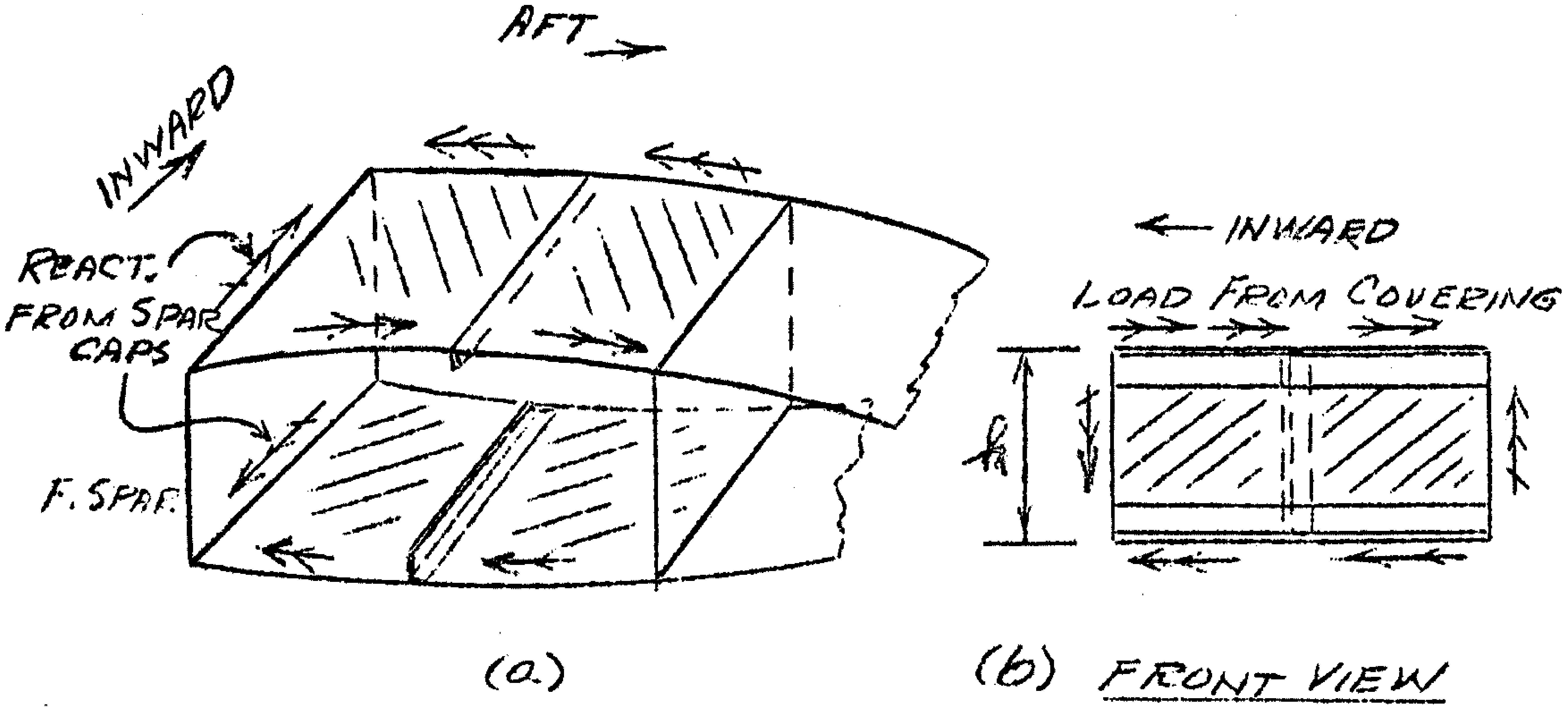

STIFFENER
VERTICAL WEB
RESISTING TORSION
INWARD
A
A
RIB
PANEL UNDER PURE TORSION
(a)
REACTIONS FURNISHED BY CONTINIOUS EXCEPT AT LAPS, WHERE IT MUST BE FURNISHED BY RIVETED CONNECTIONS
TENSION WRINKLES
LOAD
AFT
LOADS AND REACTIONS ON A PANEL
(b)
B
BALANCE OF LOADS AT LEADING EDGE
SECT A-A
(c)

Fig. 4



(a)

(b) FRONT VIEW FRONT SPAR

Fig. 5

(Increment of shear load per inch in spar due to torsion equals pull of skin load per inch. Shear wrinkles in the sheet are assumed to act at 45° with the applied shear load. Then the load per inch spanwise equals the load per inch chordwise.)

Fig. 4 (b) shows the balance of forces on a single panel of wing covering bounded by ribs and spanwise stiffeners. The thin sheets used for wing covering wrinkle under shear loads so that instead of making a transfer by pure shear the load is carried in diagonal tension. This phenomenon is covered completely in Assignment 10 on the design of Wagner Beams. Fig. 4 (c) shows the balance of forces at a section where the wrinkles change direction as at a leading or trailing edge or at a corner connection to a spar as in Fig. 3 (b).

Fig. 5 (a) shows the wrinkle pattern when a spar forms the leading or trailing edge of the torque box. Fig. 5 (b) illustrates the method whereby the torque shear adds to or subtracts from the vertical shear load in the spar.

Actually when the wing is complete with leading and trailing edges beyond the spars as in Fig. 4 (a) some load will be transferred from top to bottom by the spars. This amount may be shown to be small by both theory and test and, except for large airplanes, is generally disregarded.

Shear Due to Chordwise Loads.

Assume the wing section is a beam in the chordwise direction and use the beam formula for shear at any web section from the Basic Course, Assignment 9. The web for bending in the chordwise direction consists of the top and bottom covering. For exact values each side, top and bottom, would be considered separately.

$$f_s \text{ from drag shear} = \frac{SQ}{bI} \text{ psi.}$$

or the shear load per inch in the covering at any point is $\frac{SQ}{I}$ where

S = chordwise shear on the section.
Q = static Moment of Inertia of all parts on one cover on the side away from the neutral axis (= $\sum Ax$)
I = chordwise Moment of Inertia of entire section.

Note again that the shear stress due to chordwise loads will be a maximum at the neutral axis for chordwise bending.

After the chordwise shear stress is found it combines directly with the shear stress due to torsion adding to it on one surface of the wing and subtracting from it on the other. The shear stress from chordwise shear load is generally so low that very little error would be introduced by considering it to be uniformly distributed, in which case the unit shear stress would equal the shear load divided by the area of the covering.

Shear Due to Wing Bending.

If we again apply the beam theory of Assignment 9 of the Basic Course we may compute the shear stresses in the covering introduced by the fact that increment of loads in the bending material in the stiffeners must be transferred by the skin. Consider the stiffeners with the skin effective with them as a part of the cap strips of a beam. When bending is applied in the vertical plane a "herringbone" effect of shear wrinkles is introduced as shown in Fig. 6. The longitudinal loads produce diagonal wrinkles which give chordwise forces also. These combine with those due to drag, adding in some cases and subtracting in others. The surest method of keeping the combinations correct is to draw small sketches of effects of each separately.

The numerical value of the shear introduced by vertical bending would again be $\frac{SQ}{I}$ per inch where I would be the moment of inertia for the vertical web and the stiffeners acting with it, Q the statical moment of the stiffeners on one cover on one side of the spar cap and S the shear being transferred by the vertical web being investigated. An assumption is involved as to how much of the total shear is being transferred by a particular web. It is generally assumed that the webs take shear loads in proportion to their respective areas.

Instead of making calculations for the Q at each section it is possible to work as follows:

1. Calculate the bending fiber stresses and the total loads in the stiffeners to which the surface sheet is transferring load such as 1, 2, and 3 of Fig. 3, at Station A.
2. Calculate the same values at the next station inboard. Station B.
3. Take the differences between the loads in the stiffeners at the two stations. These differences represent the loads transferred by the sheet in the longitudinal length between stations A and B. Call them $\Delta 1$, $\Delta 2$, and $\Delta 3$. Then the sheet between stiffeners 3 and 2 transfers Δ_3 and its unit load per inch is Δ_3 divided by the distance A to B. The sheet between 1 and 2 transfers $\Delta_3 + \Delta_2$, and the sheet between 1 and the cap strip transfers $\Delta_3 + \Delta_2 + \Delta_1$.

Thus the shear values from any load may be determined by fundamental applications of the beam formula.

Loads on Riveted Seams.

Where the sheet is continuous across spar caps or stiffeners, the only purpose of the rivets is to pick up the increment of load in the bending material as the moment increases. The rivet spacing required for this purpose is so large as to be impractical if any of the sheet is to act effectively with the stiffeners as compression members. The maximum practical allowable is based on panel tests for the sheet and stiffener combinations used. Where the sheet is discontinuous, that is where laps occur, the rivets transfer the loads from one sheet to another.

With thin skin covering, where the sheet carries the shear load in diagonal tension, the loads on the rivets are 1.414 times the running load per inch shear. See Fig. 7. Rivet spacing should always take into account the effect on the net tension strength of the sheet remaining after rivet holes are deducted from the area. A balanced joint is desirable with the sheet somewhat stronger in tension than are the rivets in shear or bearing to allow for tension stress concentrations around the rivet holes. Where two or more lines of rivets are used in a splice, the distance between lines of rivets should be sufficient for the sheet to develop the full rated strength of the rivet. Longitudinal splices are designed by shear only, but chordwise splices must develop the full compression or tension strength of the sheet stiffener combination.

Where a large number of rivets carry the load no extra "fitting" margin of safety is used. Where a few rivets carry the entire load as at the end connection of main members to a fitting, a margin of safety of 20 per cent is maintained. Actually, in all practical cases, the assumptions of a full Wagner effect are sufficient to insure some additional margins of safety, since all sheet does have some resistance to compressive forces while the Wagner theory assumes that all shear load is transferred by diagonal tension.

Fig. 8 illustrates a method of sign convention which automatically takes care of the sign convention in combining torsion and chordwise shear stresses with those due to bending.

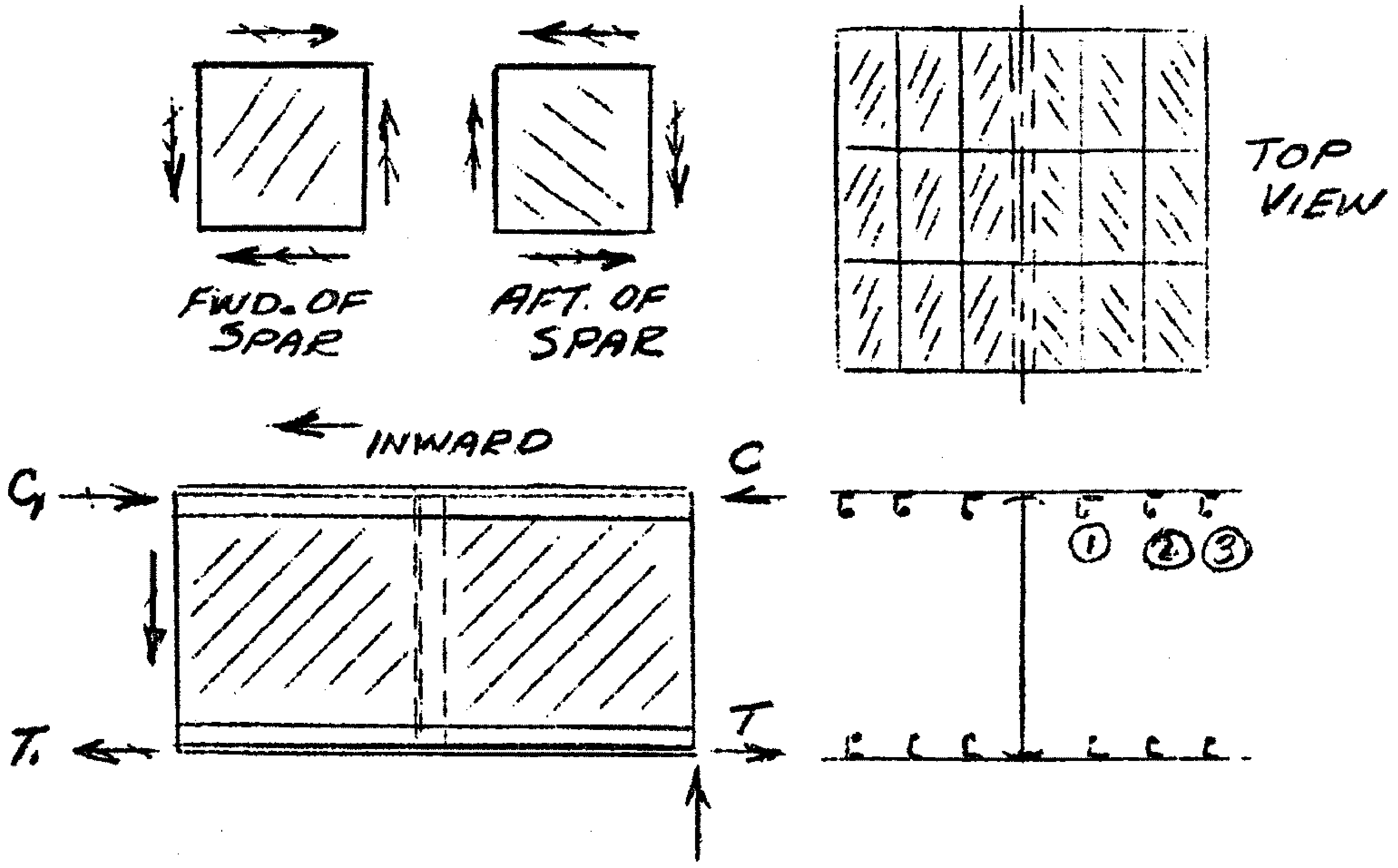

FWD. OF SPAR
AFT. OF SPAR
TOP VIEW
INWARD
C_1
C
T_1
T
①
②
③

FIG. 6

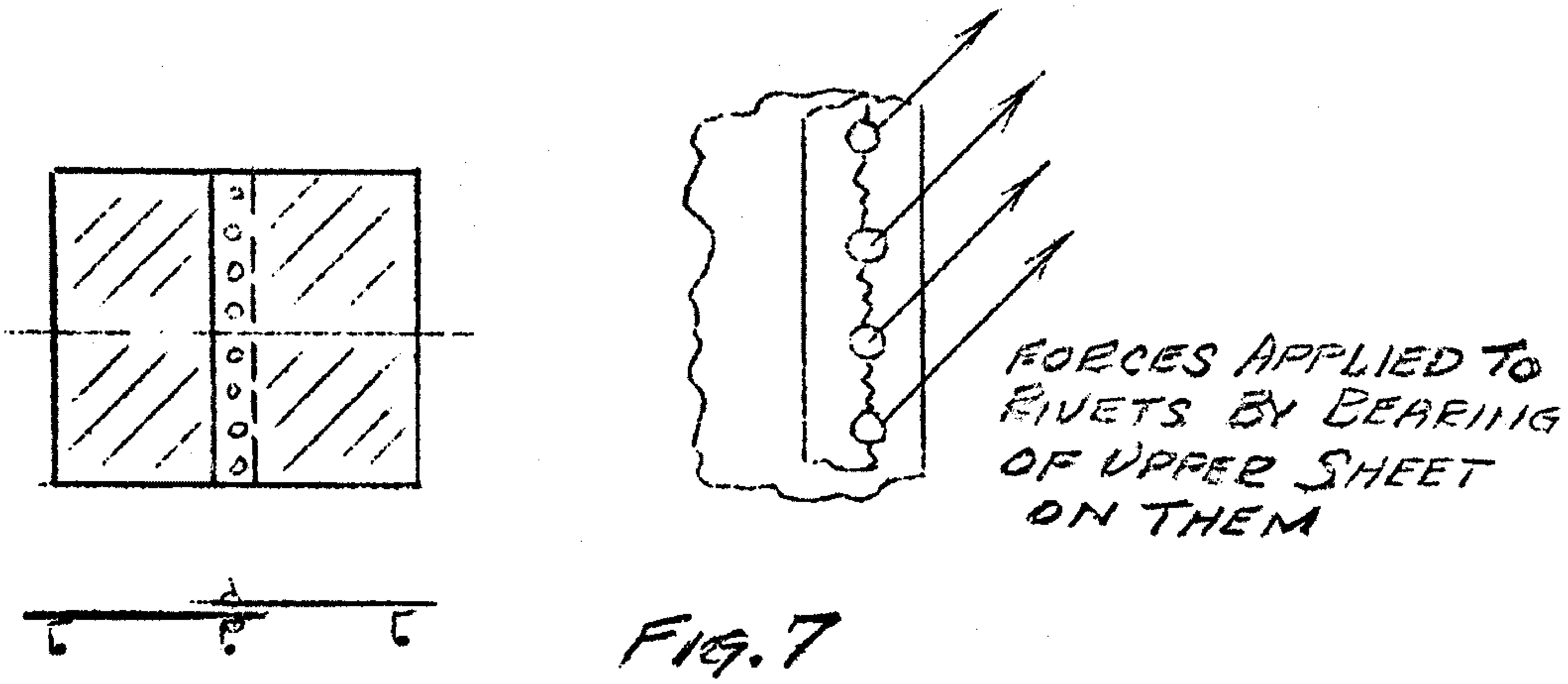

FORCES APPLIED TO RIVETS BY BEARING OF UPPER SHEET ON THEM

FIG. 7

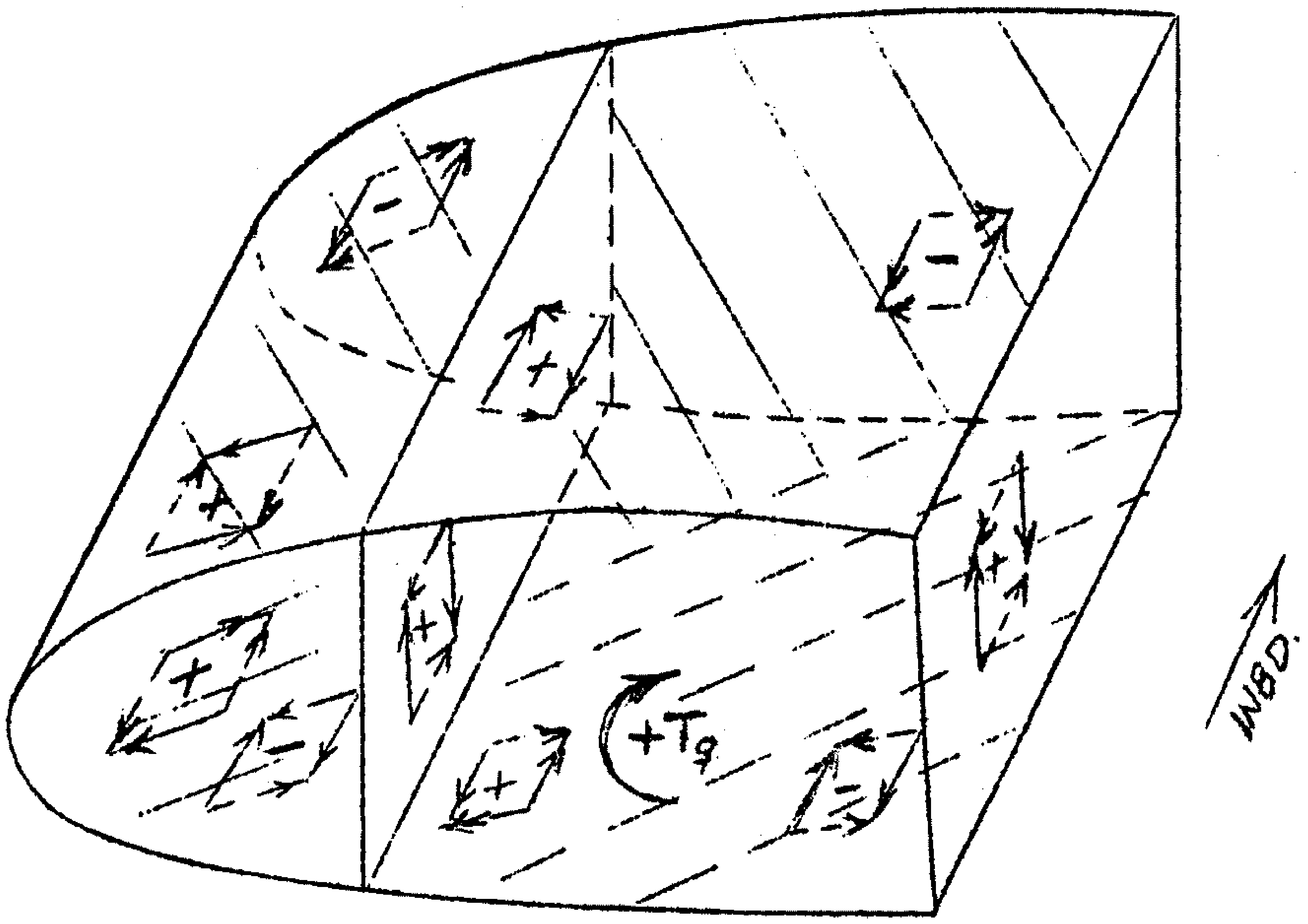


Fig. 8

Sign Convention for Shears Due to Torque and Bending

For positive torque shear wrinkles would appear on the top of the wing in the direction of the solid lines, and on the bottom in the direction of the broken lines. These sets of lines as drawn show plus shear in the covering.

Plus shear in the spar webs due to up load results in shear in the covering as indicated by the small vector diagrams.

ADVANCED STRESS ANALYSIS

ASSIGNMENT 9 A

Assignment 9-A

DESIGN OF STRESSED SKIN WINGS - CONTINUED

The Effect of Inverted Loads on the Bending Characteristics of a Wing Section.

We have been working on the assumption that all of the "stressed-skin" covering is effective in bending on the tension side of the wing, and that only a part of the skin, at each cap or stiffener, is effective on the compression side. For inverted loadings the compression side of the cantilever wing is changed from the top to the bottom. The effect on the change in section properties will be illustrated by calculating the section properties for station 115 with compression on the bottom surface. Refer to Assignment 8.

TABLE I

SECTION PROPERTIES

Station 115 - Compression on Bottom Surface

No.	A	y	Ay	x	Ax	y_o	x_o	Ay_o^2	Ax_o^2	Ix_o	Iy_o	Ax_oy_o
1	2.27	-14.6	-33.2	0	0	-8.8	20	176	908			-399
2	2.34	-16.0	-37.5	12	28.1	-10.2	8.0	243	150		78	-191
3	1.94	-16.1	-31.2	29.8	57.9	-10.3	-9.8	206	186		46	196
4	2.27	-15.9	-36.1	40.0	90.9	-10.1	-20.0	232	908			459
5	2.12	9.1	19.4	40.0	85.0	14.9	-20.0	471	848			-632
6	.79	9.3	7.3	29.3	23.1	15.1	-9.3	68	68		22	-111
7	.79	9.1	7.2	11.0	8.7	14.9	9.0	62	64		22	106
8	2.12	8.2	17.4	0	0	14.0	20.0	416	848			593
Σ =	14.64	-5.8	-85.7					1874	3980		168	+21

$I_{x-x} = 1874$ $\qquad$ $I_{y-y} = 3980 + 168 = 4148$

Areas in the preceding table are computed as follows:

(1) and (4) - A = 2.27 from Assignment 8.
(2) - A = (5.5)(4.65)(.072) + (20)(.025) = 2.34 sq. in.
(3) - A = (5)(4.65)(.064) + (18)(.025) = 1.94
(5) and (8) - A = 2.12 from Assignment 8.
(6) and (7) - A = (4)(30)(.040)(.040) + (4)(.15) = .79

Comparison with the values for compression on the top surface from Assignment 8 shows the shift in the neutral axis and the change in the moment of inertia. The student should investigate the strength of the section shown for the maximum inverted bending moment at this station.

****

The Effect of Rotation of the Principal Axes.

In Assignment 7 we developed an equation to calculate the angle between the principal axes of a wing section and any other set of reference axes through the center of gravity of the section. So far this equation has been of little importance because we have disposed the material in the wing so that our reference axes (the chord line and the spars) are nearly parallel to the principal axes. To illustrate our point -

On page 7-4 we developed the following equation for the solution for θ, the angle between the principal axes and the reference axes.

$$\tan 2\theta = \frac{2\int xydA}{Iy - Ix}$$

Using the nomenclature of our tables

$$\tan 2\theta = \frac{2 \sum x_o y_o A}{I_{y-y} - I_{x-x}}$$

If we apply this equation to the sections calculated in Assignment 8 and this assignment we obtain the following results:

For station 145 with compression on the top surface

$$\tan 2\theta = \frac{(2)(+13)}{3604 - 1832} = \frac{26}{1772} = .0147$$

$$2\theta = 0.84 \text{ degrees}, \quad \theta = 0.42 \text{ degrees}$$

For station 115 with compression on the top surface,

$$\tan 2\theta = \frac{(2)(4)}{4184 - 2223} = \frac{8}{1961}, \quad \theta = \text{zero approx.}$$

For station 115 with compression on the bottom surface,

$$\tan 2\theta = \frac{(2)(21)}{4148 - 1874} = \frac{42}{2274} = .0184$$

$$2\theta = 1.06 \text{ degrees}, \quad \theta = 0.53 \text{ degrees}$$

All of the foregoing shifts of the principal axes can be neglected without an appreciable effect on the stress distribution. It will now be shown that for a different balance of areas the effect of the shift of the principal axes may become very large.

Assume that for station 115 with compression on the top surface we make the following changes in distribution of area:

Add 0.5 sq. in. to parts (1) and (5).
Deduct 0.5 sq. in. from parts (4) and (8).

With the above changes there are no appreciable changes in the basic values except $\sum Ax_o y_o$. That is $I_{x-x} = 2223$, $I_{y-y} = 4184$, etc.

The change in $\sum Ax_o y_o$ is

$$\begin{array}{lll}
\text{For (1)} & (.5)(-10.35)(20) & = -(.5)(207) \\
\text{For (5)} & (.5)(13.35)(-20) & =. -(.5)(267) \\
\text{For (4)} & (-.5)(-11.65)(-20) & = -(.5)(233) \\
\text{For (8)} & (-5)(12.45)(20) & = \underline{-(.5)(249)} \\
 & \qquad\qquad\qquad\sum & = -(.5)(956) = -478
\end{array}$$

New value of $\sum Ax_o y_o = +4 - 478 = -474$

$$\text{Tan } 2\theta = \frac{(2)(-474)}{4184 - 2223} = \frac{-948}{1961} = -.482$$

$2\theta = -25^\circ 42'$ $\qquad\theta = -12^\circ 51'$ $\qquad\cos 2\theta = .90$

Using the equations from pages 7-4 and 7-8 we may write

$$I_{x_1} = I_x \left(\tfrac{1}{2} + \frac{1}{2 \cos 2\theta}\right) + Iy \left(\tfrac{1}{2} - \frac{1}{2 \cos 2\theta}\right)$$

$$\text{Then } I_{x_1} = (2223)(\tfrac{1}{2} - \frac{1}{(2)(.90)} + 4184 \left(\tfrac{1}{2} - \frac{1}{(2)(.90)}\right)$$

$$= 2223\ (.5 + .555) + (4184)(\tfrac{1}{2} - .555)$$

$$= 2350 - 232 = 2118$$

Since $I_x + I_y = I_{x_1} + I_{y_1}$

$$Iy_1 = 2223 - 2118 + 4184 = 4289$$

Checking Iy_1 by the equation at the bottom of page 7-8,

$$I_{y_1} = Iy_o \cos^2\theta + 2 \sin\theta \cos\theta \sum Ax_o y_o + Ix_o \sin^2\theta$$

$$\sin\theta = -.222, \quad \cos\theta = .975$$

$$I_{y_1} = (4184)(.975)^2 + (2)(-.222)(.975)(-474) + (2223)(.222)^2$$

$$= 3978 + 204 + 109 = 4291 \text{ against } 4289.$$

****

If we consider the moments in the beam and chord directions as vectors we may resolve them into components parallel to the principal axes. See Fig. 1.

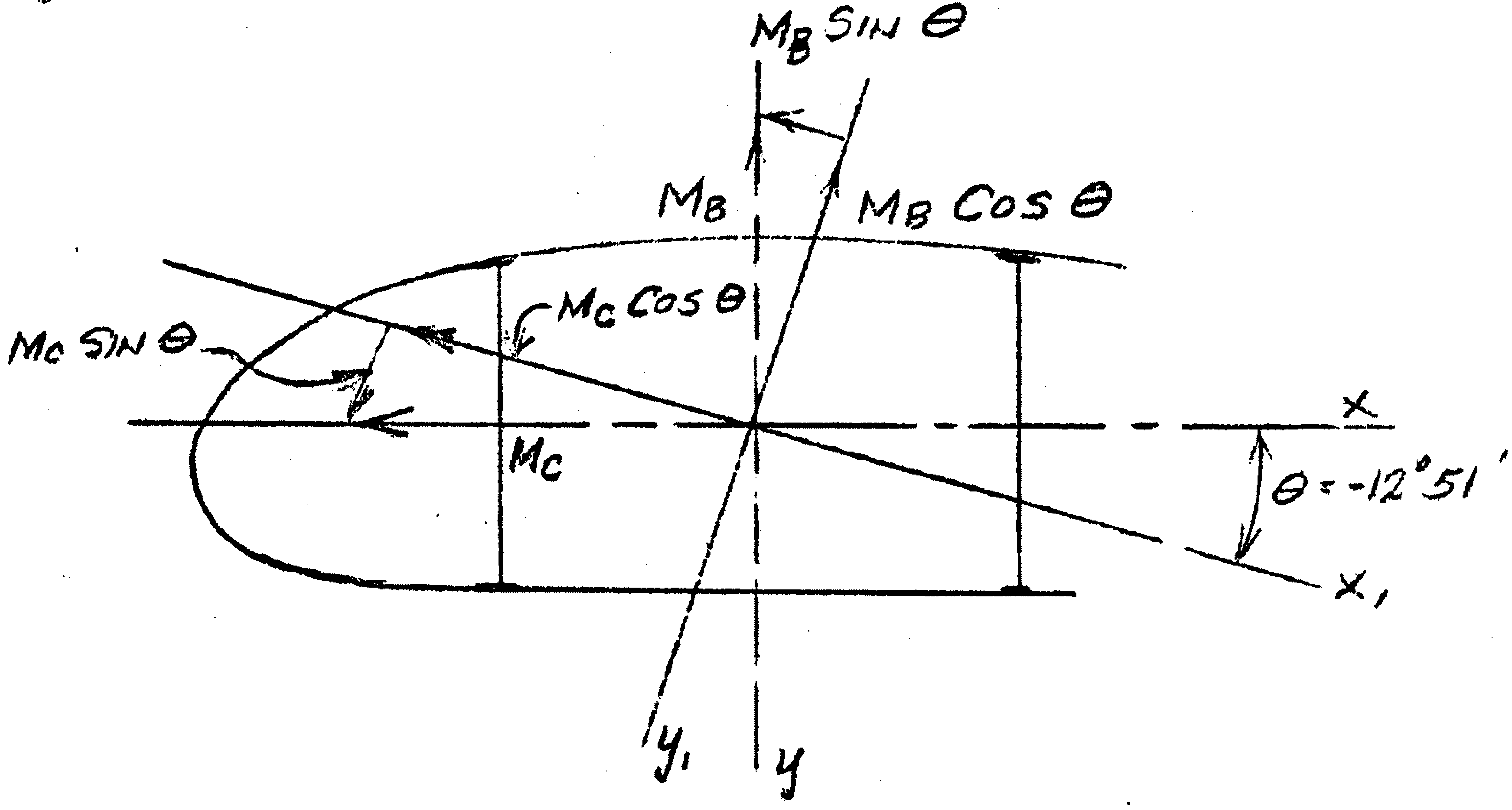


Fig. 1

Bending Components Along the Principal Axes
Condition I

For condition 1 in which the beam force is upward and the chord force is forward the moments about the principal axes are as follows:

$$M_{x_1-x_1} = M_B \cos \theta - M_c \sin \theta$$

$$M_{y_1-y_1} = -M_B \sin \theta - M_c \cos \theta$$

In the above equations the signs are already corrected to account for proper direction of the moment vectors. Then the absolute values of the sin θ, cos θ, M_B and M_c may be entered in the above equations without regard to their sign.

Where $\theta = -12°51'$, $\sin \theta = -.222$
$\cos \theta = .975$

From Table IV, page 8-8, for Condition 1

$M_B = 6,700,000$ inch lbs.
$M_c = -1,331,000$ inch lbs. (from chord load acting forward)

Then $M_{x_1-x_1} = (6,700,000)(.975) - (1,331,000)(.222)$

$= 6,530,000 - 296,000 = 6,234,000$ inch lbs.

$M_{y_1-y_1} = -(6,700,000)(.222) - (1,331,000)(.975)$

$= -1,490,000 - 1,300,000 = -2,790,000$ inch lbs.

****

We will now compare the unit fiber stresses obtained with and without correction for the principal axes. The fiber stresses on parts (1), (4), and (5) at station 115 will be investigated for Condition 1.

TABLE II

COMPARISON OF UNIT STRESSES

Calculated at Station 115 for Condition I

Part		Uncorrected		Corrected		Corrected for Lift Moment Only	
		About x-x	About y-y	About $x_1 - x_1$	About $y_1 - y_1$	About $x_1 - x_1$	About $y_1 - y_1$
	M	6,700,000	-1,331,000	6,234,000	-2,790,000	6,530,000	-1,490,000
	I	2223	4184	2118	4289	2118	4289
	M/I	2010	-318	2940	-650	3080	-347
	y	-11	+20	-6.2	+22	-6.2	+22
(1)	f_b	-33,100	-6,360	-18,200	-14,300	-19,200	-7,630
	f tot.	-39,460		-32,500		-26,830	
	y	-12.2	-20	-16.4	-17	-16.4	-17
(4)	f_b	-36,700	+6,360	-48,200	+11,100	-50,500	+5,900
	f tot.	-30,340		-37,100		-44,600	
	y	+13.6	-20	+9.0	-22.5	+9.0	-22.5
(5)	f_b	+41,000	+6,360	+26,500	+14,600	+27,700	+7,820
	f tot.	+47,360		+41,100		+35,520	

Shear Distribution.

Fig. 8 of Assignment 9, page 9-17, shows a convention by which confusion may be avoided in combining the effects of shear forces from bending and from torsion. The actual values of the shears at any section in the covering may be determined by applying the methods given in Basic Stress analysis and repeated in Assignment 9.

Shear Due to Bending Stresses.

The shear stress across any section due to bending loads is given by the formula

$$f_s = \frac{SQ}{bI}, \text{ where}$$

S = the shear on the section

Q = the statical moment (first moment) of the areas of the parts of the cross-section on the side away from the neutral axis

b = the thickness of the shear-carrying member

I = the moment of inertia of the entire cross-section.

Where the shear carrying members taper in area as in a tapered wing a satisfactory method of attack is to compute the loads in different members or parts of a cross-section at different stations along the span and by computing the difference in loads in the same members, calculate the force transmitted in a given length. The unit shear force, per inch of length, is then the total difference in load between a set of wing stations divided by the difference between the stations in inches. The unit shear stress in lbs. per sq. inch is then the stress per inch divided by the thickness of the shear-carrying member. The method will be illustrated by computing the running shear loads per inch due to bending loads, between stations 115 and 145. Using again the bending moments from Table IV, page 8-8, for Condition I at Station 115

M_B = 6,700,000 inch lbs.

M_C = -1,331,000 inch lbs.

Tabulating the areas and lever arms for the values of Q, etc.

The unit stress in any portion of a cross-section due to bending,

$$f_b = \frac{My}{I}.$$

The load in any portion of a section whose area is A is $(f_b)(A)$. or $\left(\frac{My}{I}\right)(A)$, but $(y)(A) = Q$. Therefore the load in any portion of the cross-section due to bending is $\frac{MQ}{I}$.

TABLE III

BENDING LOADS

Station 115 - Condition I

	About x-x Axis				About y-y Axis				Total
	M = 6,700,000 I = 2,223 M/I = 3,010				M = -1,331,000 I = 4.184 M/I = -318				
No.	A	y_o	$Q = Ay_o$	$\frac{MQ}{I}$	A	x_o	$Q = Ax_o$	$\frac{MQ}{I}$	$\frac{MQ}{I}$
1	2.27	-10.35	-23.5	-70,800	2.27	20.0	45.4	-14,500	-85,300
2	1.90	-11.75	-22.3	-67,200	1.90	8.0	15.4	-4,900	-72,100
3	1.55	-11.85	-18.4	-55,200	1.55	-9.8	-15.1	4,800	-50,400
4	2.27	-11.65	-26.5	-79,800	2.27	-20.0	-45.4	+14,500	-65,300
5	2.12	13.35	28.3	85,200	2.12	-20.0	-42.4	+13,500	+98,700
6	1.34	13.55	18.1	54,400	1.34	-9.3	-12.4	+3,900	+58,300
7	1.34	13.35	17.9	53,900	1.34	9.0	12.1	-3,800	+50,100
8	2.12	12.45	26.4	79,500	2.12	20.0	42.4	-13,500	+66,000
Σ			-90.7	-273,000			-115.3	-36,700	+273,100
			+90.7	+273,000			+115.3	+36,700	-273,100

TABLE IV

BENDING LOADS

Station 145 - Condition I

	About x-x Axis				About y-y Axis				Total
	M = 5,877,000 I = 1,832 M/I = 3,200				M = -1,146,000 I = 3,604 M/I = -318				
No.	A	y_o	Q	$\frac{MQ}{I}$	A	x_o	Q	$\frac{MQ}{I}$	$\frac{MQ}{I}$
1	2.07	-10.16	-21.0	-67,300	2.07	19.2	39.8	-12,700	-80,000
2	1.54	-11.16	-17.2	-55,100	1.54	9.2	14.2	-4,500	-59,600
3	1.54	-11.36	-17.5	-56,100	1.54	-9.0	-13.9	4,400	-51,700
4	2.07	-11.26	-23.4	-75,000	2.07	-19.4	-40.2	12,800	-62,200
5	1.92	12.34	23.7	+76,000	1.92	-19.4	-37.3	11,800	87,800
6	1.32	12.64	16.7	+53,500	1.32	-9.0	-11.9	3,800	57,300
7	1.32	12.34	16.3	+52,200	1.32	9.2	12.1	-3,900	48,300
8	1.92	11.64	22.4	+71,800	1.92	19.2	36.9	-11,700	60,100
Σ			79.1	-253,500			+103.0	+32,800	+253,500
			-79.1	+253,500			-103.3	-32,800	-253,500

The summation of $\frac{MQ}{I}$ at any cross-section represents the value of the tension-compression couple resisting bending at that section. The distribution of the load to elements of the cross-section is taken from the value $\frac{MQ}{I}$ for that section in which M is the external moment on the total section, I is the moment of inertia of the total section and Q is the static moment of the part under consideration about the neutral axis. Plotting the loads due to bending for the various elements of the cross-sections at stations 145 and 115 we obtain the values shown on Fig. 2. Fig. 2 (a) shows the loads due to bending on station 145. Fig. 2 (b) shows the same values for station 115. Fig. 3 (b) shows the differences in loads in the same members at the two stations. $\Delta \frac{MQ}{I}$ is therefore the load for a given member at station 115 less the load for the same member at station 145. Since the distance between the two stations is 30 inches the shear load per inch between the two stations for a given element may be obtained by dividing the differences in loads by 30. This assumes an equal distribution of shear per inch between the two stations which is practically correct if the stations are taken close together. Refer to Figs. 7 and 8 of Assignment 8 for the cross-sections at stations 115 and 145.

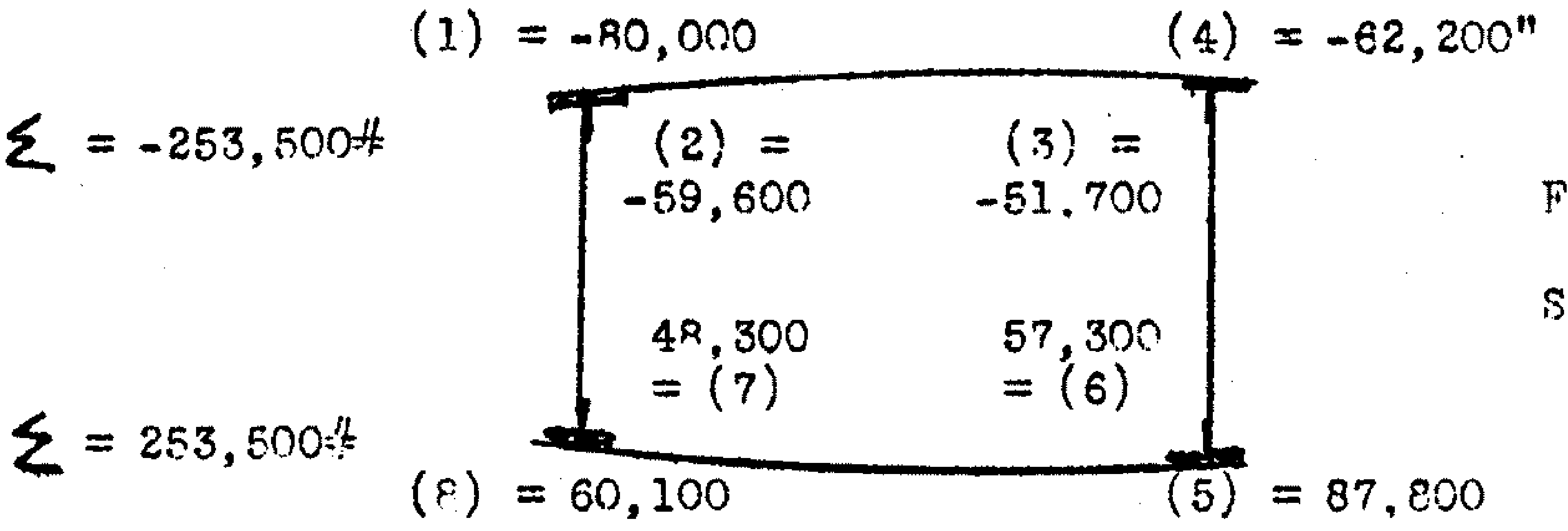


Fig. 2 (a)

Station 145

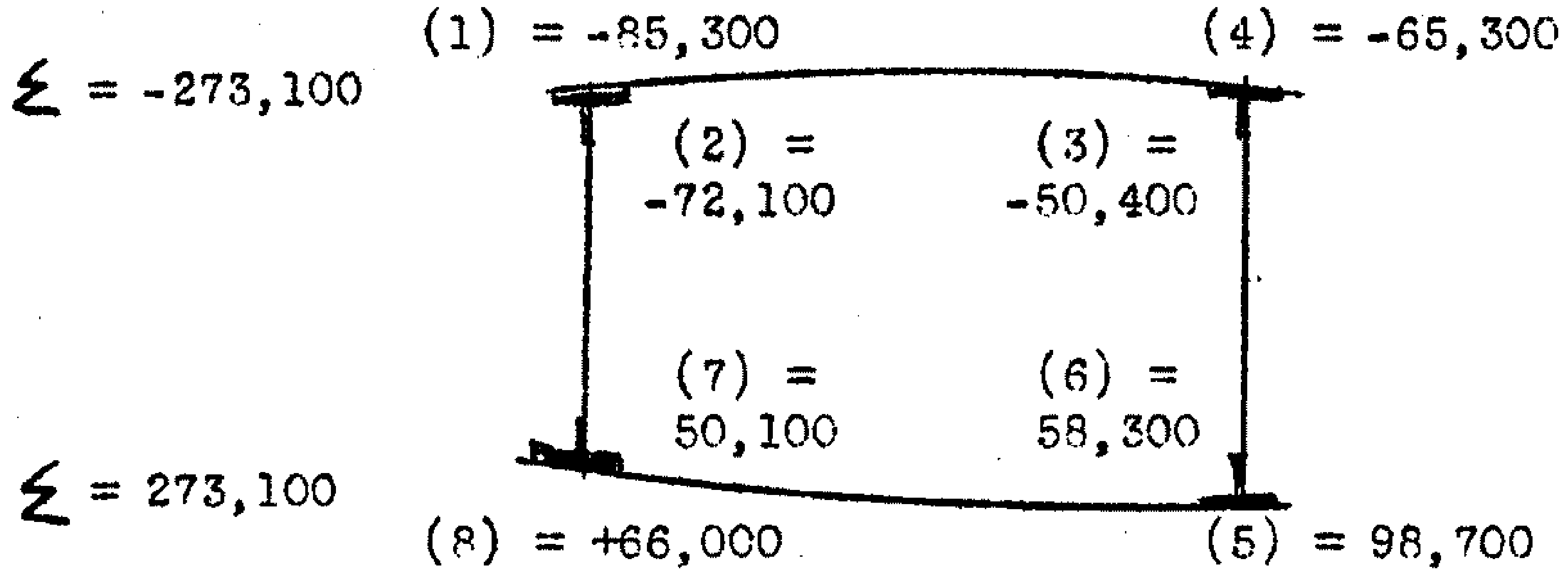


Fig. 2 (b)

Station 115

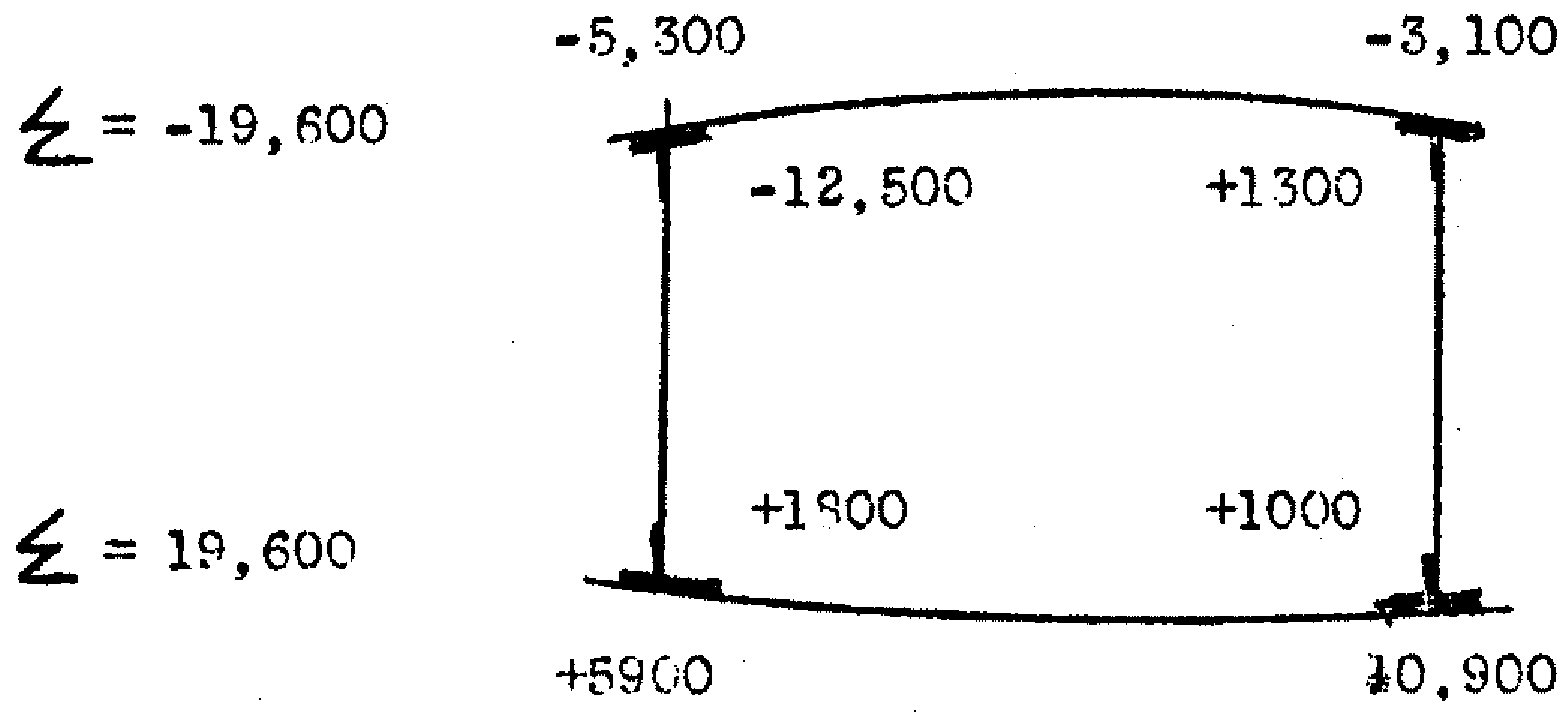


Fig. 2 (c)

$\Delta \frac{MQ}{I}$

Differences between stations 115 and 145

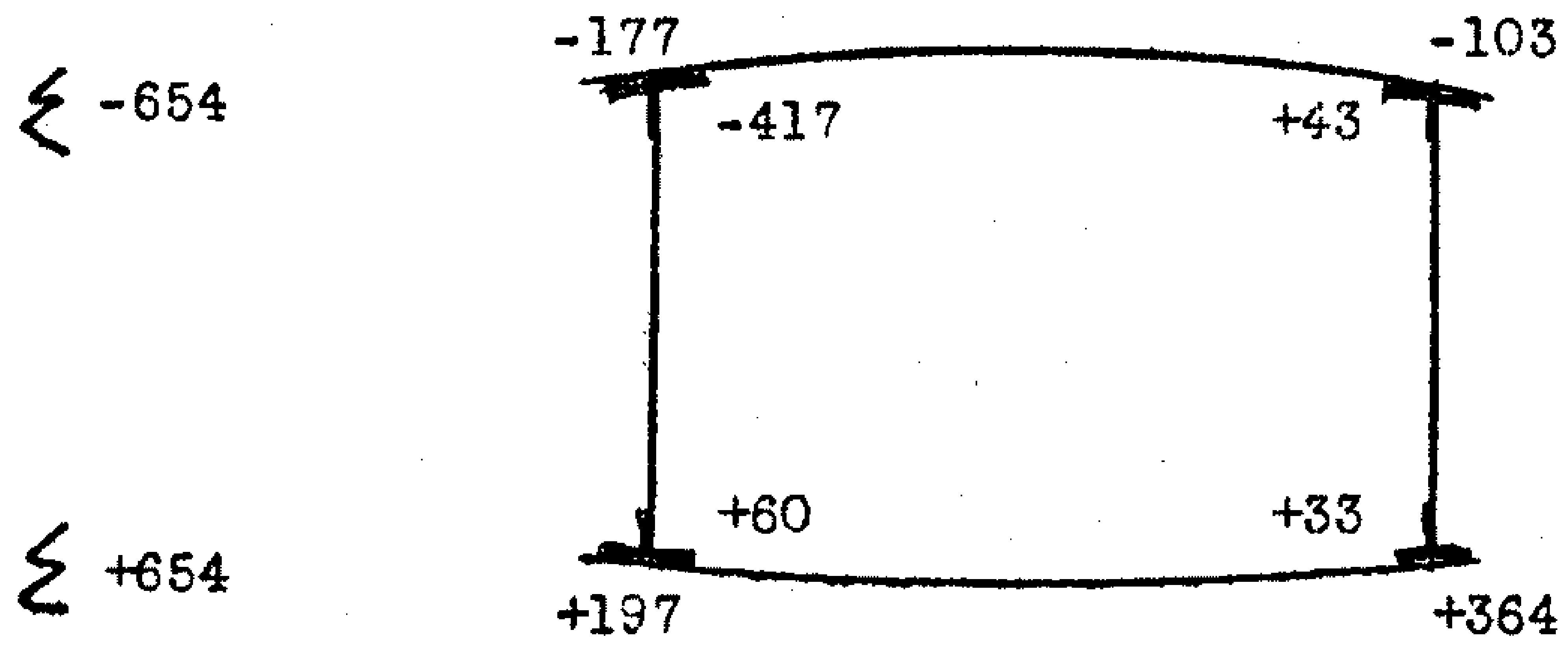


Fig. 2 (d)

Shear load per inch between stations 145 and 115 from main components of section.

Fig. 2

Shear Loads Due to Bending between Stations 145 and 115

Assuming that we are using .036 guage webs at the front and rear spars and that the vertical shears are divided equally between them:

The shear increment per inch on each spar is 654/2 or 327#. The picture of the distribution of shear due to bending would then be as shown on Fig. 3.

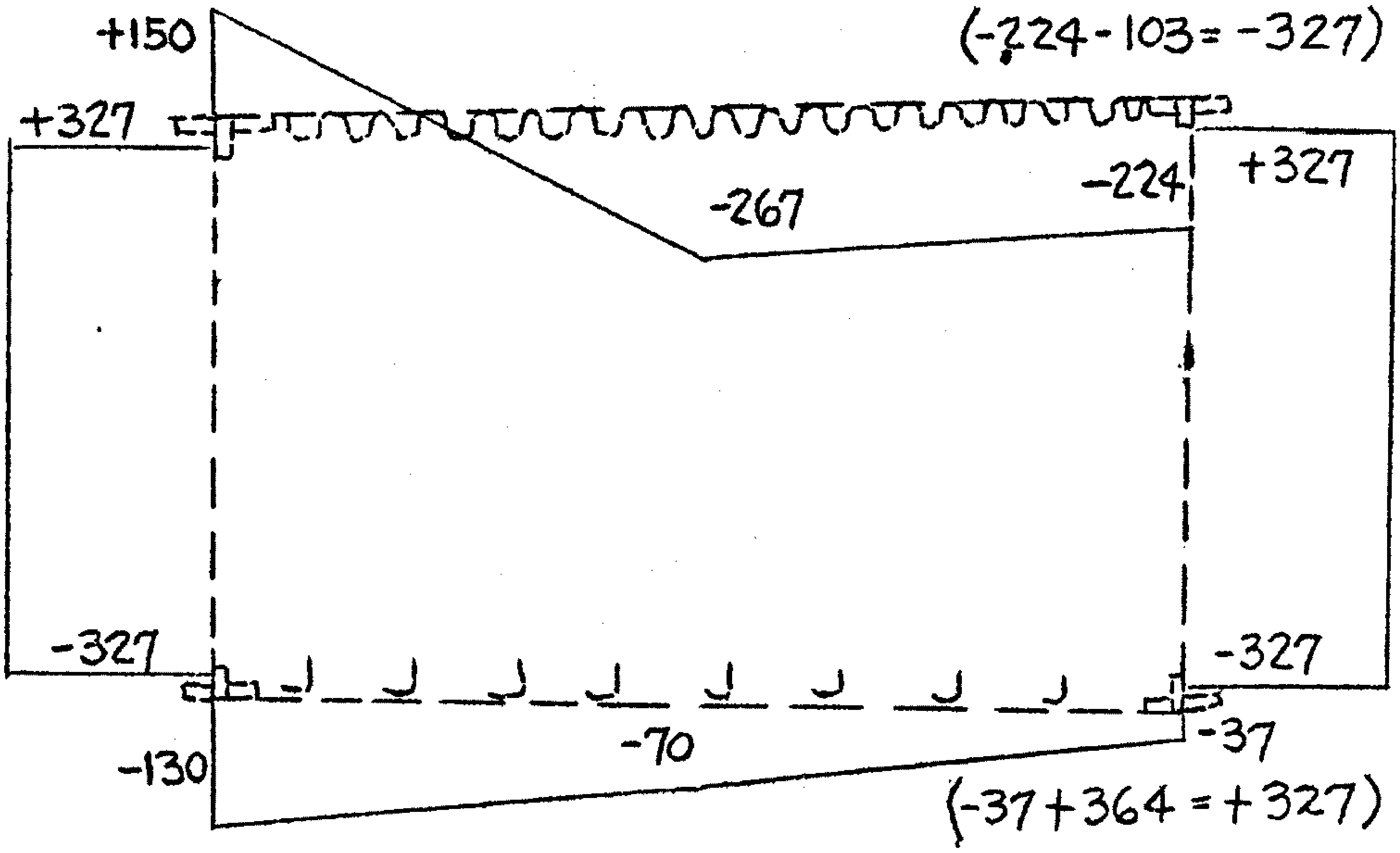


Fig. 3. SHEAR LOADS FROM BENDING

With regard to Fig. 3 the following points should be noted:

(1) The shears in the spars may be considered as reactions for the shears in the covering and the caps. Thus at the front upper cap the spar load of 327 lbs. per inch run is balanced by 177 lbs. per inch increment from the spar flange, part 1 and 150 lbs. per inch increment from the covering part 2.

(2) The 417 lbs. per inch increment in the upper corrugations, part 2 goes 150 lbs. to the front web and 267 lbs. aft of which 43 lbs. per inch is balanced in the rear covering, part 3, and 224 lbs. per inch is balanced by the shear in the rear spar. The shear in the rear spar, 327 lbs. per inch, balances 224 lbs. per inch from the covering plus 103 lbs. per inch from the rear spar upper cap.

(3) In the actual design of the front spar it will be found necessary to add a doubler between stations 115 and 145 to carry loads from the landing gear. The effect on the transfer of loads due to bending will be considered negligible.

From our membrane theory of transfer of torsional shear, the torque shear per inch of periphery is $M/2A$, where A is the enclosed area of the torque box, and M is the torque moment.

$f_s = \frac{M}{2At}$ and load per inch

equals $(f_s)(t)$ Therefore load per inch equals $\frac{M}{2A}$.

The torsional moment is -648,000 inch lbs. on the inboard side of station 145. Including the area forward of the front spar in the torque box the Area is 1353 sq. inches.

The torque shear per inch is then $\frac{-648{,}000}{(2)(1353)} = -239$ lbs. per running inch.

The negative sign indicates a twisting moment tending to depress the leading edge. This indicates that the wrinkles will run opposite to the direction shown on Fig. 4 of Assignment 9, see page 9-11.

Combining the effects according to the sign convention shown on Fig. 8, page 9-17, we obtain the combined shear values shown on Fig. 4.

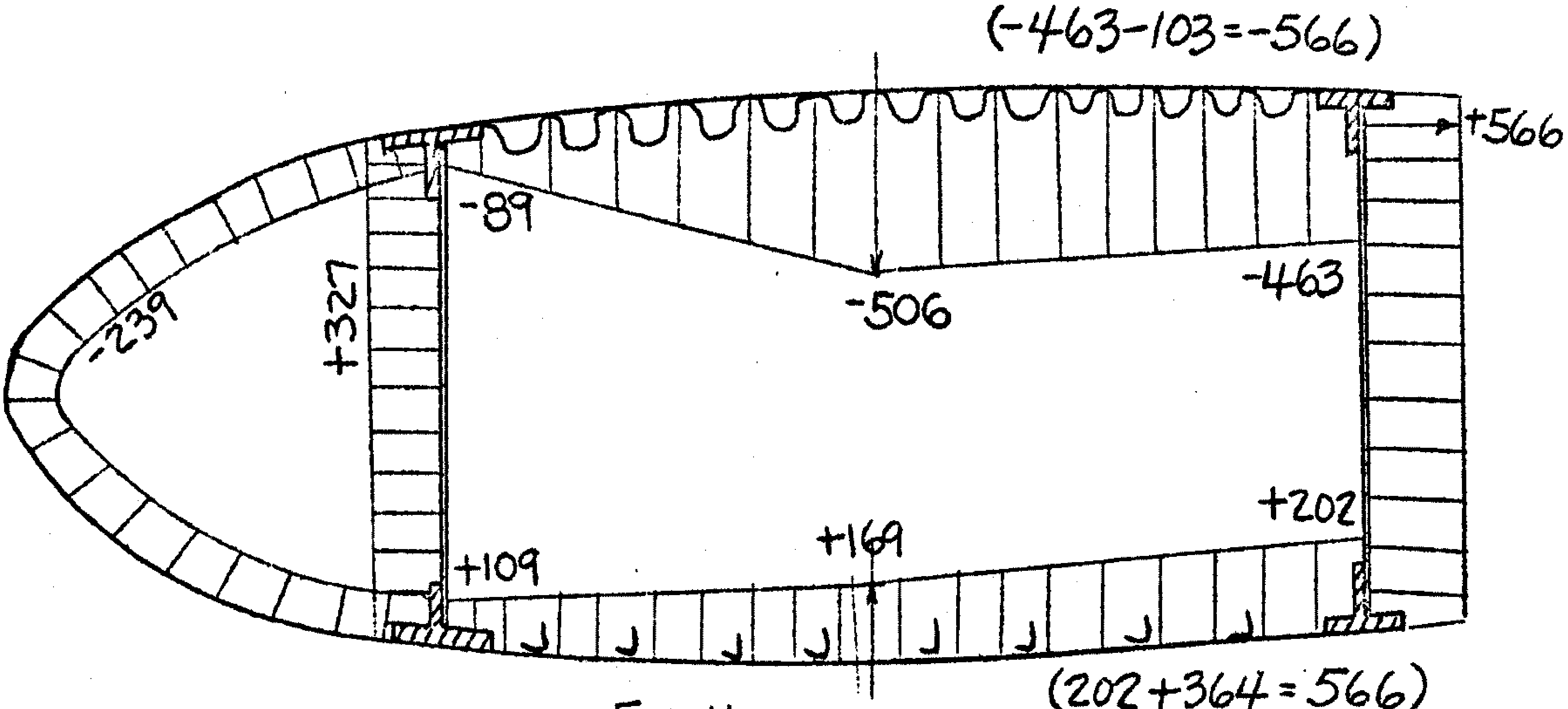


FIG. 4

ADVANCED STRESS ANALYSIS

ASSIGNMENT 10

Assignment 10

DESIGN OF TENSION FIELD OR WAGNER BEAM WEBS

There are two possible ways in which shear may be transmitted by a beam web. First, it may be carried by the web acting in pure shear, that is without the formation of buckles in the web. Second, it may be transmitted by the web after buckling has occurred, in which case it is no longer pure shear but is more accurately described as diagonal tension. In other words, of the two principal stresses induced in the web by shear, the tension component has become far more important. In the majority of cases in aircraft design it is inefficient from the standpoint of weight to use a web thick enough and stiffeners spaced closely enough together to prevent buckling at all shear loads up to the design value. The design of webs and stiffeners for the loads existing after the buckling strength of the sheet has been exceeded is therefore of great importance.

Wagner Beam Theory

The theory of the design of webs which have exceeded their buckling strength is based on two major assumptions. The first and most important is that the buckling strength is negligible, and the second is that the web makes no contribution to the bending strength of the beam. Actually neither of these is strictly true, even in extreme designs, and in all practical cases allowance should be made for this fact. Although, as will be shown later, the error introduced by the first assumption is large and in some respects non-conservative, that caused by the second assumption is small and conservative and hence this assumption will be carried throughout. For study of the effects of exceeding the buckling strength of the sheet it is most convenient to make both assumptions and this will be done in the first part of this discussion.

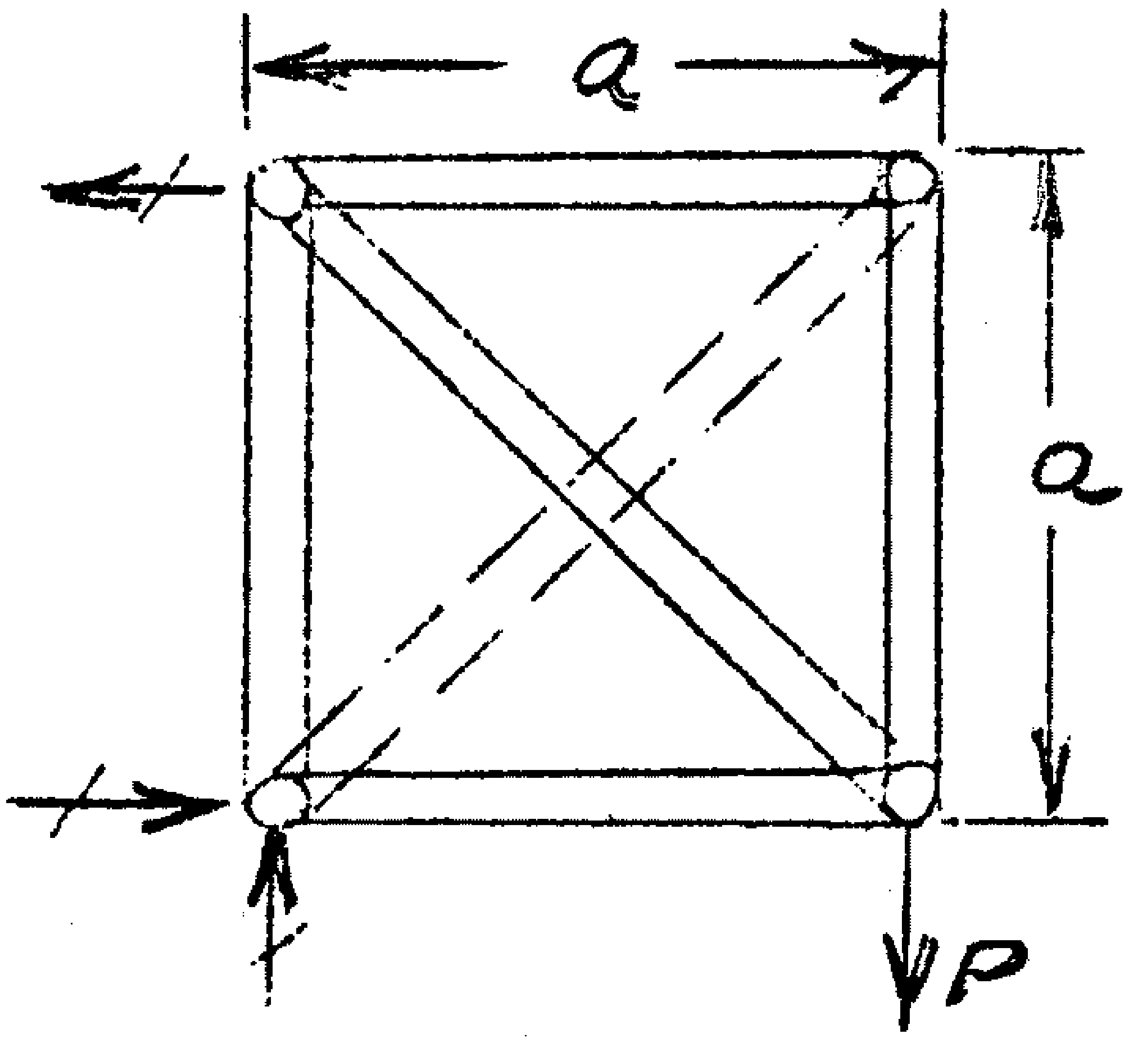


Fig. 1

Consider first a truss of the type shown in Fig. 1. Both diagonal members are thin straps of metal. As load P is applied both members will carry equal portions of the load (assuming the members to be of the same size) up to that point at which the one shown with dashed lines fails in compression as a long column. After this occurs the tension member must resist all increases in load. If the compressive strength of the dashed strap is assumed to be zero it may be neglected altogether.

In Fig. 2 we have a thin webbed beam. The web of this beam may be thought of as a number of diagonal straps, side by side, all at 45° to the chord members. (Diagonal tension wrinkles usually form at this angle or are very close to it.) Typical straps A and B will be examined separately to determine the stress conditions.

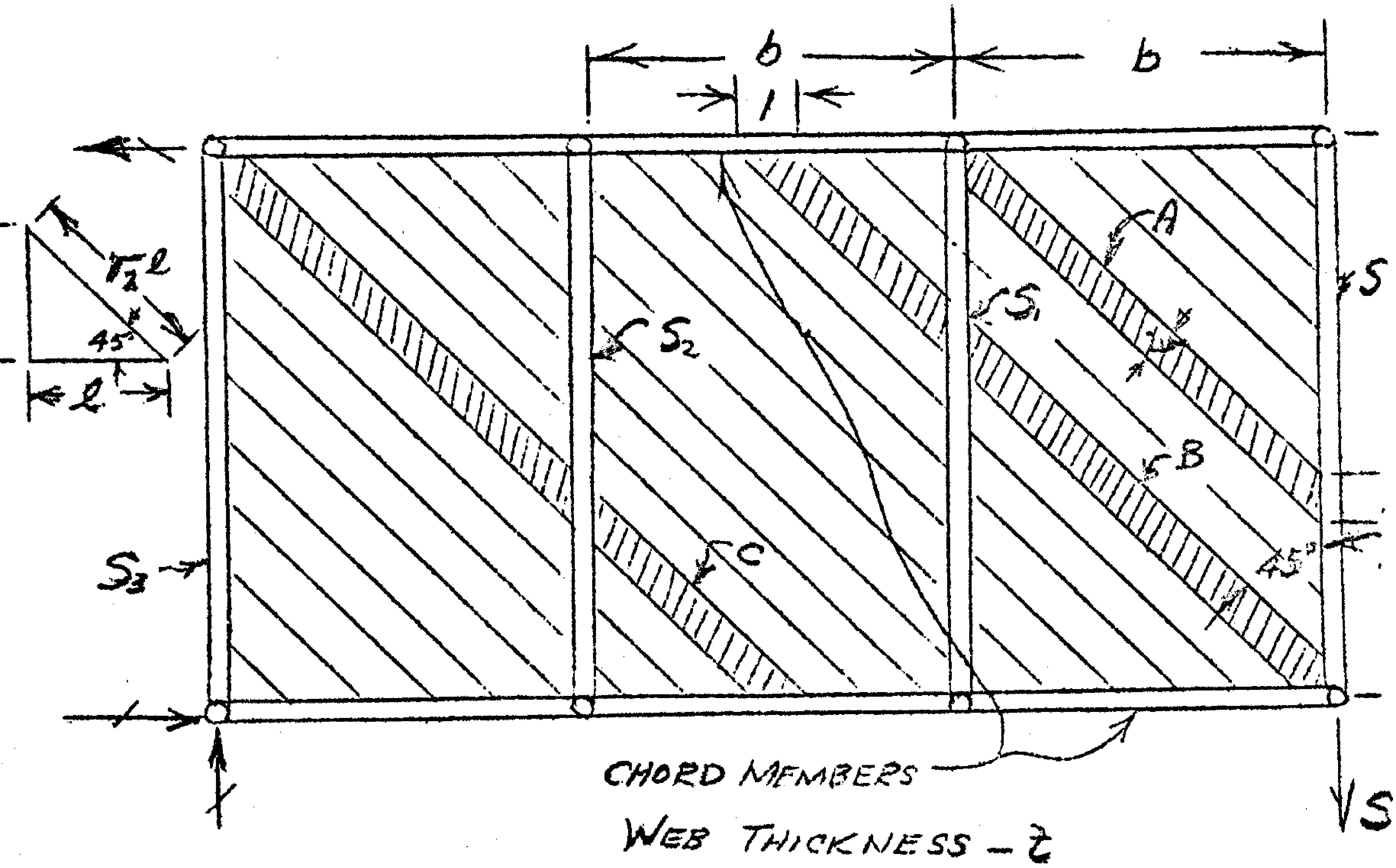


Fig. 2

First determine the tension in Strap "A". Assuming all straps of uniform width each will take an equal amount of load, or the shear, "S", will be distributed uniformly over the height "h". If the width of the strap "w" is $\frac{1 \text{ inch}}{\sqrt{2}}$ then its intersection with end stiffener S_0 will be 1 inch long. The vertical component of the tension load in that strap must be equal to the shear load per inch or $\frac{S}{h}$. The total tension in the strap is then $\frac{S}{h}\sqrt{2}$. Since the area of the strap is tw or $\frac{t}{\sqrt{2}}$, the tensile stress, $f_t = \frac{S/h\,\sqrt{2}}{t/\sqrt{2}} = \frac{2S}{th}$. But the nominal shear stress $f_s = \frac{S}{th}$. Therefore $f_t = 2f_s$, or the tensile stress in the web is twice the nominal shear stress.

The horizontal component of the load in strap "A" is resisted by the end stiffener S_0 and therefore this member is subjected to shear and bending loads. The load applied to this stiffener is uniform and is equal to the horizontal components of the tension loads in the straps, which in turn are equal to the vertical components or S/h pounds per inch. The total load is carried by stiffener S_0 to the top and bottom chord members, which carry it in compression to the inner end of the beam where an equal and opposite load exists in S_3. The total load is, of course, S/h times h or S, whence the load in each of the two chord members is $-\frac{S}{2}$. This adds to the compression load in the bottom member due to bending and subtracts from the tension in the top member.

Considering now strap B we find that at its upper end the tension load has two components, vertical and horizontal. Assuming the strap to be the same width as strap A each of these components is equal to S/h. The horizontal component increases the tension load in the upper chord member in the same manner as the shear web in any beam would do. The vertical component, however, induces bending in the upper member and compression loads in stiffeners S_1 and S_2. The compression loads in the stiffeners are resisted by equal and opposite loads in the bottom chord member resulting from the tension in strap C. Expressed quantitatively, the load on the chord members is S/h lbs. per inch run, and the load per stiffener is approximately $S(\frac{b}{h})$, assuming all stiffeners except the two end members to be loaded evenly. The maximum bending in the chord members, assuming them to be continuous beams over a number of supports, is $\frac{S}{h}\frac{b^2}{12}$ at the stiffeners and $\frac{S}{h}\frac{b^2}{24}$ midway between the stiffeners.

A final theoretical consideration is the deflection of the beam due to the diagonal tension. Considering Strap A we note that its total length is $b\sqrt{2}$. Since its tensile stress, f_t, is equal to $2f_s$, the total elongation of strap, ΔL, is equal to

$$\frac{f_t L}{E} \quad \text{or} \quad \frac{2f_s b\sqrt{2}}{E}.$$

As shown in Fig. 3 (much exaggerated) the vertical deflection due to the elongation ΔL will be greater than ΔL by the ratio of length of the hypotenuse of the small triangle abc to the side ab or ΔL. Therefore the shear deflection, d, due to the elongation of Strap A is equal to

$\Delta L\sqrt{2} = \left(\frac{2f_s b\sqrt{2}}{E}\right)\sqrt{2} = \frac{4f_s b}{E}$. Over the entire length of the beam the shear deflection would be proportionally greater and hence the formula may be written

$$\text{Shear Deflection} = D_s = \frac{4f_s}{E} = \frac{f_s}{E/4}$$

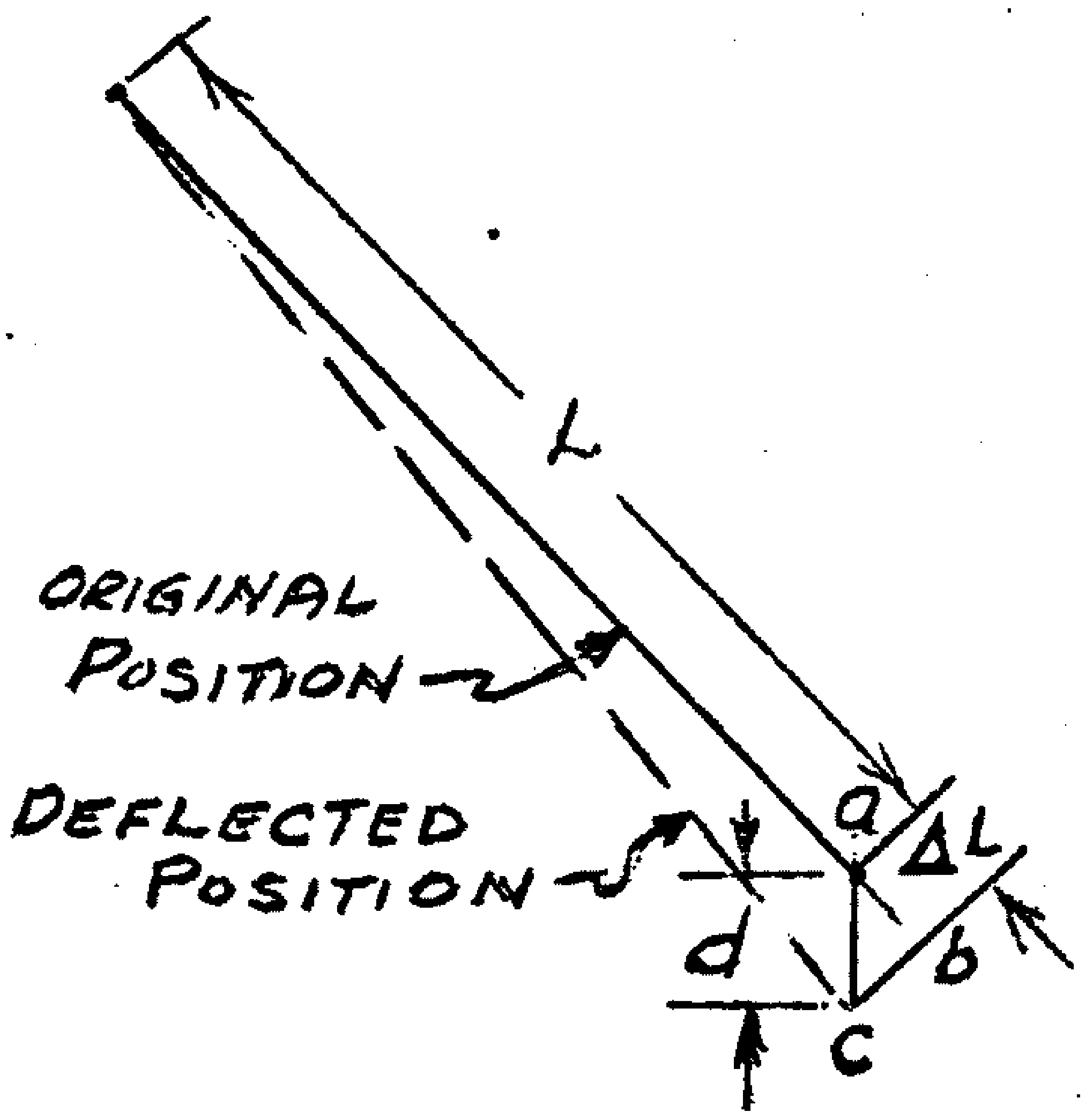


Fig. 3

where f_s = nominal shear stress = $\frac{S}{h}$

ℓ = length of the beam

and E = Modulus of Elasticity

It will be noted that this is similar to the ordinary equation for shear deflection,

$$D_s = \frac{f_s \ell}{G}$$

where G = Modulus of Rigidity.

It appears therefore that the effective Modulus of Rigidity, G, of a diagonal tension field beam is theoretically equal to .25E, instead of .38E as is the case for pure shear.

Any shortening of the vertical stiffeners S_1 and S_2 would increase the deflection computed above and hence would reduce the Modulus of Rigidity even further.

The student who wishes to go into the theory more thoroughly is referred to NACA Technical Note No. 469.

Practical Design of Tension Field Beams

The theory outlined in the preceding paragraphs is very useful in studying the effects of exceeding the buckling strength of a shear web. It is the measuring stick to which most experimental work on the subject is compared. The theory, however, cannot be used in actual design without first modifying it to conform more nearly to true conditions as indicated by test results. The most complete published data on actual test results with tension field webs is given in NACA Technical Memo. No. 809, "Tests for the Determination of the Stress Condition in Tension Fields" by R. Lahde and H. Wagner. The figures and conclusions which will be found in the following pages were taken principally from this source. The figures are not intended to be exact since the tests themselves were made using spring-hard brass sheet instead of aluminum alloy. They do, however, give a basis on which a design may be made, the actual strength to be proven later by tests on a section of the actual beam.

Web Stresses

As previously emphasized, the theory is based on the assumption that the buckling strength of the web is negligible. This is by no means true in most cases. The load at which the sheet buckles may be exceeded by a ratio of only two or three to one. Extreme cases will seldom exceed a ratio of fifty to one, whereas the theory assumes an infinite ratio. Since the web continues to carry the diagonal compression stress at which buckling occurred long after the diagonal tension wrinkles have formed, the tension stresses should be corrected to allow for this effect in all cases where the buckling load is exceeded by a ratio of less than, say 10 to 1.

At the buckling stress, $f_t = f_c = f_{s_{crit.}}$

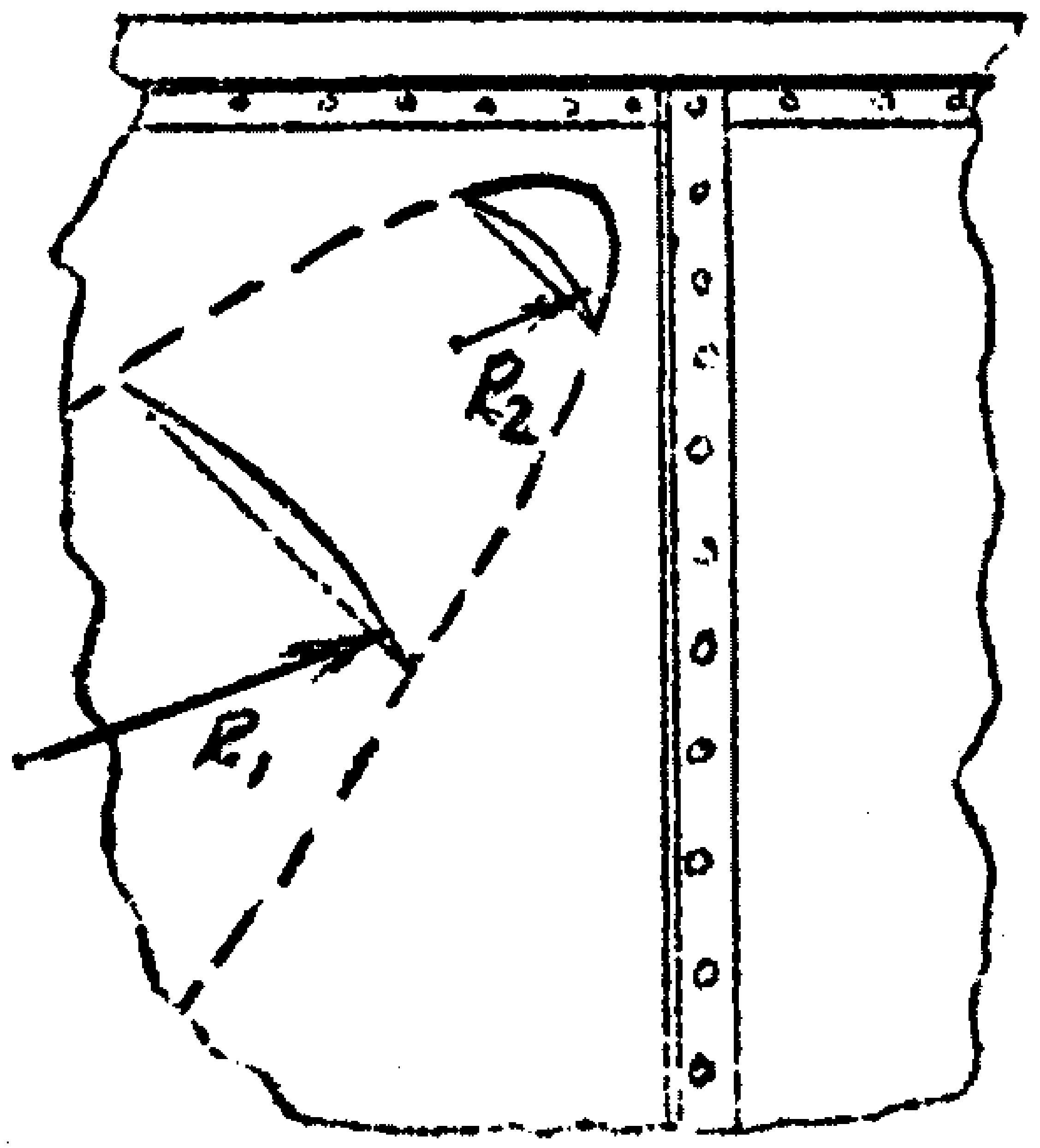
As the load is exceeded, $\Delta f_c = 0 \qquad \Delta f_t = 2\,\Delta f_s$

Therefore at maximum load $f_t = f_{s_{crit.}} + 2(f_s - f_{s_{crit.}})$

$$= 2f_s - f_{s_{crit.}}$$

The buckling stress, $f_{s_{crit}}$, may be predicted according to the methods given in ANC-5.

Thus far the implication has been that the only limitation on the ultimate shear strength of a given web is the ultimate tensile strength of the sheet (probably computed across the net section where it is riveted to the beam flange). This would be true if the bending stresses in the web introduced by wrinkling could be neglected (and provided that the web stiffeners had ample strength). These bending stresses are, however, of considerable importance since they tend to cause regions of relatively high stress where permanent set may occur at a shear stress somewhat below the yield point of the web due to the diagonal tension stresses. The tests reported in NACA Technical Memorandum No. 809 showed that although the tension stress at the neutral axis of the sheet compared rather closely with theoretical values, the stress at the outside fiber was from 50% to 80% higher than the theoretical stress. This was most pronounced either midway between two stiffeners or near the edges of the sheet where the effect of the edge restraint was to introduce higher bending stresses in local regions. Although the data given in the referenced Memo. is concerned principally with tension stresses the same effect is found in the case of compression stresses. At the intersection of the stiffener with the beam flange there is a region where the web is very well supported so that it tends to carry the loads in pure shear. The wrinkle which has formed in the center of the panel, however, comes closer and closer into the corner as the stress increases. The farther into the corner it extends, the shorter the wave length becomes, and so the smaller the radius of curvature and the higher the bending stresses.



(Note: Change in inclination of a beam $\Delta\theta = \frac{\Delta L}{R}$, where R = Radius of curvature.

But $\Delta\theta = \frac{M\,\Delta L}{EI} = \frac{f_b\,\Delta L}{Ey}\left(\frac{f_b}{y} = \frac{M}{I}\right)$

$$\frac{\Delta L}{R} = \frac{f_b\,\Delta L}{Ey} \quad \text{or} \quad f_b = \frac{Ey}{R}$$

Therefore as R decreases, f_b increases.)

Fig. 4

As the wrinkles become sharper and sharper, the stresses become higher until the yield point of the metal is passed. The result is permanent set in the web evidenced by wrinkles visible in only the corners after the removal of the load. The stress at which permanent set will occur will vary somewhat with web gauge and stiffener spacing, but will in general lie in the region of 10,000 to 15,000 psi. for 24ST material in gauges between .020 and .091, those most commonly used in aircraft beam webs. Since the design load may not exceed 1.5 times the yield point of the material, the design stresses of tension field webs will lie in the region of 15,000 to 22,500 psi, provided the stiffeners do not fail before this stress is reached. Since 22,500 psi is well below half the ultimate tensile strength of the material there is little chance of a tension failure in the web.

Referring once more to Fig. 2 it will be remembered that the tension in each strap was $\frac{S}{h}\sqrt{2}$. The rivets attaching each strap must be capable of transmitting this load to the spar cap, if theoretical conditions are assumed. This would mean increasing the strength of the rivets to 1.414 times their required strength for transmitting shear only. It has been found in actual practice that this value may be considerably reduced, due to the restraint furnished the web by the beam flange. A value such as 1.2 may be assumed for design purposes, the actual strength of the attachment to be proven in the test of the beam.

Stiffeners

The compression stresses in the stiffeners of a tension field beam are much lower than the theory would indicate. This is best illustrated by a diagram (Fig. 5) showing the vertical component of the stress in a plate web after buckling. The web adjacent to the stiffener is riveted to that stiffener and hence must be subjected to the same compression stress. That midway between stiffeners still has a vertical tension component. The load which the stiffener is required to resist is equal to the difference in areas, that is, T-C. This is obviously much smaller than the load indicated by the tension field theory, shown in Fig. 5 as the area under the dashed line. And it may also be seen that as the size of the stiffener is reduced the compression stress will tend to increase in the web as well as the stiffener which results in a further reduction of the load which the stiffener will be required to carry.

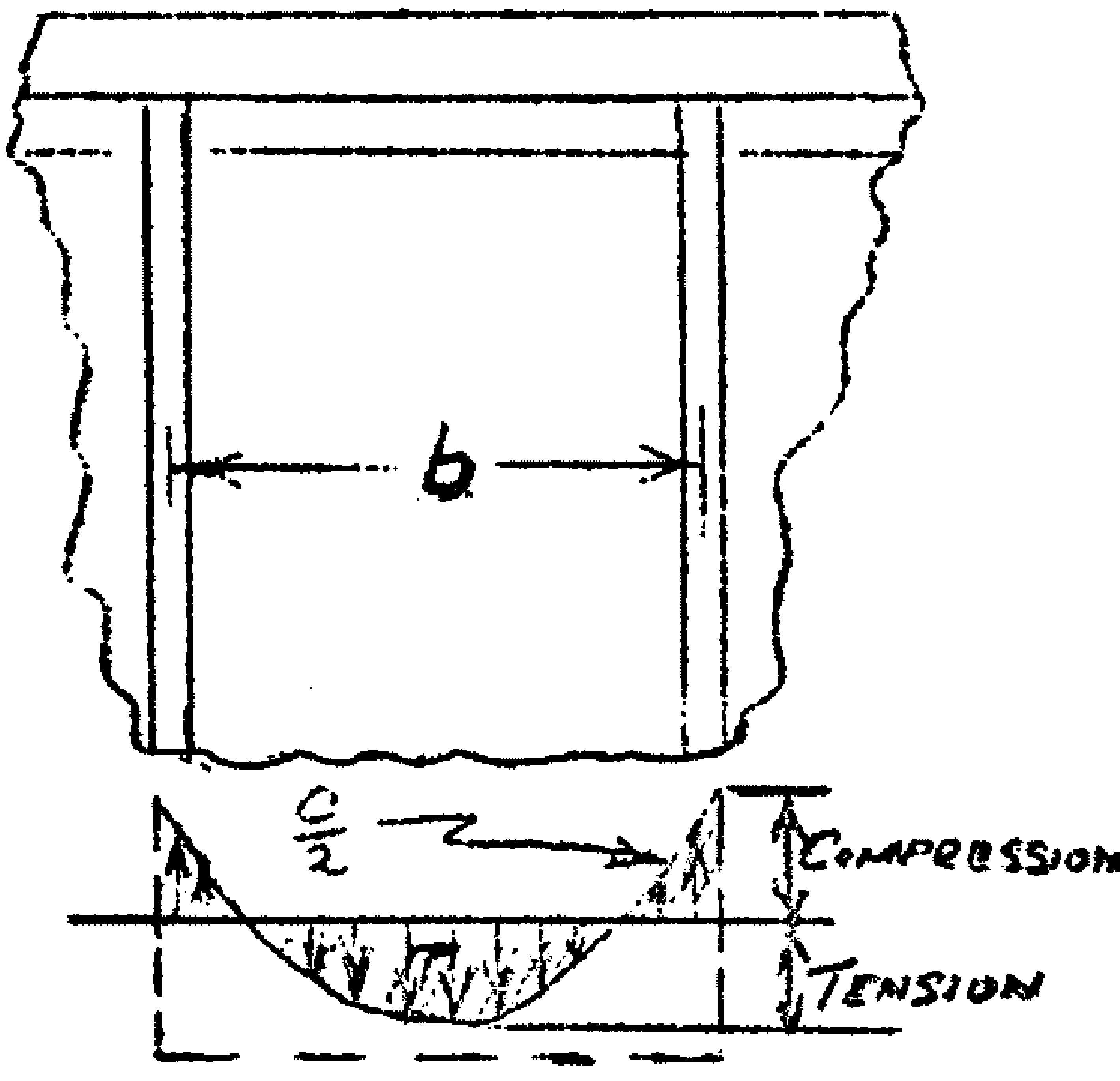


Fig. 5

Although no definite method of computing required stiffener areas can be referred to certain general rules of design may be listed:

1. The stiffener area should be proportional to the web area. The web is the primary load carrying member. The stiffener is a reinforcement. The ratio of stiffener area to web area (A_s/bt, where A = area of each stiffener or pair of stiffeners if back to back; b = stiffener spacing; and t = web thickness) should in all cases be less than 1.0 and may in most cases be less than 0.5.

2. Stiffeners should if possible be spaced in such a manner as to avoid square panels since shear wrinkles are apt to be more pronounced in panels.

3. The stiffener spacing should be the minimum consistent with economical design and with a stiffener size great enough to prevent buckling of the entire web-stiffener combination. A method of computing the buckling strength of a web stiffener

combination is given in NACA Technical Memo. No. 606. The stiffener must also be of sufficient size to prevent local buckling under the bending loads induced by the web. This consideration will usually require a stiffener slightly thicker than the beam web and stiffeners on both sides of the web for gauges greater than .040.

It will be noted that in the above no mention has been made of the effect of spar height, that is, column length, on stiffener size. This is due to the fact that the diagonal tension wrinkles in the web stabilize the stiffener against lateral failure unless the entire web stiffener combination fails as a unit before buckling of the web only takes place.

Bending and End Loads in Beam Flanges

The effect of the diagonal tension in the web on the stresses in the beam flanges approaches the theoretical value as the degree by which the buckling stress $f_{s_{crit}}$ is exceeded becomes larger. These effects may therefore be plotted against $\frac{f_s}{E}\left(\frac{b}{t}\right)^2$. Note: $f_{s_{crit.}} = KE\left(\frac{t}{b}\right)^2$.

$$\frac{f_s}{f_{s_{crit.}}} = \frac{f_s}{KE\left(\frac{t}{b}\right)^2} = \frac{1}{K}\,\frac{f_s}{E}\left(\frac{b}{t}\right)^2$$

The following figures are plotted from Technical Memo. No. 809, Figs. 7 and 8.

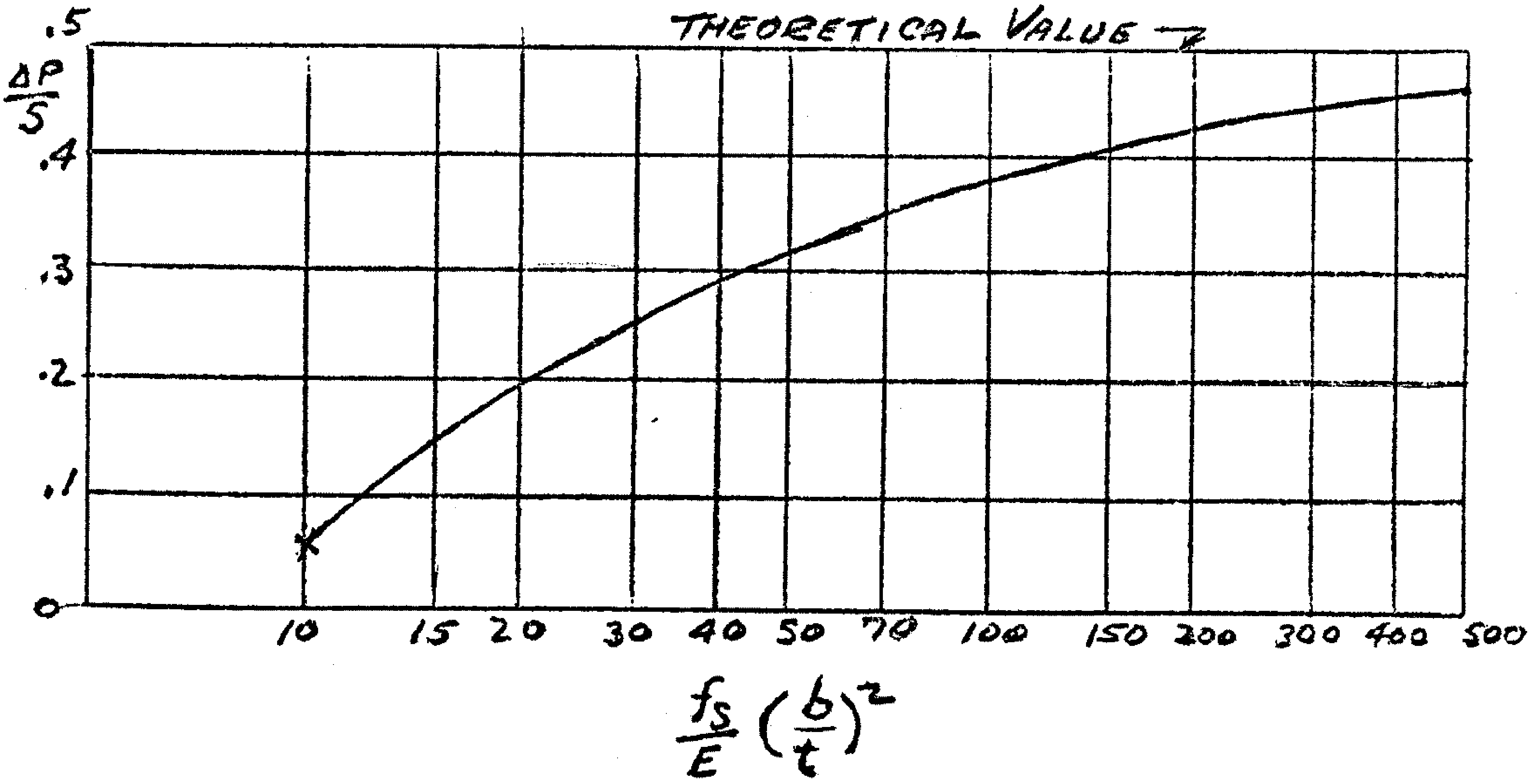


FIG. 6

ΔP = INCREMENT OF LOAD IN BEAM FLANGE DUE TO DIAGONAL TENSION IN WEB.

S = TOTAL SHEAR IN WEB.

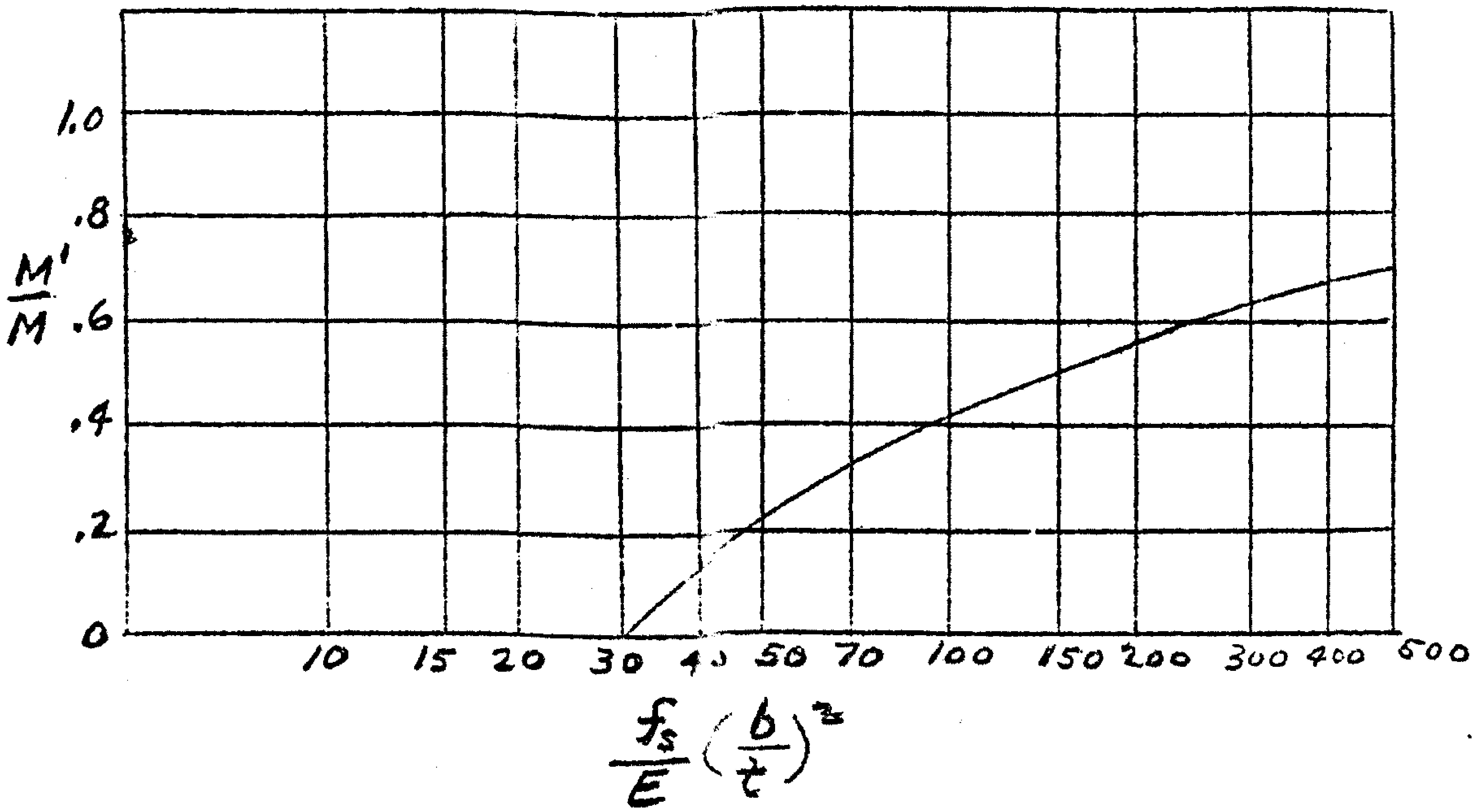


FIG 7

M' = Actual bending moment in flange

M = Theoretical bending moment in flange

$= \frac{S}{h}\frac{b^2}{12}$ over the stiffeners

$= -\frac{S}{h}\frac{b^2}{24}$ midway between stiffeners

The above values are approximate only since there is some variation with stiffener stress but they will serve as a guide for design purposes. In many cases the effect of the web tension on the stresses in the beam flanges may be entirely neglected but in some special cases where the primary bending loads are small and the value $\frac{f_s}{E}\left(\frac{b}{t}\right)^2$ is fairly large the effects of web tension may play a major part in the design of the beam flange. Such a case might be found in a wing or fuselage bulkhead.

End Stiffeners

Although no mention of the actual stresses in the end stiffeners (S_0 and S_3 in Fig. 2) is made, it is considered good design practice to reinforce these members by diagonal stiffeners from the centers of these members to the intersections of the next adjacent stiffeners (S_1 and S_2, respectively in Fig. 2) with the beam flanges. This reinforcement may, of course, be omitted if the end members are comparatively rigid as a result of some other design consideration.

Modulus of Rigidity

The Modulus of Rigidity as computed on page 4 is fairly close to test results and may be used in the majority of cases. Fairly large errors in this value may be permissible in least work computations if, as is often the case, the internal work due to shear is only a small portion of the total and the errors are well allowed for by over-lapping assumptions. However, if compression is combined with shear, as in a sheet stiffener combination on top of a wing, the Modulus of Rigidity may change radically, dropping to as low as 1,000,000 in many cases. Such a variation should be accounted for.

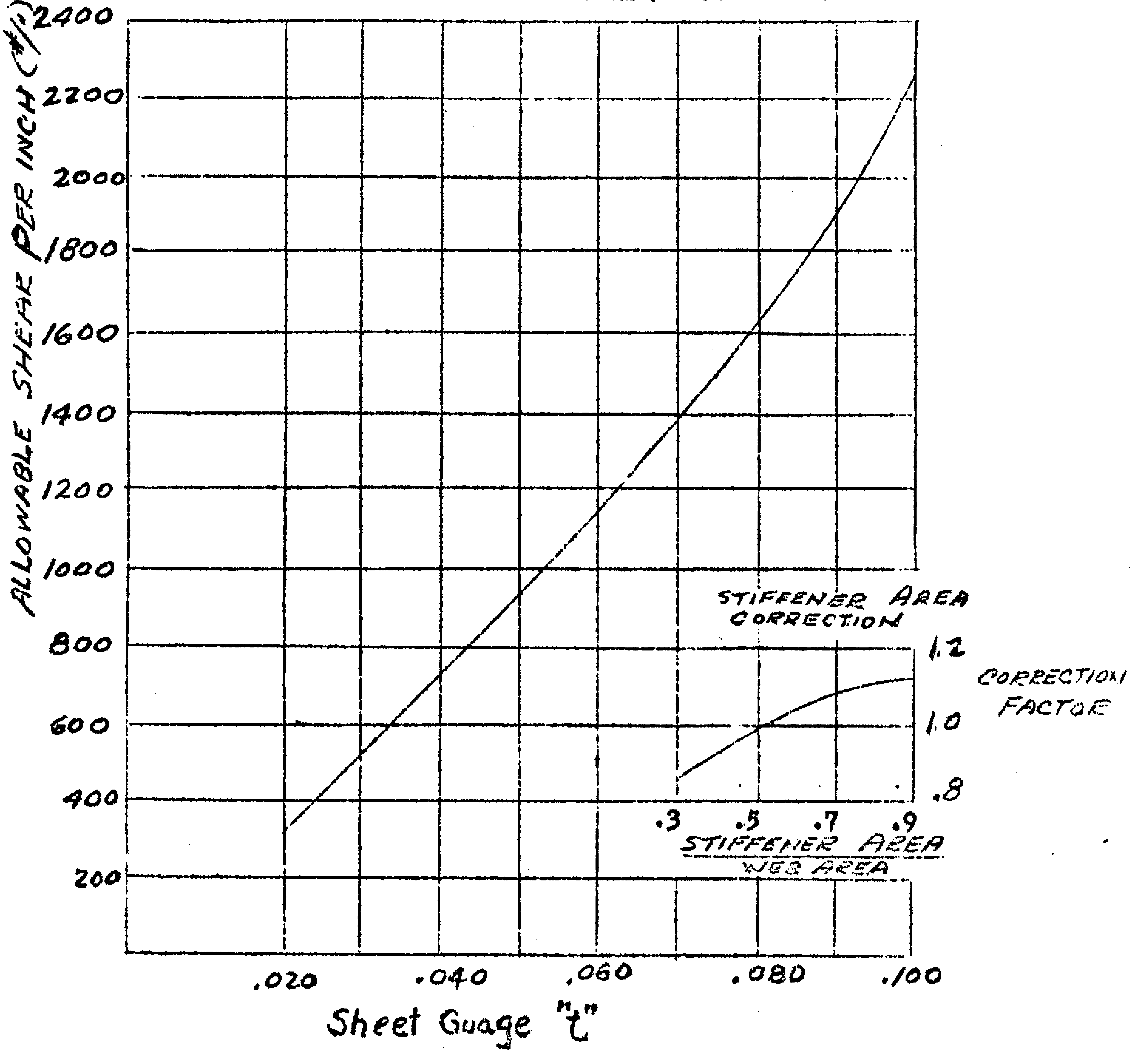
Design Curves

The final allowables for use in design must necessarily be based on the results of actual static tests on beams similar to those to be used. These may be summarized in a design chart similar to Fig. 8. Various designs from a number of different models may have similar stiffener spacings and stiffener-to-web area ratios so that the results of many tests may be combined on a single design curve by making small corrections for such variations.

The effect of curvature, as in the side of a fuselage, on the allowable shear stresses will be to raise the buckling stress slightly in most cases but to reduce the allowable ultimate stress to a very marked degree. The presence of flanged lightening holes or access holes will also result in a redistribution of stress and a reduction in the allowable, even when the reduction in area at the minimum section due to the hole has been taken into account. Both of these cases will call for additional static tests to obtain the required design information. However the same qualitative effects will be found and the same method of approach may be used in studying the results of the tests.

DESIGN CURVE

MATERIAL - 24 ST   STIFFENER SPACING = 10"



Problem Assignment.

Design the web of the center spar of the type airplane analyzed in previous assignments, between stations 115 and 145.

Given: (1) Design shear = 566 lbs. per inch;
(2) Rib spacing = 15 in. O.C.

Determine:
(1) Web thickness
(2) Stiffener spacing
(3) Stiffener size
(4) Attachment of web to spar cap flange
(5) Attachment of stiffener to spar cap flange
(6) Attachment of stiffener to web
(7) Actual bending moment in flange midway between stiffeners
(8) Margins of safety for web-stiffener combination and for web-spar rivets.

Use Fig. 7 and the Design Curve on page 10-11.

AIRPLANE
STRESS
ANALYSIS

PART ELEVEN

ADV. STRESS 11

ADV. STRESS 12

ADV. STRESS 13

ADVANCED STRESS ANALYSIS

ASSIGNMENT 11

Assignment 11

STRUCTURAL DESIGN REQUIREMENTS FOR CONTROL SURFACES

Horizontal Tail Surfaces

The data from assignments 3 and 4 were applicable to flight loads on the wings only. In assignment 5 we calculated flight balancing loads on the tail, and the inertia (or dead weight) factors to be applied to all the weights in the airplane to satisfy the equations of equilibrium, $\Sigma V = 0$, $\Sigma H = 0$, $\Sigma M = 0$. The balancing load on the tail was assumed to act at the 20% point on the mean chord of the horizontal tail. It is still necessary in order to design the tail surfaces that we determine the proportion of load on elevator and stabilizer and the direction in which it acts on each. Further, the tail surfaces must be investigated for loads imposed by the pilot in maneuvering, for the effects of gusts in flight, and for the effects of gusts in any direction when the ship is parked or taxiing on the ground.

Balancing Loads in Flight

The limit (yield point) load acting on the horizontal tail surface is the maximum balancing load calculated from flight conditions, I, II, III, IV, VII, or VIII. The distribution of load is according to Fig. 1. The elevator load, P, is taken as 40% of the total tail load, T with the load directed opposite to that on the stabilizer. This means that the stabilizer load would be 140% of the net tail load. However there are cases where 40% of the balancing load on the tail corresponds to an elevator load much higher than the pilot could apply. In such a case the limit load on the elevator, P, is reduced to that corresponding to 150 lbs. exerted by the pilot on the controls. The load on the stabilizer is still T + P. (Note that there is no reduction of load on the stabilizer except that due to the reduction in load P on the elevator.)

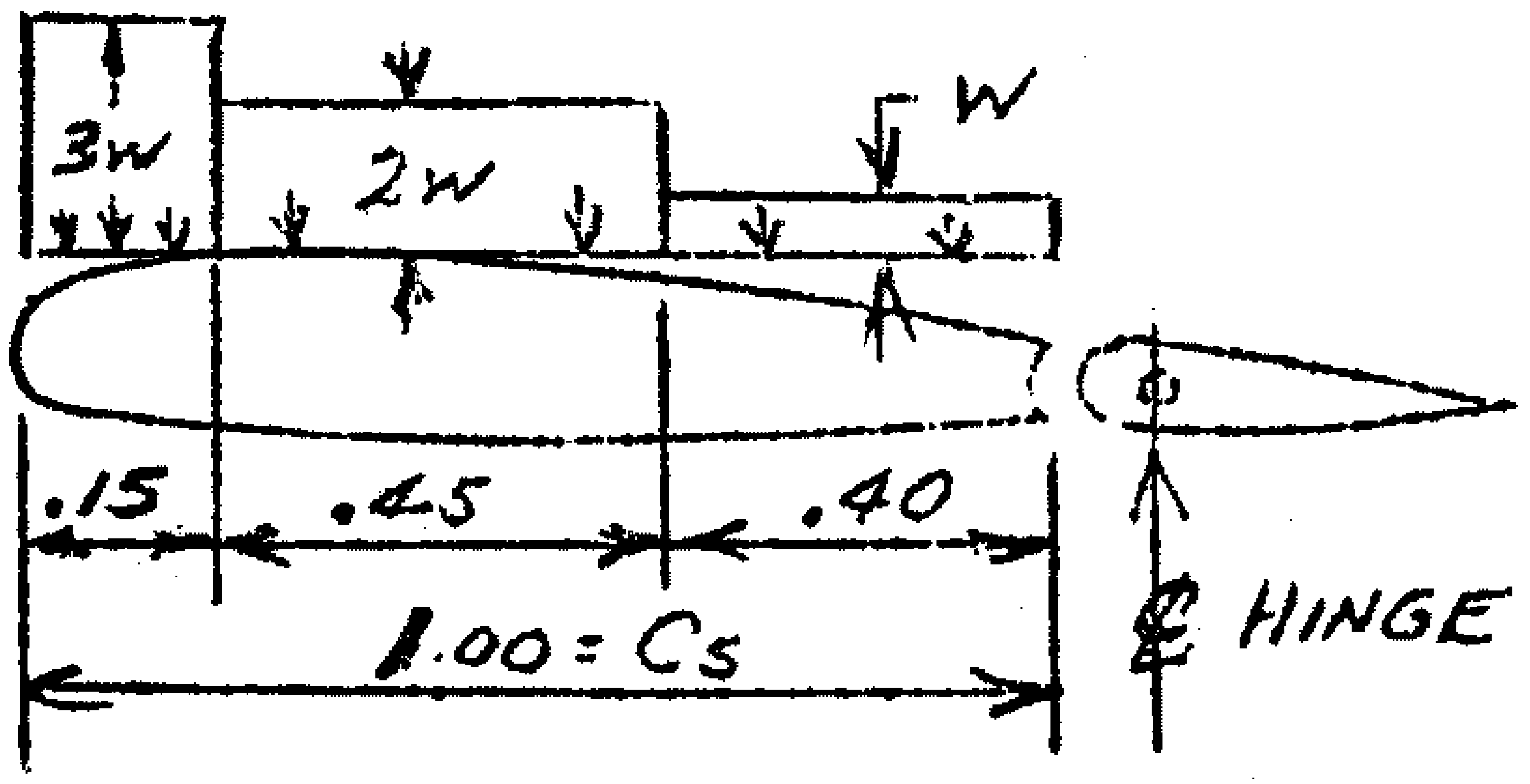


Fig. 1

Balancing Load

Fig. 1 represents the distribution of load on an average strip of chord of the horizontal surface.

For loads on the fixed surface, $W = \frac{\text{total load on fixed surface}}{1.75 \text{ x area of fixed surface}}$

A constant value of C_N is assumed along the span. The above loading designs the forward portion of the stabilizer and the structure transmitting the stabilizer torque to the fuselage.

Maneuvering Loads

Horizontal tails must also be designed for limit unit loads not less than those corresponding to pullup speed V_p and the following normal force coefficients:

$C_N = -0.55$ downward

$C_N = +0.35$ upward

$V_p = V_s + K_p\ (V_L - V_s)$

V_s = stalling speed with flaps up = $19.77 \left[\frac{W/S}{C_L \text{ Max.}}\right]^{\frac{1}{2}}$

C_L Max. = maximum lift coefficient for the normal wing with flaps up (in our case use C_L Max. = 1.41, see Assignment 3).

V_s is in miles per hour.

V_L = maximum level flight high speed = 220 mph.

$K_p = 0.15 + \frac{5400}{W_g + 3300}$, but shall not be less than 0.5.

W_g = gross design weight = 20,000 lbs.

The average limit (yield point) unit down load over the entire surface is

$-\overline{w} = -0.55\ q_p$ and the average limit up load $+\overline{w} = 0.35\ q_p$.

$q_p = .002558\ V_p^2$ = dynamic pressure at pullup speed

V_p in mph.

Where the unit loads obtained by the above equations give elevator torques in excess of those corresponding to 200 lbs. load on the controls in the cockpit they may be reduced to those which do correspond to the above control load, except in no case shall they be less than 15 lbs. per sq. ft. The distribution for this loading condition is according to Fig. 2. The maneuvering condition designs the rear portion of the stabilizer.

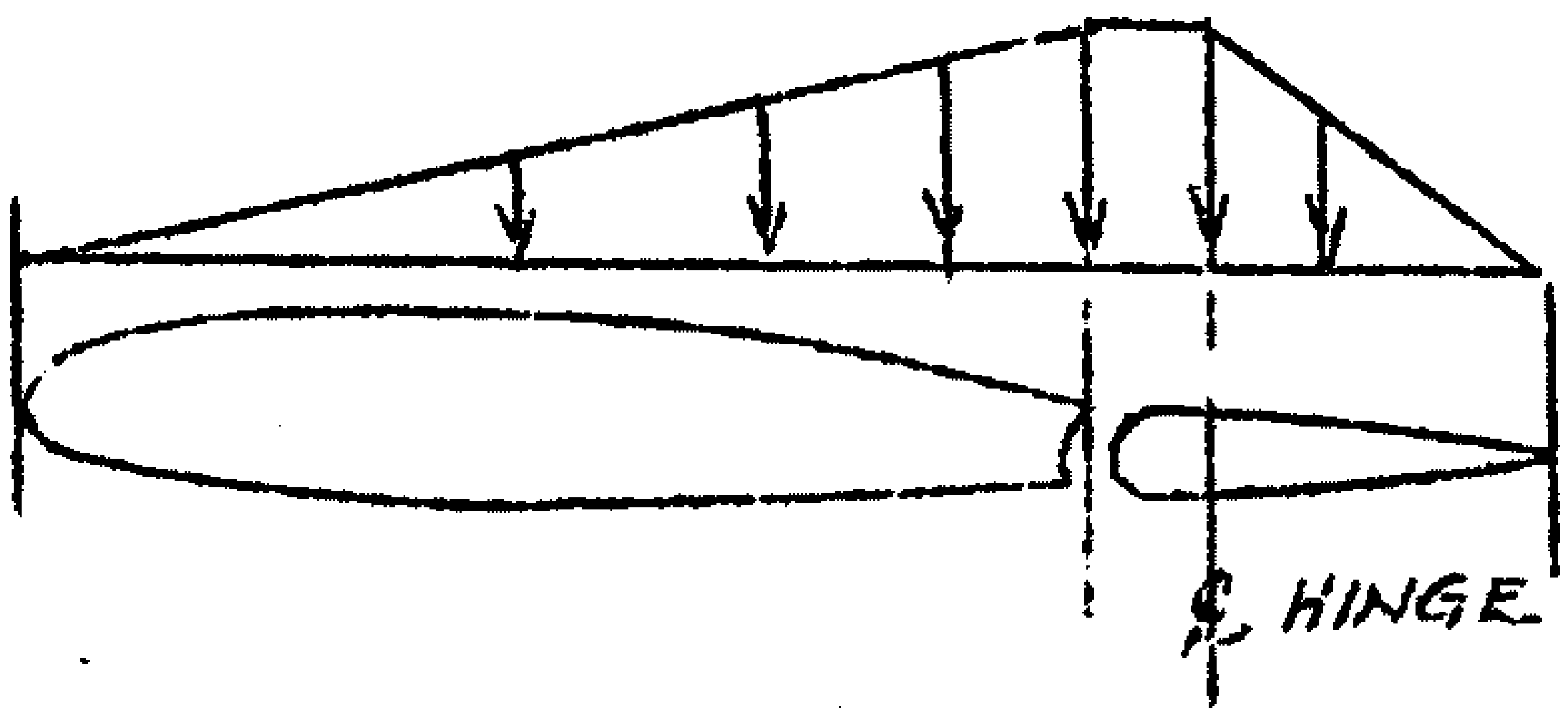


Fig. 2

Maneuvering Load

The maneuvering condition designs the rear portion of the stabilizer.

"Damping" Condition

To simulate a damping condition from the stabilizer the total limit load acting on this surface is assumed to be redistributed according to Fig. 1, except that no load whatever is assumed to act on the elevator.

Effect of Tabs on Elevators

When tabs are installed on the elevators, either to trim the ship or to aid the pilot in moving the surfaces, their effect must be accounted for. Under such conditions the minimum limit load over the entire horizontal surfaces shall not be less than that corresponding to the torque applied on the elevator when the pilot exerts a force of 200 lbs. in addition to the maximum aiding effect from the trim tab when fully deflected with relation to the elevator. When the tabs aid the pilots the loads are computed as follows:

1. Determine the normal force coefficient, (C_N) for the tab fully deflected, assuming the deflection and angle of attack of the elevator to be zero. (See NACA TR 360).
2. Determine the applied unit load, $\overline{w}$, on the tab corresponding to the normal force coefficient, C_N, and the high speed of the airplane in level flight, V_L.

$$\overline{w} = C_N q_L$$

The load distribution is assumed uniform over the tab.

3. Determine the loading which when distributed over the elevator in accordance with Fig. 3 will balance the sum of the hinge moments produced by the tab load and a force of 200 lbs. applied by the pilot. The loading over the main surface aft of the hinge line is assumed triangular while that over the tab is uniform and in the opposite direction.

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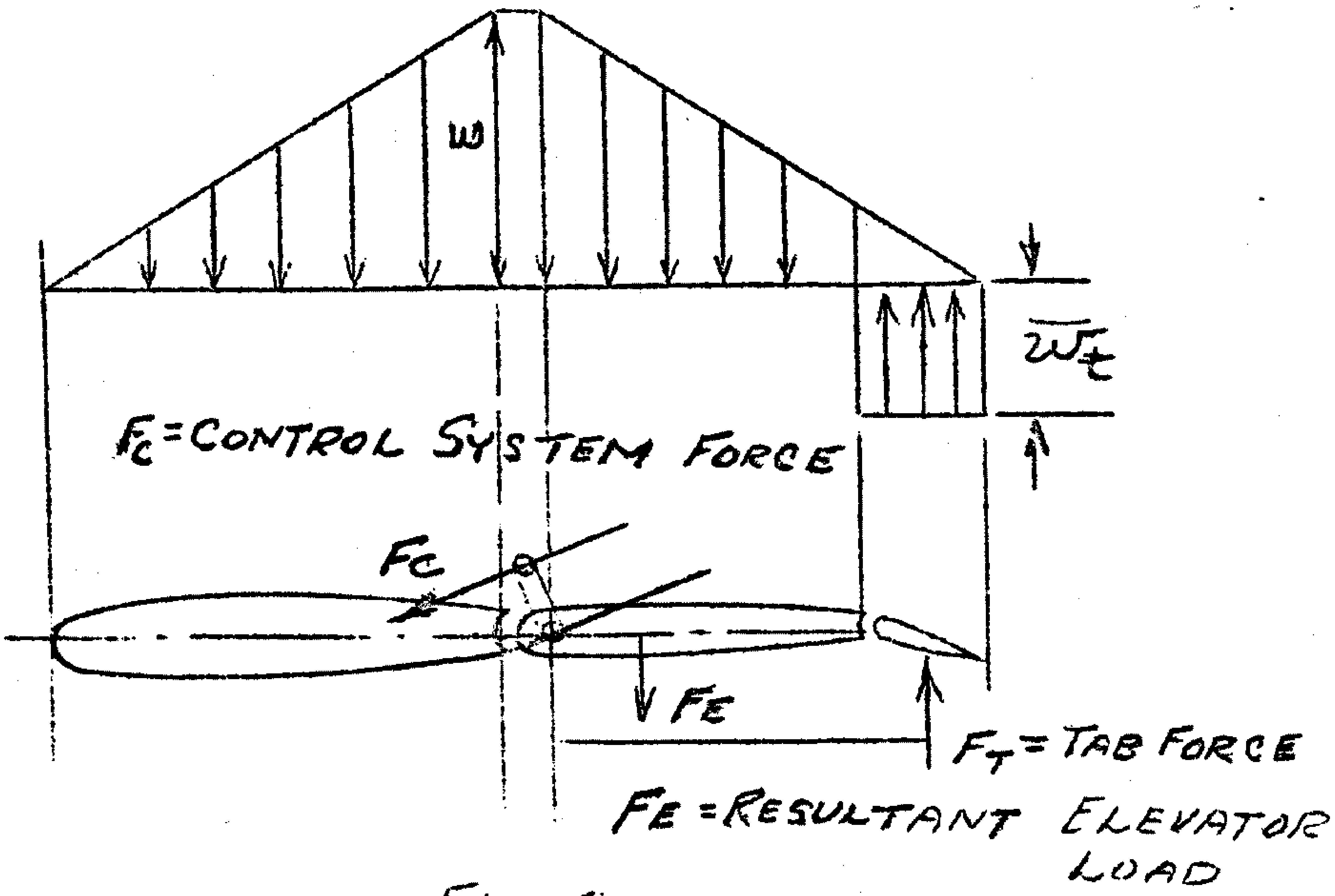


FIG. 3

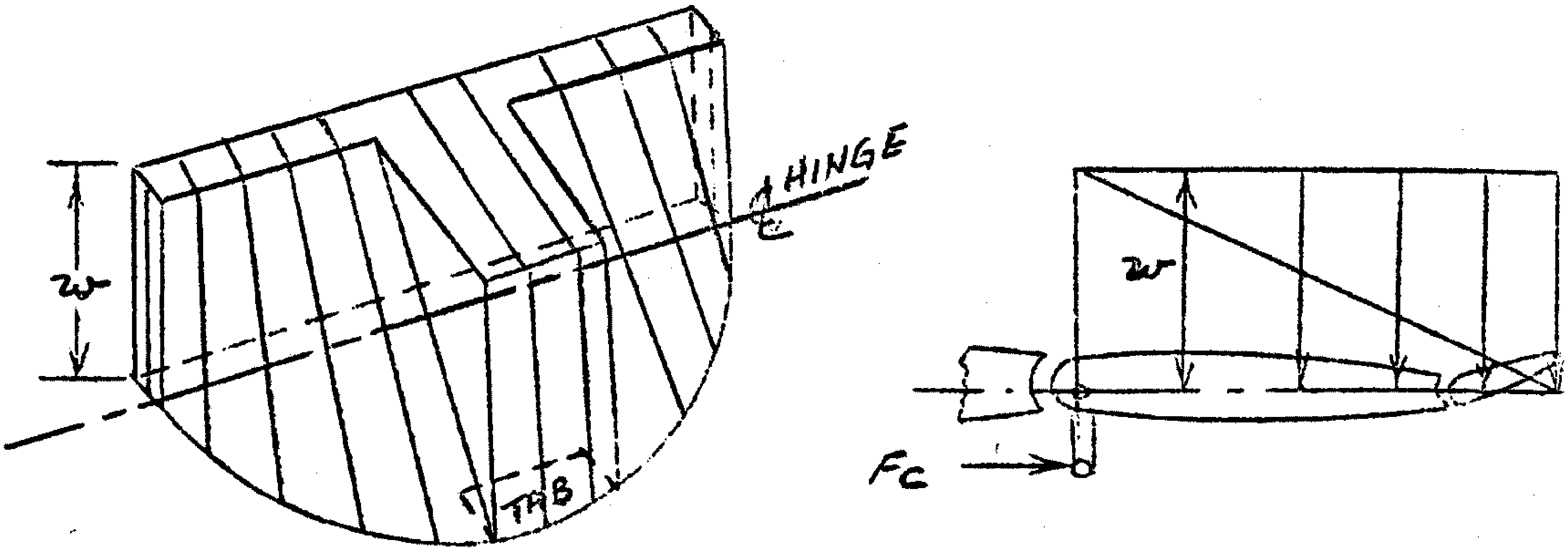


FIG. 4

Fig. 4 shows the loading over the elevator when the tab and elevator are deflected in the same direction. Because of the effect of the tab on the surface in front of it this condition will usually design the ribs forward of the tab, and may be critical for the elevator torque member. The loads for this condition are obtained as follows:

1. For all portions of the movable surface except the area between the hinge line and the tab, assume the usual distribution of Fig. 2, with an unknown value of unit loading, w, at the hinge line.

2. For that portion of the movable surface included between the hinge line and the trailing edge of the tab, assume a uniform distribution. The unit load is w, the same value as at the hinge line.

3. Compute the moment about the main hinge line in terms of w, and equate this value to the moment produced by a 200 lb. load applied in the cockpit by the pilot, to solve for the unknown value of w.

Note: All the foregoing values have been given in terms of limit or yield point loads. Design loads are 1.5 times limit loads.

******

Vertical Tail Surfaces

Maneuvering Loads

Where the propeller axes are in the plane of symmetry the vertical surfaces are designed for an average normal force coefficient, $C_N = 0.45$ and for speed, V_p, then Avg. $\overline{w} = 0.45\ q_p$. The load distribution is the same as that of Fig. 2.

If the propeller axes are not in the plane of symmetry the vertical surfaces must be investigated for the following condition:

(a) The design speed shall not be less than the maximum speed in level flight with one engine inoperative.

(b) The limit load need not be greater than that corresponding to 200 lbs. force applied by the pilot except that in no case may it be less than 12 lbs. per sq. ft.

Damping Loads

The total limit load acting on the fin in the maneuvering condition shall be applied in accordance with the load distribution of Fig. 1, except that no load need be assumed on the movable surface.

Flight Gust on Vertical Tail

The following gust condition shall be assumed for the vertical tail:

Gust speed $U = 30$ fps

Flight speed $= V_L$ (same as Condition I for wing)

Avg. $\bar{w} = \frac{UVm}{575}$ where m is slope of lift curve for vertical surface, C_L per radian, corrected for aspect ratio. An aspect ratio of at least 2 shall be used. Where "m" for the vertical surfaces is not available from wind tunnel tests the following approximate formula may be used:

$$\bar{w} = .226\, V_L \left(\frac{4}{3 + \frac{6}{R}}\right)$$

V_L = mph
R = aspect ratio (not less than 2)

The gust condition applies only to that portion of the vertical surface which has a well defined leading edge.

The chord distribution shall be that for a symmetrical airfoil where data are available. The distribution of Fig. 1 may be used for the fin where desired.

Tabs on the Rudder

Where the propeller axes are in the plane of symmetry the loads on the surfaces are such as to balance the load from a maximum tab deflection at speed V_L with no force applied by the pilot.

Where propeller axes are not in the plane of symmetry the loads are such as to balance the effect of 200 lbs. force applied by the pilot, plus the effect of maximum tab deflection, not at the speed V_L, but at a reduced speed V_L' corresponding to the maximum speed in level flight with one engine inoperative. By approximate formula $V_L' = 0.9\, V_L \, {\frac{n-1}{n}}^{1/3}$, where n is the number of engines.

Loads are distributed along the chord in the same manner as for the horizontal surfaces. See Figs. 3 and 4.

Ailerons

Maneuvering

If the propeller axes are in the plane of symmetry the ailerons are designed for normal force coefficient, $C_N = 0.45$ at the pullup speed V_p. Then the limit average loading $\overline{w} = 0.45\ q_p$.

The distribution is according to Fig. 5, which applies for all aileron loading conditions. If the propeller axes are not in the plane of symmetry the design speed shall not be less than the maximum speed in level flight with one engine inoperative.

The limit unit load in either direction need not exceed that corresponding to 80 lbs. acting at the stick or an 80 couple acting at the rim of a control wheel considered to be resisted by one aileron only, except that in no case shall it be lower than 12 lbs. per sq. ft.

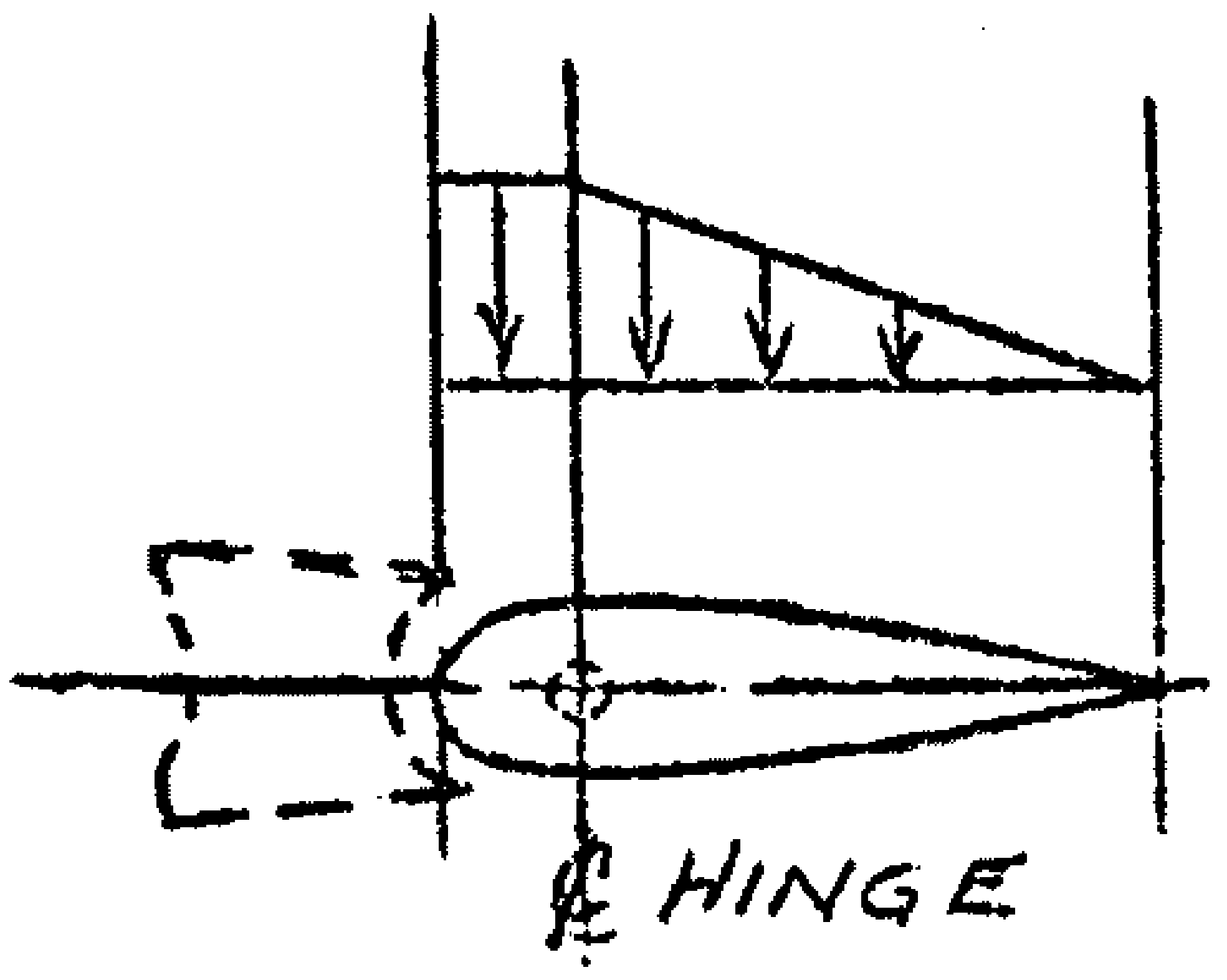


Fig. 5

Ailerons (Tab Effects)

Only those airplanes with propeller axes not in the plane of symmetry need be checked for the effect of tabs. When a tab is installed on one or both ailerons so that it can be used by the pilot to assist in moving the ailerons, the limit unit loading over both ailerons shall be of sufficient magnitude and in such a direction as to hold the ailerons in equilibrium with the tab or tabs deflected to their maximum throw. For this condition the loads on the tabs are computed at maximum level flight speed with one engine inoperative.

Ailerons--Flying Conditions

We must not lose sight of the fact that the ailerons are also part of the wing area. They must therefore be checked to insure that they meet all the requirements of the symmetrical load conditions for the wings. To check this condition reference is made to plots of chordwise distribution of pressure for the airfoil. The chordwise distribution varies with the angle of attack and is available in NACA Reports as a plot of p/q against C_L. The pressure distribution for our wing is given on page 6 of Assignment 4.

Values of C_L and q are taken from the wing flight conditions. Having these values the pressure across the aileron is obtained. These values are limit values. As in all conditions described the design values are 1.5 times those obtained from the basic data.

Wing Flaps

Flaps are designed for Conditions VII and VIII described in Assignment 3. The minimum design speed V_f is $2V_{s_f}$ (twice the stalling speed with flaps down). The distribution of load over the flap is assumed uniform unless wind tunnel data are available to show the actual distribution. For aileron type flaps the normal force coefficients may be taken from NACA TR 360. For split flaps for which the upper portion is a fixed part of the wing the value of C_N may be given.

$C_N = .273 + .021\,\delta$, where δ is the flap opening in degrees.

Where no data on C_N are available it is assumed to be 1.60, which is a very conservative value.

Control Surface Tabs

The loads previously given for control surface tabs are for the purpose of designing the surfaces to which they are attached. Actually the tabs themselves may carry higher loads when they are partially deflected at higher speeds. For design of the tabs only all usable combinations of speed and tab deflection up to design speed V_G must be considered. Tab loads are considered to be uniformly distributed.

Effect of Horizontal Ground Gusts

For large airplanes it has been found that the most severe loads on the tail surfaces may be encountered when the airplane is parked on the ground or taxiing in a cross wind. For large airplanes in addition to the requirements of the Civil Aeronautics Authority it is good practice to assume that the tail surfaces may have to withstand a ground gust load up to 60 mph velocity applied in any direction up to 30° from either side for the elevator, or up to 45° from either side for the rudder. A satisfactory approximate solution is as follows:

1. Assume that the trailing edge of the movable surface is the leading edge of a rectangular flat plate, that the surfaces are locked in neutral, and that the wind may come from the rear at any angle up to those noted in the preceding paragraph.
2. Take the normal force coefficient and center of pressure data from basic Flap Plate data. (Fig. 7 of this assignment)

3. Calculate the average normal force at the value of C_N for the maximum angle of attack (30° for elevator, 45° for rudder).

Values of C_N are .895 at 30°
and 1.30 at 45°

It will be noted that the center of pressure lies between 34 and 35 per cent in the range from 30 to 45 degrees angle of attack. It is therefore assumed that the distribution is triangular over the entire tail surface with the maximum value at the trailing edge.

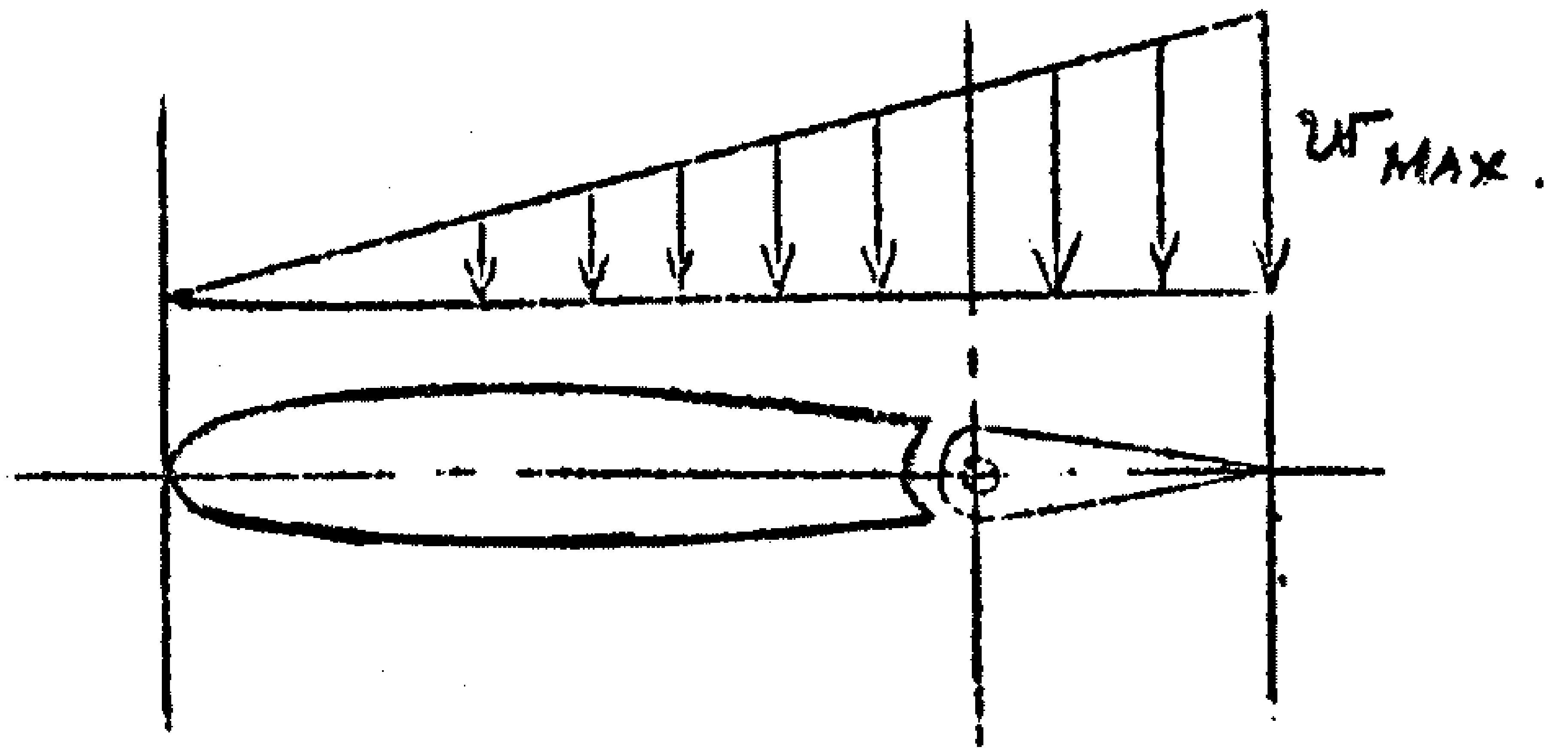


Fig. 6

Ground Gusts

Average $\bar{w} = C_{N_q}$ where $q = .002558 \times 60^2$

Maximum w at trailing edge = 2 x $\bar{w}$

The values given above are limit loads. Design loads are 1.5 times the above values.

Problem Assignment.

Given the horizontal tail surface areas shown by the attached line diagram (Fig. 8) and the maximum balancing tail load from Assignment 5 determine the limit loads as follows:

1. Maximum balancing load on stabilizers and elevators.
2. Maneuvering load on stabilizer and elevator
   - (a) With tab aiding pilot.
   - (b) With tab opposing pilot.

   For tab load assume C_N max. = .264 at a maximum speed of V_L = 220 mph.

   An applied load of 150 lbs. on the elevator control column gives a moment about the elevator hinge line of 3240 inch lbs. applied to two elevators.
3. Check also the arbitrary maneuvering unit loads at the design speed V_p.

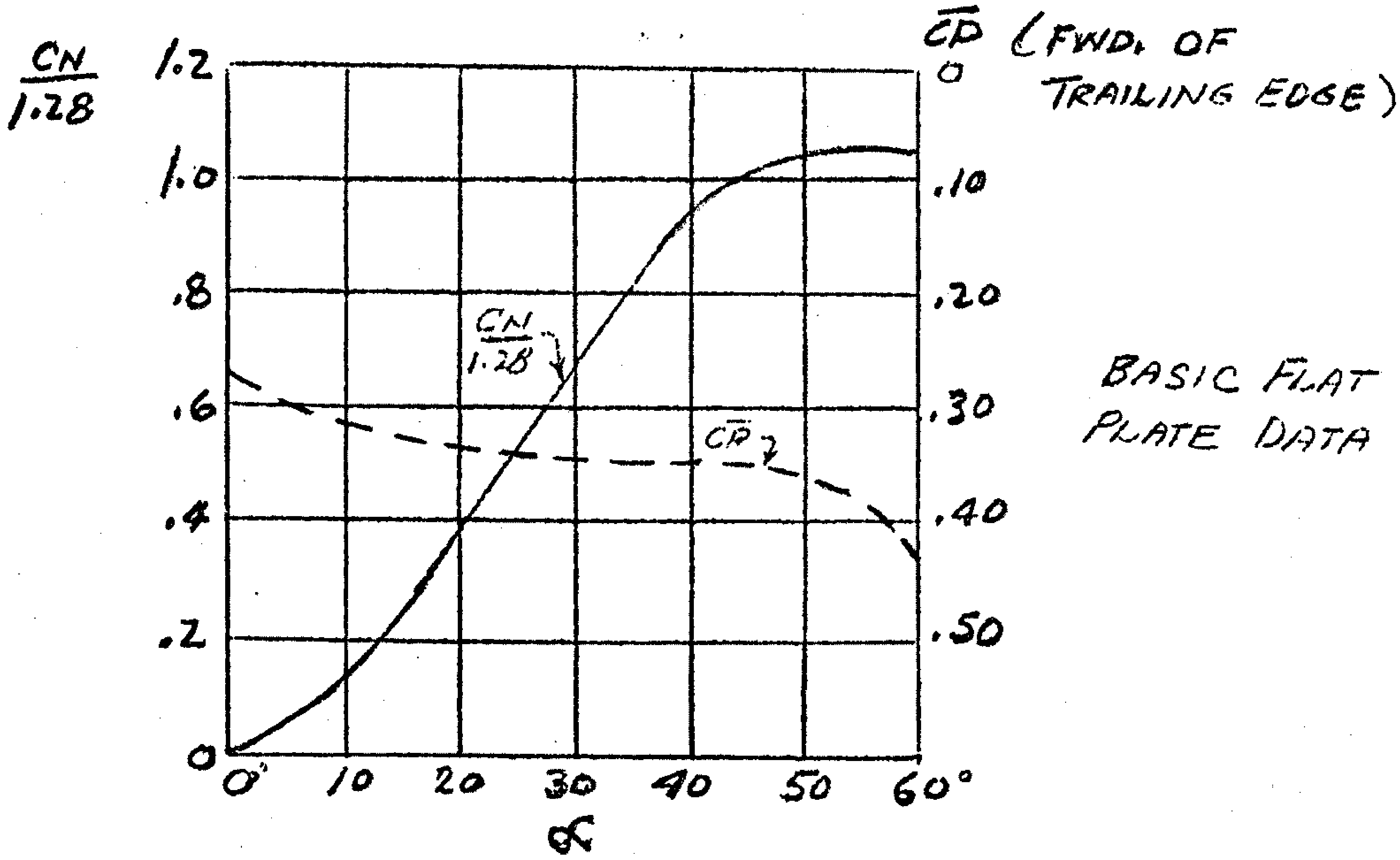
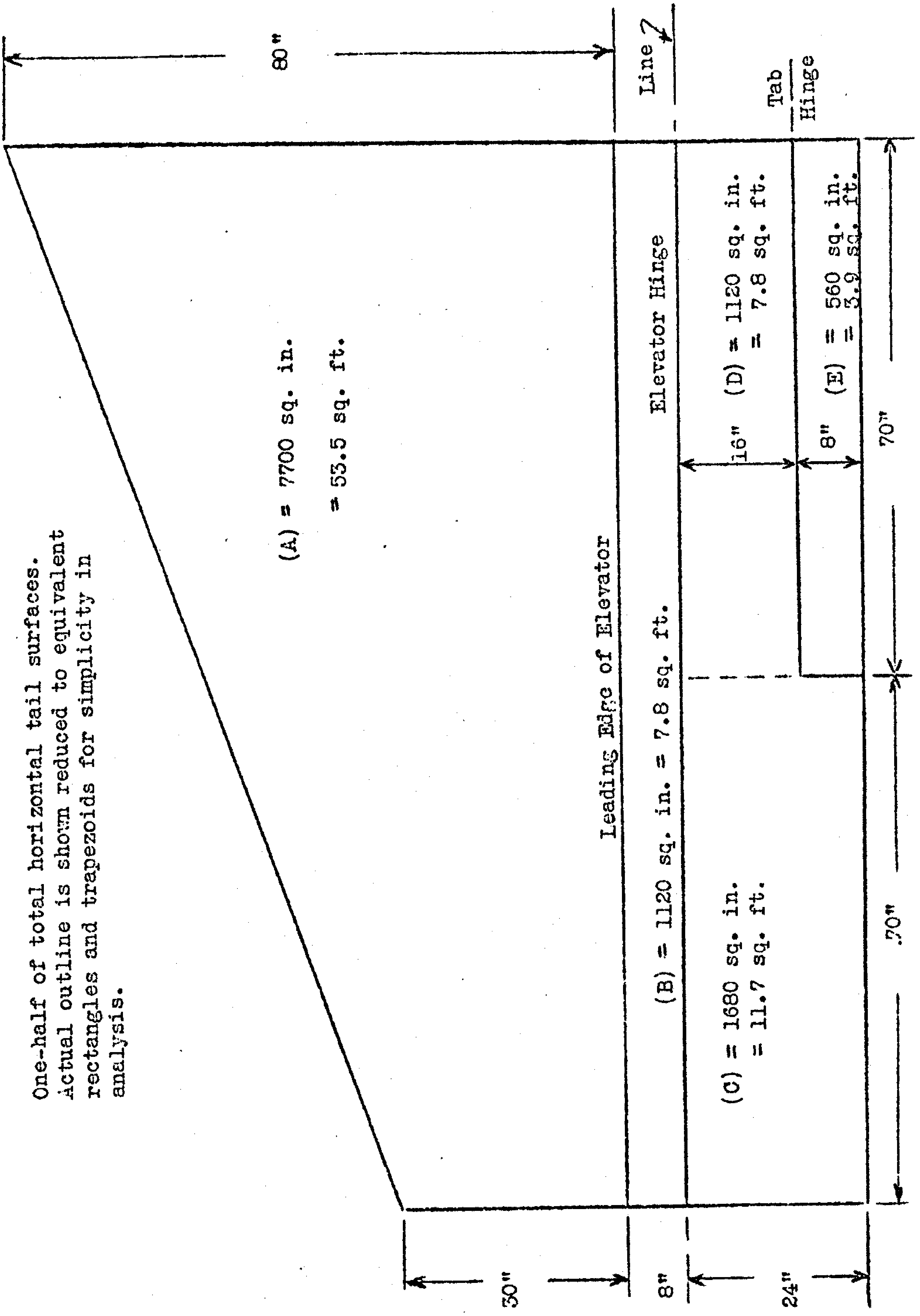


FIG. 7



One-half of total horizontal tail surfaces. Actual outline is shown reduced to equivalent rectangles and trapezoids for simplicity in analysis.

Fig. 8

ADVANCED STRESS ANALYSIS

ASSIGNMENT 12

Assignment 12

The design conditions prescribed by the Civil Aeronautics Authority in 6 CFR 04 will be illustrated in this assignment. These conditions apply to designs which incorporate tail wheels. The recommended requirements for gears incorporating nose wheels, commonly known as tricycle landing gears, are given in CAAM-04. The requirements for nose wheel gears are not mandatory but are recommended practice. Those given in 6 CFR for gears with tail wheels are the minimum requirements which will be accepted by the Civil Aeronautics Authority. All of the tricycle gear arrangements which have been approved to date have been covered by special rulings from the Authority. Loads for the tricycle gears have been based on rational equations which take into account the potential and dynamic energies involved. For a conventional tail wheel type gear the same energy considerations are approximated by using a set of semi-arbitrary landing conditions as follows:

CONVENTIONAL (TAIL-WHEEL TYPE) LANDING GEARS

1. Level Landing

The minimum "limit" (yield point) load factor is the lower value from either of the following equations except that in no case is it necessary to exceed 4.33.

(a) $n = 2.80 + \frac{9000}{W + 4000}$, where W = gross weight of airplane

(b) $n = 3.00 + 0.133\left(\frac{W}{S}\right)$, where $\frac{W}{S}$ = wing loading in lbs. per sq. ft. at a load factor of 1.0.

(b) applies only on planes where the wing loading is less than 10 lbs. per sq. ft. The factor is a vertical load factor.

The resultant ground reactions are assumed to be taken entirely by the main chassis, equally distributed to the two wheels and in such a direction that they lie in a plane which includes the axles and the center of gravity of the airplane less landing gear; except that the aft component of the resultant load need not exceed 25 per cent of the vertical component. The basic weight is that for full gross less chassis weight. The propeller axis or fuselage reference line is taken as a horizontal reference line. The shock absorber unit and tires are assumed deflected to half their total travel, unless some other position is shown to be more critical for the landing gear. The minimum factor of safety required is 1.5; that is the maximum "n" from equation (a) or (b) is multiplied by 1.5 to give the design load factor. Balancing loads in the airplane are supplied by inertia loads acting opposite to the ground reactions. This condition simulates a tail high landing.

2. 3-Point Landing

The design load factor is the same as for level landing. The sum of the static ground reactions (load factor = 1) is the gross weight of the airplane less chassis. These loads act normal to the ground line. The total load is divided between the chassis and tail wheel in inverse proportion to the distances, measured parallel to the ground line, from the center of gravity of the airplane less chassis to the points of contact with the ground. The load on the main chassis is divided equally between the wheels (assuming a conventional chassis with two main wheels). All shock absorbers and tires are deflected to the same degree as in level landing. The tail wheel is also investigated for this condition. This condition simulates a three point landing.

3. Side Load on Main Chassis - (Ground Loop Condition)

The minimum "limit" load factor is 0.667. The gross weight of the airplane is assumed to act on one wheel in a direction perpendicular to the ground, airplane remaining level (not tipping sidewise). In addition a side component of equal magnitude is assumed to act inward and normal to the plane of symmetry at the point of contact of the wheel on the ground. An aft component equal to 0.55 times the vertical component is assumed to act parallel to the ground at the point of contact. The airplane is assumed to be in the three-point attitude with shock absorbers deflected to their static position and the tire deflected to one-quarter the nominal diameter of its cross-section. The minimum ultimate factor of safety is 1.5. This condition simulates a turn on the ground executed by applying brakes to one wheel only.

4. One Wheel Landing

An investigation of the fuselage structure is required for a one-wheel landing, in which only those loads obtained on one side of the fuselage in the level landing condition are applied. The resulting dead weight load factor is one-half that for the level landing condition of item 1. (This condition is identical with level landing for loads on the loaded wheel.) Its purpose is to design the fuselage for unsymmetrical landing loads. The minimum ultimate factor of safety is 1.5.

5. Braked Landing

The minimum "limit" load factor is 1.33. The airplane is investigated for loads developed when a landing is made with the wheels locked and the airplane in an attitude such that the tail wheel just clears the ground. The weight of the airplane less chassis is assumed to act on the wheels in a direction perpendicular to the ground line in this attitude. In addition, a component parallel to the ground line shall be assumed to act at the point of contact of the wheels and the ground, the magnitude of the component being equal to .55 times the vertical component. The tire is assumed to have deflected not more than one-fourth the nominal diameter of its cross-section, and the deflection of the shock absorbers is the same as in level landing. The minimum factor of safety is 1.5. This condition simulates the sudden application of brakes on landing.

Shock Absorption

The shock absorption of the landing gear unit must be demonstrated by a drop test. The level landing loads are assumed to be produced by a free drop in inches equal to 0.36 times the stalling speed in miles per hour, except that the height of free drop must not be less than 18 inches for ships employing flaps or other high lift devices and need not be more than 18 inches when such devices are not employed. This requirement accounts for the steeper glide when flaps are used. The height of free drop is measured from the bottom of the tire to the ground, with the landing gear extended to its extreme unloaded position.

*****

The method of analysis will be demonstrated for a conventional braced cantilever landing gear.

Design data:

$$\begin{aligned} \text{Gross weight} &= 6200 \text{ lbs.} \\ \text{Land gear wt.} &= \underline{\ 400\ } \\ \text{Net weight} &= 5800 \text{ lbs.} \end{aligned} \qquad \text{Wing area} = 332 \text{ sq. ft.} \quad \frac{W}{S} = 18.67$$

Level Landing

Vertical "limit" factor n

$$\text{(a)} \quad n = 2.80 + \frac{9000}{6200 + 4000} = 2.80 + .88 = 3.68$$

Since $\frac{W}{S}$ is greater than 10 lbs. per sq. ft. equation (b) does not apply. If the load factor calculations are made by equation (b) for this loading $n = 3.0 + 0.133\,(18.67) = 3.0 + 2.48 = 5.48$.

Since the lower value from the two criteria is to be used n = 3.68 and the design value = 3.68 x 1.5 = 5.52.

Net weight = 5800 lbs.

Vertical Design Load per wheel = $\frac{5800}{2}$ x 5.52 = 16,000 lbs.

Fig. 1 shows the line diagram of the chassis and the location of the most rearward c.g. with respect to the axles. Since this location would give an aft component greater than 25% of the vertical component, if the resultant load passes through axle and c.g., it is permissible to reduce the aft component to 25% of the vertical component. Then aft load per wheel = .25 x 16,000 = 4000 lbs. Reaction points are at E, the shock absorber attachment point; B, the axle attachment point; and C, the brace strut attachment point. The complete solution for loads and reactions is given on Fig. 1.

Fig. 2 shows the ship in the 3-point landing attitude. This attitude may also be used for the braked landing condition.

Fig. 3 shows the side load condition. It is required that this condition be checked with the gear deflected to its static position only, but since it is obvious with the gear shown that higher loads will be obtained with the gear extended a check should also be made in this condition. Such a check covers also loads from an accidental side landing with one wheel carrying most of the load.

Assignment

It is advisable that the student complete the solution for the 3-point landing and ground loop conditions indicated on Figs. 2 and 3 to obtain practice in landing gear analysis. The axial loads should be obtained in members BC, CD, BD, ED, EB and in the axle below D for level landing, 3-point and ground loop conditions.

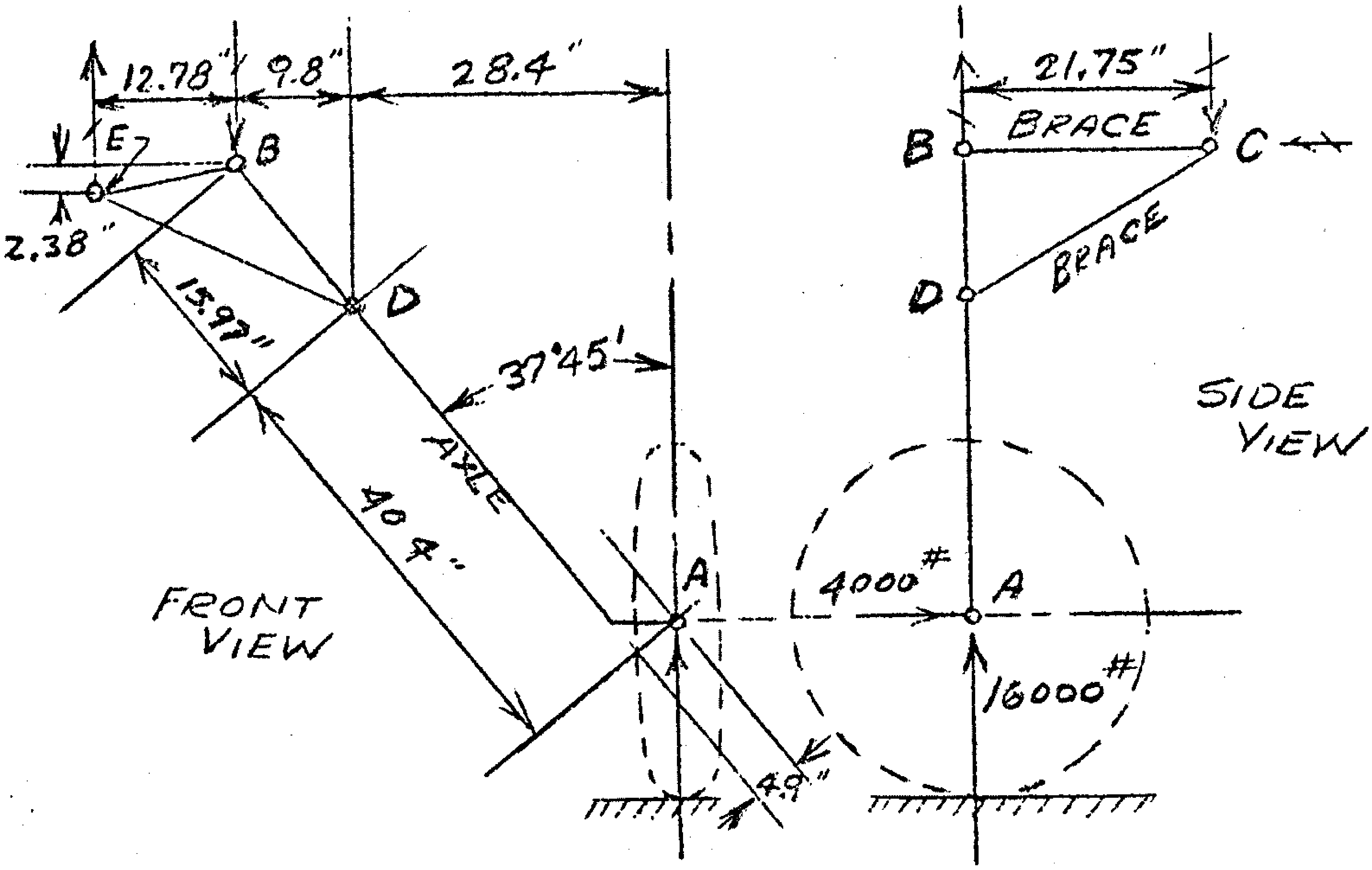


Fig. 1 LEVEL LANDING

Solution for Level Landing

Reactions at B, C, and E. "E" is shock strut attach. point and can take only vertical reaction. Gear pivots about axis "BC". Vertical load taken in plane "BDE". Drag load taken in plane "BCD".

Vertical Load

$$\Sigma M_B = 0 \qquad V_E = \frac{(16,000)(38.2)}{12.78} = 48,000 \text{ lbs.} \uparrow$$

$$V_B = 48,000 + 16,000 = 64,000 \text{ lbs.} \downarrow$$

Horizontal Load

Reactions in plane "BCD".
Forward reaction at C = 4000 lbs. ←

Couple at B and C = $\frac{(4000)(56.37)}{21.75}$ = 10,350 lbs. ↖ B ↘ C

Reactions $\perp$ to plane "BCD".

Couple at B and C = $\frac{(4000)(4.9)}{21.75}$ = 900 lbs. ↙ B ↗ C

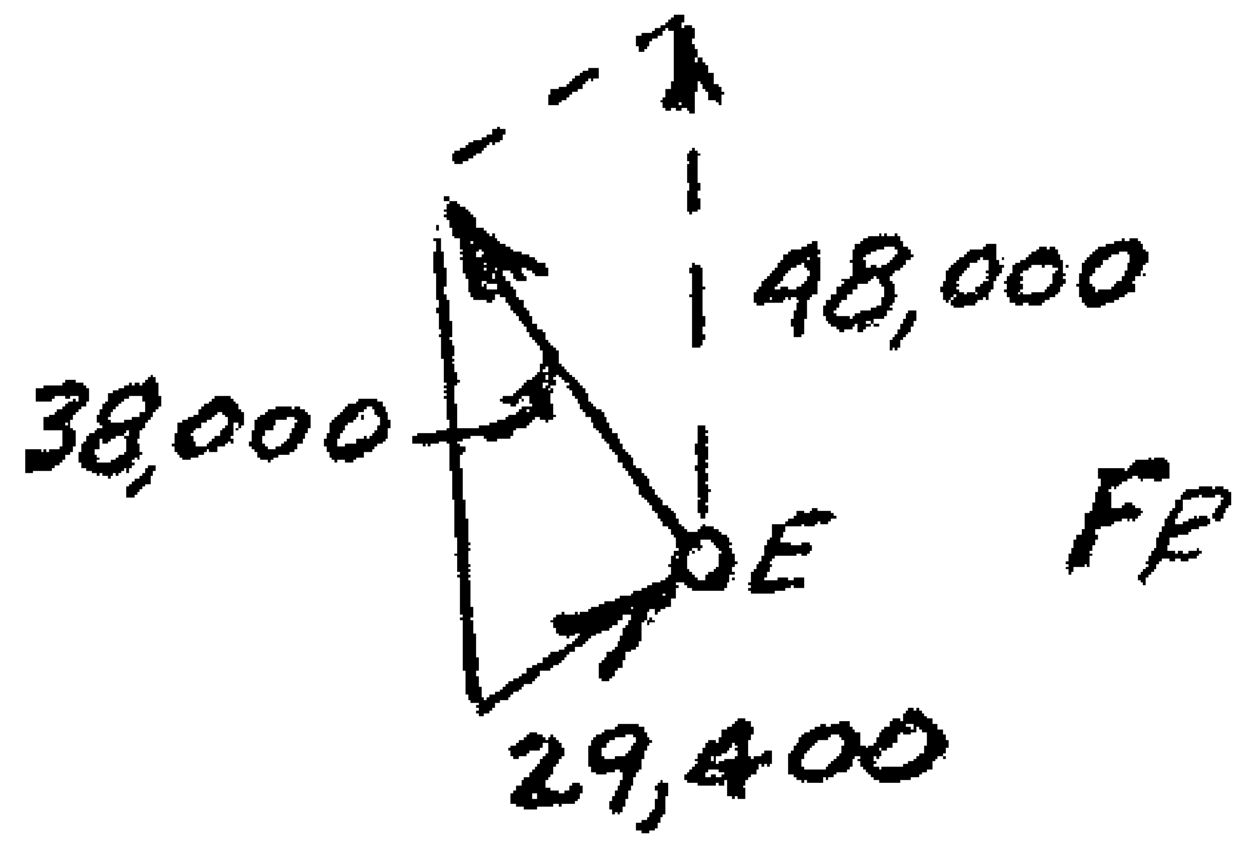
*****

Resolution of Reactions $\perp$ and $\parallel$ to plane "BCD".

Joint "E"

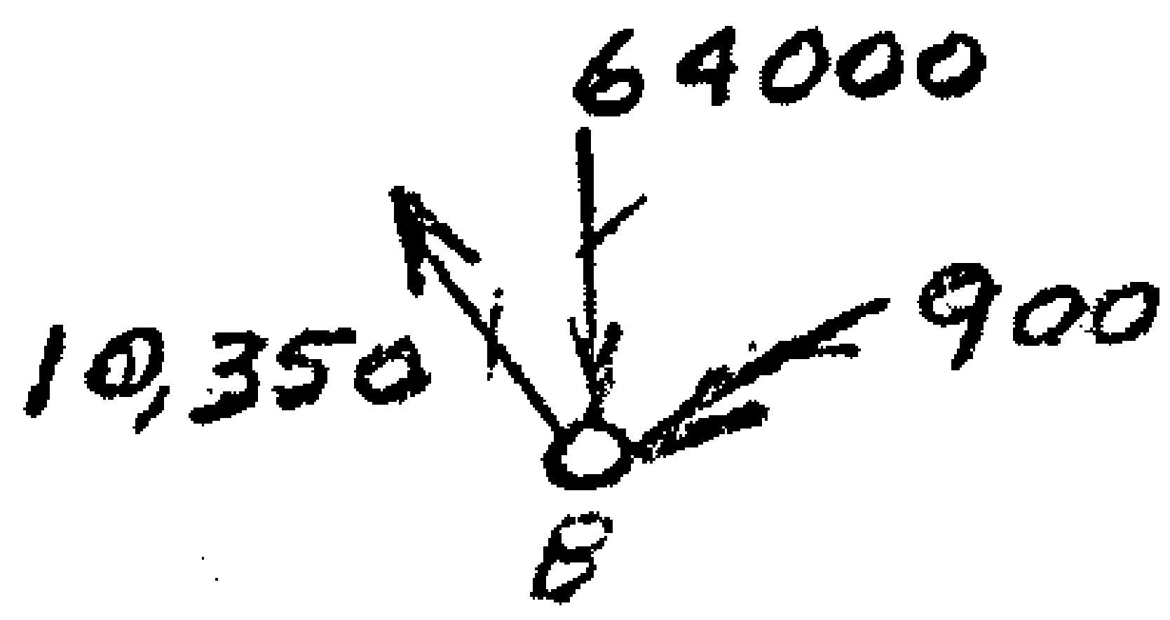
$\perp$ to plane BCD = 48,000 x sin 37°45' = 29,400 lbs.

$\parallel$ to plane BCD = 48,000 x cos 37°45' = 38,000 lbs.



FRONT VIEW

Joint "B"



FRONT VIEW

64,000 x cos 37°45' = 50,700 lbs.

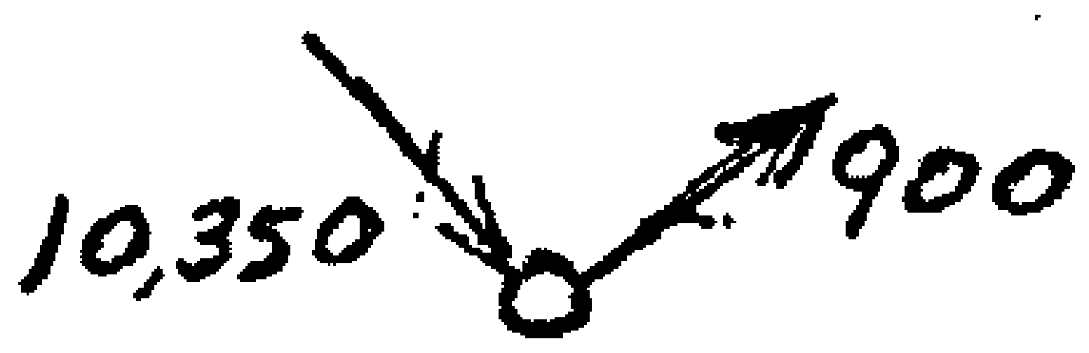
64,000 x sin 37°45' = 39,200 lbs.

Totals at "B"

$\perp$ to plane BCD = 39,200 + 900 = 40,100 ↙

$\parallel$ to plane BCD = 50,700 - 10,350 = 40,350 ↘

Joint "C"



$\parallel$ to plane BCD = 10,350 ↘

$\perp$ to plane BCD = 900 ↗

Forward = 4000 lbs.

Check by Taking Moments about Point "D"

Moment about "D" in plane "BCD" from wheel reactions

= 4000 x 40.4 = 161,600"# ↺ → aft

Moment about "D" in plane "BDE" = 16,000 x 28.4 = 454,400"# ↺ out →

From reactions at fuselage at B, C, and E

Mom. in plane "BCD" = 10,350 x 21.75 - 4000 x 15.97 = 161,600"# (Check)

Mom. in plane "BDE" = 48,000 x 22.58 - 40,100 x 15.97 + 900 x 15.97

= 457,800"# (error = 3,400"# in 0.7%)

*****

Torsional moment on axle at "D" = 4000 x 4.9 = 19,600"#

= 900 x 21.75 = 19,600"# (Check)

*****

Sequence of Operations: To obtain Axial Loads in Members.

1. Obtain loads in $\overline{EB}$ and $\overline{ED}$ from reaction at "E"
2. Obtain loads in $\overline{BC}$ and $\overline{CD}$ from reaction at "C"
3. Axial load in axle below "D" = component of 16,000 lbs.
   Load along axle = 16,000 x cos $37^{\circ}45'$
4. Axial load in axle, part $\overline{BD}$ from reactions at "B"
5. Check by summation of axial loads at "D", considering components along $\overline{BD}$ of loads in $\overline{ED}$ and $\overline{CD}$ as well as axial load in lower axle below "D".

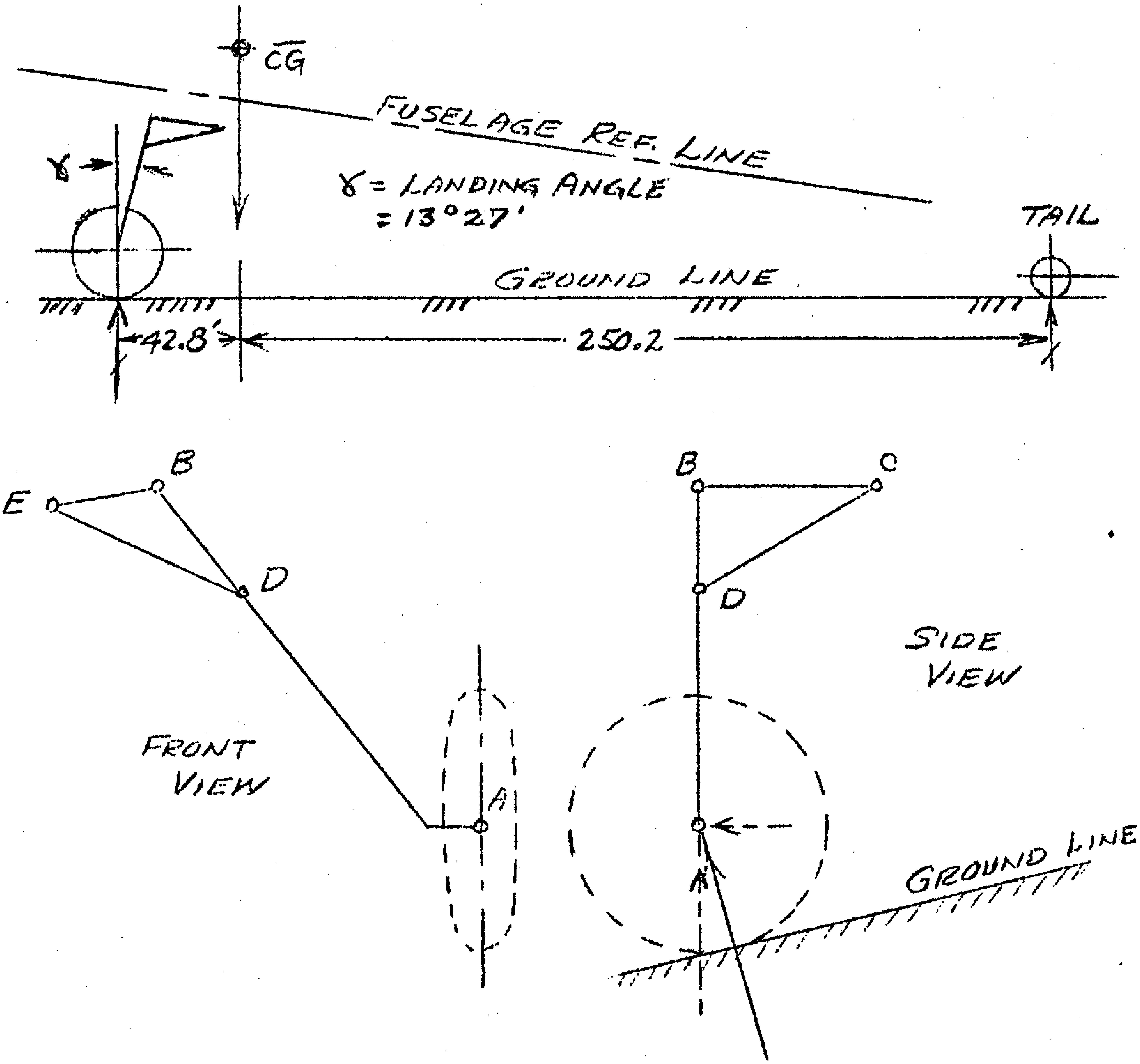


FIG.2 . 3-POINT LANDING.

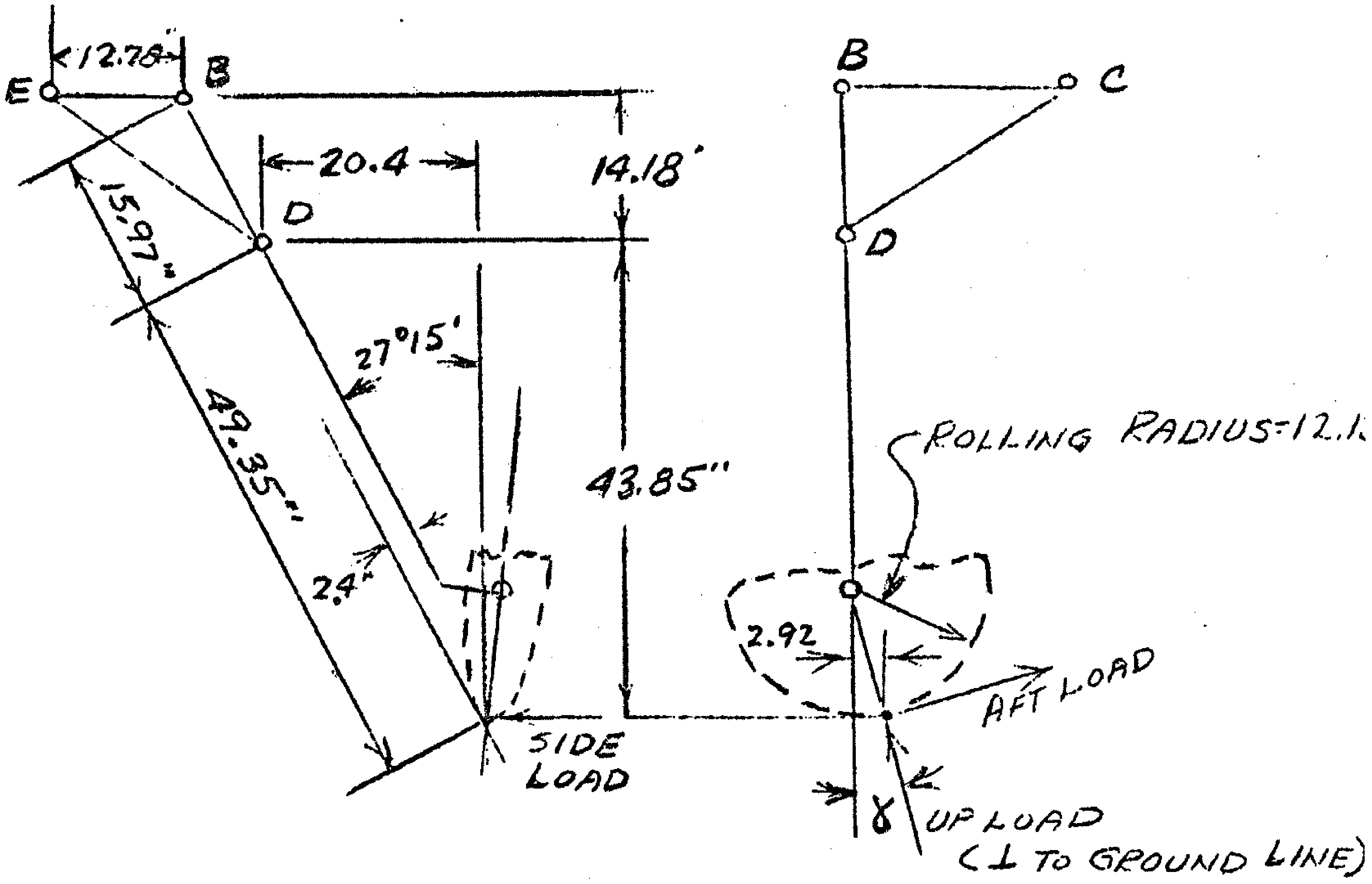


FIG.3 - GROUND LOOP

TRICYCLE (NOSE-WHEEL TYPE) LANDING GEARS

The minimum conditions recommended for the design of Tricycle Landing gears are as follows:

1. Three Wheel Landing with Vertical Reactions.

The minimum limit load factor is the same as that used in level landing with tail wheel type gear. That is it is taken from the formulas given on page 12-1. The sum of the static ground reactions (one load factor on the airplane) is equal to the gross weight of the airplane less the weight of the landing gear. The total load is divided between the main wheels and nose wheel inversely proportional to the distances, measured parallel to the ground line, from the center of gravity of the airplane less landing gear to the points of contact with the ground. The load on the main gear is equally divided between wheels. The dead weight loads and the ground reactions are assumed to act perpendicular to the ground line, with the ship in the 3-wheel landing attitude with all shock absorbers and tires deflected to one-half their total travel unless it is apparent that some other more critical combinations of deflection could occur. Where the center of gravity may vary the most critical c.g. locations must be investigated for each gear; that is maximum aft c.g. for the main gear and maximum forward c.g. for the nose gear. The ultimate factor of safety required is 1.5. That is the ultimate load factor is 1.5 times the applied load factor. This condition is illustrated in Fig. 4 and simulates a straight drop in making ground contact.

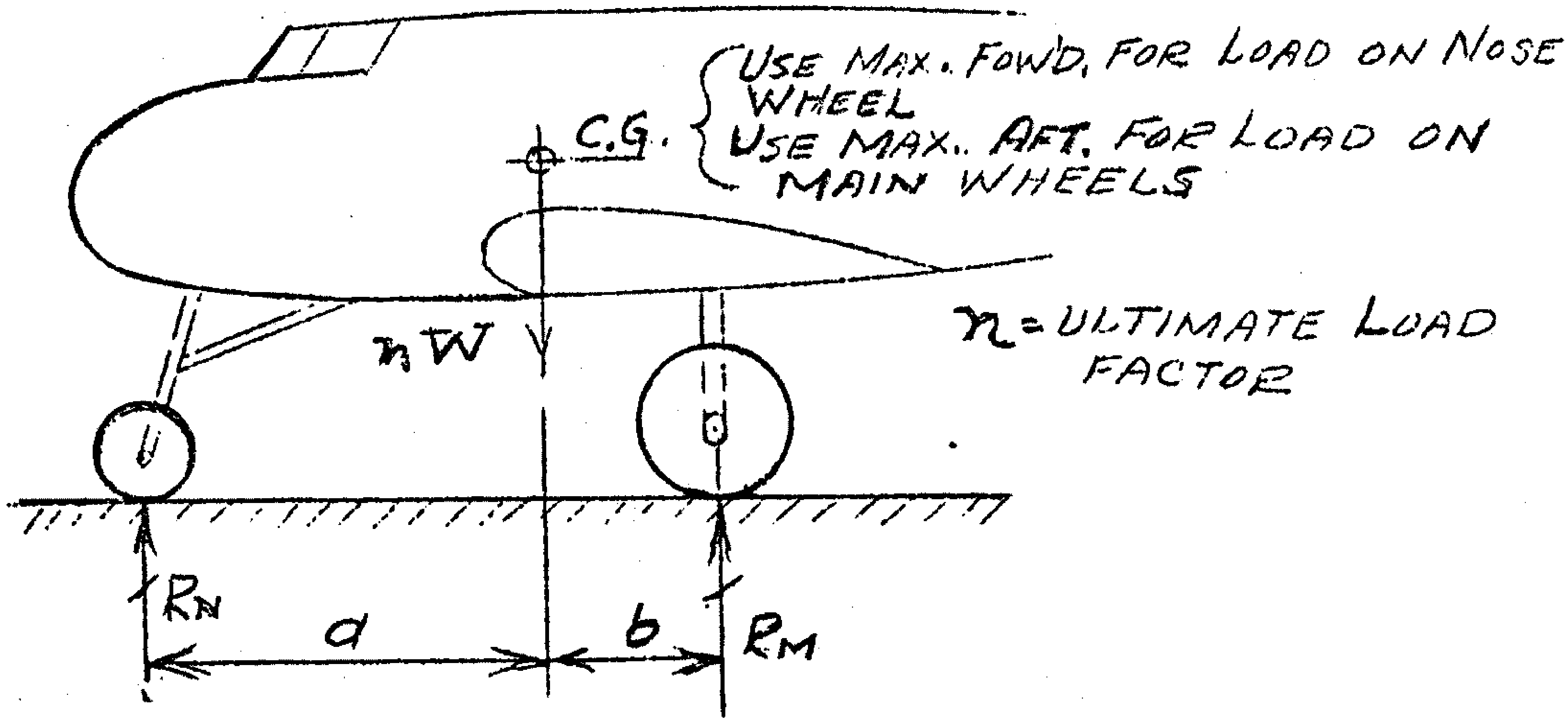

C.G.
USE MAX. FOWD. FOR LOAD ON NOSE WHEEL
USE MAX.. AFT. FOR LOAD ON MAIN WHEELS
nW
n = ULTIMATE LOAD FACTOR
R_N
a
b
R_M

FIG.4. LANDING WITH VERTICAL REACTIONS:

$$R_N = \frac{(nW)(b)}{a + b}, \qquad R_M = \frac{(nW)(a)}{a + b}$$

Net design load on nose wheel equals
R_N - (n)(unsprung weight of nose wheel)

*****

Net design load on two main wheels equals
R_M - (n)(unsprung weight of main chassis)

Where the dead weight of the landing gear is high, as on a large airplane, it is sometimes advisable to calculate the dead load at the panel points at which the landing gear dead weight is carried and account for it item by item. It is of course permissible and conservative to neglect the effect of the landing gear dead weight in computing the landing loads in the chassis members.

*****

2. Three Wheel Landing with Inclined Reactions.

The minimum limit load factor is the same as that for condition 1. The resultant of the ground reactions is a force lying in the plane of symmetry and passing through the center of gravity of the airplane less chassis. The value of the vertical dead weight force for a load factor of one, (1), is equal to the weight of the airplane less landing gear. The horizontal component of the dead weight factor acts forward and is 25 per cent of the vertical component. The ground reaction opposing this component is 25 per cent acting aft. The total force acting on the nose gear and main gear is divided so that the resultant moment on the airplane is zero. The shock struts and tires are deflected the same as for condition 1. The most critical center of gravity positions must be investigated. These are again forward c.g. for the nose gear and aft c.g. for the main gear. The loads on the main gear are equally divided between wheels. See Fig. 5. The minimum factor of safety is 1.5. This condition represents a landing with considerable forward speed, decelerated by contact with the ground.

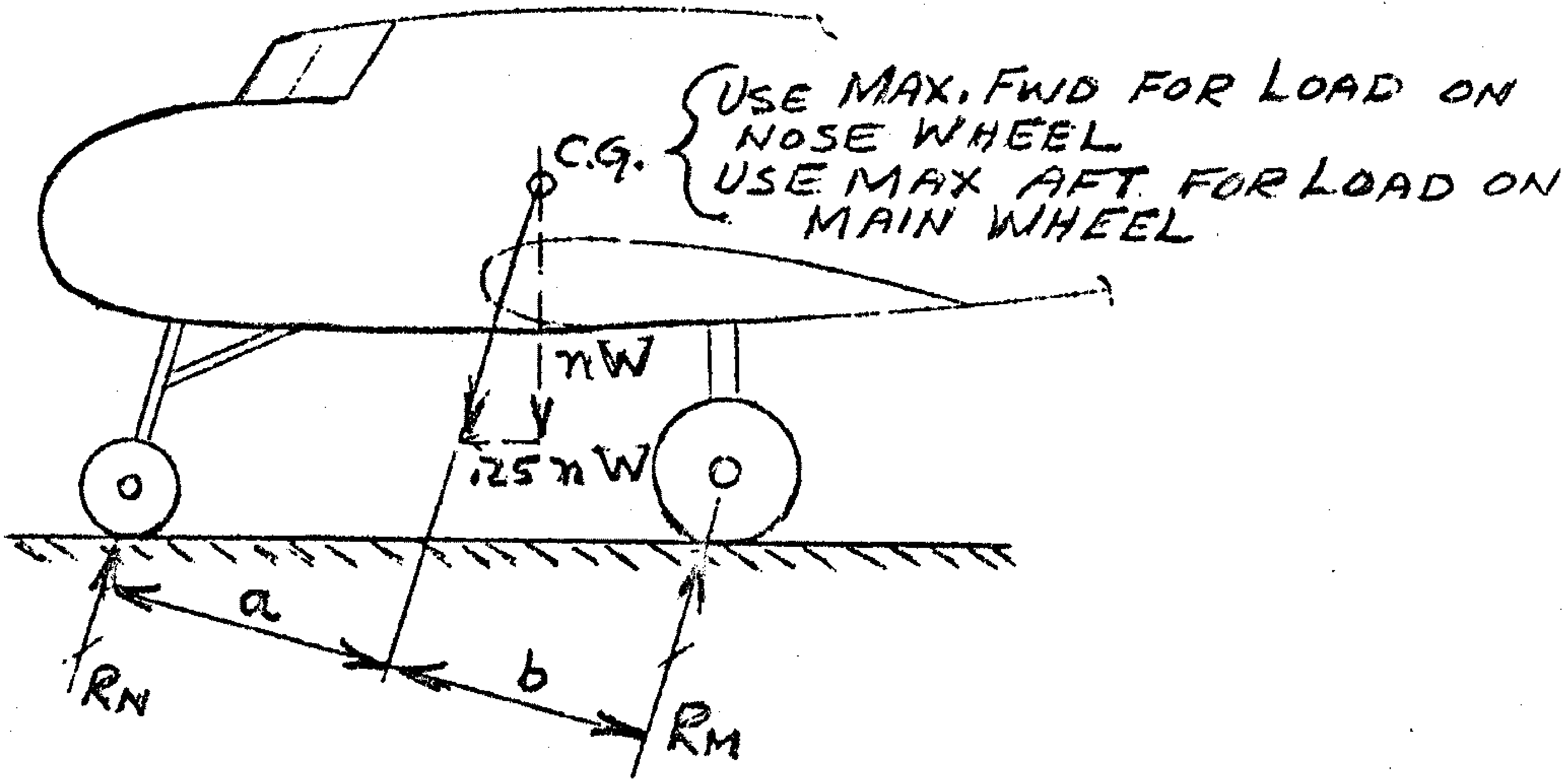

USE MAX. FWD FOR LOAD ON NOSE WHEEL
USE MAX AFT FOR LOAD ON MAIN WHEEL
C.G.
nW
.25 nW
a
b
R_N
R_M

FIG.5. 3-WHEEL LANDING WITH INCLINED REACTIONS

3. Two Wheel Landing with Vertical Reactions--Nose Up.

The minimum limit vertical load factor is the same as for conditions 1 and 2. The airplane is assumed to be in the extreme nose-up attitude, governed either by ground clearance at the tail or by the maximum angle of attack for the airfoil. This condition therefore represents a stalled landing with brakes off. The gross weight of the airplane less the weight of the main chassis gear is assumed to act at the rear wheels in a direction perpendicular to the ground line. The total load is to be divided equally between the two rear wheels.

A couple will be set up between the dead weight and the ground reactions which must be balanced by the rotational inertia forces on the airplane. See Fig. 6. Shock absorbers and tires are deflected the same as in conditions 1 and 2. The ultimate factor of safety is 1.5.

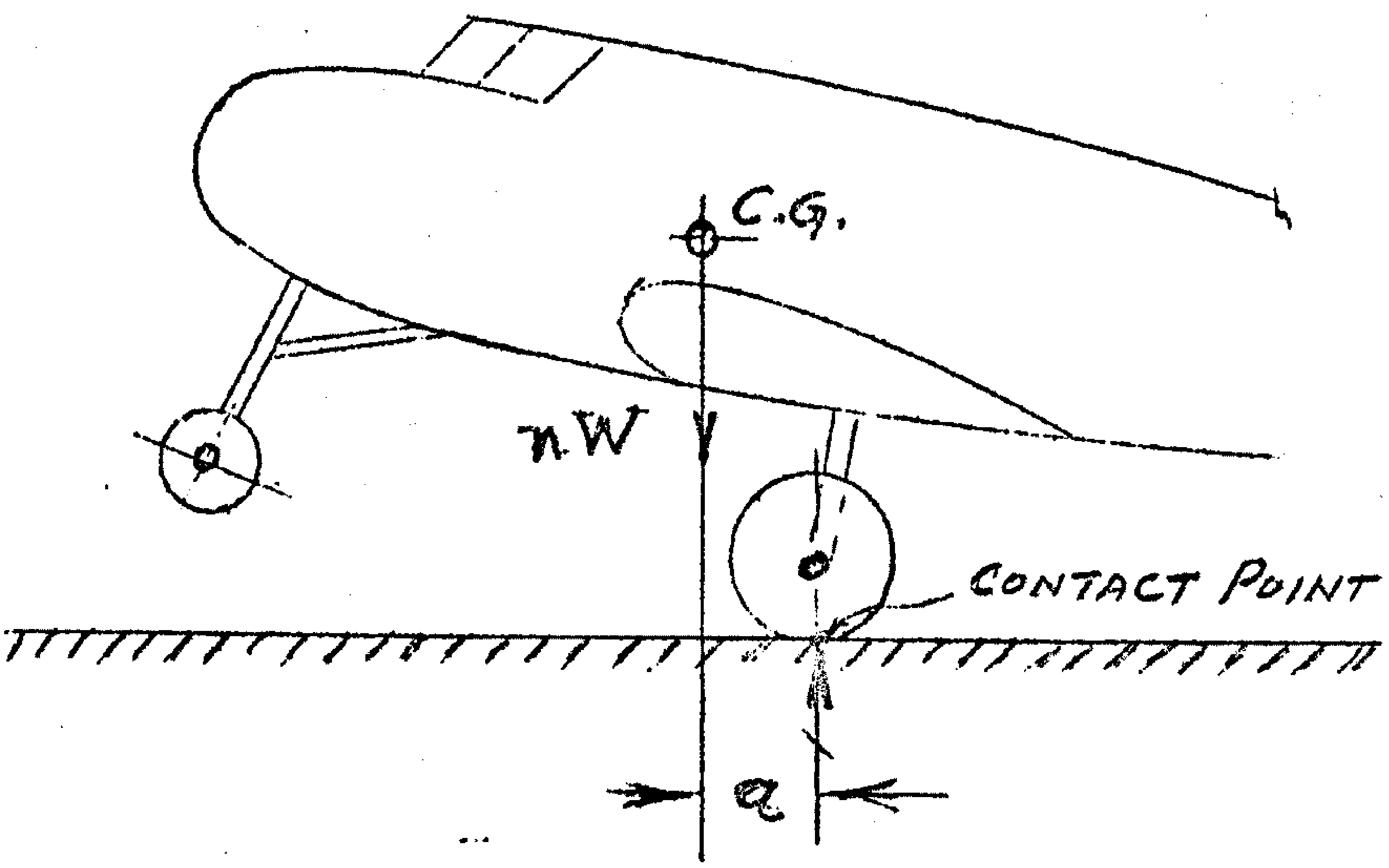


FIG. 6. TWO WHEEL LANDING
VERTICAL REACTIONS
NOSE UP

The torque causing rotation is (nW)(a). The center of rotation is the point of contact with the ground. The inertia forces may be calculated from the standard formulas to be found in any physics handbook. Items of mass may be grouped in a small number of major assemblies in calculating the angular acceleration and the damping inertia loads. The formulas to be used are as follows:

Torque = (Moment of Inertia)(angular acceleration)

$$T = I \alpha$$

$$\text{where } I = \sum \frac{Wd^2}{g}$$

For consistent units the torque should be in foot pounds, d in feet, W in lbs., and g is the acceleration of gravity in ft. per sec. per sec. Given the angular rate of acceleration it is a straight forward process to obtain the maximum accelerations on dead weight items in the airplane. These will of course be maximum at points farthest removed from the point of rotation. As far as the design of the landing gear is concerned the actual angular acceleration and rotational inertia are not required, the necessary forces necessary to balance the moment on the landing gear being supplied as reaction couples at the support points. Shock absorbers and tires are deflected the same as for conditions 1 and 2.

4. Two-Wheel Landing with Inclined Reactions--Nose Up.

The load factor is the same as for conditions 1, 2, and 3. The airplane is in the extreme nose up position shown in Fig. 6. The resultant force on the main wheels acts in the direction shown in Fig. 5, (25% aft component), and is equal to the gross weight of the airplane less the unsprung weight of the rear gear, at one load factor. The total ground reaction is divided equally between the two rear wheels. The resultant moment on the airplane is balanced by inertia forces, as in condition 3. The shock absorbers and tires are deflected the same amount as for condition 1. This condition represents a nose-up two wheel landing with brakes. The usual factor of safety of 1.5 is maintained.

5. Two-Wheel Landing with Inclined Reactions--Nose Down.

The load factor is the same as for the four previous conditions. The airplane is considered to be in a nose down attitude with the front wheel just clearing the ground. The resultant forces at the ground act with 25% aft component as in Fig. 7. It will be noted that the basic (one load

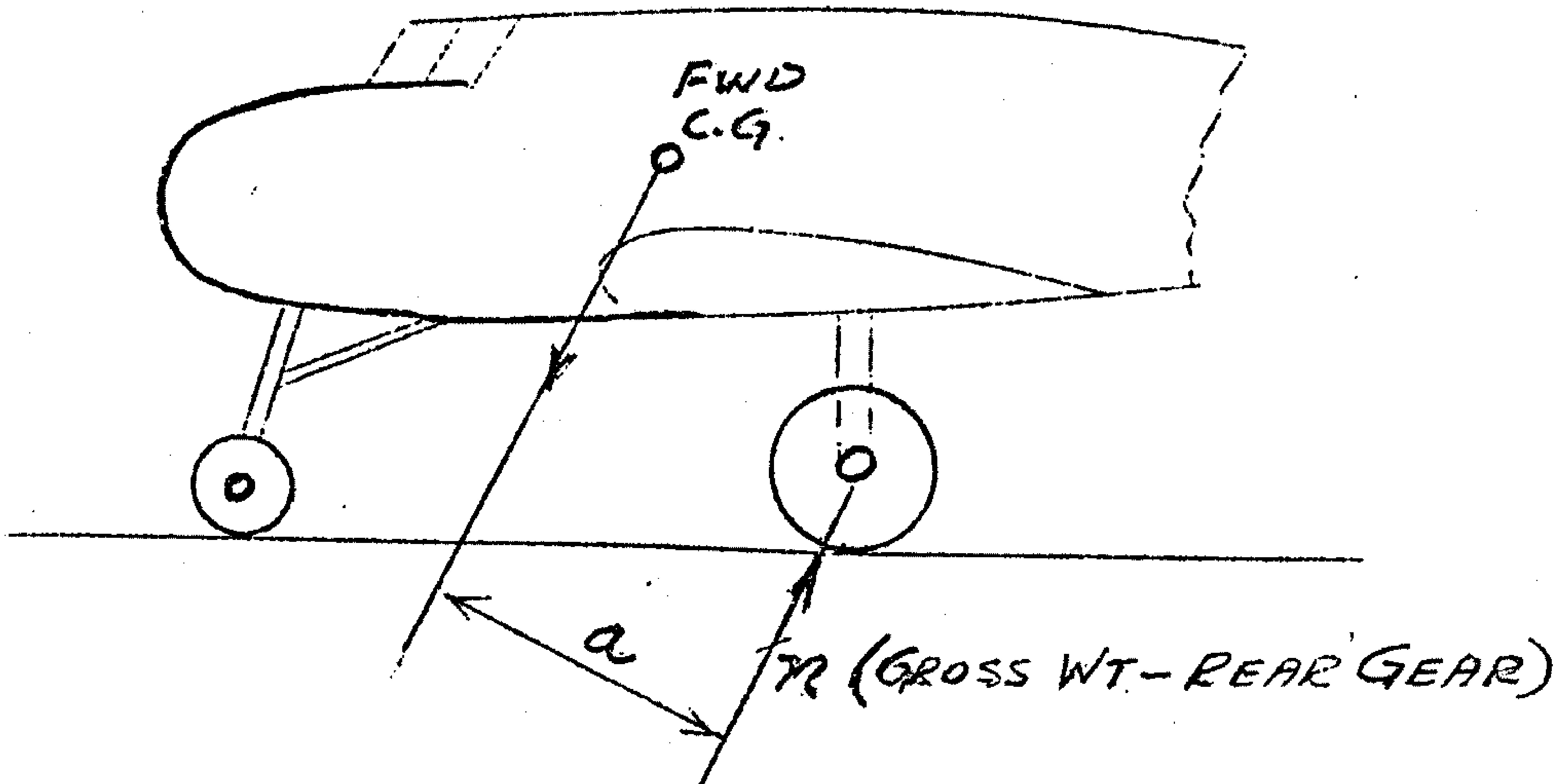


FIG.7. TWO WHEEL LANDING WITH INCLINED REACTIONS. NOSE DOWN

factor) weight is that of the ship less rear gear. There is a couple between the wheel reactions and the center of gravity of the airplane which is balanced by the rotational inertia of the airplane. Shock absorbers and tires are deflected as in condition 1. The critical center of gravity position must be investigated. This will be maximum forward c.g. for highest rotational accelerations. This condition represents the transition point from condition 4 to condition 2 when the nose wheel is just ready to contact the ground. The usual factor of safety of 1.5 is maintained.

6. Two-Wheel Landing with Brakes--Nose Down.

The minimum limit load factor is 1.33. The minimum design load factor is therefore 2.0. The airplane is in the attitude shown in Fig.7 but the gross weight of the airplane less the rear gear is assumed to act at the rear wheels in a direction perpendicular to the ground line. In addition there is a horizontal aft component of .55 times the vertical component which is applied at each wheel at the point of contact with the ground. The total load is divided equally between the two rear wheels. The resultant moment is balanced by inertia forces on the airplane. The deflection of the shock absorbers is considered to be the same as for condition 1, but the tires are assumed to be deflected not more than one-fourth their nominal diameter. Factor of safety = 1.5.

7. Side Drift Landing.

The load factor is the same as for condition 1. The attitude of the airplane, the vertical components of the landing gear reactions, and the deflections of the shock absorbers and tires are the same as for condition 1. In addition a horizontal aft component and a side component, each equal to .25 times the vertical component, are applied at each wheel at the point of contact with the ground. The resultant moment on the airplane is balanced by inertia forces. The factor of safety is 1.5. The side forces act inward on one main wheel and outward on the other. The side load on the nose wheel acts in the same direction as the side loads on the main wheels.

8. Side Drift Landing with Brakes.

The minimum limit load factor is 1.0. The attitude of the airplane, the static ground reactions (one load factor reactions) on the front and rear gear, and the deflections of the shock absorbers and tires are the same as for condition 1. However the total load on the rear gear is assumed to act on one wheel in conjunction with an inward side component .75 times the vertical component and an aft component along the ground line of .55 times the vertical component. The side load on the nose gear is .75 times the vertical component and acts in the same direction as the side load on the main gear. (Thus one rear wheel is not loaded). The resultant moments on the airplane are assumed balanced by inertia forces. The minimum ultimate factor of safety is 1.5. This condition represents a short turn with one main wheel braked and the other just swung clear by centrifugal force, or a side slip landing with brakes. See Fig. 8.

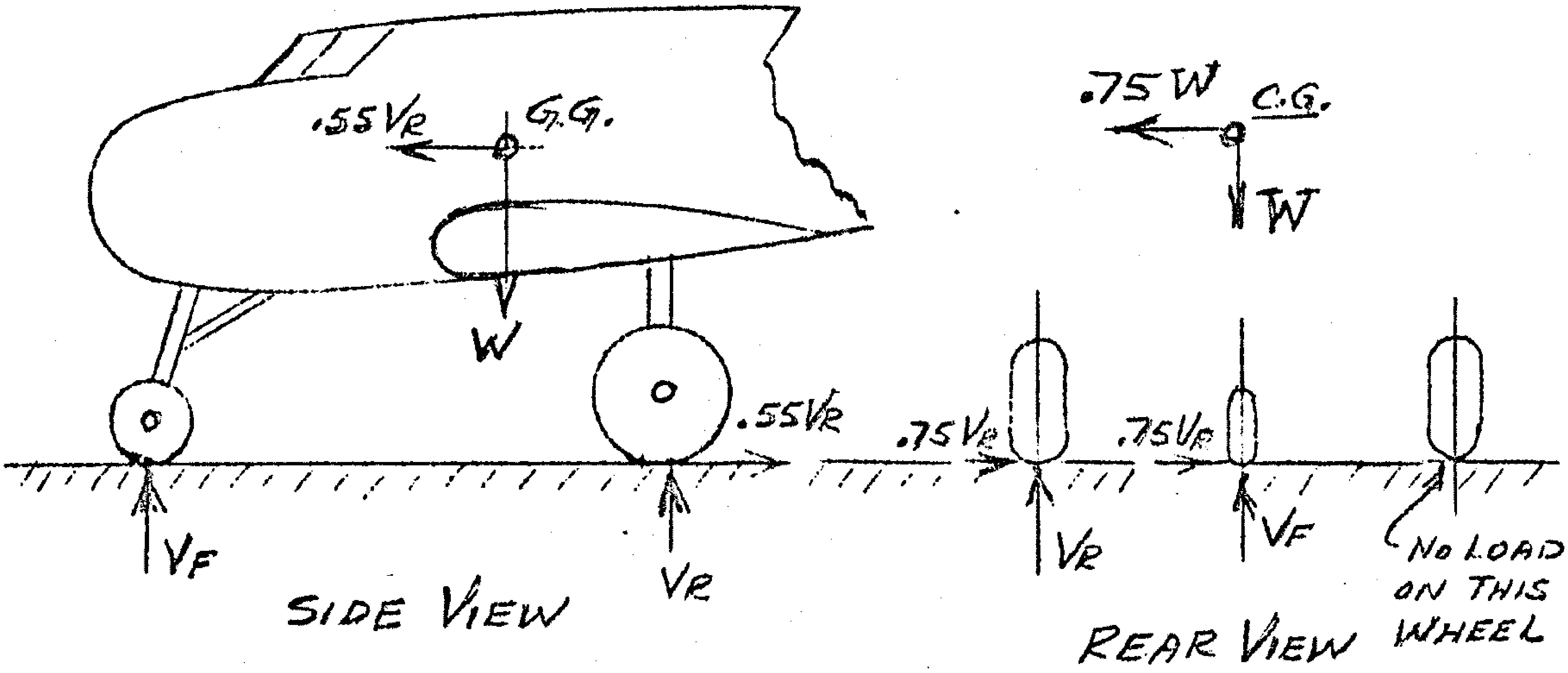


FIG. 8. LIMIT LOADS FOR SIDE DRIFT LANDING WITH BRAKES:

9. One-Wheel Landing.

An investigation of the fuselage structure is required for a one-wheel landing in which only those loads obtained on one side of the fuselage in condition 5 are applied. This condition is intended to design the fuselage for wracking loads in a one-wheel landing. The loads on one side of the main landing gear are exactly as for condition 5, while the other side is not loaded. The loads acting at the center of gravity of the airplane are therefore one-half of those for condition 5.

ADVANCED STRESS ANALYSIS

ASSIGNMENT 13

Assignment 13

DESIGN LOADS FOR CANTILEVER LANDING GEAR OF TRANSPORT AIRPLANE

In this assignment we will compute the design loads for a landing gear suitable for the 20,000 lb. transport airplane used as an example in this course. All critical conditions will be considered. The method illustrated completely covers the analysis requirements for a conventional landing gear, using a tail wheel. Tabulated data are used wherever possible because of the advantages of conciseness, and ease in checking the results. The landing gear and its support points are shown on Fig. 1.

Ground Loads

Reference: 6 CFR 04.241 - 04.245
ACM 04.24 - Fig. 24

Landplane Landing Conditions for
Tail Wheel Type Landing Gear (6 CFR 04.24)

Condition	Level	3-Point	Side
Reference 6 CFR 04	.241	.242	.243
Load Factor = n	$2.8 + \frac{9000}{20000 + 4000} = 3.18$	3.18	.667
Attitude	Prop. Axis Horiz.	3-Point	3-Point
Comp. Normal to Ground	nW (1) = 3.18 (20000 - 900) = 60,750	60,750	.667 (20000) = 13,340 (3)
Comp. Parallel with Ground	60,750 (.44) = 26,800# .25 x 60,750 = 15,200# Use (2)	Zero	.55 (13340) = 7,340 aft.
Side Comp. Parallel With Ground	Zero	Zero	nW = 13,340 inward
Shock Strut Deflection	50% Travel = 5" of Strut Extended	50% = 5" of Strut Extended	Static = 2" of Strut Extended
Tire Deflection	50% = 16" Roll. Radius	50% = 16" Rolling Radius	25% = 18½" Rolling Radius

Notes:
(1) W = Gross Wt. (20,000) - Chassis Wt. (900)
(2) With C.G. aft, the resultant would make an angle whose tangent = .44. This aft force is greater than the maximum required = .25V.
(3) Side landing loads given for one wheel only.

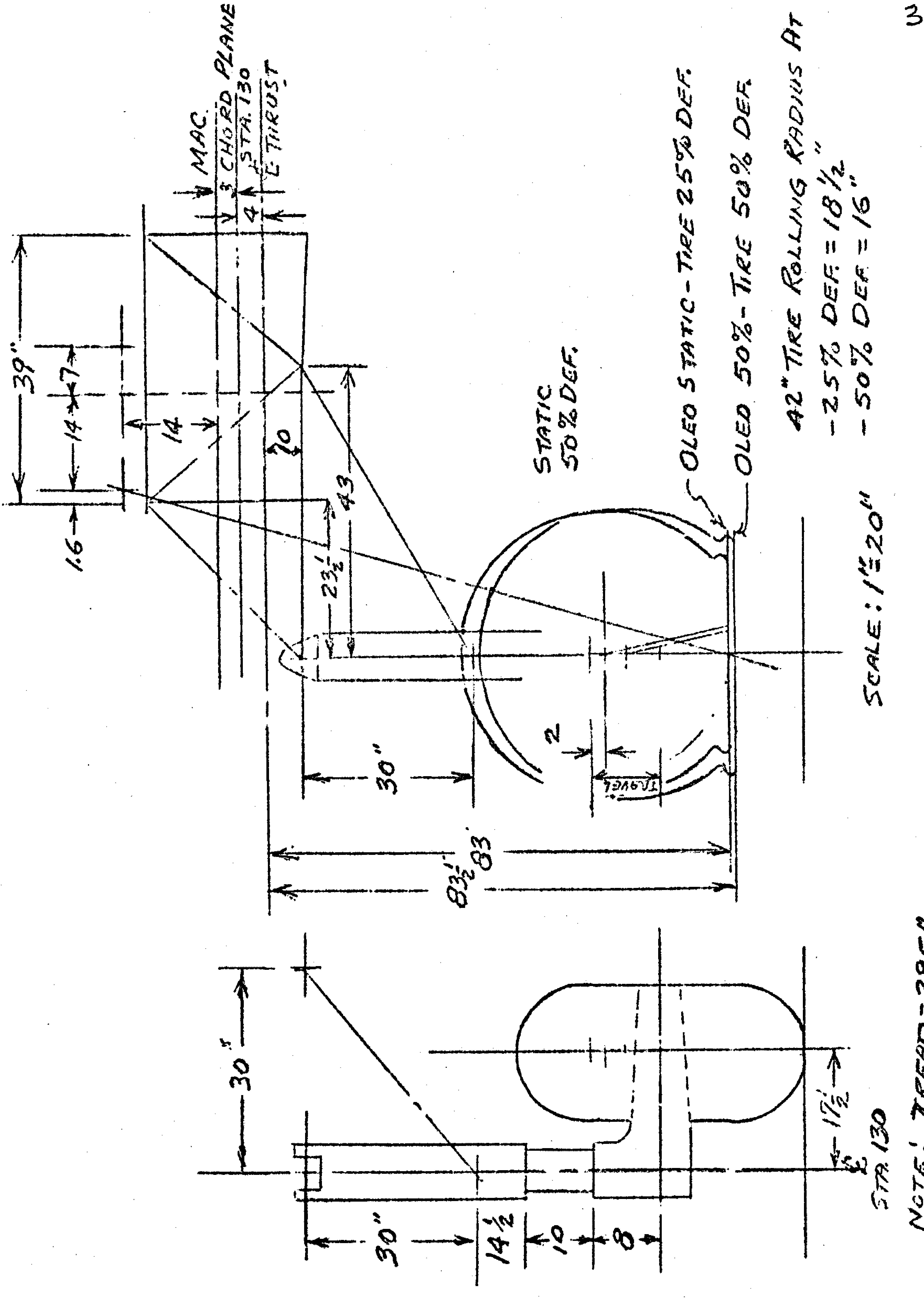

MAC.
3 CHORD PLANE
℄ STA. 130
℄ THRUST
4
39"
7
14
14
1.6
10
43
23½
STATIC
50% DEF.
OLEO STATIC - TIRE 25% DEF.
OLEO 50% - TIRE 50% DEF.
42" TIRE ROLLING RADIUS AT
-25% DEF. = 18½"
-50% DEF. = 16"
2
TRAVEL
30"
83½
83
SCALE: 1" = 20"
30"
30"
14½
10
8
17½
℄ STA 130
NOTE! TREAD = 295"

FIGURE 1

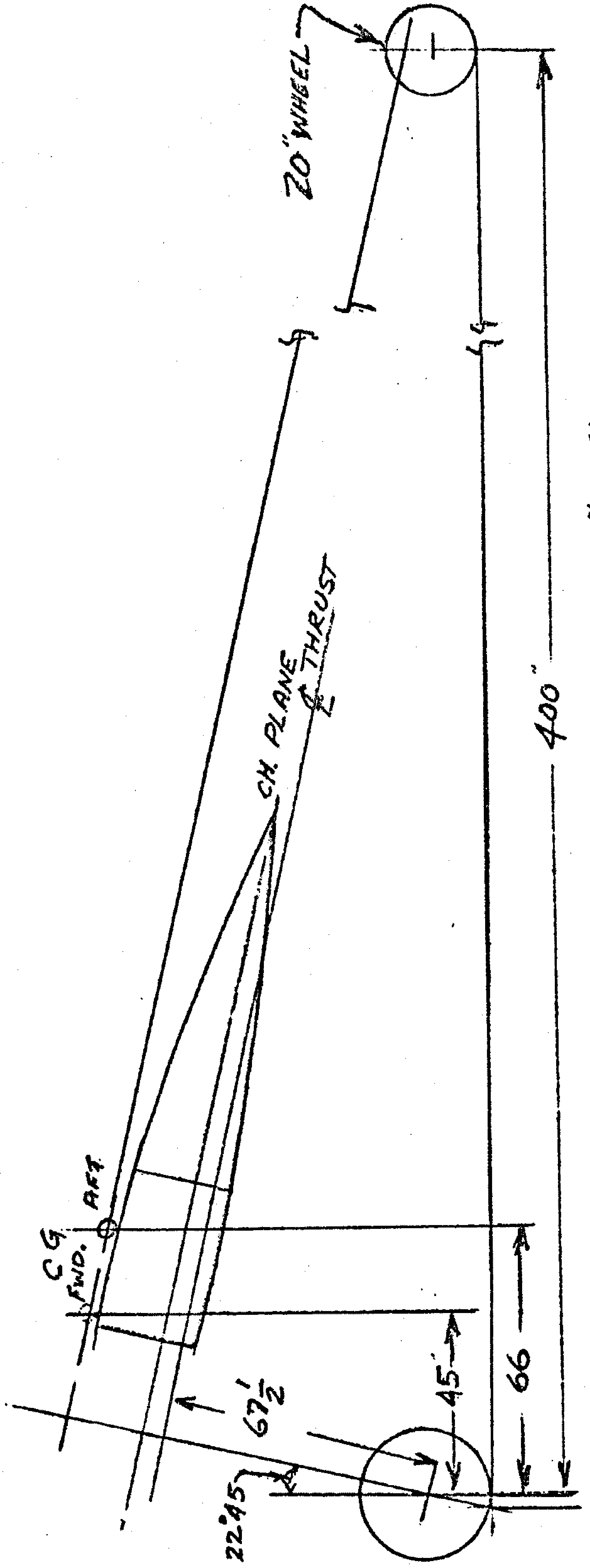

20" WHEEL
400"
SCALE 1"=40"
CH. PLANE
℄ THRUST
CG
FWD.
AFT
66
45
$67\frac{1}{2}$
22°45'

Fig. 2

Ground Loads on Main Chassis

Sign Convention to be Used

V = Forces parallel to shock strut.
Plus is Up.

H = Forces normal to ℄ shock strut and normal the plane of symmetry of Fuselage.
Plus is used as inboard in the following analysis.

S = Forces normal to ℄ shock strut and parallel to the plane of symmetry of fuselage.
Plus is Aft.

Level Landing

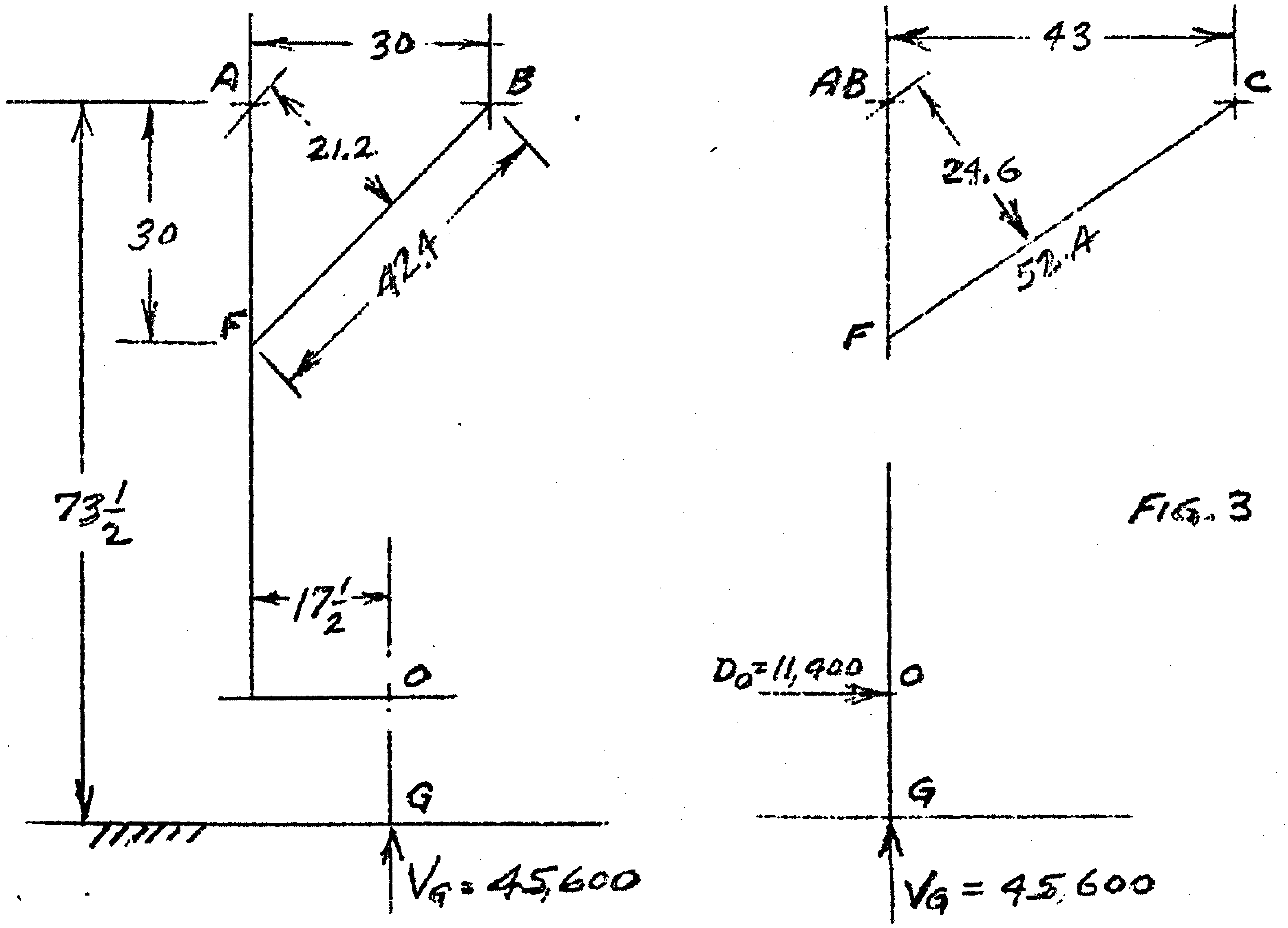


FIG. 3

Loads at C.G. are equally divided between main wheels.
Loads include 1.5 min. F.S. on the ultimate.

$V_G = 1.5 \times 60,750 \times \frac{1}{2} = 45,600$ lbs.

$D_O = 1.5 \times 15,200 \times \frac{1}{2} = 11,400$ lbs.

Level Landing

Side Brace Member (ΣM_A in V-S plane)

$P_{BF} = \frac{17.5}{21.2}(45600) = -37,550$ lbs. Comp.

Drag Brace Member (ΣM_A in V-D plane)

$P_{FC} = \frac{57.5}{24.6}(11400) = -26,600$ lbs. Comp.

Reactions

Point B

$V_B = 37,550 \times \frac{30}{42.4} = -26,500$ Down

$S_B = 37,550 \times \frac{30}{42.4} = +26,500$ Inbd.

Point C

$V_c = 26,600 \times \frac{30}{52.4} = -15,200$ lbs. Down

$D_c = 26,600 \times \frac{43}{52.4} = -21,800$ lbs. Fwd.

Point A

$V_A + V_c + V_B + 45,600 = 0$ $\quad$ $V_A = 26,500 + 15,200 - 45,600 = -3,900$ Down

$D_A + D_c + 11,400 = 0$ $\quad$ $D_A = 21,800 - 11,400 = +10,400$ Aft.

$S_A + S_B = 0$ $\quad$ $S_A = -26,500$ Out-Bd.

Moment about V axis at A = $M_V = 17.5 \times 11,400 = -199,500$"#

Loads on Shock Strut from Brace Members

Point F

$V_F = V_c + V_B = -26,500 - 15,200 = -41,700$ lbs. Down

$D_F = D_c = -21,800$ lbs. Fwd.

$S_F = S_B = +26,500$ lbs. Inbd.

Level Landing

Axle

Bearing Loads

$V_J = 45,600\ (\frac{5}{13}) = 17,500$

$V_K = 45,600 - 17,500 = 28,100$

$D_J = 11,400\ (\frac{5}{13}) = 4,390$

$D_K = 11,400 - 4,390 = 7,010$

Section J-J

$\text{Shear} = \sqrt{(17,500)^2 + (4390)^2} = 18,100$

Section K-K

$M = 18,100\ (13) = 235,000''\#$

$S = \sqrt{(45,600)^2 + (11,400)^2} = 47,000\#$

Section L-L

$M = 47,000 \times 10 = 470,000''\#$

$S = 47,000$ lbs.

Inner Oleo Tube - Section M-M

$M_S = 11,400 \times 16 = 182,500''\#$

$M_D = 45,600 \times 17.5 = 798,000''\#$

$M_{Resultant} = \sqrt{(182,500)^2 + (798,000)^2} = 817,500''\#$

Oleo Bearing Loads

$D_M = 11,400\ (\frac{49.5}{33.5}) = 16,900\#$ Fwd. on inner tube.

$D_N = 16,900 - 11,400 = 5,500$ lbs. Aft. on inner tube.

S_M & $S_N = 45600\ (\frac{17.5}{33.5}) = 23,700\#$ Inbd. & Outbd.

Shear at M-M = 11,400 lbs.

Section N-N

$$\text{Shear} = \sqrt{(23,700)^2 + (5,500)^2} = 24,300\#$$

Loads, Shears, and Moments in Shock Strut & Axle.

Level Landing

Moment Convention

M_V, M_D, M_S : V, D, S correspond to the axis about which moment acts.

Plus Moment = when moment is clockwise as you look in plus direction of axis.

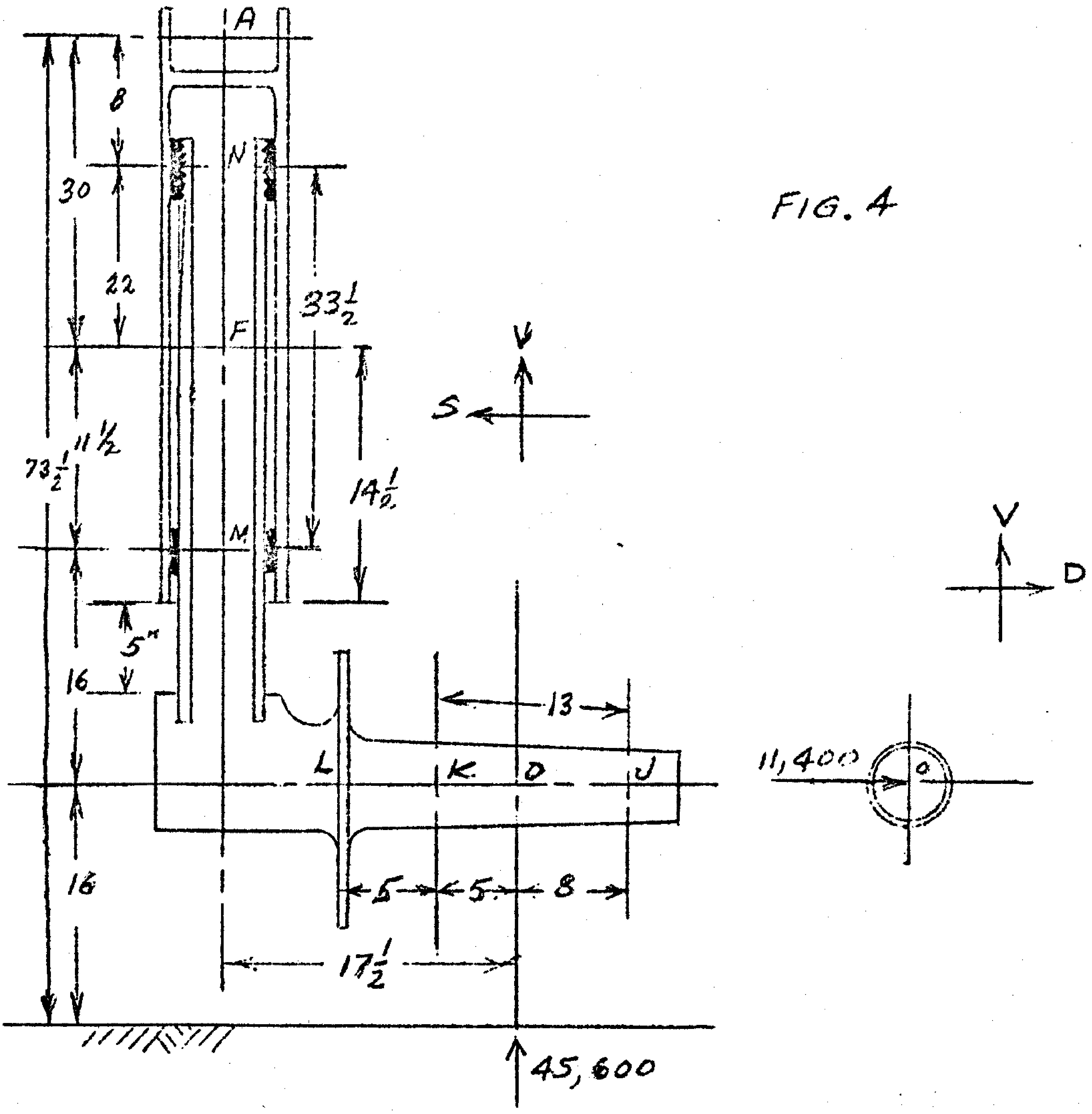


FIG. 4

Level Landing

Oleo Fluid

Load on column of oil = 45,600# Compression

Outer Oleo Tube

Section M-M

$$S = \sqrt{(D_M)^2 + (S_M)^2} = \sqrt{(16,900)^2 + (23,700)^2} = 29,100\#$$

Torque = M_V = 17.5 (11,400) = 199,500"#

Section F-F

Torque = 199,500"#

Shear = $S_F - S_M$ = 26,500 - 23,700 = 2,800# Inbd.

= $D_F - D_M$ = -21,800 + 16,900 = 4,900# Fwd.

Shear Resultant = $\sqrt{(2,800)^2 + (4,900)^2}$ = 5,640# Fwd-Inbd. (Above F-F)

M = 29,100 (11.5) = 334,500"#

Axial Load = V_F = 41,700# Tension

Section N-N

Torque = 199,500"#

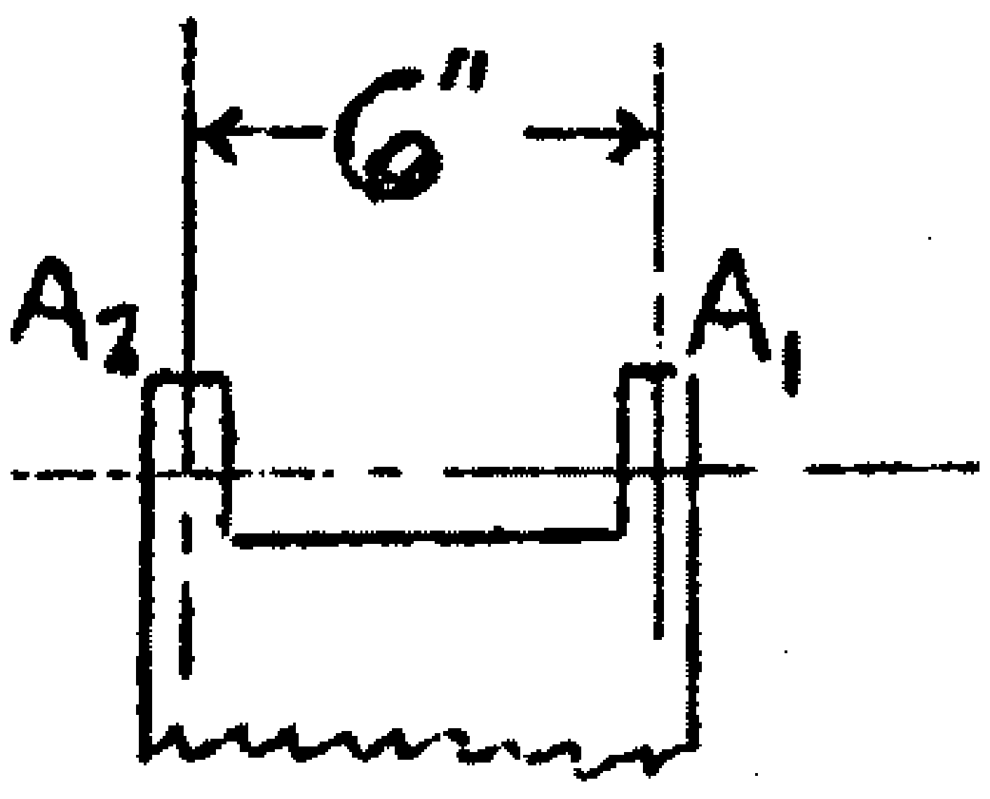
M_D = 23,700 (33.5) - 26,500 (22) = -211,000"#

M_S = 16,900 (33.5) - 21,800 (22) = -86,000"#

$M_{Resultant} = \sqrt{(211,000)^2 + (86,000)^2}$ = 228,000"#

Axial Load = V_F = 41,700# Tension

Lugs - Point A - 6" ℄ to ℄ Lugs.



Out-Bd. Lug - Torque taken by two Lugs.

$V_{A_1} = -3900 \times \frac{1}{2} = -1950\#$ Down

$S_{A_1} = -26,500 \times \frac{1}{2} = -13,250\#$ Out-Bd.

$D_{A_1} = 10,400 \times \frac{1}{2} - \frac{199,500}{6} = 5,200 - 33,250 = -27,050\#$ Fwd.

In-Bd. Lug

$V_{A_2} = -1950^{\#}$ Down

$S_{A_2} = -13{,}250^{\#}$ Outbd.

$D_{A_2} = 5200 + 33{,}250 = 38{,}450^{\#}$ Aft.

Three-Point Landing

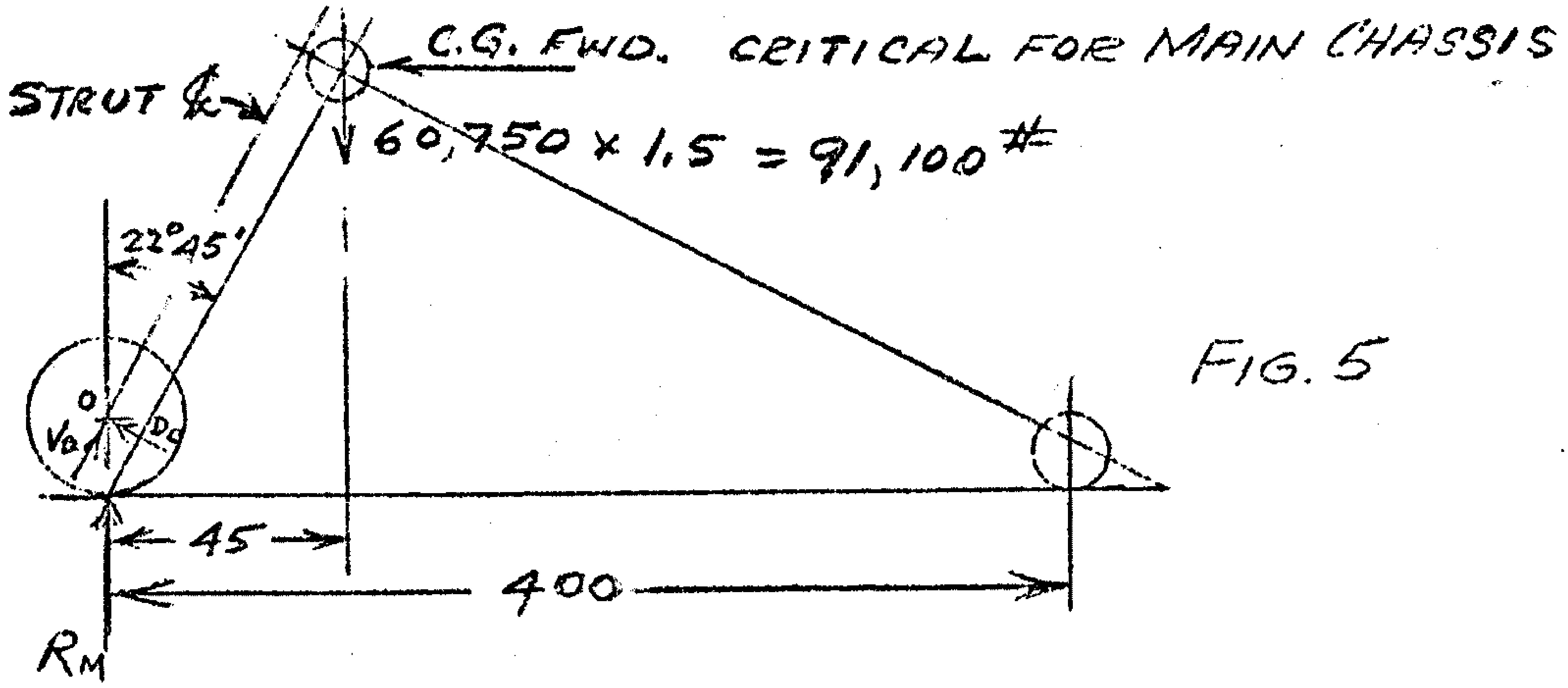


FIG. 5

$$R_M = 91{,}100 \left(\frac{400 - 45}{400}\right) \times \frac{1}{2} = 40{,}400\#$$

Loads equal to each wheel.

Loads at Axle

$$V_o = 40{,}400 \left(\cos \overset{.922}{22^\circ 45'}\right) = 37{,}300^{\#} \text{ Up}$$

$$D_o = 40{,}400 \left(\sin \overset{.387}{22^\circ 45'}\right) = -15{,}650^{\#} \text{ Fwd.}$$

*****

Line Diagram for Gear same as for Level Landing (See page 13-2)

Follow Similar Procedure as in Level Landing.

Side Brace Member

$$P_{BF} = 37{,}300 \left(\frac{17.5}{21.2}\right) = -30{,}800 \text{ lbs. Comp.}$$

Drag Brace

$P_{FC} = 15,650\ (\frac{73.5}{24.6}) = 46,700\#$ Tension

Reactions

Point B

$V_B = 30,800\ (\frac{30}{42.4}) = -21,700$ Down

$S_B = 30,800\ (\frac{30}{42.4}) = 21,700$ Inbd.

Point C

$V_c = 46,700\ (\frac{30}{52.45}) = 26,700$ Up

$D_c = 46,700\ (\frac{43}{52.45}) = 38,300$ Aft

Point A

$V_A = -26,700 + 21,700 - 37,300 = -42,300$ Down

$D_A = -38,300 + 15,650 = -22,650\#$ Fwd.

$S_A = -21,700$ Outbd.

Point F - Loads on Shock Strut

$V_F = 26,700 - 21,700 = 5000\#$ Up

$D_F = 38,300$ Aft

$S_F = 21,700$ Inbd.

*****

Axle

Bearing Loads

$V_S = 37,300\ (\frac{5}{13}) = 14,350$ Up

$D_J = 15,650\ (\frac{5}{13}) = 6,020$ Fwd.

$V_K = 37,300 - 14,350 = 22,950$ Up

$D_K = 15,650 - 6,020 = 9,630$ Fwd.

Section J-J

Shear = $\sqrt{(14,350)^2 + (6,020)^2}$ = 15,550# Up-Fwd.

Section K-K

Shear = $\sqrt{(37,300)^2 + (15,650)^2}$ = 40,400#

Moment = 40,400 x 5 = 202,000"#

Section L-L

M = 10 x 40,400 = 404,000"#

S = 40,400#

Inner Oleo Tube

Section M-M

Shear = 15,650 Fwd. Below M-M

M_S = 15,650 (16) = 250,500"#

M_D = 37,300 (17.5) = 653,000"#

$M_{Resultant}$ = $\sqrt{(653,000)^2 + (250,500)^2}$ = 698,000"#

Oleo Bearing Loads (On Inner Tube)

$D_M = 15,650 \left(\frac{49.5}{33.5}\right) = 23,100$ Aft.

D_N = 23,100 - 15,650 = 7,450# Fwd.

S_M = $S_N = 37,300 \left(\frac{17.5}{33.5}\right) = 19,450$# Inbd. ℄ Outbd.

Resultant Gearing Load = $\sqrt{(23,100)^2 + (19,450)^2}$ = 30,200#

Section N-N

Shear = $\sqrt{(7,450)^2 + (19,450)^2}$ = 20,850#

Oleo Fluid

Load = 37,300# Compression

Outer Oleo Tube

Section M-M

Shear = 30,200#

Torque = 17.5 (15,650) = M_V = -274,000"#

Section F-F

Torque = -274,000"#

Axial Load = -5,000# Compression

Shear (Below F-F) = 30,200#

Shear (Above F-F)

= $S_F - S_M$ = 21,700 - 19,450 = 2,250# Inbd.

= $D_F - D_M$ = -38,300 + 23,100 = -15,200# Fwd.

Shear Resultant (Above F-F) = $\sqrt{(2,250)^2 + (15,200)^2}$ = 15,350#

M = (11.5) 30,200 = 347,000"#

Section N-N

Torque = -274,000"#

Axial Load = -5,000# Comp.

Shear = 15,350#

M_D = 19,450 (33.5) - 21,700 (22) = 173,500"#

M_S = 23,100 (33.5) - 38,300 (22) = -68,000"#

$M_{Resultant}$ = $\sqrt{(68,000)^2 + (173,500)^2}$ = 186,500"#

Lugs - Point A

$V_{A_1} = -21,150\#$ Down

$S_{A_1} = -10,850\#$ Outbd.

$D_{A_1} = -\frac{22,650}{2} + \frac{274,000}{6} = 34,300\#$ Aft

$V_{A_2} = -21,150\#$ Down

$S_{A_2} = -10,850\#$

$D_{A_2} = -\frac{22,650}{2} - \frac{274,000}{6} = -56,985\#$ Fwd.

Side Landing Condition

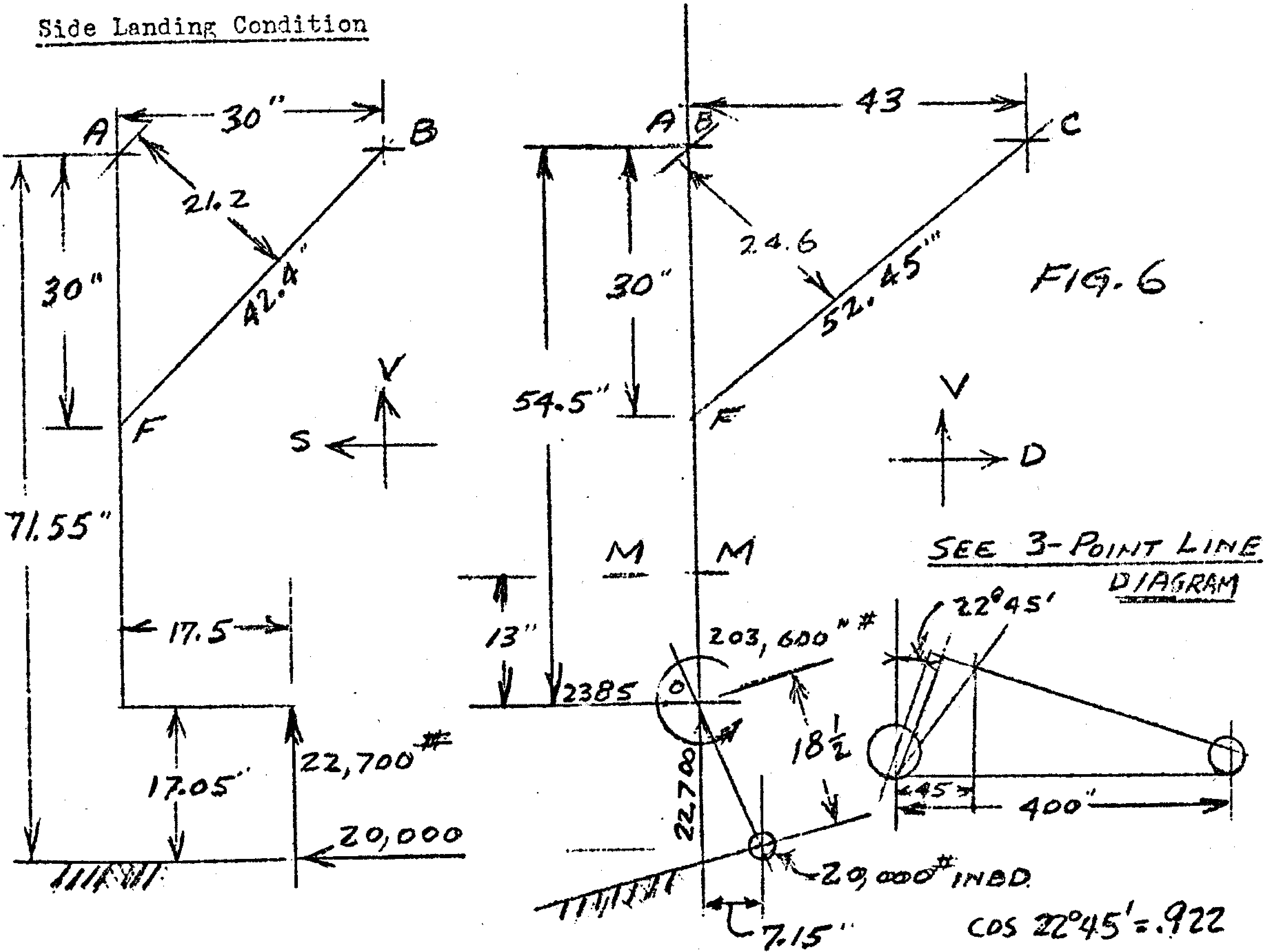


FIG. 6

SEE 3-POINT LINE DIAGRAM

COS 22°45' = .922
SIN 22°45' = .387

Loads Axle - Point O

$$M_S = 7,340\ (18.5) \times 1\tfrac{1}{2} = 203,600''\#$$

$$V_o = \left[13,340 \times (.922) + 7,340\ (.387)\right] \times 1.5 = 22,700\#$$

$$D_o = \left[7,340\ (.922) - 13,340\ (.387)\right] \times 1.5 = 2,385\#$$

$$S_G = 13,340 \times 1.5 = 20,000\#$$

$$M_V = 7.15\ (20,000) = 143,000''\#$$

Side Brace

$$P_{BF} = \frac{20,000\ (71.55) - 22,700\ (17.5)}{21.2} = 48,800\#\ \text{Tension}$$

Drag Brace

$$P_{FC} = \frac{2,385\ (54.5) + 203,600}{(24.6)} = -13,560\#\ \text{Comp.}$$

****

Reactions

Point B

$$V_B = 48,800\ \left(\frac{30}{42.4}\right) = 34,500 \quad \text{Up}$$

$$S_B = 48,800\ \left(\frac{30}{42.4}\right) = -34,500 \quad \text{Outbd.}$$

Point C

$$V_c = 13,560\ \left(\frac{30}{52.45}\right) = -7,760 \quad \text{Down}$$

$$D_c = 13,560\ \left(\frac{43}{52.45}\right) = -11,100 \quad \text{Fwd.}$$

Point A

$$V_A = -22,700 - 34,500 + 7,760 = -49,440\# \quad \text{Down}$$

$$D_A = -2,385 + 11,100 = 8,715 \quad \text{Aft.}$$

$$S_A = -20,000 + 34,500 = 14,500 \quad \text{Inbd.}$$

Point F

$V_F = 34,500 - 7,760 = 26,740\#$ Up

$S_F = -34,500\#$ Outbd.

$D_F = -11,100\#$ Fwd.

*****

Axle

Bearing Loads

$$V_J = \frac{-20,000\ (17.05) + 22,700\ (5)}{13} = -17,500\# \quad \text{Down}$$

$V_K = 40,200\#$ Up

$$D_J = \frac{(5)\ 2,385 + 143,000}{13} = 11,900\# \quad \text{Aft.}$$

$D_K = 2,385 - 11,900 = -9,515\#$ Fwd.

Section J-J

$$\text{Shear} = \sqrt{(17,500)^2 + (11,900)^2} = 21,150\#$$

Section K-K

$$\text{Shear} = \sqrt{(22,700)^2 + (2,385)^2} = 22,800\#$$

$M_D = (17.05)\ 20,000 - (5)\ 22,700 = 227,500"\#$

$M_V = 2,385\ (5) + 143,000 = 154,900"\#$

$$M_{Resultant} = \sqrt{(227,500)^2 + (154,900)^2} = 275,000"\#$$

Axial Load = 20,000# Comp.

Section L-L

Shear = 22,800#
Torque = 203,600"#

$M_D = (17.05)\ 20,000 - (10)\ 22,700 = 114,00"\#$

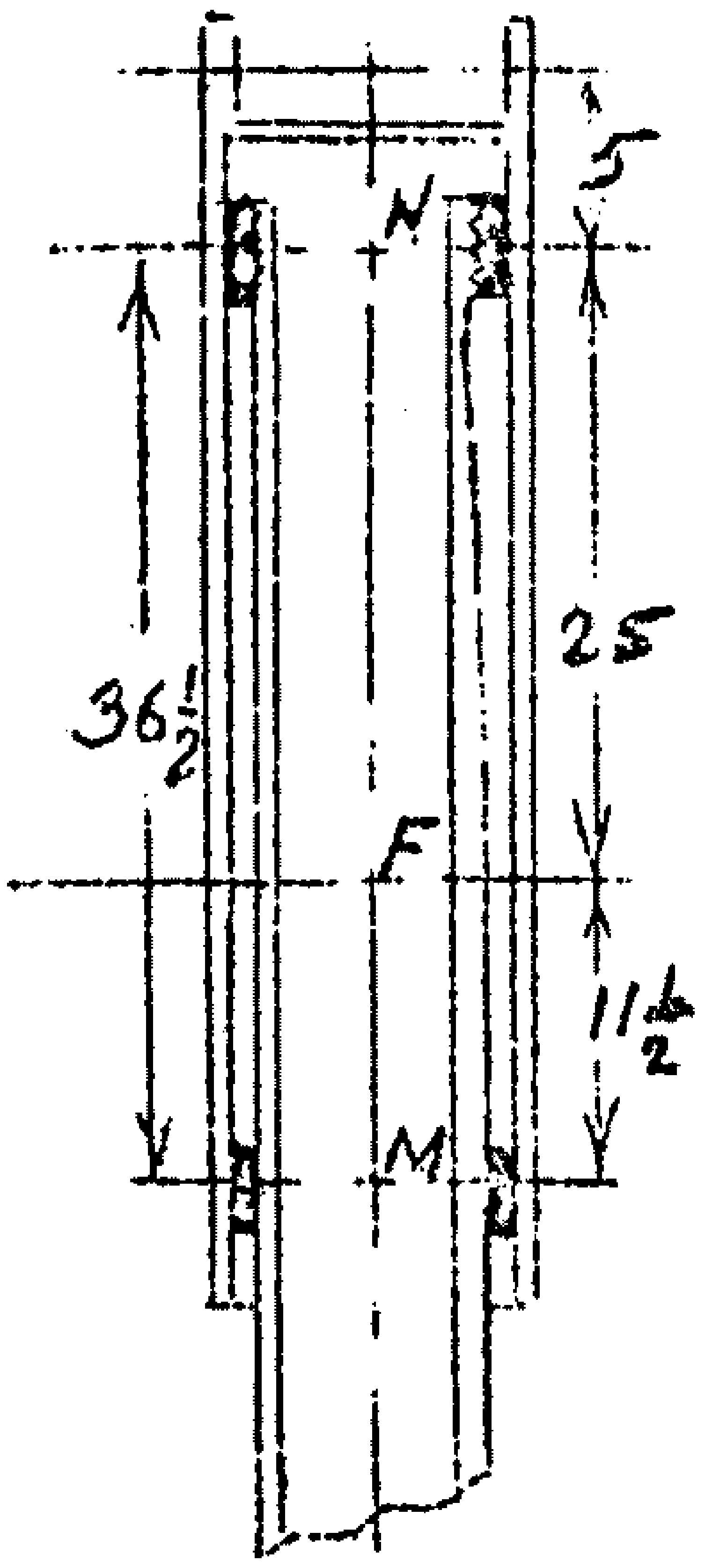
$M_V = 10\ (2,385) + 143,000 = 166,850"\#$

$$M_R = \sqrt{(114,000)^2 + (166,850)^2} = 202,000"\#$$

Axial Load = -20,000 Comp.

Inner Oleo Tube

Section M-M



STATIC POSITION

FIG. 7

$$\text{Shear} = \sqrt{(20{,}000)^2 + (2{,}385)^2} = 20{,}150\#$$

$$M_S = 203{,}600 + 2{,}385\ (13) = 234{,}600''\#$$

$$M_D = (30.05)\ 20{,}000 - (17.5)\ 22{,}700 = 205{,}000''\#$$

$$M_{Resultant} = \sqrt{(205{,}000)^2 + (234{,}600)^2} = 312{,}000\#$$

Oleo Bearing Loads on Inner Tube

$$D_M = \frac{203{,}600 + (49.5)\ 2{,}385}{36.5} = -8{,}810\# \quad \text{Fwd.}$$

$$D_N = -8{,}810 + 2{,}385 = 6{,}425 \quad \text{Aft.}$$

$$S_M = \frac{(66.55)\ 20{,}000 - (17.5)\ 22{,}700}{36.5} = -25{,}600\# \quad \text{Outbd.}$$

$$S_N = 25{,}600 - 20{,}000 = 5{,}600\# \quad \text{Inbd.}$$

Section N-N

$$\text{Shear} = \sqrt{(5{,}600)^2 + (6{,}425)^2} = 8{,}530\#$$

Oleo Fluid

Load on Column of Oil = 22,700# Comp.

Outer Oleo Tube

Section M-M

$$\text{Torque} = 143{,}000 + (17.5)\ 2{,}385 = 184{,}700''\#$$

$$\text{Shear} = \sqrt{(25{,}600)^2 + (8{,}810)^2} = 27{,}100\#$$

Section F-F

Shear (below F-F) = 27,100

$M = 27{,}100\ (11.5) = 312{,}000''\#$

Torque = M_V = 184,700"#

Axial Load = $V_F = -26{,}740\#$ Comp.

Section N-N

Torque = 184,700"#

Axial Load = -26,740# Comp.

Shear: $S_F - S_M = -34,500 + 25,600 = -8,900$#

$D_F - D_M = -11,100 + 8,810 = -2,290$#

Shear = $\sqrt{(8,900)^2 + (2,290)^2} = 9,200$#

$M_D = -34,500\ (25) + 25,600\ (36.5) = +72,500$"#

$M_S = +11,100\ (25) - 8,810\ (36.5) = -45,000$"#

Check: $M_D = 5 \times S_A = 5\ (14,100) = -70,500$"#

$M_S = 5 \times D_A = 5\ (8,715) = 43,600$"#

$M_{Resultant} = \sqrt{(70,500)^2 + (43,600)^2} = 83,000$"#

Lug - Point A

$V_{A_1} = \frac{1}{2}\ (-49,440) = -24,720$# Down

$S_{A_1} = \frac{1}{2}\ (14,500) = 7,250$# Inbd.

$D_{A_1} = \frac{1}{2}\ (8,715) + \frac{184,700}{6} = -35,130$# Fwd.

$V_{A_2} = -24,720$# Down

$S_{A_2} = 7,250$# Inbd.

$D_{A_2} = \frac{1}{2}\ (8,715) - \frac{184,700}{6} = 26,430$# Aft.

Problem Assignment

1. Check all computations in the text being sure that you thoroughly understand each set of data.

2. Make a summary of design axial loads, shears, moments and torsion by completing the table attached. This sheet will give a complete resume of load data for the landing gear.

Condition	Point A			Point B			Point C		
	V	S	D	V	S	D	V	S	D
Level									
Three-Point									
Side									

Condition	Side Brace	Drag Brace	Axle				Axle		
	Axial Load	Axial Load	Sec. J-J	Section K-K			Section L-L		
	P_{BF}	P_{FC}	Shear	Shear	Moment	Axial Load	Shear	Moment	Torsion
Level									
Three-Point									
Side									

Condition	Axle	Inner Oleo Tube				Outer Oleo Tube			
	Sec. L-L	Section M-M		Fluid	Sec. N-N	Section M-M		Section F-F	
	Axial Load	Shear	Moment	Load	Shear	Shear	Torsion	Shear	Moment
Level									
Three-Point									
Side									

Condition	Oleo Outer Tube					
	Section F-F		Section N-N			
	Torsion	Axial Load	Axial Load	Shear	Torsion	Moment
Level						
Three-Pt.						
Side						

AIRPLANE
STRESS
ANALYSIS

PART TWELVE

ADV. STRESS 14

ADV. STRESS 15

ADV. STRESS 16

ADVANCED STRESS ANALYSIS

ASSIGNMENT 14

Assignment 14

DESIGN OF LANDING GEAR MEMBERS

The previous assignment dealt with complete loads, shears, moments and torques on the landing gear members. This assignment will concern the detail design of the members themselves. Fig. 1 is a sketch of the design arrangement for the gear.

The allowable unit loads are based on the values given in ANC-5.

In the following example we adhere to our standard nomenclature which is as follows:

I = Moment of Inertia of a cross-section.

Q = static moment of a cross-section.

O.D. = outside diameter.

I.D. = inside diameter.

A = cross-sectional area.

$Z = \frac{I}{y}$ = section modulus.

f_b = fiber stress in bending.

f_c = fiber stress in compression.

f_s = fiber stress in shear.

R = stress ratio = $\frac{\text{Actual load per sq. inch}}{\text{Allowable load per sq. inch}}$

S = Shear on a cross-section.

P = Compression end load on a cross-section.

Only representative critical sections are investigated in the sample analysis to follow. In an actual analysis it would be necessary to investigate a great number of additional points to secure an adequate analysis. For the purpose of illustration only those most critical will be investigated. The sample calculations are representative of an actual analysis.

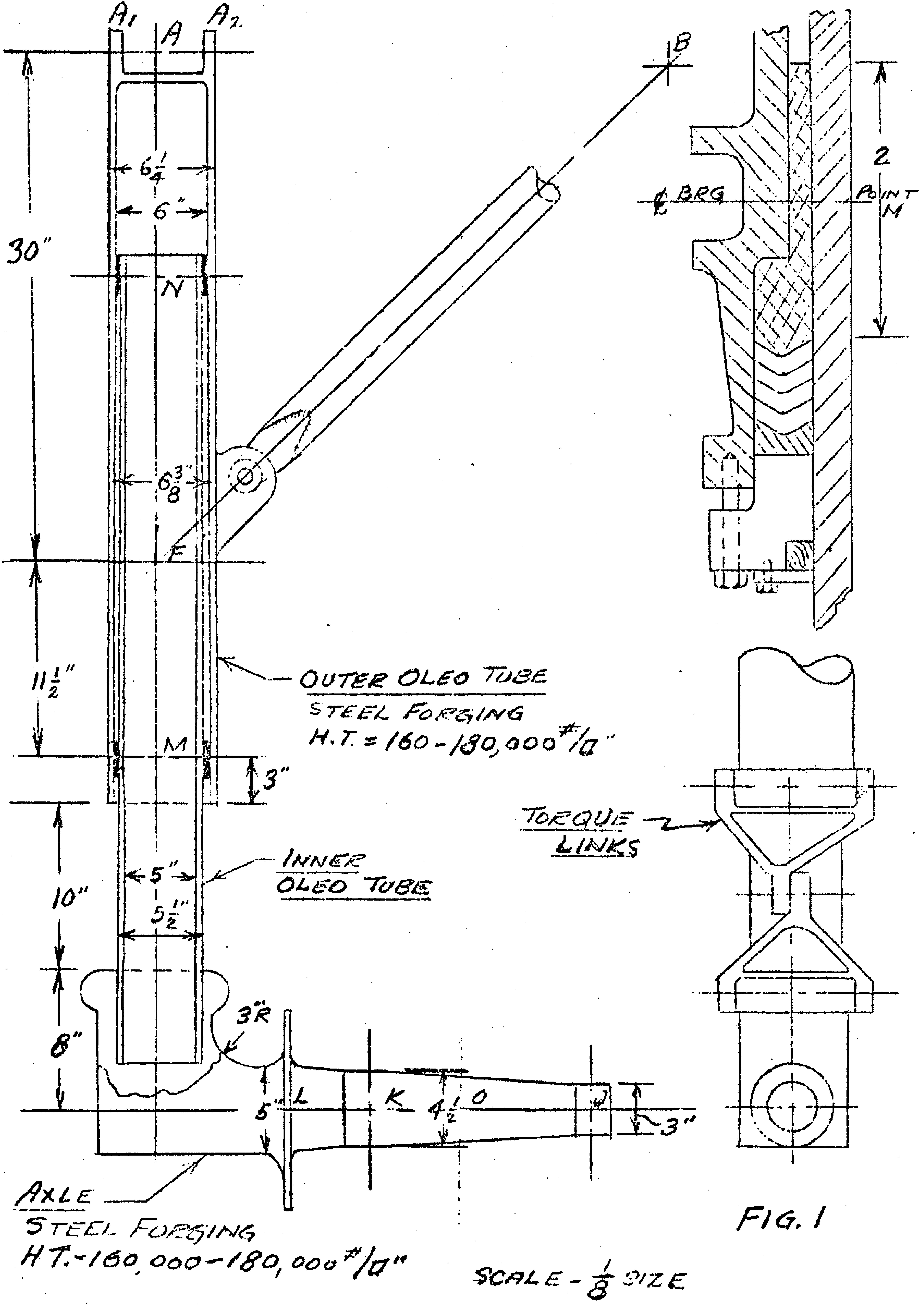


FIG. 1

SCALE - $\frac{1}{8}$ SIZE

Design of Axle - C.M. Steel Forging.

Section J-J

Load - Shear = 21150 - Side Landing

O.D. = 3"　　A = 1.396 sq. in.　　$Q = .632$
I.D. = 2-11/16"　　$I = 1.41$　　$t = 5/32"$

$$f_s = \frac{SQ}{bI} = \frac{21150 \times .632}{2 \times (\frac{5}{32}) \times 1.41} = 30,300 \text{ lbs. per sq. in.}$$

H.T. = 160,000 - 180,000　　All. Shear = 95,000 lbs. per sq. in.

$$M.S. = \frac{95,000}{30,300} - 1 = 2.13$$

Section K-K

Loads - Shear = 47,000 lbs. - Level Landing

$M = 275,000"\#$　　- Side Landing

$P = -20,000\#$

O.D. = 4½"　　$I = 5.035$　　$D/t = 28.8$
I.D. = 4-3/16"　　$A = 2.13$　　$Q = 1.47$
$Z = 2.24$

$$f_b = \frac{275,000}{2.24} = 123,000 \text{ lbs. per sq. inch}$$

$$f_c = \frac{20,000}{2.13} = 9,400 \text{ lbs. per sq. inch}$$

All. Mod. of Rupture = 167,500 lbs. per sq. inch. ANC-5, page 4-22.

$$R = \frac{123,000}{167,500} + \frac{9,400}{160,000} = .734 + .059 = .793$$

$$M.S. = \frac{1}{.793} - 1 = \underline{.26}$$

$$f_s = \frac{SQ}{bI} = \frac{47,000 \times 1.47}{2 \times (\frac{5}{32}) \times 5.035} = 44,000 \text{ lbs. per sq. inch}$$

$$M.S. = \frac{95,000}{44,000} - 1 = \underline{1.16}$$

Section L-L

Loads - $S = 22,800\#$
$T = 203,600''\#$
$M = 203,000''\#$ Side Landing
$P = -20,000\#$

$S = 47,000\#$
$M = 470,000''\#$ Level Landing

O.D. = 5" $I = 10.55$ $Q = 2.82$
I.D. = 4½" $A = 3.73$ $Z = 4.22$
$D/t = 20$

$$f_b = \frac{470,000}{4.22} = 111,500 \text{ lbs. per sq. inch - Level Landing}$$

All. Modulus of Rupture = 188,000 lbs. per sq. inch.

$$\text{M.S.} = \frac{188}{111.5} - 1 = \underline{.68}$$

$$f_s = \frac{47,000 \times 2.82}{\frac{1}{2} \times 10.55} = 25,200 \text{ lbs. per sq. inch - Level Landing}$$

****

Side Landing

$$f_b = \frac{202,000}{4.22} = 48,000 \text{ lbs. per sq. inch.}$$

$$f_{s_T} = \frac{TR}{2I} = \frac{203,600 \times 2.5}{2 \times 10.55} = 24,100 \text{ lbs. per sq. inch.}$$

$$f_{s_{\text{Horiz. Shear at N.A.}}} = \frac{SQ}{bI} = \frac{22,800 \times 2.82}{\frac{1}{2} \times 10.55} = 12,200 \text{ lbs. per sq. inch}$$

$$f_c = \frac{20,000}{3.73} = -5,360 \text{ lbs. per sq. inch. Comp.}$$

$$f_{s_{(\text{at N.A.})}} = 24,100 + 12,200 = 36,300 \text{ lbs. per sq. inch}$$

$$\text{M.S.} = \frac{95}{36.3} - 1 = 1.61$$

At Extreme Fiber

$$\tau_{max.} = \sqrt{(24,100)^2 + \left(\frac{48,000 + 5,360}{2}\right)^2} = 36,000 \text{ lbs. per sq. in.}$$

All. Tor. Mod. of Rupture = .655 (160,000) = 105,000 lbs./sq. in.
ANC-5

M.S. = L.G.

$$\sigma_{max.} = f_b - \tfrac{1}{2}(f_b + f_c) + \sqrt{(f_s)^2 + \left(\frac{f_b + f_c}{2}\right)^2}$$

$$= 48,000 - 26,680 + (36,000) = 57,300 \text{ lbs. per sq. in.}$$

$$M.S. = \frac{188}{57.3} - 1 = L.G.$$

Design Inner Oleo Tube

Section M-M

Loads - S = 11,400# - Level Landing
M = 817,500"# - Level Landing
Fluid Column Load = 45,600# - Level Landing

O.D. = $5\tfrac{1}{2}$"
I.D. = 5"

I = 5.178
A = 4.12 sq. in.
Q = 3.48

t = 1/4
$D/t = \frac{5.5}{1/4} = 22.$
$L/D = \frac{41.5}{5.5} = 7.5$

$$f_b = \frac{817,500}{5.178} = 158,000\text{"\#}$$

$$f_s = \frac{11,400 \times 3.48}{5.178 \times \frac{1}{2}} = 15,350 \text{ lbs. per sq. inch}$$

Hoop Tension Fluid Pressure = $\frac{45,600}{\frac{\pi}{4}(5)^2}$ = 2,320 lbs. per sq. in.
Level Landing

$$f_{H.T.} = \frac{2,320 \times 5}{2 \times 1/4} = 23,200 \text{ lbs. per sq. inch}$$

$$\tau = \sqrt{(15,350)^2 + \left(\frac{158,000 - 23,200}{2}\right)^2} = 69,500 \text{ lbs. per sq. in.}$$

All. Tor. Mod. of Rupture = .62 (160,000) = 99,200 lbs. per sq. in.
ANC-5, page 4-12

$$M.S. = \frac{99,200}{69,500} - 1 = \underline{.43}$$

All. Bending Modulus of Rupture = 184,000 lbs. per sq. in.

$$M.S. = \frac{184,000}{158,000} - 1 = \underline{.16}$$

Section N-N

Loads - Shear = 24,300 lbs.

O.D. = 5 1/4 I = 6.612 in. Z = 2.52
I.D. = 5 A = 2.02 sq. in. f = 1.81
Q = 1.64

$$f_s = \frac{24,300 \times 1.64}{.25 \times 6.61} = 24,100 \text{ lbs. per sq. in.}$$

$$M.S. = \frac{95,000}{24,100} - 1 = L.G.$$

Outer Oleo Tube

Section M-M

Loads - S = 30,200# 3 PT.
T = -274,000"#
Fluid Column Load = -37,300#

$$p = \frac{37,300}{\frac{\pi}{4}(5)^2} = 1,900 \text{ lbs. per sq. in.}$$

O.D. = 6 1/4" I = 12 Q = 2.37
I.D. = 6" A = 2.43 D/t = 50

$$f_s = \frac{30,200 \times 2.37}{.25 \times 12} = 24,800 \text{ lbs. per sq. in.} \quad \left(\frac{SQ}{bI}\right) \quad \text{At N.A.}$$

$$f_{s(Torsion)} = \frac{274,000}{2 \times 12}(3.12) = 35,600 \text{ lbs. per sq. in.}$$

$$f_{s(Total)} = 35,600 + 24,800 = 60,400 \text{ lbs. per sq. in.}$$

$$M.S. = \frac{95,000}{60,400} - 1 = .57$$

$$f_{H.T.} = \frac{1,900 \times 6}{2 \times .125} = 45,600 \text{ lbs. per sq. in.}$$

$$\tau = \sqrt{\left(\frac{45,600}{2}\right)^2 + (60,400)^2} = 64,600 \text{ lbs. per sq. in.}$$

All. Tor. Mod. of Rupture = .5 (160,000) = 80,000 lbs. per sq. in.
D/t = 50 L/D = 4.9

$$\text{M.S.} = \frac{80,000}{64,600} - 1 = \underline{.24}$$

Section F-F

Loads - M = 347,000"#
P = -5,000# Comp.
T = 274,000"#
p = 1900 lbs. per sq. in.
S = 30,200 lbs. per sq. in. 3-Point Landing

O.D. = 6 3/8" A = 3.73 Q = 3.67
I.D. = 6" I = 17.7 D/t = 34

$$f_b = \frac{347,000}{17.7} \times 3.187 = 62,500 \text{ lbs. per sq. in.}$$

$$f_{s(\text{Torsion})} = \frac{274,000}{2 \times 17.7} \times 3.187 = 24,700 \text{ lbs. per sq. in.}$$

$$f_{s(\text{Horiz})} = \frac{SQ}{bI} = \frac{30,200 \times 3.67}{.375 \times 17.7} = 16,700 \text{ lbs. per sq. in.}$$

$$f_c = \frac{-5000}{3.73} = -1340 \text{ lbs. per sq. in.}$$

$$f_{H.T.} = \frac{1900 \times 6}{2 \times .187} = 30,400 \text{ lbs. per sq. in.}$$

$$\tau_{(\text{Max.})} = \sqrt{(24,700)^2 + \left(\frac{1340 + 62,500 + 30,400}{2}\right)^2} = 52,000 \text{ lbs. per sq. in.}$$

All Tor. Mod. of Rupture = .575 (160,000) = 92,000 lbs. per sq. in.

$$\text{M.S.} = \frac{92,000}{52,000} - 1 = .77$$

Section N-N

Loads - M = 186,500"#
S = 15,350"#
P = -5,000
T = -274,000"#
p = 1900 lbs. per sq. in.   3-Point Landing

O.D. = 6 1/4"   I = 12   A = 2.43
I.D. = 6"   Q = 2.37   D/t = 50
L/D = 4.8

$$f_b = \frac{186,500 \times 3.12}{12} = 48,500 \text{ lbs. per sq. in.}$$

$$f_c = \frac{-5000}{2.43} = -2060 \text{ lbs. per sq. in.}$$

$$f_{H.T.} = \frac{1900 \times 6}{.125 \times 2} = 46,400 \text{ lbs. per sq. in.}$$

$$f_s = \frac{SQ}{bI} = \frac{15,350 \times 2.37}{.25 \times 12} = 12,100 \text{ lbs. per sq. in.}$$

$$f_{s(\text{Torsion})} = \frac{274,000 \times 3.12}{2 \times 12} = 35,600 \text{ lbs. per sq. in.}$$

$$\tau = \sqrt{(35,600)^2 + \left(\frac{48,500 + 46,400 + 2,060}{2}\right)^2} = 60,000 \text{ lbs./sq. in.}$$

All. Tor. Mod. of Rupture = .5 (160,000) = 80,000

$$M.S. = \frac{80}{60} - 1 = .33$$

****

Side Brace Tube - C.M. Steel Tube - Welded Lug Ends
H.T. after Welding = 160,000 lbs. per sq. in.
All. = .80 x 160,000 = 128,000 lbs. per sq. in.
Use Column Yield Stress = 100,000 lbs. per sq. in.

Use Curves Columns
ANC-5, page 4-16

$P_{BF} = -37,350$ Use full length = 42.4" (conservative)

Tube 2" x .120"   = .666 $L/\rho = 63.8$ A = .7087 sq. in.

Allow. $P/A = 65,000$ lbs. per sq. in.

Actual $P/A = \frac{-37,350}{.7087} = -53,000$ lbs. per sq. in.

$$M.S. = \frac{65}{53} - 1 = \underline{.22}$$

$P_{BF} =$ 48,800 Tension $f_t = \frac{48,800}{.7087} = 69,000$ lbs. per sq. in.

$$M.S. = \frac{160}{69} - 1 = \underline{1.31}$$

Drag Brace Tube

C.M. Steel - Tube - Welded Ends (see previous page)

$P_{FC} = -26,600\#$
$46,700\#$ L = 52.4" (conservative)

2 1/4" x .083" A = .565 sq. in. P = .766 L/P = 68.4

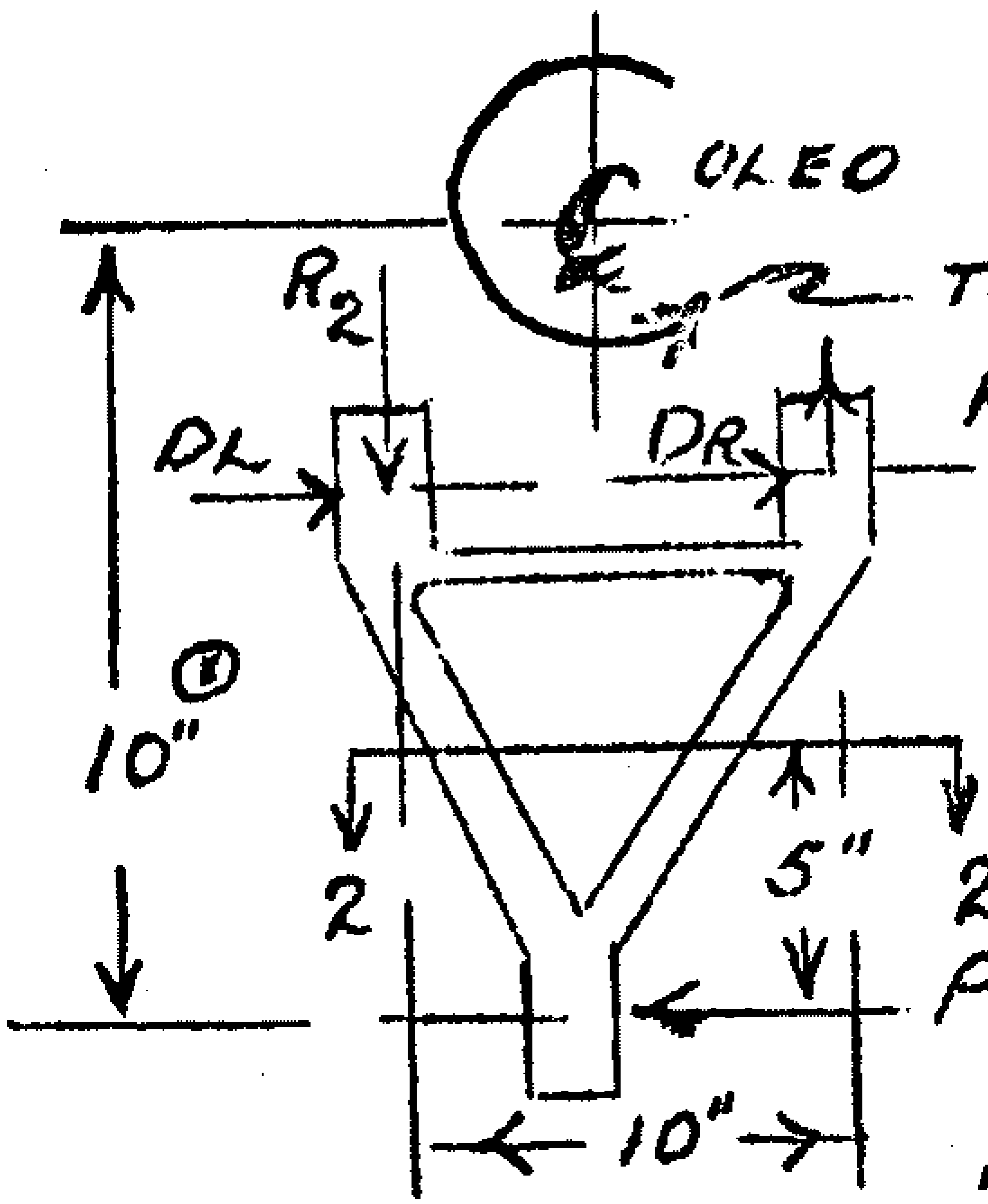
All. P/A = 57,800 lbs. per sq. in.

Actual $P/A = \frac{-26,600}{.565} = 47,000$ lbs. per sq. in.

$$M.S. = \frac{57,800}{47,000} - 1 = \underline{.23}$$

Torque Link - Alum. Alloy - 17ST Processed Billet Forging

ULT = 50,000 #/□" } with
Y.P. = 28,000 #/□" } grain
Elon. = 16%



TORQUE = -274,000"# MAX. - 3 PT. LANDING

① NOTE: THIS DISTANCE

$$\rho = \frac{274000}{10} = 27,400\text{"\#}$$

$$R_L = 27,400 \times \left(\frac{10}{10}\right) = 27,400$$

$$R_R = 27,400\#$$

$$D_L \text{ \& } D_R = \frac{27,400}{2} = 13,700\#$$

FIG. 2

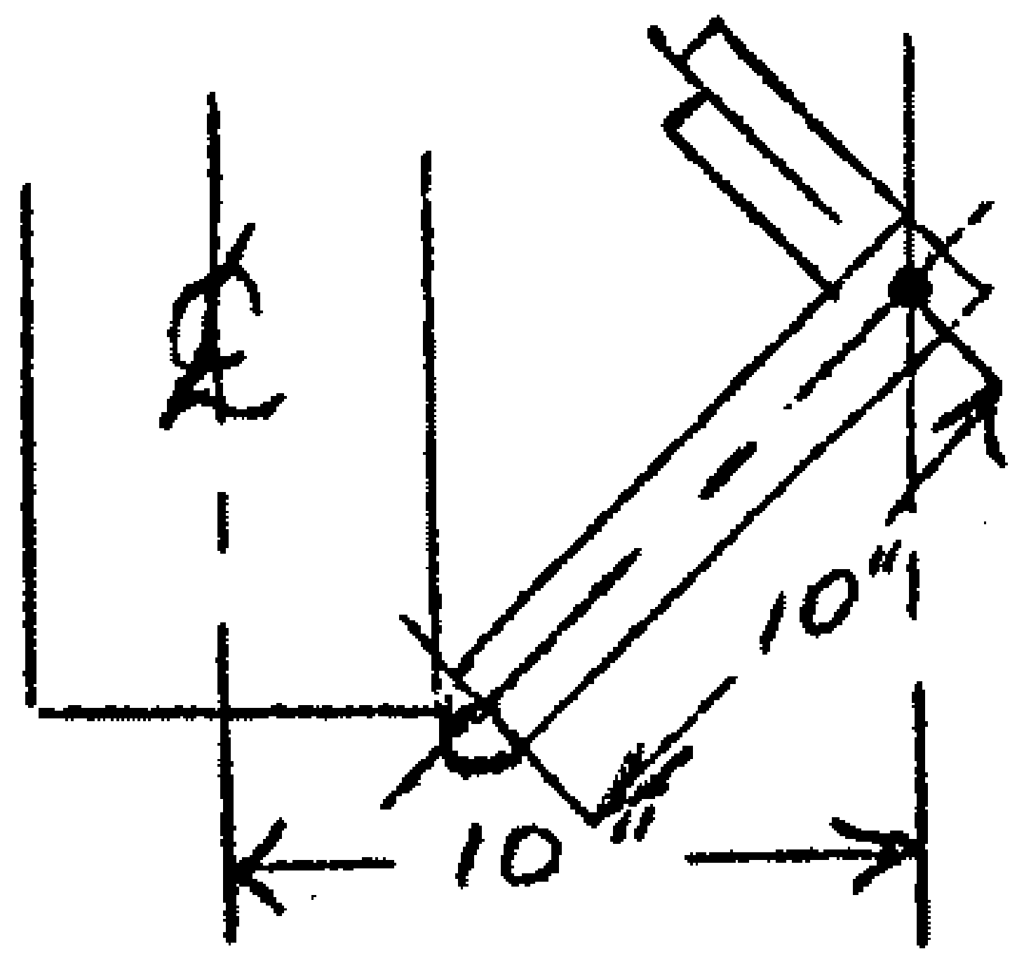


FIG. 3

Section Z-Z

Bending Mom. = 5 x 27,400 = 137,000"#
Shear = 27,400#

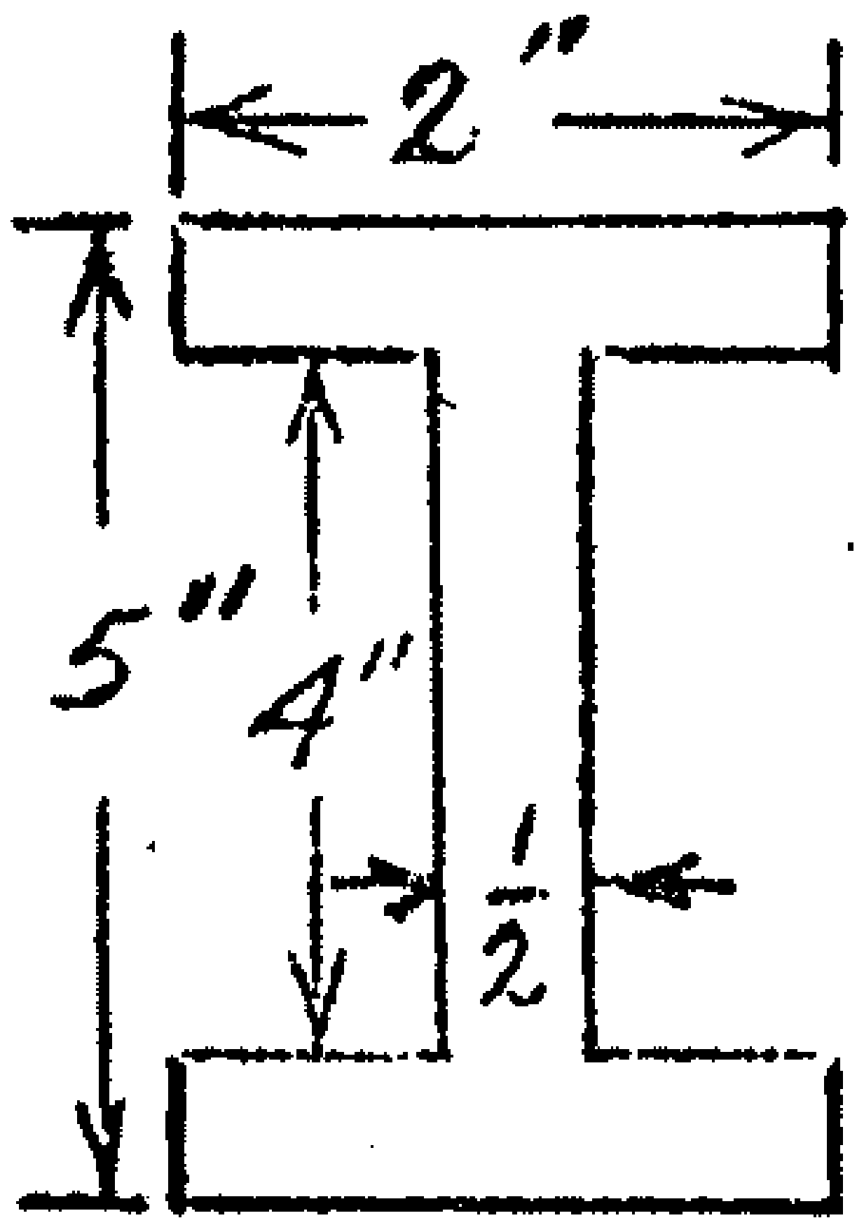


FIG. 4

$$I = \frac{1}{12} \times 2 \times (5)^3 - \frac{1}{12} \times 1.5 \times (4)^3 = 12.8 \text{ in.}^4$$

$$f_b = \frac{137,000}{12.8} \times 2.5 = 26,800 \text{ lbs. per sq. in.}$$

$$\text{M.S.} = \frac{1.5 \times 28,000}{26,800} - 1 = \underline{.57}$$

$$Q_{NA} = (2 \times \tfrac{1}{2})\ 2.25 + 2 \times \tfrac{1}{2} \times 1 = 3.25$$

$$f_s = \frac{27,400 \times 3.25}{.5 \times 12.8} = 13,900 \text{ lbs. per sq. in.}$$

All Shear = 30,000 lbs. per sq. in.

M.S. = L.G.

Problem Assignment.

(1) Make a complete check of the design of members given in the text. Note that for the heat-treated steels used in the landing gear, the Moduli of Rupture in shear, bending and torsion are used, based on the allowable values from ANC-5.

ADVANCED STRESS ANALYSIS

ASSIGNMENT 15

Assignment 15

ANALYSIS OF WELDED STEEL TUBE ENGINE MOUNT

Note: The method given for engine mount analysis is also applicable to a trussed fuselage.

1. Space Diagram.

The first step in the solution of a truss type engine mount is to draw a center line diagram of the members to scale. This type of structure is best solved graphically so that an accurate space diagram showing side, top, bottom and cross-sectional views is essential. For the purpose of graphic analysis the projections of the members in the side truss are used. It is then necessary to correct the loads in members which do not appear in their true length in a side view and to account for the effect of their side components in loading the top and bottom trusses.

2. Weight and Balance of Dead Load.

The second step in the solution involves the determination of the dead loads which are to be carried by the structure. For this purpose it is necessary to have the weights and locations of the different items which are to be supported by the truss. The weights and centers of gravity of purchased parts such as propellers, engines, engine accessories, starters, etc. may be obtained from their manufacturers. Structure, cowling, oil tanks, the oil system and controls weights must be estimated or calculated. For the airplane as a whole a complete weight and balance estimate is required. This is arrived at by choosing a set of coordinate axes perpendicular to each other and so located that they lie forward of and above (or below) all parts of the airplane, for convenience in keeping all lever arms to weights of the same sign. Each main item of dead load is located to scale on a balance diagram. For convenience items are generally grouped into assemblies, and a number of miscellaneous items in the same locality will generally be consolidated into one main item before any attempt is made to distribute the loads to the respective panel points. For the engine mount the same reference axes are used as in the weight and balance estimate, thus simplifying location of loads. The weight and balance diagram for the engine mount which we will analyze are shown on fig. 2, page 15-4.

3. Distribution of Dead Load to Panel Points.

For the method of distributing loads inside the structure to panel points refer to Fig. 1. Panel points are A, B, C, and D. The Load to be distributed is represented by W.

Along AC: $(W)\left(\frac{b}{a+b}\right)$

At A: $(W)\left(\frac{b}{a+b}\right)\left(\frac{d}{c+d}\right)$

At C: $(W)\left(\frac{b}{a+b}\right)\left(\frac{c}{c+d}\right)$

Along BD: $(W)\left(\frac{a}{a+b}\right)$

At B: $(W)\left(\frac{a}{a+b}\right)\left(\frac{f}{e+f}\right)$

At D: $(W)\left(\frac{a}{a+b}\right)\left(\frac{e}{e+f}\right)$

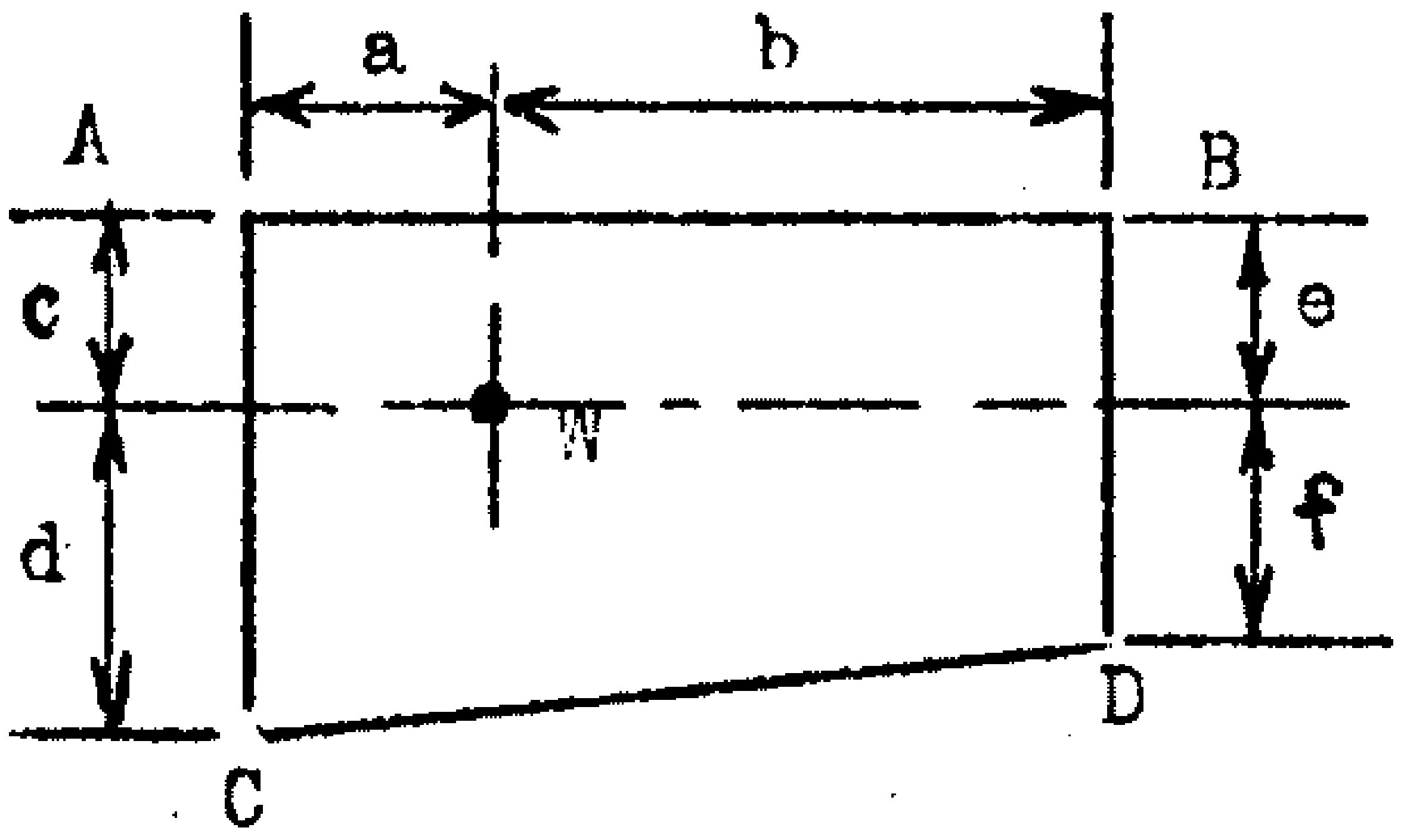


Fig. 1

For loads inside the structure the general case covering distribution to any number of panel points is as follows:

	Panel Point	Weight	Horizontal		Vertical	
			Arm	Moment	Arm	Moment
(1)	All	W	x	Wx	y	Wy
(2)	A	w_a	x_a	$w_a\ x_a$	y_a	$w_a\ y_a$
(3)	B	w_b	x_b	$w_b\ x_b$	y_b	$w_b\ y_b$
(4)	C	w_c	x_c	$w_c\ x_c$	y_c	$w_c\ y_c$
			etc.			
(5)		W	x	Wx	y	Wy

w_a, w_b, etc. must be chosen such that line (5) which is the sum of lines (2), (3), (4), etc. is equal to line (1) as to original weight and horizontal and vertical balance.

Loads forward of the engine ring are grouped for convenience into a single resultant load, acting at the center of gravity of the entire group. See Table 1.

Tables 2 and 3 illustrate the method of distributing the loads to panel points. The horizontal and vertical distances to the panel points are tabulated for reference. Their use is illustrated in the apportionment of the structure and cowling to the panel points. For convenience in taking the weights from the weight and balance report for the entire airplane it is customary to use the total for the entire ship and to divide the final values to correspond to the loads for one truss of one engine mount. In our example we will use the loads for one nacelle complete and divide by two to get the final loads for one truss.

In addition to the dead loads an engine mount is subjected to loads from engine torque and in certain cases to loads from landing chassis. Also for the flight conditions it is necessary to take into account the thrust of the propeller, the drag of the nacelle proper, and the fore and aft components of the inertia loads.

Design Conditions (Civil Aeronautics Authority)

The engine mounts and nacelles must be designed to meet the loading conditions corresponding to any of those computed for the wing, tail surfaces and fuselages in the preceding assignments. Since the weights in the engine section remain practically constant while the weight of the airplane as a whole is varied, it is obvious that the worst gust loads will be encountered at minimum flying weight for the airplane as a whole. The following are the minimum conditions which must be investigated.

1. High Angle of Attack at Light Load. (Condition I)

(a) The vertical and fore and aft dead weight factors are determined in the manner given in preceding assignments. The propeller thrust and nacelle drag are taken for a speed of V_L. The torque loads which must be combined with the dead weight loads are based upon the normal rated engine RPM and power which gives the speed V_L.

(b) Seventy-five per cent of the dead weight, thrust and drag loads from Condition I are combined with torque loads obtained by using the full rated take-off engine RPM and power.

(c) Torque. Torque loads are computed by the formula given on page 15-7. The torque from this formula is actual or basic torque. Design load factors must be added as follows:

For a 2-cylinder engine, 6
For a 3-cylinder engine, 4.5
For a 4-cylinder engine, 3.0
For 5 or more cylinders, 2.25 (our case)

2. Side Load on Engine Mount.

The design load factor is one-third of that for Condition I (light load gust). The loads are assumed to be due to sideward inertia forces. The design factor shall in no case be less than 2.0. No other loads are considered acting. This condition is used to insure rigidity in the cross bracing members of the top and bottom truss.

3. Landing Conditions.

The engine mount must be designed to withstand any landing condition. For land planes it is generally true that light load gust factors are greater than the landing factors so that landing loads are not critical. For sea-planes and flying boats the landing factors are in general considerably higher than the flight factors, so that landing loads design the engine mounts.

4. Up Load on Engine Mount.

The engine mount must be designed for the maximum inverted flight load. Where the inverted flight load factors are smaller than one-half the landing load factor, the load in each member shall arbitrarily be assumed to be 50 per cent of that for level landing but opposite in sign.

5. Nosing Over.

Where the engine will strike first in nosing over an investigation should be made for the loads sustained when the ship is on the main wheels and the nose, with the load factor being the same as for level landing. For this condition the gross weight of the airplane acting through the center of gravity is balanced by parallel acting forces on the nose of the airplane and the main wheels. The attitude of the airplane is determined by its geometry.

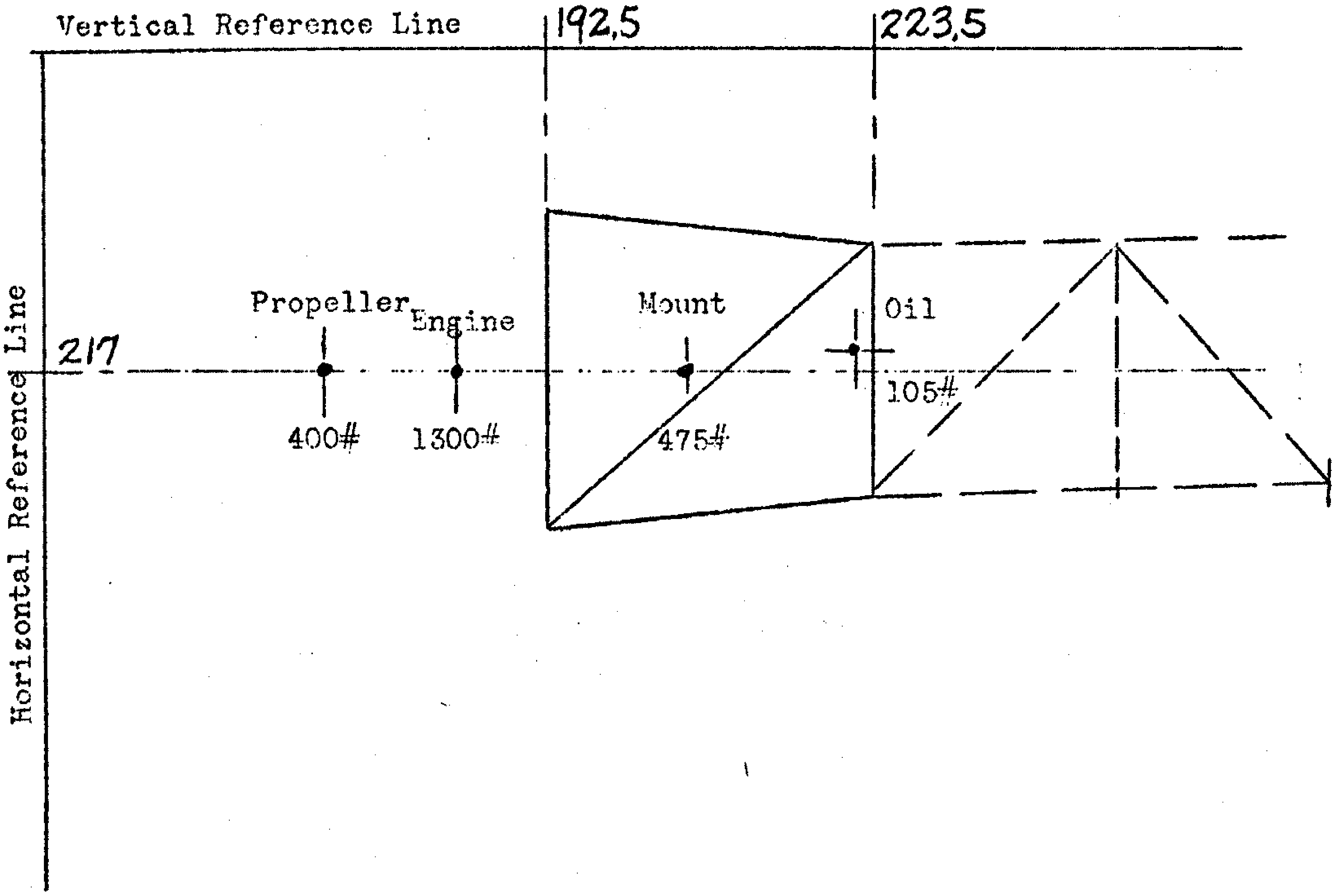


Fig. 2

Weight and Balance Diagram for Engine Mount

Note: Reference lines for vertical and horizontal distances agree with those on page 16-9, Assignment 16.

The items shown on Fig. 2 have already been condensed into a relatively few groups. A full weight and balance diagram would normally include a much larger group of items, which would condense to approximately the values shown here. Note that the items ahead of the engine mount, (that is, the engine and propeller), have far more effect on the design loads in the members than do those behind the ring. For preliminary design it is possible to consider the engine and propeller only, neglecting the effect of structure weight, in preparing an estimate of sizes for the mount. For final design the weight of the mount structure may then be taken from calculated weights.

The engine mount structure must be laid out carefully to use the available attachment points on the wing or fuselage and still clear all accessories on the engine. The line design shown in the example is for analysis illustration purposes only and is not intended to work with any particular engine.

TABLE I

ITEMS FORWARD OF THE STRUCTURE

Item	Weight	Horizontal Arm	Horizontal Moment	Vertical Arm	Vertical Moment
Propeller	400	170.5	68,200	217	
Engine, including Accessories	1300	183.5	238,550	217	
Total Forward of Structure	1700	180.5	306,750	217	
ITEMS IN THE STRUCTURE					
Engine Mount and Nacelle	475	205.5	97,600	217	103,000
Oil and Tank	105	221.5	23,200	215	22,600
Total in Structure	580		120,800		125,600

TABLE II

DISTRIBUTION OF ITEMS IN STRUCTURE TO PANEL POINTS

Item	Panel Point	Weight	Horizontal Arm	Horizontal Moment	Vertical Arm	Vertical Moment
Engine Mount and Nacelle	1_u	138	192.5		202	
	1_L	138	192.5		232	
	2u	100	223.5		205	
	2_L	100	223.5		229	
Oil and Oil System	1u	4	192.5		202	
	1_L	3	192.5		232	
	2u	57	223.5		205	
	2_L	41	223.5		229	

TABLE III

TOTAL PANEL POINT LOADS - ONE NACELLE

Panel Point	Load	Horizontal Arm	Horizontal Moment	Vertical Arm	Vertical Moment
1u	138 + 4 = 142	192.5	27,300	202	28,700
1	138 + 3 = 141	192.5	27,100	232	32,700
2u	100 + 57 = 157	223.5	35,100	205	32,200
2	100 + 41 = 141	223.5	31,500	229	32,300
Total	= 581		121,000		125,900
			against 120,800 Table I		against 125,600 Table I

*****

Basic Loads for 1-side of 1-nacelle are one-half the values shown on Table III.

Where a number of conditions must be investigated it is convenient to work with unit loads. This is handled simply for members forward of the structure by considering separately in turn the effect of say 1000 lbs. acting vertically, fore and aft, and sideward at the centroid of weights forward of the structure. In this case the weights at panel points lu and 1_L are also to be considered. The panel points 2u and 2_L are attachment points for the engine mount and their loads need not be included in analyzing the mount itself.

For illustration we will compute the actual loads for Light Load Gust, Condition I. Load factors are calculated as follows:

The minimum flying weight from Assignment 3, page 3-18, is 13,290 lbs. The unit wing loading is therefore

$$\frac{13,290}{1,010} = 13.1 \text{ lbs. per sq. ft.}$$

Using the data from Assignments 2 and 3:

$$n_{1\ \text{light}} = \frac{KUV \quad m}{(575)(s)} = \frac{(.95)(30)(220)(4.4)}{(575)(13.1)} = 3.67$$

$$(K \text{ for } s = 13.1 = \tfrac{1}{2}(13.1)^{1/4} = .95)$$

$n_1 = 3.67 + 1.0 = 4.67$ applied wing load factor.

For the balancing tail load with any center of gravity, referring to Assignment 5:

The greatest dead weight load factor is obtained with center of gravity at 30 per cent of the mean aerodynamic chord.

$$C_N = \frac{n_1 \; s}{q} = \frac{(4.67)(13.1)}{123.7} = .494 \qquad = 4.90$$

$$C_c = -.01, \quad C_c{}^1 = -.100 = (-.20)(C_N) \qquad \overline{CP} = .27 \text{ (most fwd.)}$$

$$\overline{CP} = .25 - \frac{C_M{}^1}{C_N} \text{ or } C_M{}^1 = -(.02)(.494) = -.01$$

$$N_{x_1} = (C_c{}^1)(\frac{q}{s}) = (-.100)(\frac{123.7}{13.1}) = -.945$$

$$M_1 = (-.01)(\frac{123.7}{13.1}) = -.095$$

$N_{x_4} = -.1365$ from page 5-8.

*****

With c.g. at 30 per cent, taking lever arms from Fig. 1 of Assignment 5, page 5-6, taking counter-clockwise moments around the center of gravity as plus

$$(N_3)(2.95) + .095 - (4.67)(.05) - (.945)(.10) - (.1365)(.15) = 0$$

$$2.95N_3 = -.095 + .238 + .095 + .020 = .258$$

$$N_3 = \frac{.258}{2.95} = .0874 \text{ upward on tail (say .09)}$$

Vertical Dead Weight load factor, acting down,

$= -4.67 - .09 = -4.76$ (applied)

Horizontal Dead Weight load factor, acting aft

$= .945 + .1365 = 1.0815$ (applied)

Design Dead Weight load factors are

$-(4.76)(1.5) = -7.14$ acting down

$+(1.0815)(1.5) = 1.52$ acting aft.

*****

Design Torque.

According to requirements it is necessary to consider only 75 per cent of the light load gust factors to be acting with the torque from take off power or 100 per cent of the gust loads to be acting with torque from maximum (except takeoff) power. Usually the difference in torque is not great and some analysis can be omitted by using takeoff torque with the full light load gust loads. This practice is recommended. For our case take off power as given on page 3-18 is 1200 HP at 2600 RPM; gear ratio is 2:1.

Then the basic torque at takeoff is

$$\frac{(63{,}000)(1200)}{(2600)(\frac{1}{2})} = 58{,}100 \text{ in. lbs.}$$

The torque at normal power would be

$$\frac{(63{,}000)(1000)}{(2400)(\frac{1}{2})} = 52{,}500 \text{ in. lbs.}$$

The difference in torque if the maximum is used would have little effect on the final sizes of members.

Adding the required safety factors from 6CFR-04 the design torque is

$$(58{,}100)(1.5)(1.5) = 131{,}000 \text{ in. lbs.}$$

The thrust for this condition is 2730 lbs. for two engines or 1365 lbs. for one. Multiplying by the factor of safety the design thrust would be (1365)(1.5) = 2053 lbs. acting forward. The estimated design drag of one nacelle at this speed is 400 lbs. This figure includes the factor of safety of 1.5. The net thrust load acting on one nacelle would then be 2053 - 400 = 1653 lbs. acting forward.

*****

Resume of Design Loads and Load Factors for Light Load Gust.

Load Factors	7.14 down 1.52 aft
Torque	131,000 inch lbs.
Thrust and Drag (Resultant)	1653 lbs. acting forward

Since it is possible that under some unusual condition power may be lost, it is advisable to consider the dead loads either with or without thrust and torque whichever gives the more critical loads. This is accomplished by tabulating the increments of loads in members due to thrust and torque and to dead loads separately and taking the summations of loads in each member which give the greatest combined effect. The method will be illustrated.

Our first step is to draw the space diagram for the mount, Fig. 3, and to locate the center of gravity of loads forward of the engine and place the panel point loads. Fig. 4. At this point we will compute the correction factors for the members which do not appear in their true lengths in the side view, since the solution of truss loads is to be graphic, and based on the projected side view. Fig. 5 illustrates the method of apportioning the torque between the top and bottom trusses, and the side trusses. The loads due to torque are applied at the engine ring. Where either a top or bottom truss is cut out to clear accessories all of the torque is assumed to be carried as a couple on the side trusses. Torque loads add to the dead weight loads in members of the side truss since the effect of torque loads is to give upward loads on one side and downward loads on the other.

Where the torque loads are not easily balanced as in this case of a symmetrical truss it is possible to calculate the exact distribution by methods involving consistent deflections. (Least Work) This procedure is not usually required. Instead it is possible to make over-lapping assumptions, such as assuming two-thirds of the total torque carried as a couple on top and bottom trusses, and two-thirds carried on side trusses. It is not generally advisable to put much faith in Least Work analyses on engine mounts, because of the errors introduced by the use of the rubber shock mounts.

The foregoing method of analysis has assumed rigid attachment between engine and engine ring. Dynamic suspension, where shock mounts are used at the points of attachment of engine to the mount, are of such a variety and are being changed and improved at such a rate that no method successfully covering all of them can be given in the scope of this course. The distribution of load for such a mount depends upon detail design, ratio of the spring rates of the absorber unit in different directions, etc. Specific advice on a particular installation can usually be obtained from either the engine or the shock mount manufacturer.

Assignment:

Using the method indicated on Figs. 3, 4, and 5 obtain the loads in the members of the engine truss for Condition I. Light Load Gust. Work is to be done graphically and correction factors applied to obtain true loads in members as explained in Assignment 4 of the Basic Analysis Course.

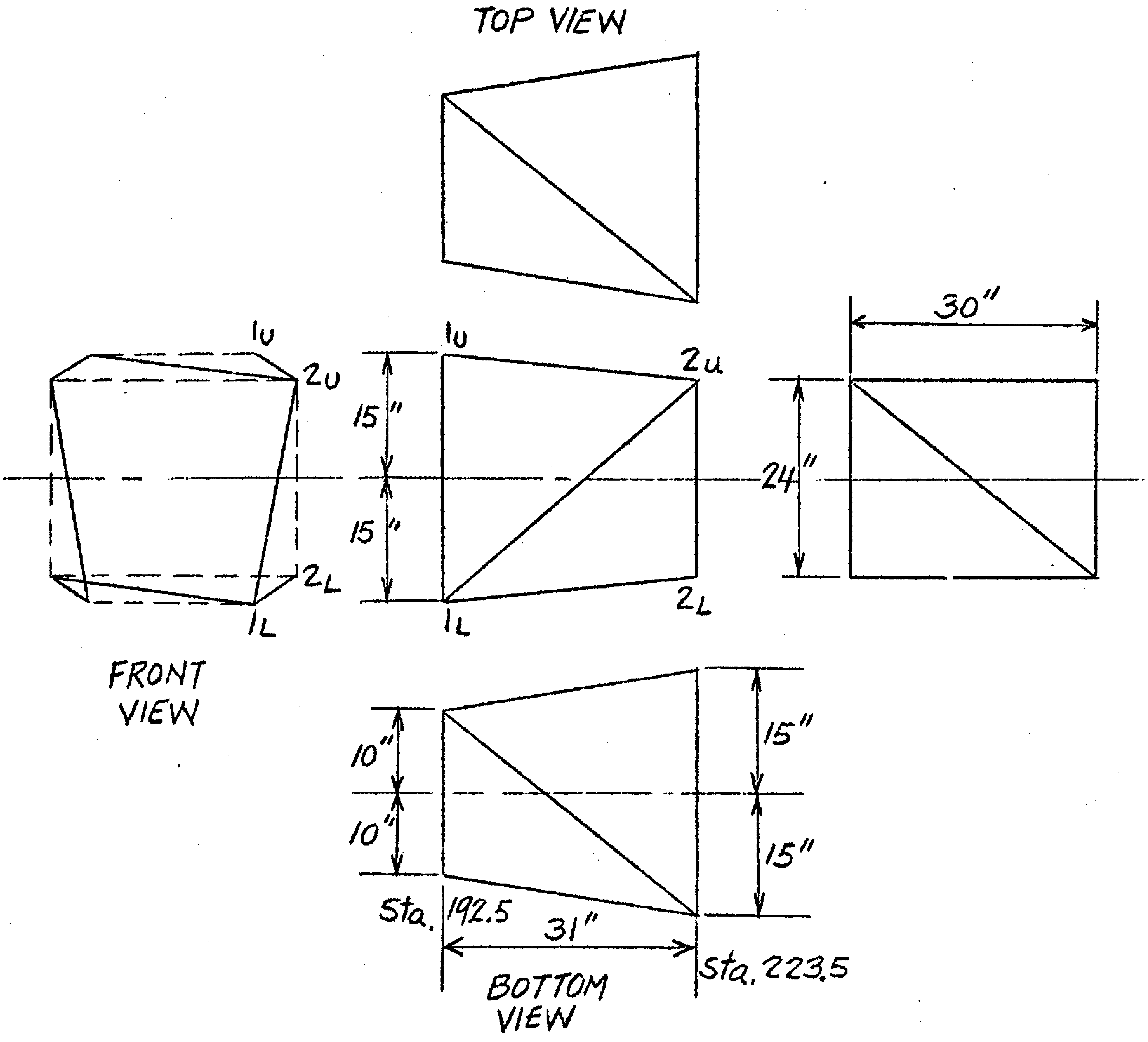
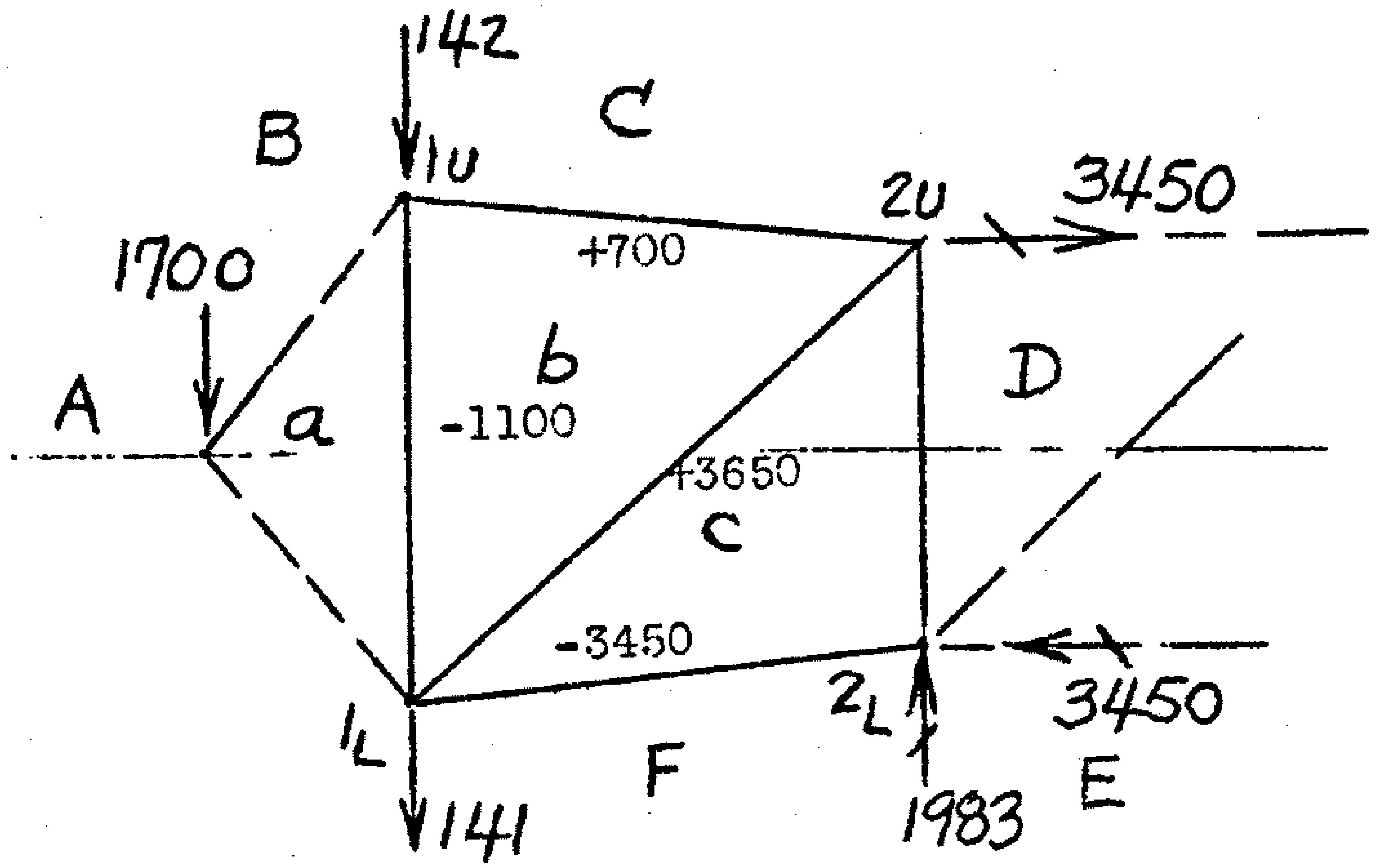


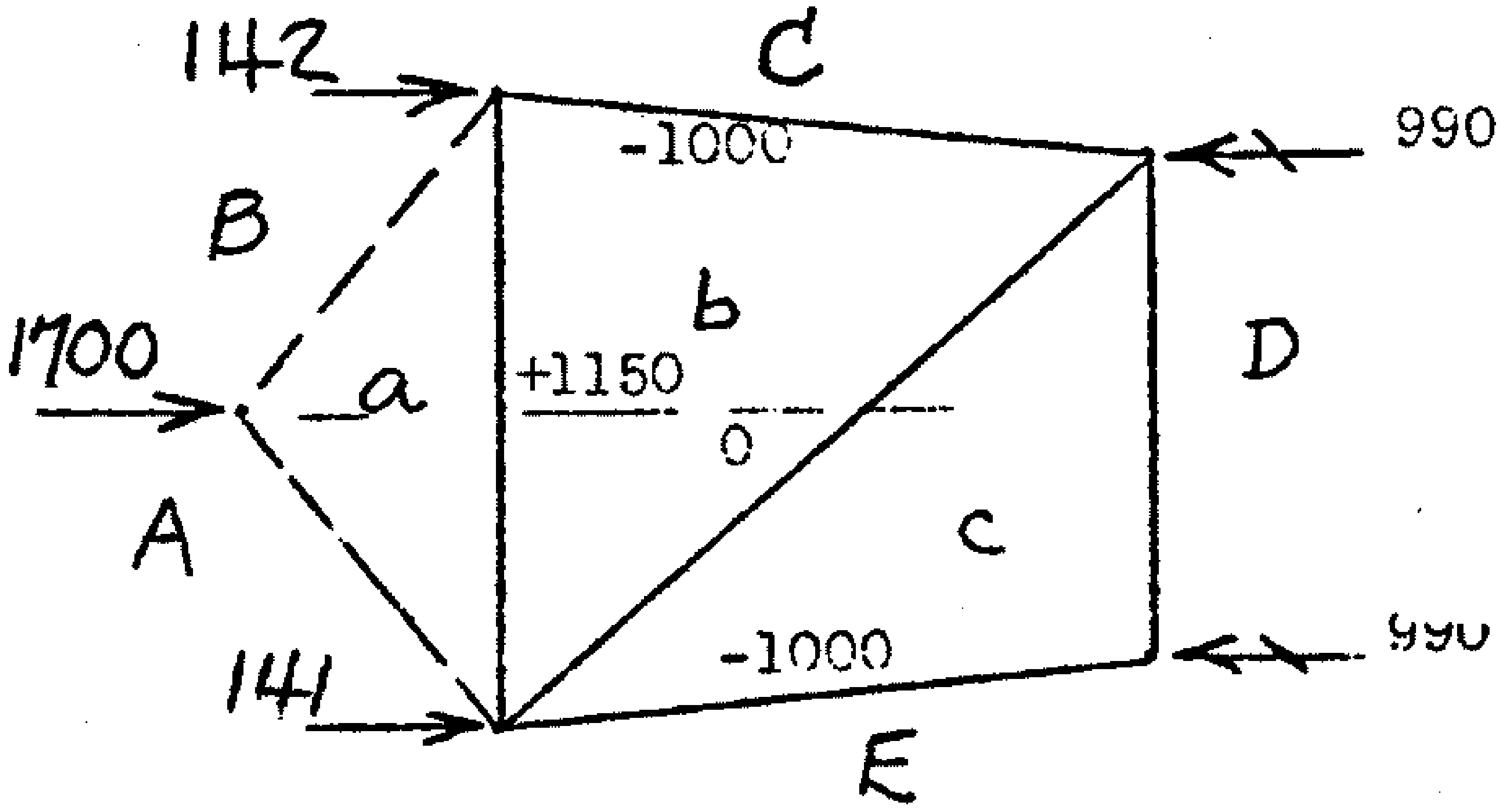
Fig. 3

Line Diagram of Engine Mount

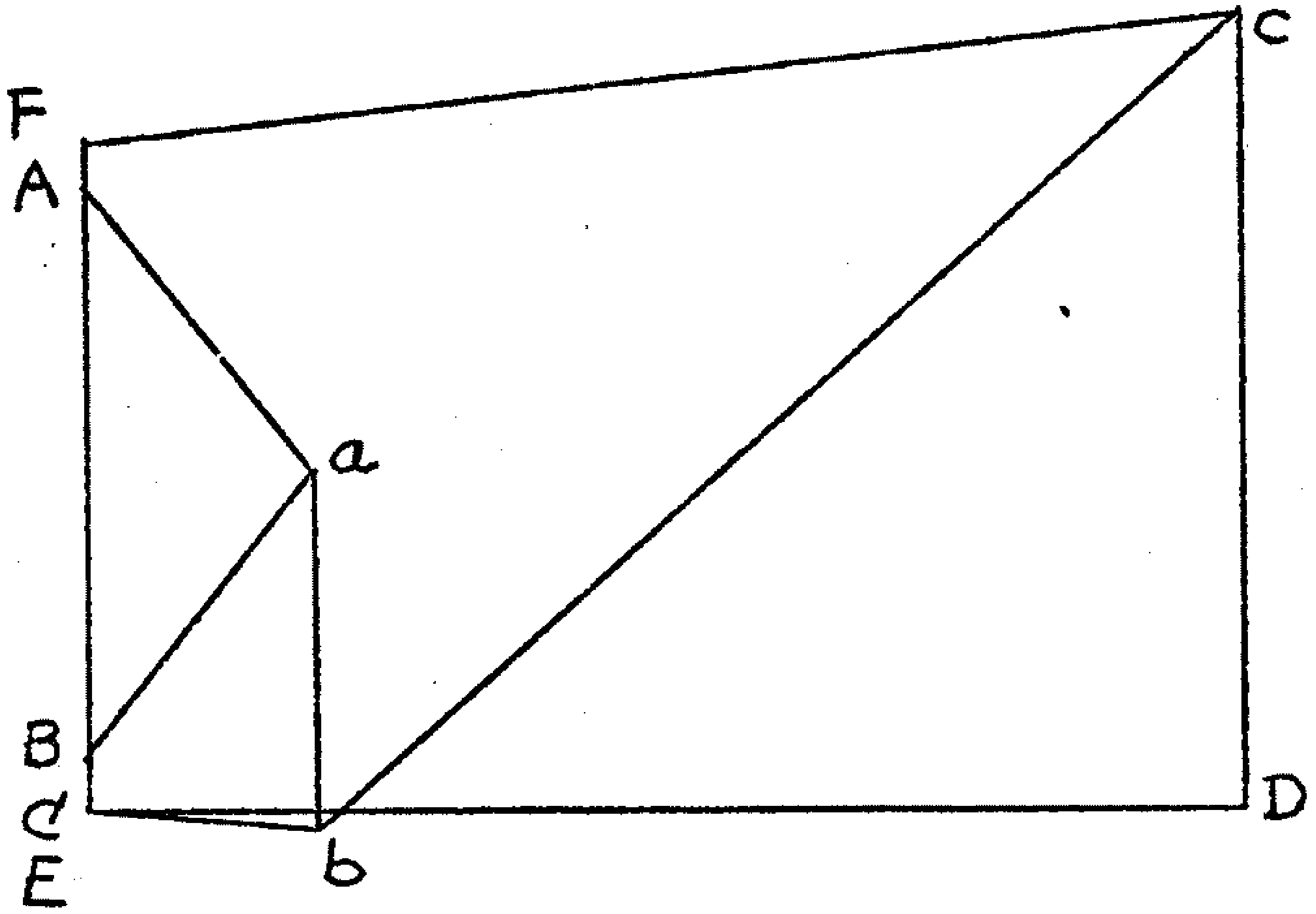
Scale 1" = 20"



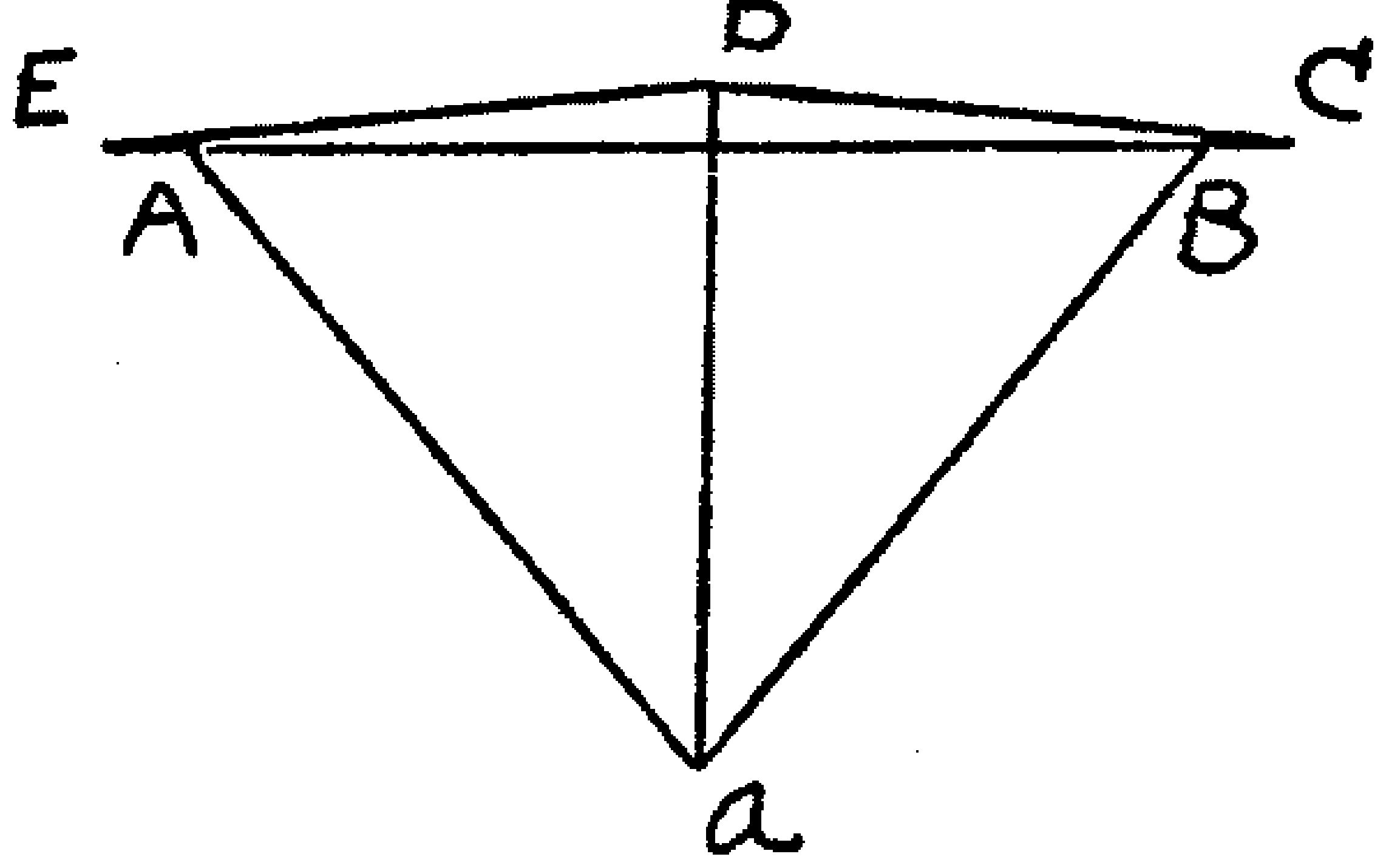
Vertical Load
Load Factor = 1.0



Horizontal Load
Load Factor = 1.0



Stress Diagram for Vertical Load
1" = 1000#



Stress Diagram for Horizontal Load
1" = 1000#

Fig. 4

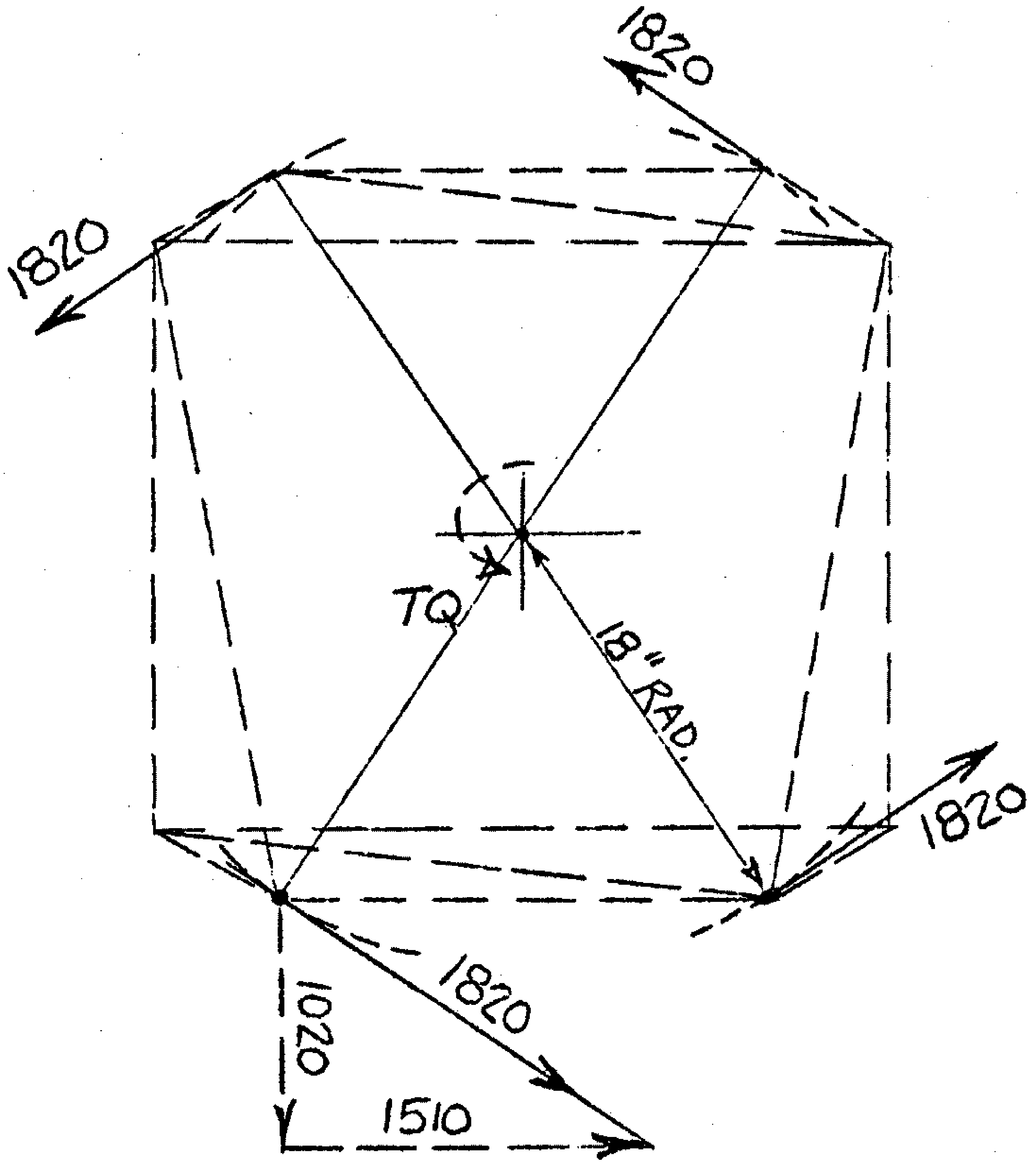


Fig. 5 - Front View

Showing method of distributing torque to top and bottom trusses and to side trusses.

*****

Vertical couple = (2)(1020) = 2,040 lbs.
Horizontal couple = (2)(1510) = 3,020 lbs.

Design torque = 131,000 lbs.
Tangential reactions at 4 points on ring $= \frac{131,000}{(4)(18)} = 1820\#$

*****

Components are 1020 vertical
1510 horizontal

Check of torque moments:

$$
\begin{aligned}
(2040)(20) &= 40,800 \\
(3020)(30) &= \underline{90,600} \\
\text{Total} &= 131,400
\end{aligned}
$$

ADVANCED STRESS ANALYSIS

ASSIGNMENT 16

Assignment 16

GENERAL OUTLINE OF FUSELAGE ANALYSIS

A. Method of Analysis.

The general steps in the analysis of a fuselage of the semi-monocoque type used on transport airplanes will be outlined here.

1. Weight and Balance and weight distribution.

The dead weight of structure and the weight and location of variable weight items such as fuel in the fuselage, passengers and cargo must be determined from a weight and balance estimate before the analysis of the fuselage structure can be made. The weight estimate gives the weights, and locations with respect to a set of horizontal and vertical reference axes of these items. It is generally necessary to make a redistribution for analysis purposes. For trussed type fuselages this redistribution consists in apportioning the loads to panel points by the same method used for engine mount analysis. For a monocoque structure the loads are assumed to be transferred to the structure at their respective centers of gravity. A series of concentrated loads having the same value are assumed to be a uniform load over a section of the fuselage. Structure weights are assumed to be a series of uniform loads or of uniformly varying loads depending on the actual weights of the fuselage cross sections at different stations. The final center of gravity of the distributed loads must check the original center of gravity in both the horizontal and vertical directions. Fig. 1 shows a typical fuselage loading diagram.

2. Range of weights in fuselage and possible c.g. positions.

All loading conditions must be studied to determine the worst loading cases for the fuselage. For example the forward portion of the fuselage which has fixed weights (the crew) will probably be designed by the light load gust condition in which the rest of the fuselage is carrying minimum weights. The aft portion of the fuselage may be designed by an intermediate weight condition in which the rear cargo compartments and rear cabin seats are loaded at the same time that there is a downward load on the tail surfaces. The critical conditions are:

(a) Maximum forward c.g.
(b) Maximum aft c.g.

with
(1) Lightest possible load in airplane (light load gust)
(2) Semi-light condition (max. fuselage load with minimum fuel load in wings) - designs fuselage proper.
(3) Full gross with maximum fuselage load.

3. Unit dead weights.

To facilitate compilation of load, shear and moment data for the different conditions of flight and landing it is advantageous to work up a dead weights load, shear and moment diagram with a load factor of one. This diagram is made for weights vertical and for weights horizontal, since in certain design conditions the dead weights have fore and aft components with respect to the horizontal fuselage reference line. For compilation of final values the resultant inertia forces acting in a given condition are resolved into horizontal and vertical components. These components are the multiplying factors to be used on dead loads to give their actual values in the design conditions.

4. Design conditions.

(a) The fuselage must be analyzed for any of the critical flight conditions for which the wings are investigated, including the effects of both inertia loads and tail loads.

(b) The fuselage must be checked for level and 3-point landing, and where such a condition is possible for a nosing over condition in which the fuselage is supported on the main wheels and the nose and the load factor, taken perpendicular to the ground line is the same as for level or 3-point landing.

(c) The fuselage must make good the side bending and twist occasioned by side loads on the vertical fin or by side loads on the tail wheel.

(d) The fuselage must be able to take the wracking loads from a landing on one wheel, or from unsymmetrical loads on the wing or horizontal tail surfaces.

5. Computations of fuselage shears and moments.

For a monocoque fuselage allowance must be made, as on a monocoque wing, for the fact that the sheet covering is only partially effective in compression. This means that the neutral axis at any fuselage station is not known when the shears and moments are calculated, and also that the neutral axis must be determined by cut and try methods.

The vertical location of the neutral axis is not important when moments are computed for vertical loads only, but must be considered when horizontal loads exist since the latter loads have an effect on the moment except in the rare cases when they act at the neutral axis. Since the neutral axes not only are unplaced, but will vary in position along the length of the fuselage a final correction must be made at each station to obtain final corrected bending moments. Then for original data, when horizontal loads are present, the moments of the horizontal loads are computed about a reference horizontal axis and at each station a correction for eccentricity is made after the neutral axis is determined. The method is illustrated on Fig. 2.

Reactions are supplied by the wings and tail surfaces in the flight conditions and by wheels in landing conditions. A check on the reactions is obtained from those required at the fuselage in any particular condition. Where small discrepancies occur they are balanced by vertical couples on the spars.

B. Design Considerations.

The design of a monocoque or semi-monocoque fuselage is analagous to that of a stressed skin wing in many respects. The same fundamental stress distribution is assumed; that is the entire fuselage is assumed to act as a beam, with the panels of the skin transferring the shear load in diagonal tension. The general method of calculation of section properties and unit stresses has already been given in the wing analysis. Certain special problems which also apply to stressed skin wings will be treated here, particularly those related to detail design.

1. From the beam theory the unit stresses in the longitudinal stiffener and sheet combinations are proportional to the distance from the neutral axis. It is advantageous therefore to place the bending material on top and bottom in so far as this is consistent with the requirements of side bending. This can be accomplished in a design where longitudinal bulb angles or T-sections are used, by placing heavy sections top and bottom and using light sections along the sides. In addition an additional economy in weight is generally gained because the heavier sections have higher allowable compressive stresses than the light ones.

2. Since the shear stresses due to bending are a maximum at the neutral axis it would be advantageous to use the maximum gauge skin near the center (in side view) and use lighter sheets top and bottom. Practically this end may be only partially attained since the thin sheet on the top and bottom, when it is under compression loads from bending, wrinkles so badly that for sake of appearance as well as actual strength it cannot be reduced to gauges much lower than those used on the sides.

3. The Wagner beam theory of shear transfer is applicable to the design of the sheet panels and rivet seams. If the theory were used to design frames and transverse stiffeners the results would be unduly conservative. The methods for design of frames carrying special loads will be

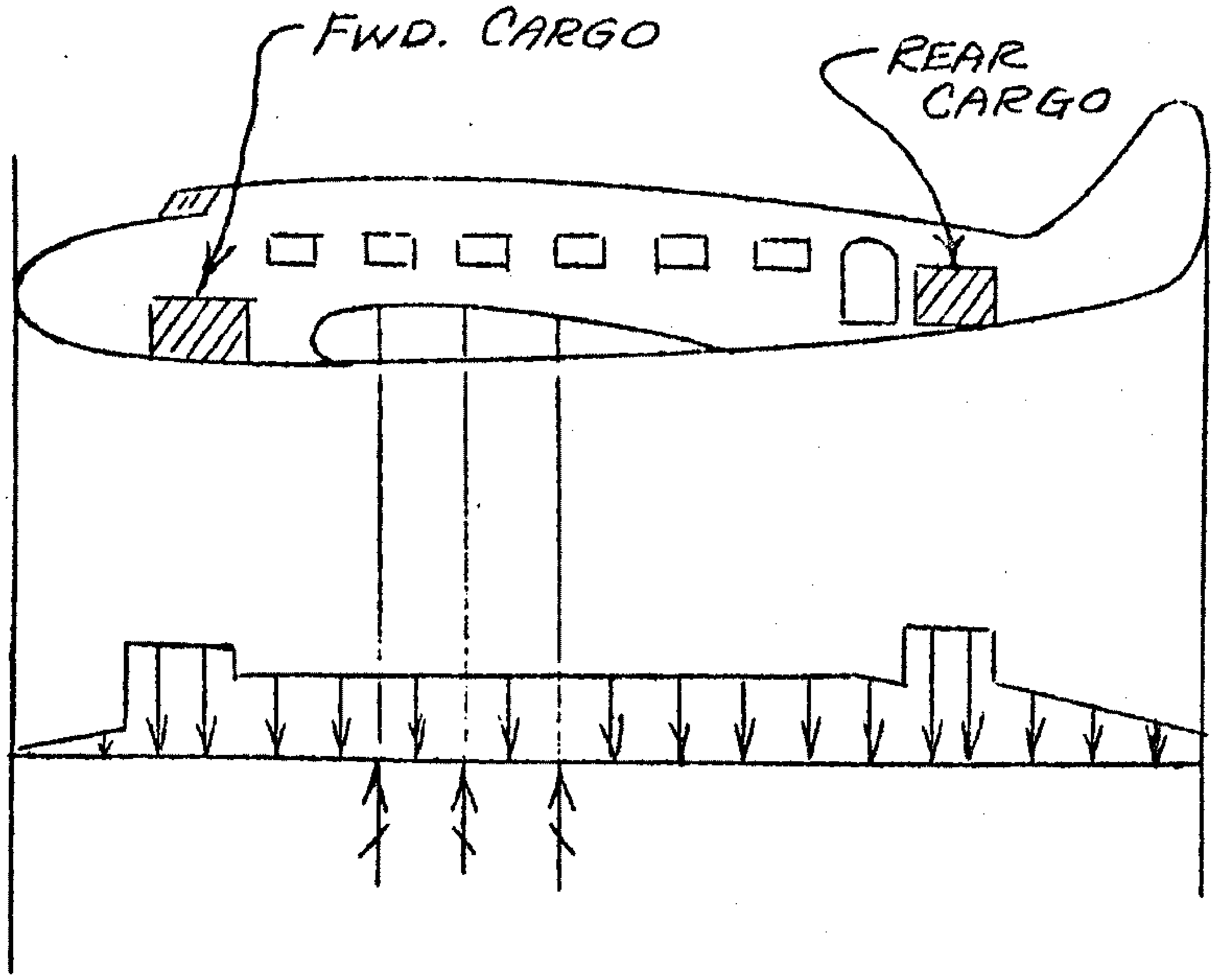
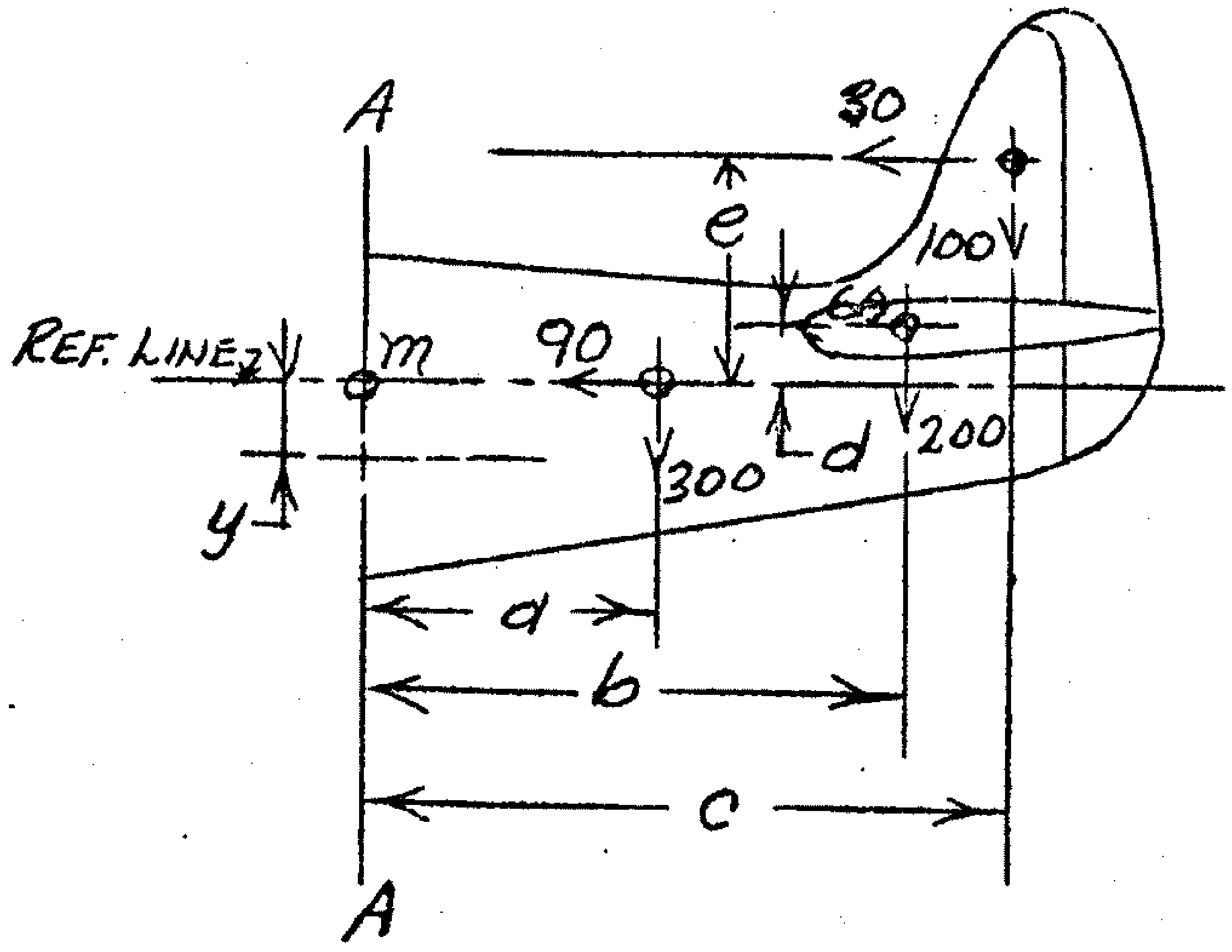


Fig. 1

Fuselage
Loading Diagram

Fig. 2

Level Landing Loads
Shears, Moments



On Section A-A

V-Shear = 100 + 200 + 300
= 600 lbs.

Compression = 30 + 60 + 90
= 180 lbs.

Mom. about Point "m"
= 300 a + 200 b + 100 c
- (90 x 0 + 60 d + 30 e)

If neutral axis is "y" inches below "m", then final moment about N.A. =

Mom. at "m" - 180 x y

Same method may be applied to any fuselage section.

given. Frames which are intermediate only, and carry no special external loads are theoretically subjected only to loads induced by the tendency of the fuselage tube to flatten during bending. It may be determined from actual deflection measurements on a fuselage test, compared to deflections of the frames when loaded alone, that the loads induced in the frames by bending of the fuselage are very slight.

4. Special problems.

(a) Longitudinal seams in sheets.

Longitudinal seams in sheets are most critical near the neutral axis, since their function is to transfer the longitudinal shear (the intensity of longitudinal shear at any point equals that of the vertical shear) from one sheet to another. The longitudinal shear may be calculated from the formula:

$$f_s = \frac{SQ}{bI}$$

where S = total vertical shear
Q = static moment of parts on the side away from the neutral axis
b = sheet thickness
I = Moment of Inertia of entire cross-section

and from torsion

$$f_s = \frac{\text{Torque}}{2\ At}$$

The values of "Q" may be taken for one side only if the sheet thickness of one side is used for "b". After the value of f_s is determined multiply it by the sheet thickness to determine the running shear per inch. A seam may then be chosen in which the rivets will develop the required strength per inch. A final check should be made to determine that the net area of sheet remaining after rivet holes are deducted will develop the diagonal tension load in the sheet. For this purpose it may be assumed that the diagonal tension per inch on the sheet is equal in intensity to twice the shear stress. Working backward from this assumption an allowable shear value for skin panels may be approximated as follows:

Assume that the skin is 24 ST Alclad for which the allowable tensile stress is 56,000 lbs. per sq. in. If the longitudinal seam were made with 1/8 rivets 3/4 inch on centers the net length of sheet per inch would be 1 - 4/3 x 1/8 = 5/6 inch. Then the allowable tension load per inch on the net area would be 5/6 x 56,000 = 46,800 lbs. per sq. in. Since from the Wagner theory the tension load is twice the shear load the shear allowable,

$$F_s = \frac{46,800}{2} = 23,400 \text{ lbs. per sq. in.}$$

For flat panels this value has been approached and even exceeded considerably in static test. For fuselages it must be greatly reduced because of the curvature in the sheet, which actually deflects in bending as well as diagonal tension. Good average allowables for shear in fuselage panels are:

10,000 lbs. per sq. in. for .025 and under
12,500 " " " " " .036 " "
15,000 " " " " " .040 " "
17,500 " " " " " .051 and over

The above values are based on average curvature where the longitudinal spacing does not exceed seven inches.

(b) Vertical and crosswise seams in sheets.

Cross seams in sheets are designed primarily by the bending tensions. Combined with shear from vertical or side bending and torsion. Since from the Wegner Beam theory the shear is transmitted through diagonal tension at approximately 45° with the longitudinals, the crosswise loads per inch from shear due to bending are the same as the longitudinal load per inch. The torsion shear load may be represented by loads acting directly along a cross-seam. The loads which act on a cross-seam are therefore the vector summation of the longitudinal loads in the skin and the loads parallel to the seam which are the algebraic summation of the bending and torsional shear loads, with the bending loads very much the more important factor. Suppose the unit tension stress due to bending to be 30,000 lbs. per sq. in. on the bottom of the fuselage, and the shear stress to be 5,000 lbs. per sq. in. Then per running inch the loads on .025 skin would be 750 lbs. longitudinally and 125 lbs. crosswise. The resultant diagonal pull would be 760 lbs. per inch. A 2-row splice of 1/8 17ST rivets - 1/2 inch on centers would give the following values: Bearing for 17ST rivets = 230 lbs. each. Value per inch of splice = 4 x 230 = 920 lbs. (o.k.)

Checking the net area of the sheet.
Net length per inch = 1 - 2 x 1/8 = 3/4 in.

$$\text{Tension value on the net area} = \frac{760}{3/4 \times .025} = 40{,}500 \text{ lbs. per sq. in.}$$

If the material is 24ST Alclad the allowable is 56,000 lbs. per sq. in. and the splice is therefore o.k.

(c) Cutouts.

Cutouts are allowed for by framing them with enough material to more than replace the parts removed. It is essential that enough stiffness be present to resist the bending loads induced when the stresses are made to flow around the opening. Openings which are round or oval are simpler to reinforce than those which are square or rectangular because of the absence of concentrations at the corners.

When cutouts, such as windows occur in the regions of highest longitudinal shear their presence is allowed for by placing sufficient area between the windows to compensate for that cutout. In general this means that the sheet between windows will be twice as thick as that above and below the windows. See Fig. 3.

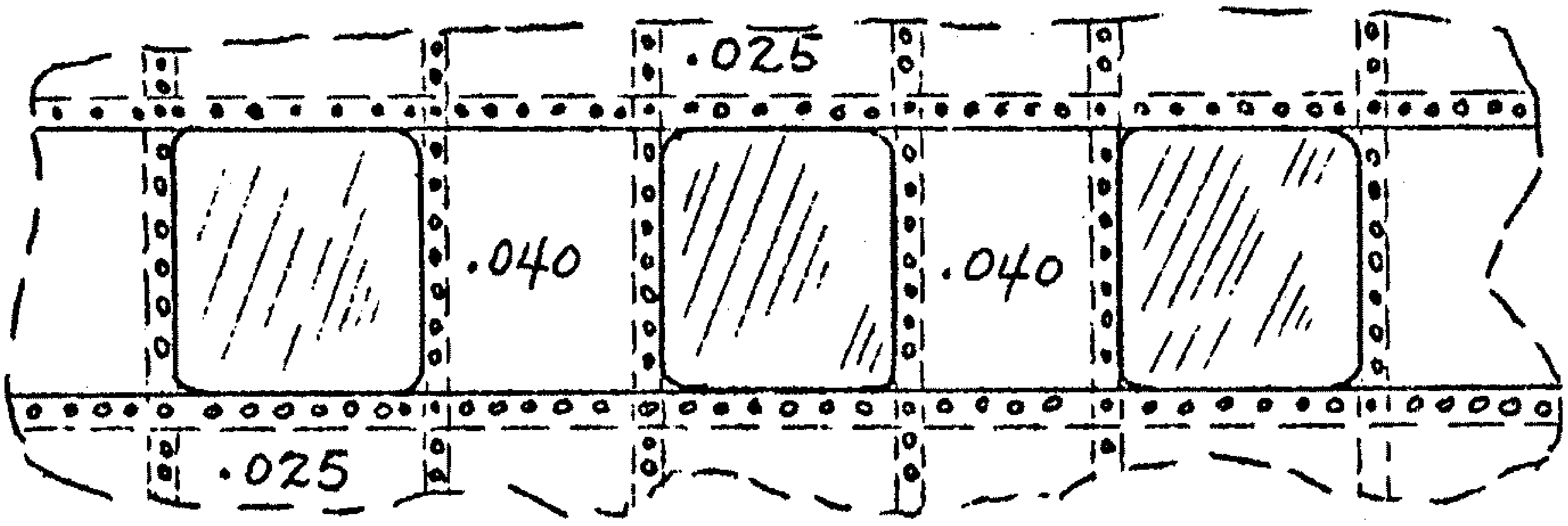


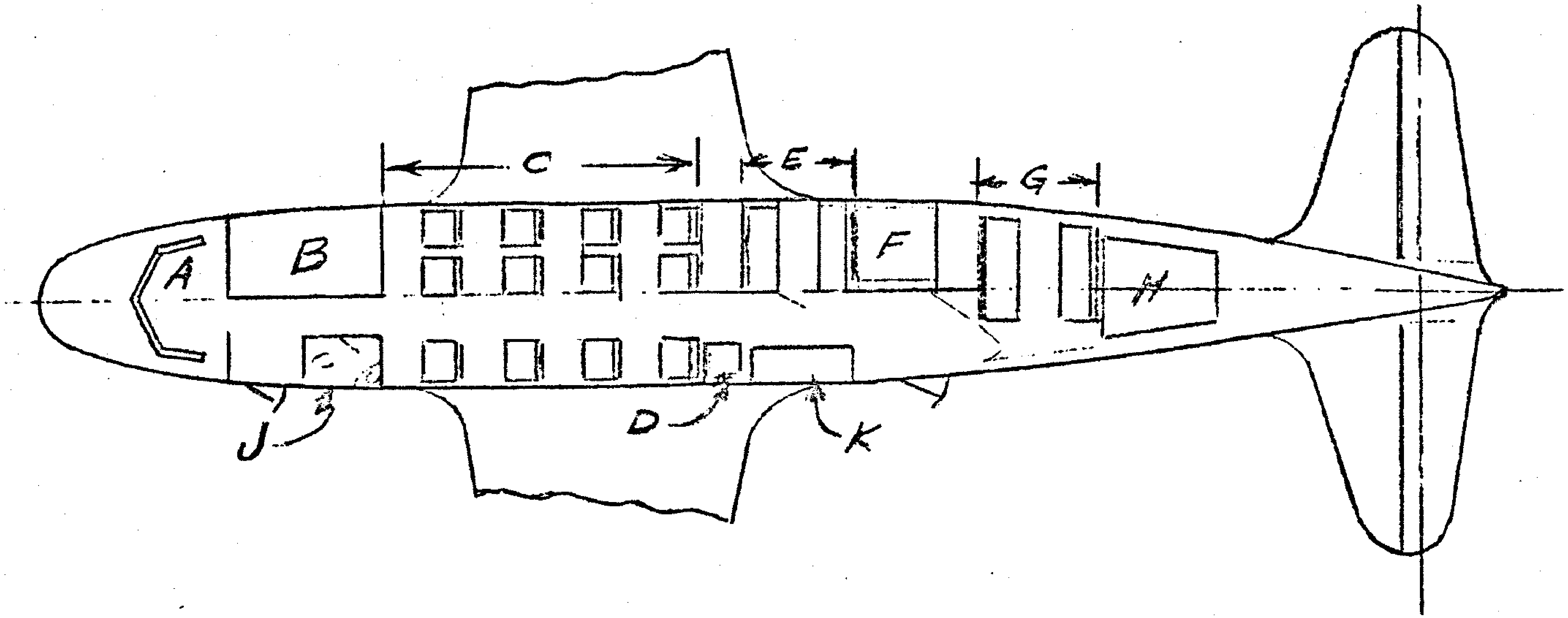
FIG.3.

If shear per inch = 300 lbs. in .025 sheet shear across sheet between windows = 600 lbs. per in.

Unit stress in .025 sheet = 12,000 lbs. per sq. in. and in .040 sheet = 15,000 lbs. per sq. in.

*****

We will now consider the foregoing points in the design of the fuselage for our type problem. The planview of the fuselage with the arrangement of passengers and cargo is shown on Fig. 4, page 16-8.. Fig. 5 shows the side view of the fuselage and the diagrams of load distribution for maximum forward and aft center of gravity locations.



A - Pilot's Compartment
B - Forward Cargo Compartment - 125 cubic feet
C - Main Cabin - 12 passengers
D - Stewardess
E - Stateroom #1 (with provision for passenger's baggage - may be converted to berths) 2 passengers
F - Baggage Compartment - for passengers in main cabin
G - Stateroom #2 (similar to #1)
H - Rear Cargo Compartment - 50 cubic feet
J - Lavatory
K - Buffet

Fig. 4

Fig. 5

REF. LINE
100
200
300
400
500
600
700
775
100
182
#247
#290
352
373
436
499
562
625
760
F.S. R.S.
100
200
#217 #190
#
#229
M.A.C.=140.2
① WING DO. WT.
② AIRPLANE - C.G. FWD
③ AIRPLANE - C.G. AFT.
④ FUSELAGE - C.G. FWD.
⑤ FUSELAGE - C.G. AFT.
USEFUL LOAD C.G. FWD.
USEFUL LOAD C.G. AFT.
DEAD WT
A
B
C
D
E
F
G
G1
G2
H
#373
5.83 #/in
3.1 #/in
9.4 #/in
2.5
85
STRUCTURE, FURNISHINGS, RADIO, ELEC. EQUIP., SURFACE CONTROLS, FLIGHT CONTROLS, ETC.
TAIL SURFACES & TAIL WHEEL

In calculation of the centers of gravity lever arms to the different components are taken from a set of horizontal and vertical reference axes. It is convenient to take the reference axis for horizontal lever arms at the nose of the fuselage and to number fuselage stages from the same reference point, so that the fuselage stations and horizontal weight stations are the same. The reference line for vertical lever arms may be taken conveniently any place outside the structure. See Fig. 5 for an illustration of the use of reference axes.

In determining the most critical loading conditions the intended use of the plane, the maximum length of flights required, and the maximum allowable loadings for the different baggage compartments must be considered. The weight distribution for the most critical cases is shown on Table I. Note that all computations are within the accuracy of a 10 inch slide rule which is considered close enough for design.

TABLE I

Weight Distribution

	C.G. Fwd.			C.G. Aft.					
	Wt.	H-Dist.	H-Mom.	Wt.	H-Dist.	H-Mom.	Wt.	V-Dist.	V-Mom.
Wing	9,860	231.7	2,285,000		231.7	2,285,000		227	2,240,000
Fuselage	10,140	269	2,715,000		309	3,135,000		179.5	1,820,000
Gross	20,000	250	5,000,000		271	5,420,000		203	4,060,000
Fuselage Misc.	3,350	299	1,000,000					194	650,000
Tail Surf & Wheel	600	690	414,000					135	81,000
Fuselage Dead Wt.	3,950	358	1,414,000		358	1,414,000		185	731,000
Pilot & Co-Pilot	340	80	27,200	340	80	27,200	A	176.5	
Cargo Forward	2,800	141.5	396,300	2,050	141.5	290,300	B		
Pass. Main Cabin	2,040	267	545,000	2,040	267	545,000	C		
Stewardess	130	362	47,000	130	362	47,000	D		
Pass. & Baggage	410	404	165,500	410	404	165,500	E		
Baggage	60	467	28,000	420	467	196,500	F		
Pass. &	340	161	54,900				G_1 &		
Baggage	70	530	37,100	410	530	217,000	G_2		
Cargo Rear		594		390	594	232,500	H	176.5	
Fuselage Useful Ld	6,190		1,301,000	6,190		1,721,000	6,190	176.5	1,089,000
	3,950		1,414,000	3,950		1,414,000	3,950		731,000
Fuselage Total	10,140		2,715,000	10,140		3,135,000			1,820,000

The Effect of a 1000 lb. unit up load on the tail will now be considered. See Fig. 6.

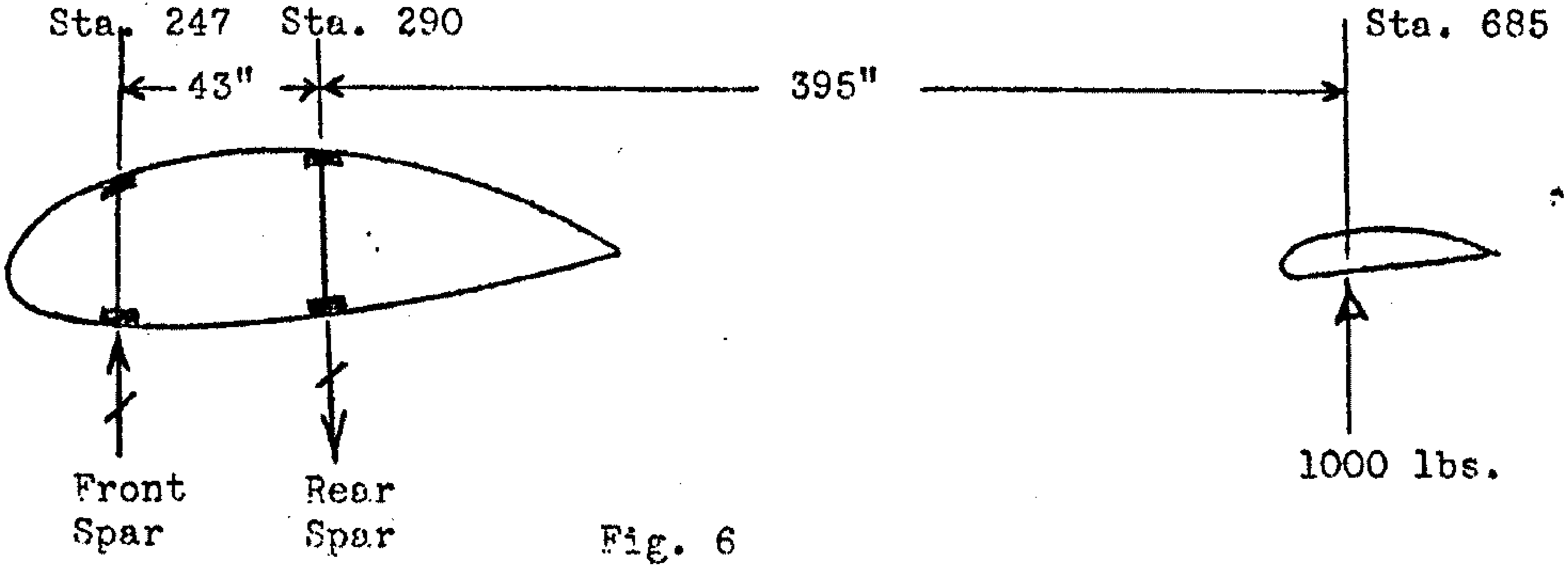


Fig. 6

Unit Tail Load

For a load of 1000 lbs. on the tail the front spar reaction is $(1000)(\frac{395}{43}) =$ 9180 lbs. and that at the rear spar is 9180 + 1000 = 10,180 lbs.

The moments at the rear fuselage stations from unit tail load are as follows:

$$M_{562} = -(1000)(685 - 562) = -123,000 \text{ in. lbs.}$$

$$M_{373} = -(1000)(685 - 373) = -312,000 \text{ in. lbs.}$$

$$M_{290} \text{ (at rear spar)} = -(1000)(685 - 290) = -395,000 \text{ in. lbs.}$$

Note that moment is considered minus when it puts tension on the bottom of the fuselage.

TABLE II

Dead Weight Shears and Moments. L.F. = 1.0

Sta.		Shear		Moment
775		0		0
	600	- 600	-600 (690 - 562)	-77,000
	$\frac{5.83}{402}\left(\frac{775 - 562}{2}\right)^2$	- 331	$-331\left(\frac{213}{3}\right)$	-23,500
562		- 931		-100,500
			-931 (562 - 373)	-176,000
	70 - (G_2)	- 70	- 70 (530 - 373)	- 11,000
	60 - (F)	- 60	- 60 (467 - 373)	- 5,600
	410 - (E)	- 410	-410 (404 - 373)	- 12,700
	3.1 (189)	- 585	-585 (94.5)	- 55,200
	$\frac{5.83}{402}\left(\frac{189}{2}\right)^2$	- 260	-260 (63)	- 16,400
373		-2316		-377,400
			-2316 (373 - 290)	-192,500
	130 - (D)	- 130	-130 (362 - 290)	- 9,400
	$\frac{2040}{170}(62)$	- 742	-742 (31)	- 23,000
	5.83 (83)	- 485	-485 (41.5)	- 20,100
290 Aft		-3673		-622,400
		+5190		
290 Fwd		+1517		
			+1517 (43)	+ 65,100
	$\frac{2040}{170}(43)$	- 516	-516 (21.5)	- 11,100
	5.83 (43)	- 251	-251 (21.5)	- 5,400
247 Aft		+ 750		-573,800
		+4950		
247 Fwd		+5700		
			+5700 (247)	+1,407,000
	12 (65)	- 780	-780 (214.5)	-167,500
	340 - G_1	- 340	-340 (161)	- 54,900
	2800 B	-2800	-2800 (141)	-396,300
	340 - A	- 340	-340 (80)	- 27,200
	247 (5.83)	-1440	-1440 (123.5)	-178,000
0		0	Error	+ 9,300

The fuselage reactions at the spars for center of gravity forward for items aside from the tail load are found as follows:

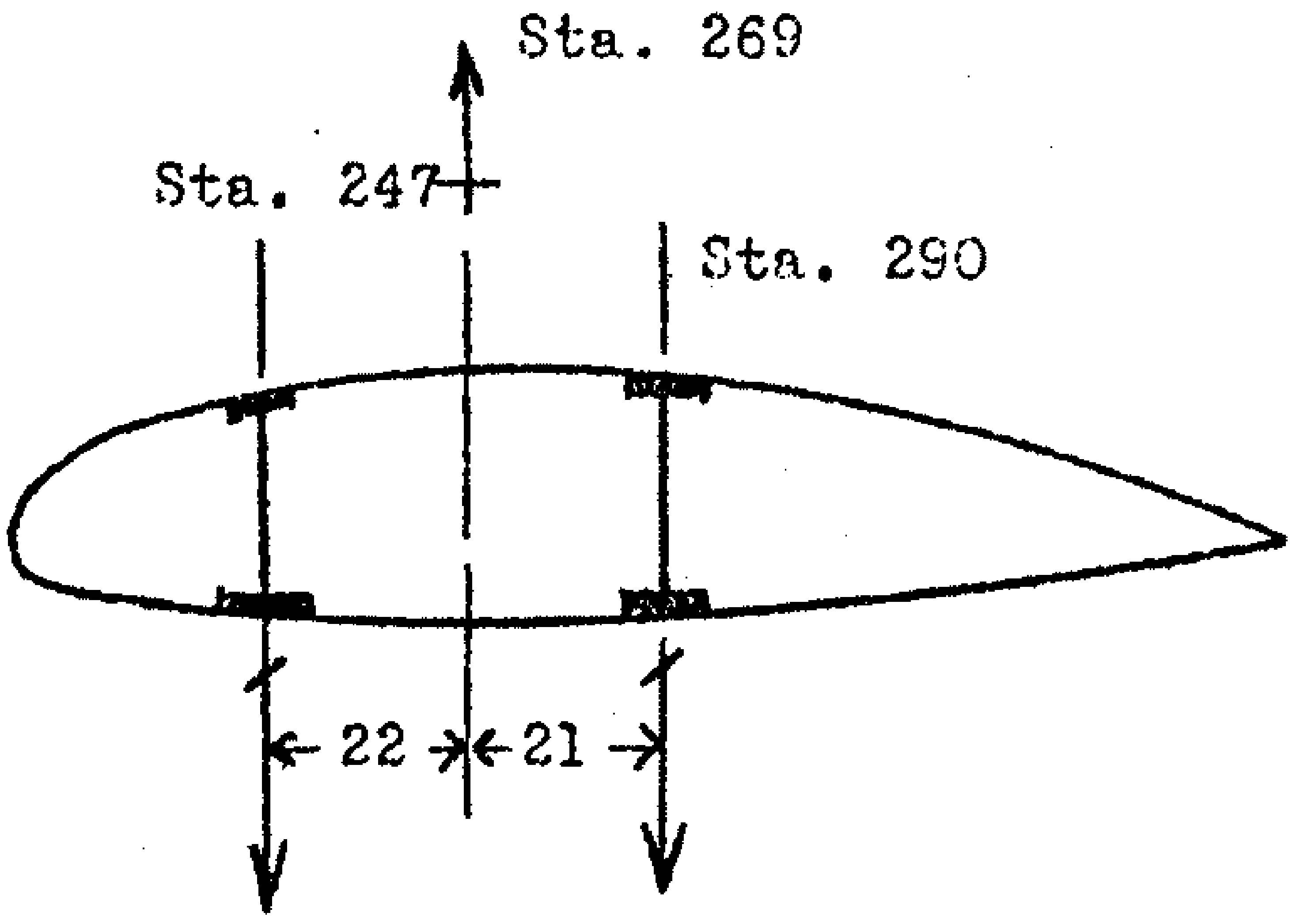


Fig. 7

Spar Reactions for Fuselage Dead Weight

The center of gravity for fuselage weights is at station 269. The weight of the fuselage and contents at one load factor is 10,140 lbs.

$$\text{Front Spar Reaction} = \left(\frac{21}{43}\right)(10{,}140) = 4{,}950 \text{ lbs.}$$

$$\text{Rear Spar Reaction} = \left(\frac{22}{43}\right)(10{,}140) = 5{,}190 \text{ lbs.}$$

****

Table II following shows the dead weight shears and moments for a load factor of 1.0. Loads are assumed acting upward and a minus moment means tension on the bottom fibers of the fuselage.

Table II covers the effect of vertical components of weights only. The horizontal components of weights will be treated separately. Aft load is considered to be plus. The actual locations of weights are taken from scale drawings of the side view of the airplane. For purposes of illustration we will assume a set of conditions for this airplane as follows:

1. All useful load items in the fuselage act along a line 176.5" below the line for vertical reference. (See Fig. 5)

2. All structure items other than the tail surfaces and tail wheel act along a line 194" below the reference line.

3. The total horizontal load is resisted between the front and rear spars along a horizontal line 217" below the reference line.

4. All moments due to horizontal components of loads will be calculated about a line 182" below the reference line. (This is the center of gravity of items 1 and 2).

5. The unbalanced moment of the horizontal loads will be resisted by a vertical couple on the spars. The spar reactions due to this couple assuming aft horizontal loads with a load factor of 1.0 are as follows:

$$\text{Spar reactions} = \frac{(10,140)(35) + (600)(47)}{43} = 8900 \text{ lbs.}$$

See Fig. 8.

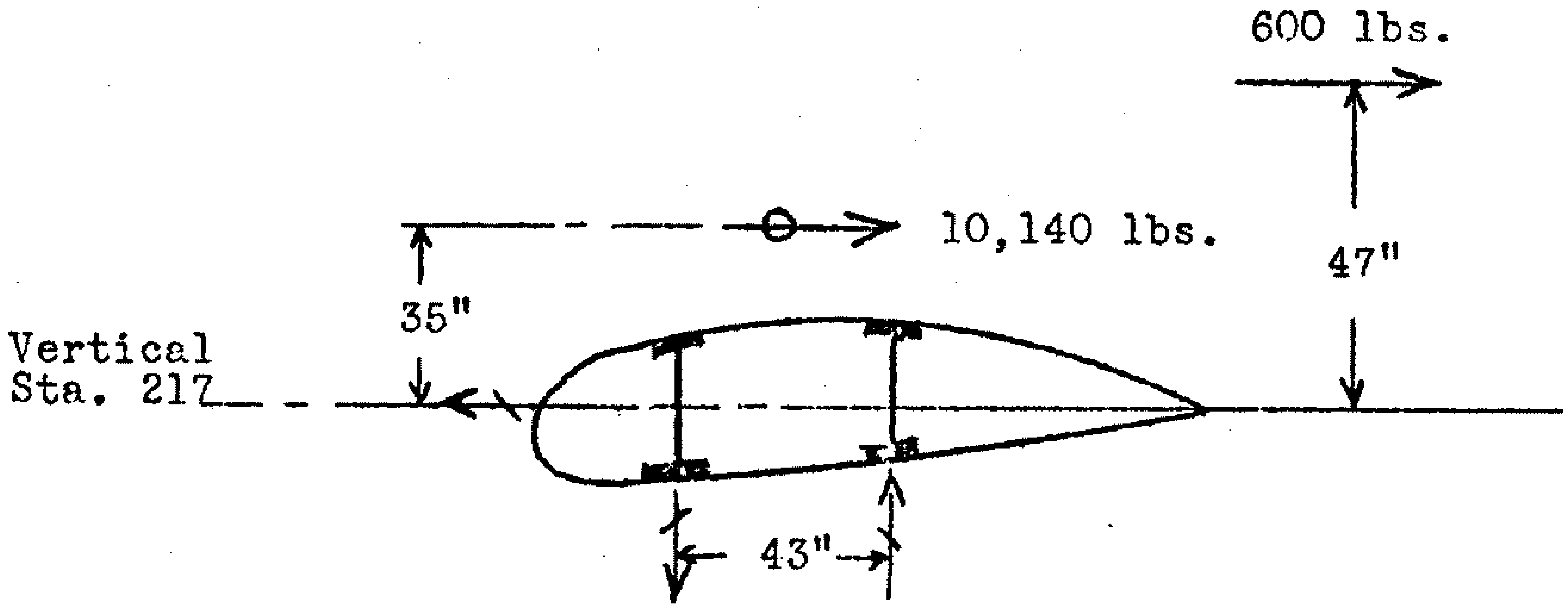


Fig. 8

Balance of Moments
Horizontal Loads

6. For a load of one factor the end load forward of the front spar or aft of the rear spar is equal to the shear for a vertical load factor of one. See Fig. 9.

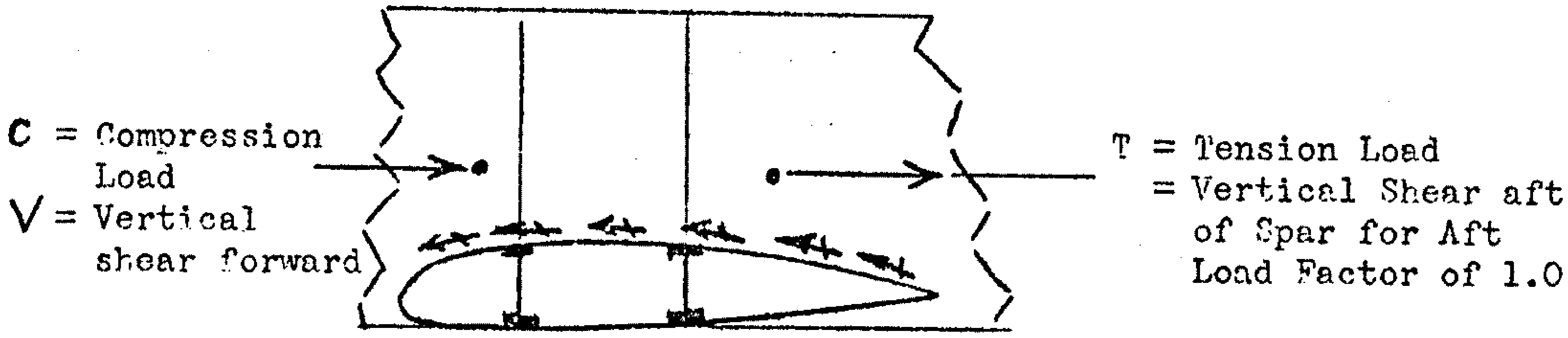

C = Compression Load
V = Vertical shear forward
T = Tension Load
= Vertical Shear aft of Spar for Aft Load Factor of 1.0

Fig. 9

TABLE III

DESIGN SHEARS AND MOMENTS ON FUSELAGE

		SHEAR						MOMENT			
Station	Factor	247 Fwd	247 Aft	290 Fwd	290 Aft	373	562	247	290	373	562
					UNIT LOADS						
Wts. Vert.	1.0	5,700	750	1517	-3673	-2316	-931	-573,800	-622,400	-377,400	-100,500
Wts. Horiz.	1.0	0	-8,900	-8,900				0	- 28,200	- 28,200	- 28,200
Tail Load	1000	0	9,180	9,180	-1000	-1000	-1000	0	-395,000	-312,000	-123,000
					CONDITION I						
Wts. Vert.	5.14	-29,300	-3,860	-7,800	19,000	11,950	4,800	2,960,000	3,210,000	1,940,000	519,000
Wts. Horiz.	1.27	0	-11,300	-11,300				0	-36,000	-36,000	-36,000
Tail Load	-3300	0	-30,300	-30,300	3,300	3,300	3,300	0	1,306,000	1,032,000	406,000
		-29,300	-45,460	-49,400	22,300	15,250	8,100	-2,960,000	4,480,000	2,936,000	889,000
					CONDITION II						
Wts. Vert.	2.31	13,200	1,740	3,500	-8,510	-5,380	-2,160	-1,328,000	-1,440,000	-872,000	-232,000
Wts. Horiz.	.205	0	-1,820	-1,820				0	-6,000	-6,000	-6,000
Tail Load	-60	0	-550	-550	60	60	60	0	24,000	19,000	7,000
		13,200	-630	1,130	-8,450	-5,320	-2,100	-1,328,000	-1,422,000	-859,000	-231,000
					CONDITION III$_1$						
Wts. Vert.	-3.61	-20,600	-2,710	-5,480	13,280	8,380	3,370	2,070,000	2,250,000	1,365,000	364,000
Wts. Horiz.	0										
Tail Load	-6360	0	-58,300	-58,300	6,360	6,360	6,360	0	2,510,000	1,985,000	668,000
		-20,600	-61,010	-63,780	19,540	14,740	10,730	2,070,000	4,760,000	3,350,000	1,032,000

AIRPLANE
STRESS
ANALYSIS

RYAN AERONAUTICAL INSTITUTE
LINDBERGH FIELD
SAN DIEGO, CALIFORNIA

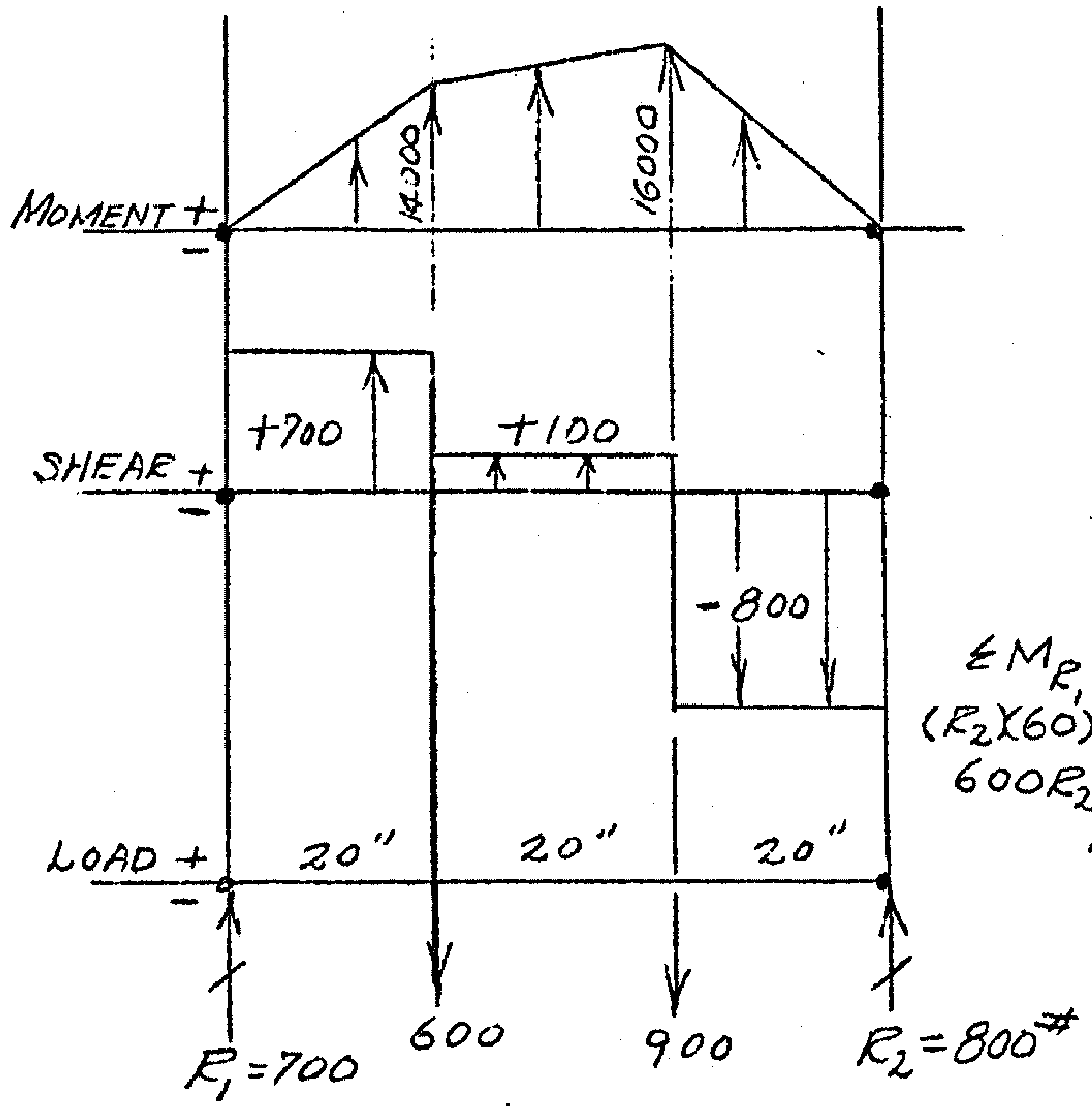
SOLUTION SHEETS TO ALL ASSIGNMENTS

BASIC STRESS ANALYSIS

SOLUTION SHEETS

Solution to Problem of Assignment 1:1.

+ shear part to the left tends to go up.
+ moment = tension on bottom fibers of beam.



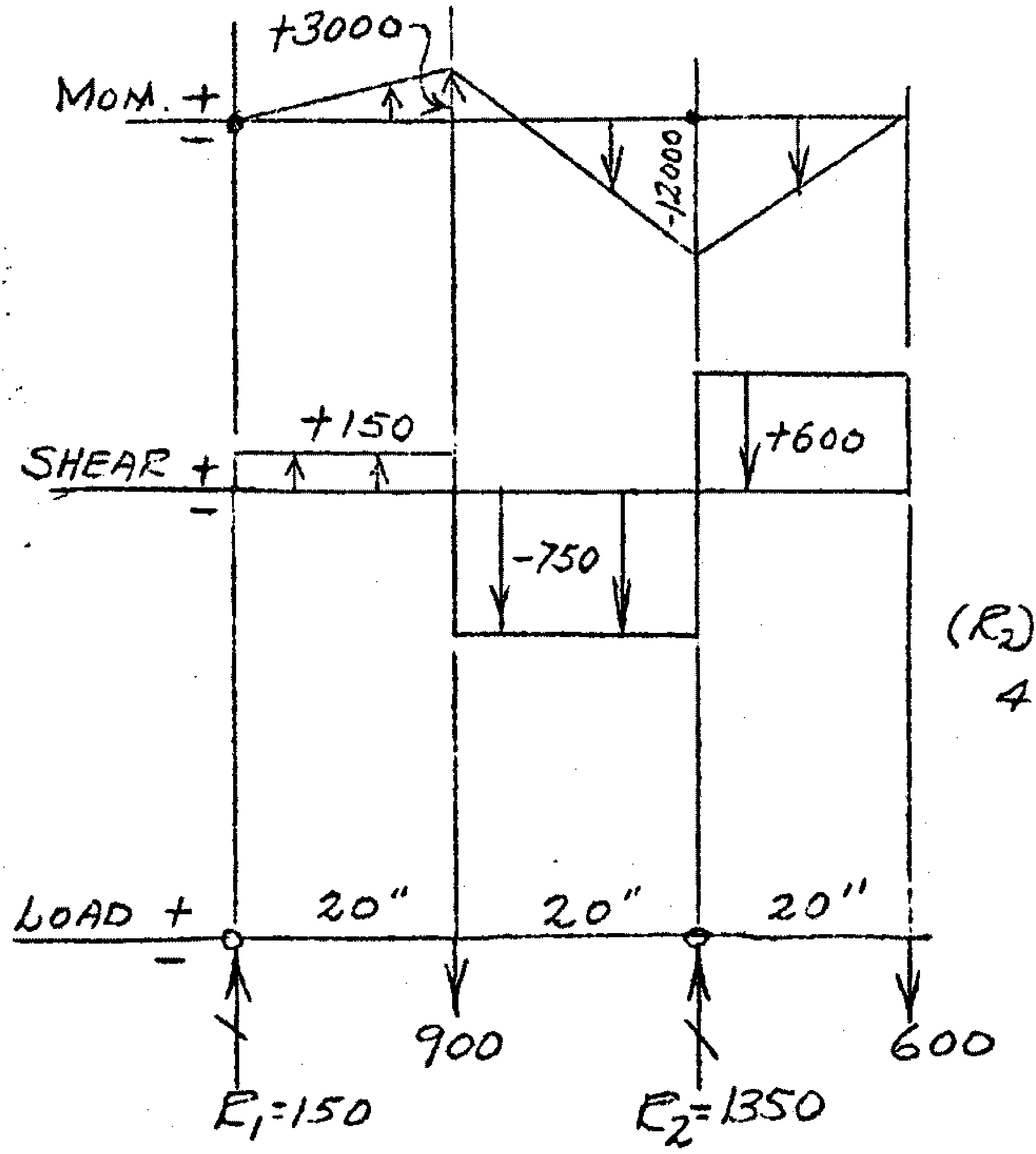
1:1
(a)

$\Sigma M_{R_1} = 0 \; +\circlearrowleft$

$(R_2)(60)-(900)(40)-(600)(20)=0$

$600R_2=3600+12000=48{,}000$

$R_2 = 800\#$



1:1
(b)

$\Sigma M_{R_1} = 0 \; +\circlearrowleft$

$(R_2)(40)-(600)(60)-(900)(20)=0$

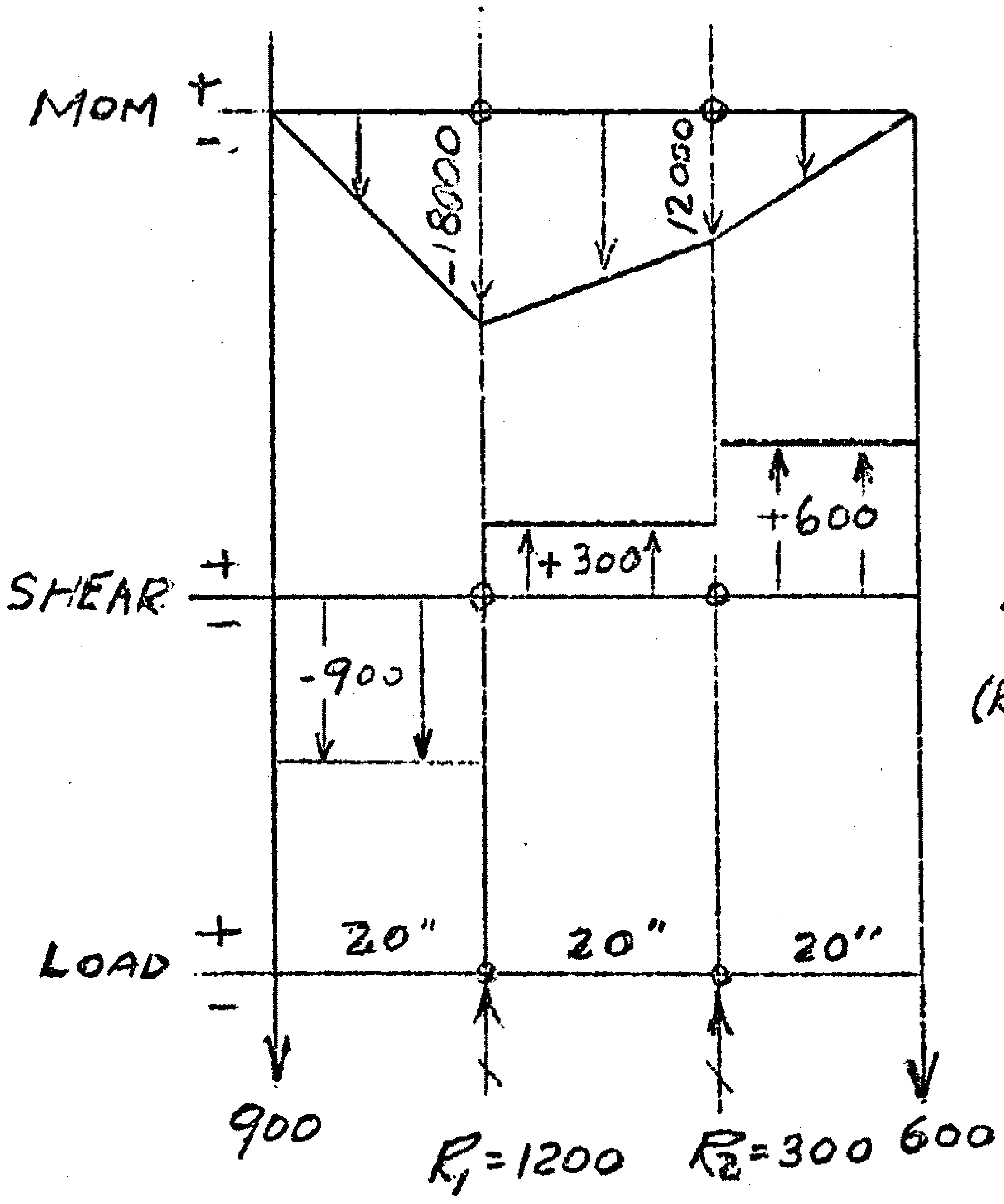
$40R_2 = 36000+18000=54{,}000$

$R_2 = 1350\#$

$R_1 = 150\#$

Solution to Problem of Assignment.

1:1



1:1

(C)

$\Sigma M_{E_1} = 0$ ↺+

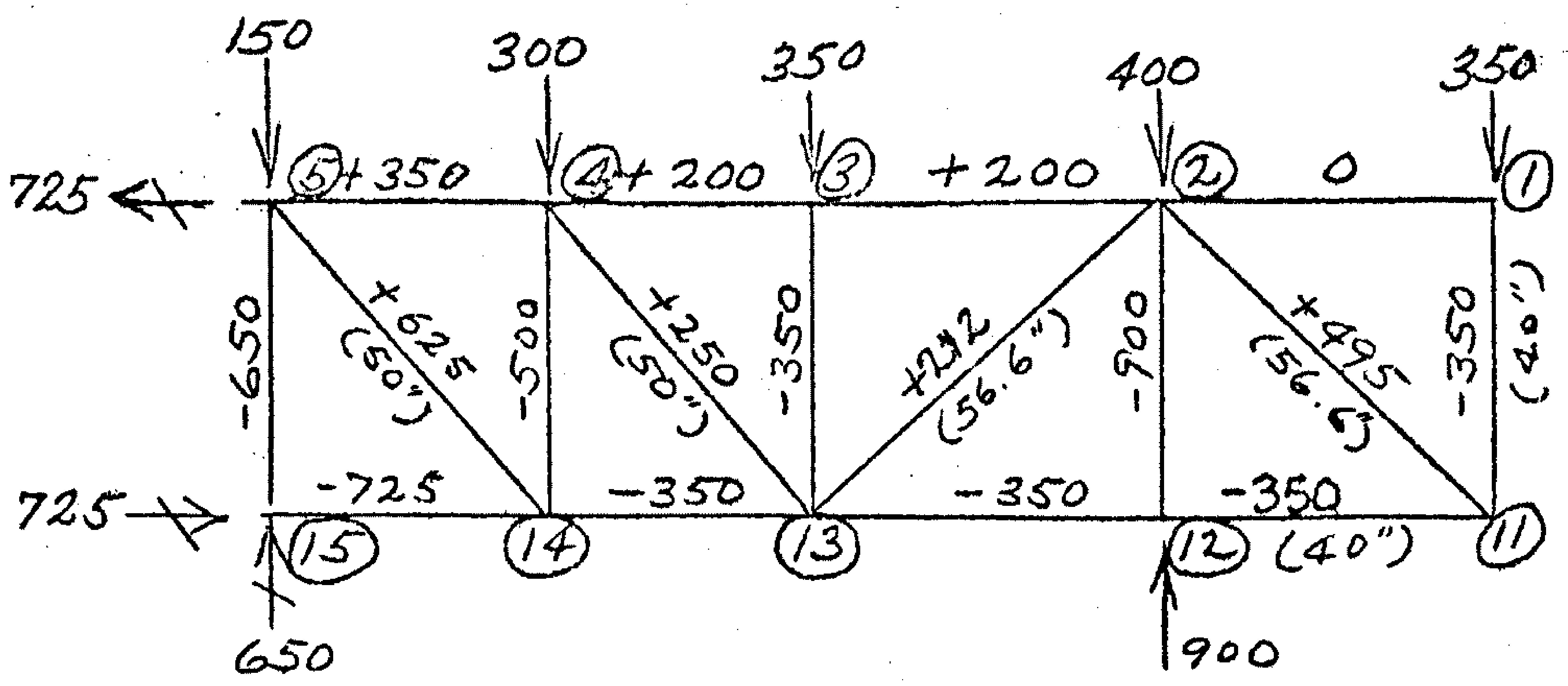
$(R_2)(20) + (900)(20) - (600)(40) = 0$

$20R_2 = 24000 - 18000$

$= 6000$

$R_2 = 300^{\#}, R_1 = 1200^{\#}$

Solution to Problems of Assignment 2.



Assume above loads are for a load factor of 1 and that the design load factor is 6. Then the table of design loads in members would show loads for a factor of 6. Only the web members, i.e. the drag struts and the tie rods are designed here. The loads shown in the chord members must be combined with the effect of lift loads to design the wing beams.

Solution to Problem of Assignment.

3:1 and 2

	Symbol	Matl.	Size	Area sq. inch	Length	Design LD.	Allow. LD.	M.S.
Tie Rods	5-14	Steel	5/16-24 Round			+ 3750	5700	.52
	4-13	"	10-32 Round			+ 1500	2100	.40
	2-13	"	10-32 Round			+ 1270	2100	.65
	2-11	"	1/4-28 Round			+ 2970	3400	.15

	Symbol	Matl.	Size	Area sq. inch	Length	Design LD.	Allow. LD.	M.S.
Drag Struts	5-15	Al. Alloy 17ST	1 1/2 x .058	.263	40"	- 3900	- 4310	
	4-14	"	1 1/2 x .049	.223	40"	- 3000	- 3650	
	3-13	"	1 1/4 x .058	.217	40"	- 2100	- 2450	
	2-12	"	1 3/4 x .058	.308	40"	- 5400	- 6190	
	1-11	"	1 1/4 x .058	.217	40"	- 2100	- 2450	

	Symbol	Matl.	Size	Area sq. inch	Length	Design LD.	Allow. LD.	M.S.
Drag Struts	5-15	C.M. Steel*	1 1/4 x .035	.134	40"	- 3900	- 4420	.13
	4-14	"	1 1/8 x .035	.120	40"	- 3000	- 3190	.06
	3-13	"	1 x .035	.106	40"	- 2100	- 2200	.05
	2-12	"	1 3/8 x .035	.147	40"	- 5400	- 5900	.09
	1-11	"	1 x .035	.106	40"	- 2100	- 2200	.05

*Heat treated to yield point = 100,000 lbs. per sq. inch.

Column formulas

Al. Alloy 17ST $\frac{P}{A}$ $42{,}500 - 3345 \frac{L}{\rho}$ Short column, pinned ends.

Critical L/ρ 85 $\frac{P}{A}$ $101{,}600{,}000/(\frac{L}{\rho})^2$ Long column, pinned ends.

Steel HT. to yield point = 100,000 lbs. per sq. inch $\frac{P}{A}$ $100{,}000 - 8.74 \, (\frac{L}{\rho})^2$ Short column

Critical L/ρ 75.6 $\frac{P}{A}$ $286{,}000{,}000/(\frac{L}{\rho})^2$ Long column

Tube areas may be taken from tables in Niles and Newell "Airplane Structures" or calculated.

Solution to Problem of Assignment.

4:1 $\Sigma M_{R_1} = 0 \qquad \curvearrowleft +$

$$(R_2)(40) + (2000)(40) + (900)(0) - (800)(40) - (300)(80) - (300)(110)$$

$$- (200)(140) - (300)(170) = 0$$

$$40R_2 + 80{,}000 - 32{,}000 - 24{,}000 - 33{,}000 - 28{,}000 - 51{,}000 = 0$$

$$40R_2 = 168{,}000 - 80{,}000 = 88{,}000$$

$$R_2 = 88{,}000/40 = 2200 \text{ lbs.}$$

$\Sigma R_2 = 0 \qquad \curvearrowright +$

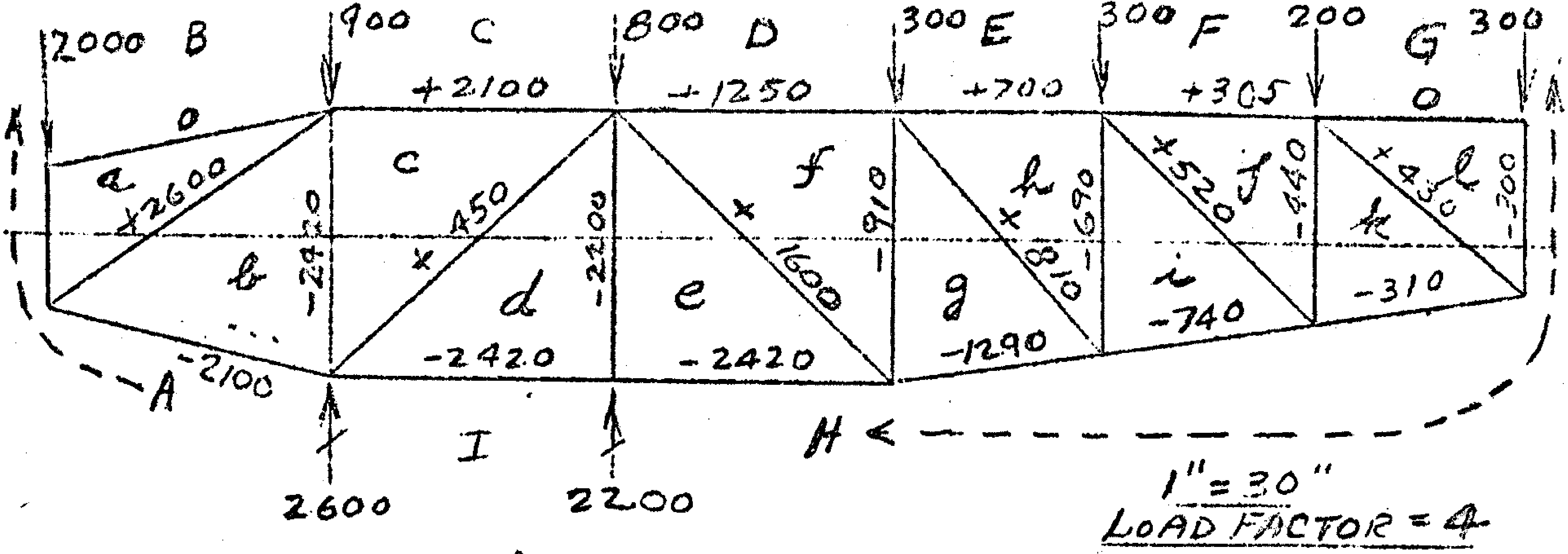
$$R_1 = 2600 \text{ lbs.}$$

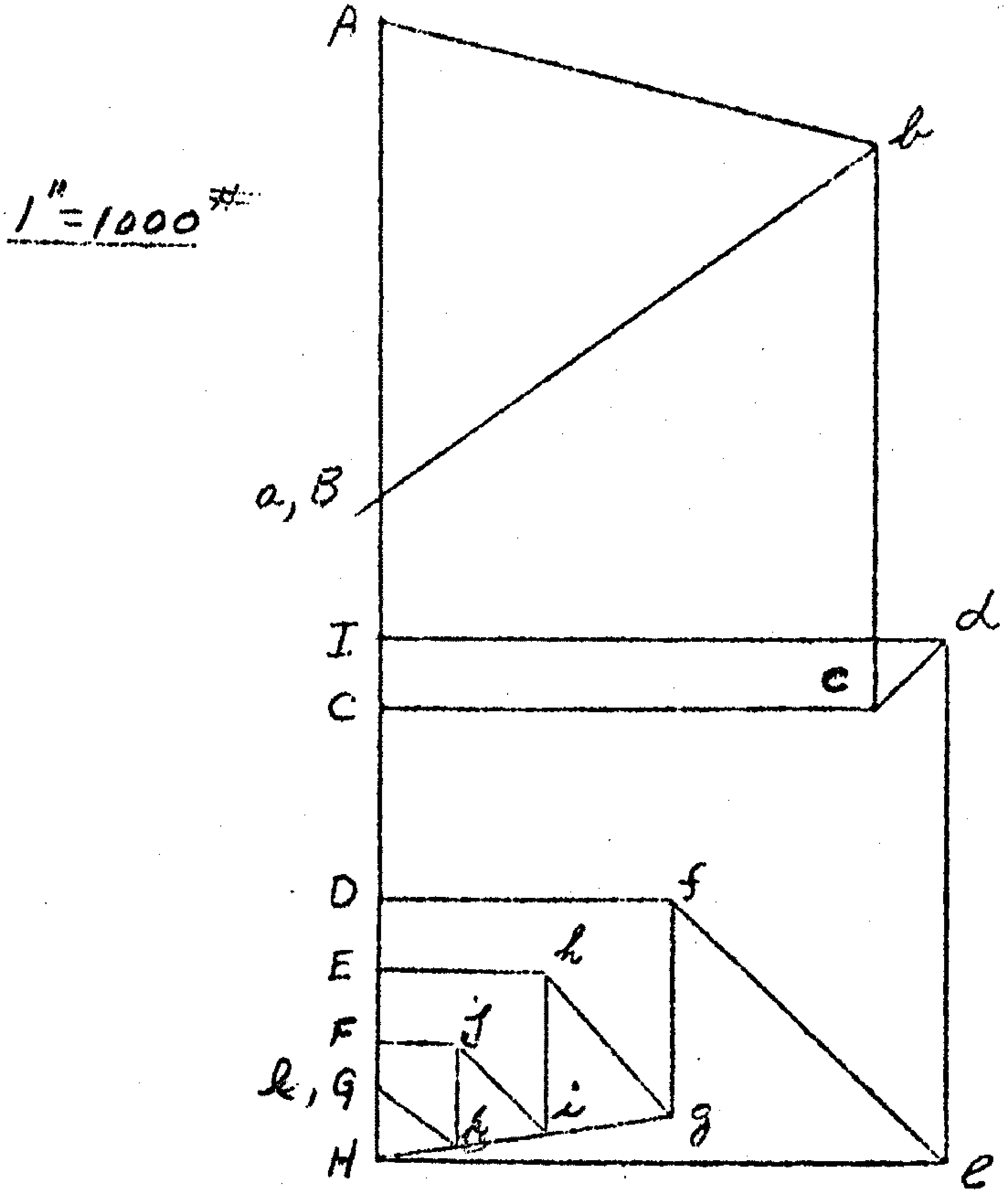
$\Sigma V = 0$, Check $R_1 + R_2 = 4800$ lbs.

$2000 + 900 + 800 + 300 + 300 + 200 + 300 = 4800$ lbs. Checks.

Solution to Problem of Assignment. (Continued)

4:1





Solution to Problems of Assignment.

5:1 A = -3800 lbs. Compression
V = .5800 x 3800 = 2200 lbs. (down)
H = .1110 x 3800 = 420 lbs. (forward)
S = .8069 x 3800 = 3060 lbs. (outward)

B = +3300 lbs. Tension
V = .7495 x 3300 = 2470 lbs. (upward)
H = .5344 x 3300 = 1760 lbs. (forward)
S = .3910 x 3300 = 1290 lbs. (inward)

C = - 4880 lbs. Compression
V = .8741 x 4880 = 4270 lbs. (down)
H = .1672 x 4880 = 820 lbs. (forward)
S = .4560 x 4880 = 2230 lbs. (outward)

$\Sigma V = 0$ $\uparrow + =$ up
$4000 + 2470 - 2200 - 4270 = ?$
$6470 - 6470 = 0$ checks.

$\Sigma H = 0$ $\rightarrow + =$ aft
$3000 - 420 - 1760 - 820 = ?$
$3000 - 3000 = 0$ Checks.

$\Sigma S = 0$ $\leftarrow + =$ in
$4000 + 1290 - 3060 - 2230 = ?$
$5290 - 5290 = 0$ Checks.

5:2 V = 4000 lbs. up, H = 3000 lbs. aft. S = 0

$\Sigma V = 0$
$$.5800A + .7495B + .8741C = -4000 \qquad (1)$$

$\Sigma H = 0$
$$.1110A - .5344B + .1672C = -3000 \qquad (2)$$

$\Sigma S = 0$
$$.8069A + .3910B + .4560C = 0 \qquad (3)$$

From (1)
$$1.000A + 1.29B + 1.505C = -6900 \qquad (1a)$$

From (2)
$$1.000A - 4.81B + 1.505C = -27000 \qquad (2a)$$

From (3)
$$1.000A + .485B + .565C = 0 \qquad (3a)$$

Subtracting equation (3a) from equation (1a)

$$\begin{array}{r} \cancel{1.000A} + 1.29B + 1.505C = -6900 \\ -\cancel{1.000A} + .485B + .565C = 0 \\ \hline .805B + .940C = -6900 \end{array} \qquad (4)$$

Solution to Problems of Assignment. (Continued)

5:2

Subtracting equation (3a) from equation 2a)

$$\begin{array}{rrrrl} & \cancel{1.00A} - 4.81B & + 1.505C & = & -27,000 \\ - & \cancel{1.00A +} .485B & + .565C & = & 0 \\ \hline & - 5.295B & + .940C & = & -27,000 \end{array} \qquad (5)$$

Subtracting equation (5) from (4)

$$\begin{array}{rrrrl} & .805B & + \cancel{.940C} & = & - 6,900 \\ - & 5.295B & + \cancel{.940C} & = & -27,000 \\ \hline & 6.100B & & = & 20,100 \end{array}$$

$B = + 3300$ lbs.
$C = - 10,180$ lbs.
$A = + 4150$ lbs.

Components check:

A = 4150 tens.	V = x .5800 = 2410 lbs.	(upward)
	H = x .1110 = 460 lbs.	(backward)
	S = x .8069 = 3340 lbs.	(inward)
B = + 3300	V = x .7495 = 2470 lbs.	(upward)
	H = x .5344 = 1760 lbs.	(forward)
	S = x .3910 = 1290 lbs.	(inward)
C = - 10,180	V = x .8741 = 8880 lbs.	(down)
	H = x .1672 = 1700 lbs.	(forward)
	S = x .4560 = 4630 lbs.	(outward)

$\Sigma V = 0$; $4000 + 2470 + 2410 - 8880 = ?$
$8880 - 8880 = 0$ checks.

$\Sigma H = 0$; $3000 + 460 - 1760 - 1700 = ?$
$3460 - 3460 = 0$ checks.

$\Sigma S = 0$; $4630 - 3340 - 1290 = ?$
$4630 - 4630 = 0$ checks.

Solution to Problem of Assignment.

6:1 (a) $y = 3x^5 + 25x^4 + 16x^3 + 12x^2 + 10x + 8$

$$\int y\,dx = \int (3x^5 + 25x^4 + 16x^3 + 12x^2 + 10x + 8)dx$$

$$= \int 3x^5dx + 25x^4dx + 16x^3dx + 12x^2dx + 10x\,dx + 8\,dx$$

$$= \frac{3x^6}{6} + \frac{25x^5}{5} + \frac{16x^4}{4} + \frac{12x^3}{3} + \frac{10x^2}{2} + 8x + C$$

$$\int y\,dx = 0.5x^6 + 5x^5 + 4x^4 + 4x^3 + 5x^2 + 8x + C$$

To find the area under this curve between x = 2 and x = 1 we substitute these values as the limits.

$$\int_1^2 y\,dx = \left[(0.5)(2)^6 + (5)(2)^5 + (4)(2)^4 + (4)(2)^3 + (5)(2)^2 + (8)(2)\right]$$

$$- \left[(0.5)(1)^6 + (5)(1)^5 + (4)(1)^4 + (4)(1)^3 + (5)(1)^2 + (8)(1)\right]$$

$$= \left[32 + 160 + 64 + 32 + 20 + 16\right]$$

$$- \left[0.5 + 5 + 4 + 4 + 5 + 8\right] = 324 - 26.5 = \underline{297.5}$$

(b) $y = 8x^3 + 9x^2 + 12x + 24$

$$\frac{dy}{dx} = (3)(8x^2) + (2)(9x) + 12 + 0$$

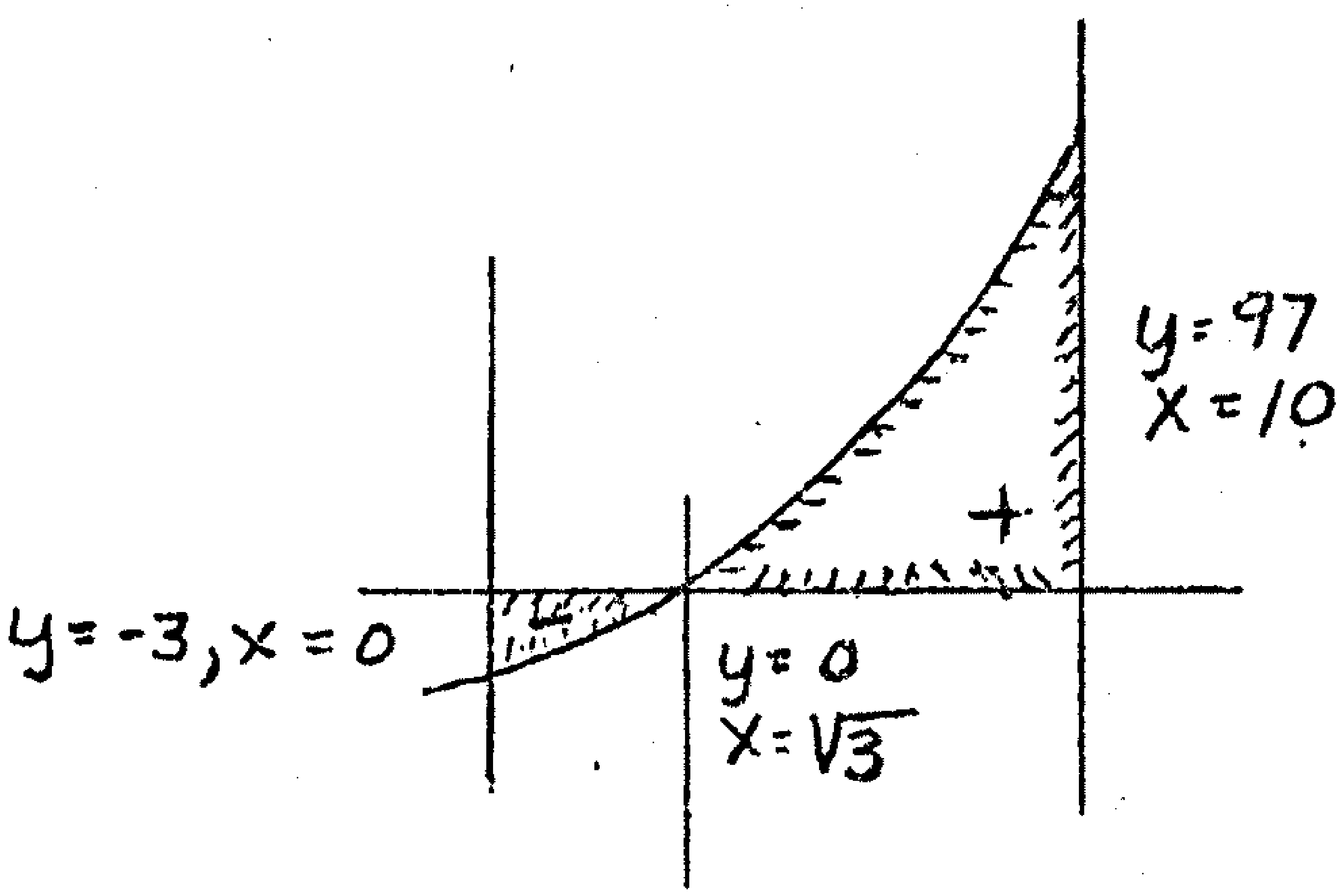
$$= \underline{24x^2 + 18x + 12}$$

(c) To find the area between x = 10 and x = 0 for the curve $y = x^2 - 3$

$$\text{Area} = \int y\,dx = \int_0^{10} (x^2 - 3)dx = \int_0^{10} x^2dx - 3dx$$

$$= \left[\frac{x^3}{3} - 3x\right]_0^{10} = \left[\frac{10^3}{3} - (3)(10)\right] - \left[0\right]$$

$$= 333.3 - 30 = \underline{303.3}$$



Graph of Areas

To check by segments, take the plus and minus areas separately. For the negative area

$$\int_{x=0}^{x=\sqrt{3}} y\,dx = \int_{x=0}^{x=\sqrt{3}} (x^2 - 3)dx = \left[\frac{x^3}{3} - 3x\right]_0^{\sqrt{3}} = \left[\frac{(\sqrt{3})^3}{3} - 3\sqrt{3}\right] - \left[0\right]$$

$$= \sqrt{3} - 3\sqrt{3} = -2(1.73) = -3.46$$

****

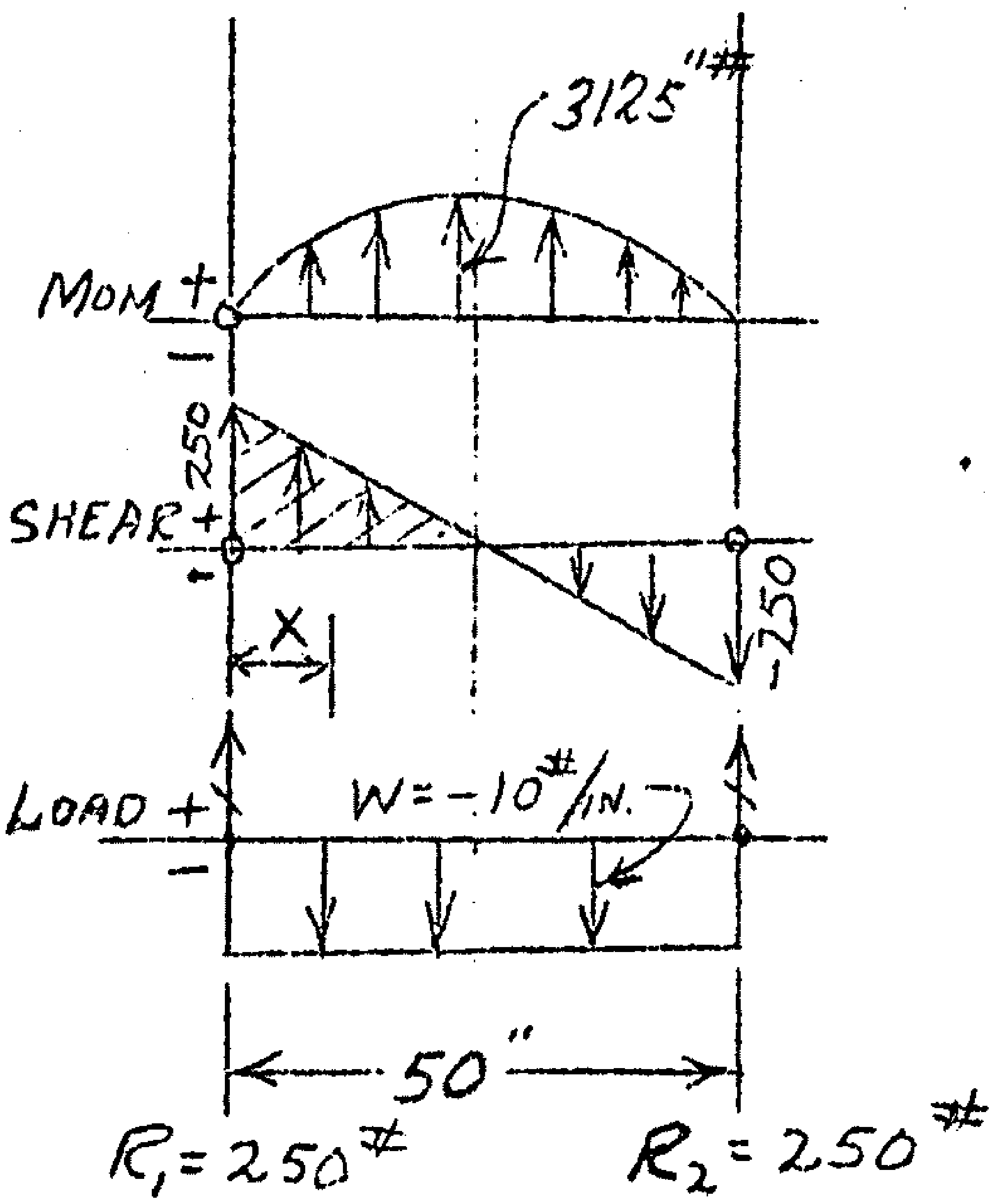
$$\int_{\sqrt{3}}^{10} y\,dx = \int_{\sqrt{3}}^{10} (x^2 - 3)dx = \left[\frac{x^3}{3} - 3x\right]_{\sqrt{3}}^{10} = \left[\frac{10^3}{3} - (3)(10)\right] - \left[\frac{(\sqrt{3})^3}{3} - 3\sqrt{3}\right]$$

$$= \left[333.3 - 30\right] - \left[-3.46\right] = 303.3 + 3.46 = 306.76$$

Total of plus and minus areas = 306.76 - 3.46 = 303.3 Checks.

Solution to Problems of Assignment.

7:1 (a)



Mom. at center = $(250)(\frac{25}{2}) = 3125''\#$

from area between R_2 and of beam in shear curve.

Equation of Mom. at any station "x". Mom. = $250x - \frac{10x^2}{2}$

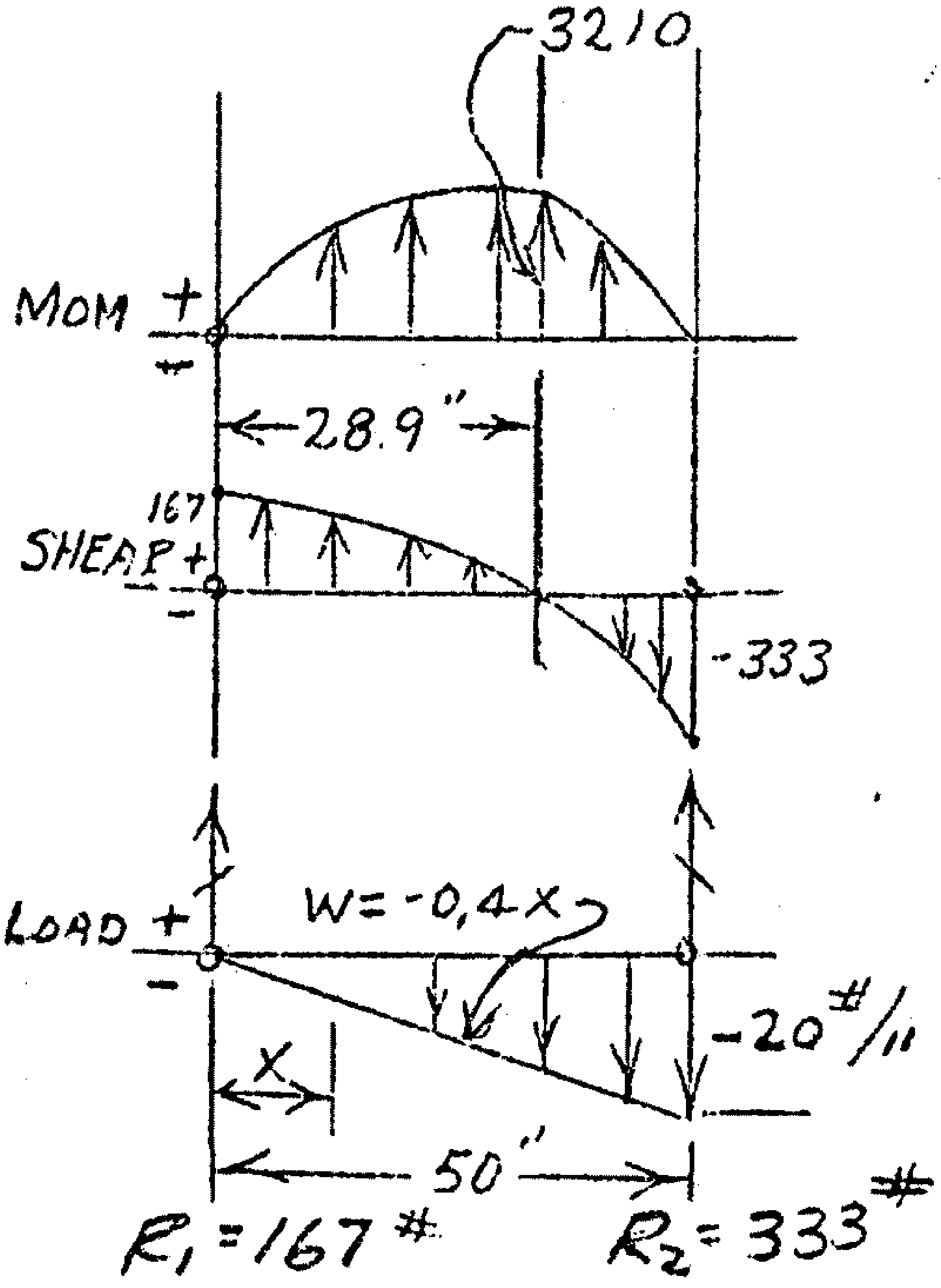
***

Shear = $+250 - 10x$

Shear at just to left of R_2 = $+250 - (10)(50) = -250$ lbs.

$R_1 = R_2 = 250$ lbs.

7:1 (b)



Mom. = $167x - \frac{0.2x^3}{3} = 167x - .0667x^3$

Max. Mom. = $(167)(28.9) - (.0667)(28.9)^3 = 3210''\#$

At R_2 where $x = 50$.

Mom. = $(167)(50) - (.0667)(50)^3$
= $8350 - 8350 = 0$ (Check)

***

Calc. "x", where shear passes through zero.

$167 - 0.2x^2 = 0$, $0.2x^2 = 167$

$x = \sqrt{835} = 28.9''$

$x^2 = \frac{167}{0.2} = 835$

***

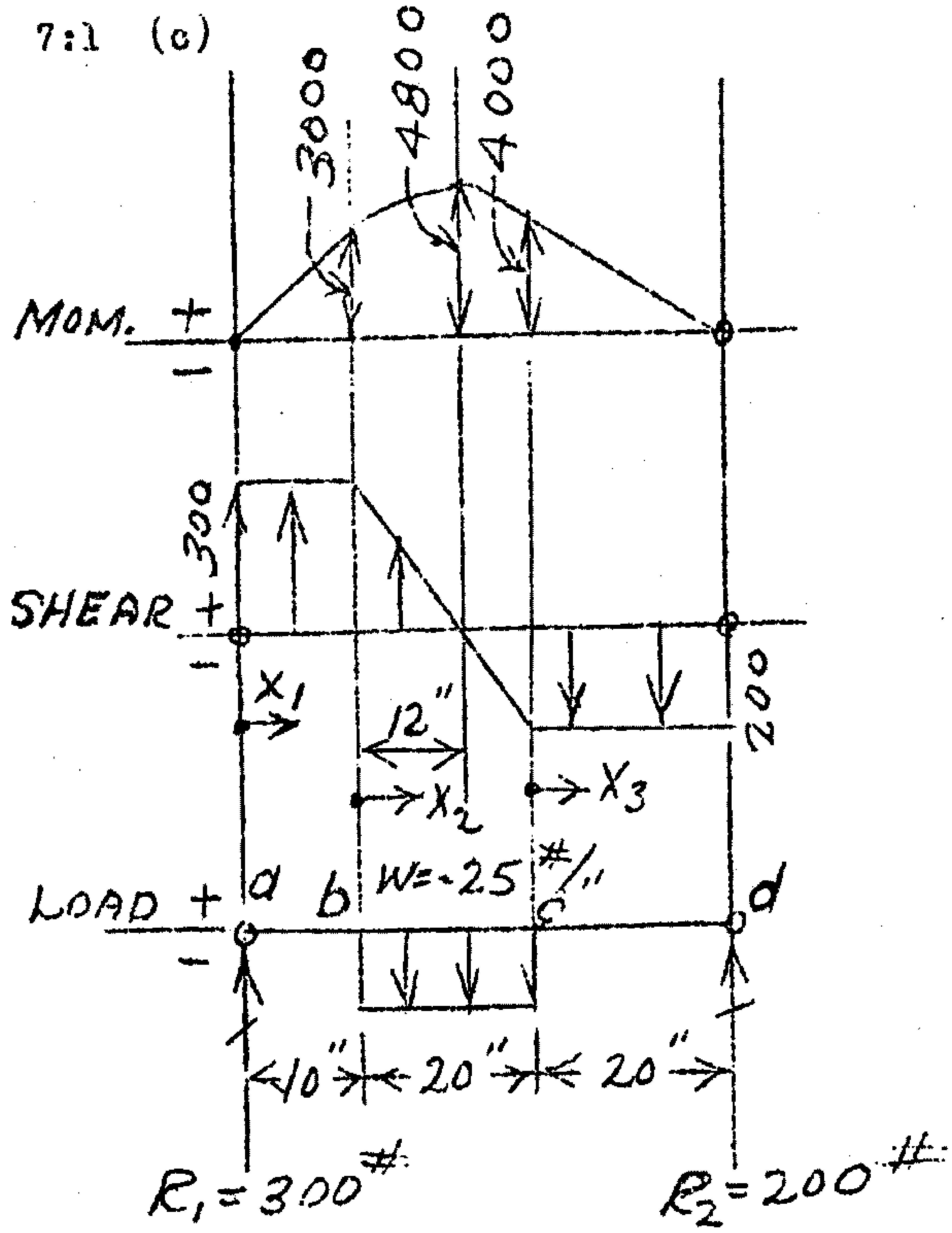
Shear = $167 - \frac{0.4x^2}{2} = 167 - 0.2x^2$

Shear at 50" = $167 - (0.2)(50)^2$

$(x = 50) = 167 - 500 = -333$ (Check)

Load = $-0.4x$

7:1 (c)



Mom. from a to b = $300x_1$

Mom. at b = (300)(10) = 3000"#

Mom. from b to c = $3000 + 300x_2 - \frac{25x_2^2}{2}$

Max. Mom. (at x_2 = 12") = $300 + (300)(12) - \frac{(25)}{2}(12)^2$ = 4800"#

Mom. at x_2 = 20" = 4000"#

Mom. at R_2 = 4000 − (200)(20) = 0 (Check)

***

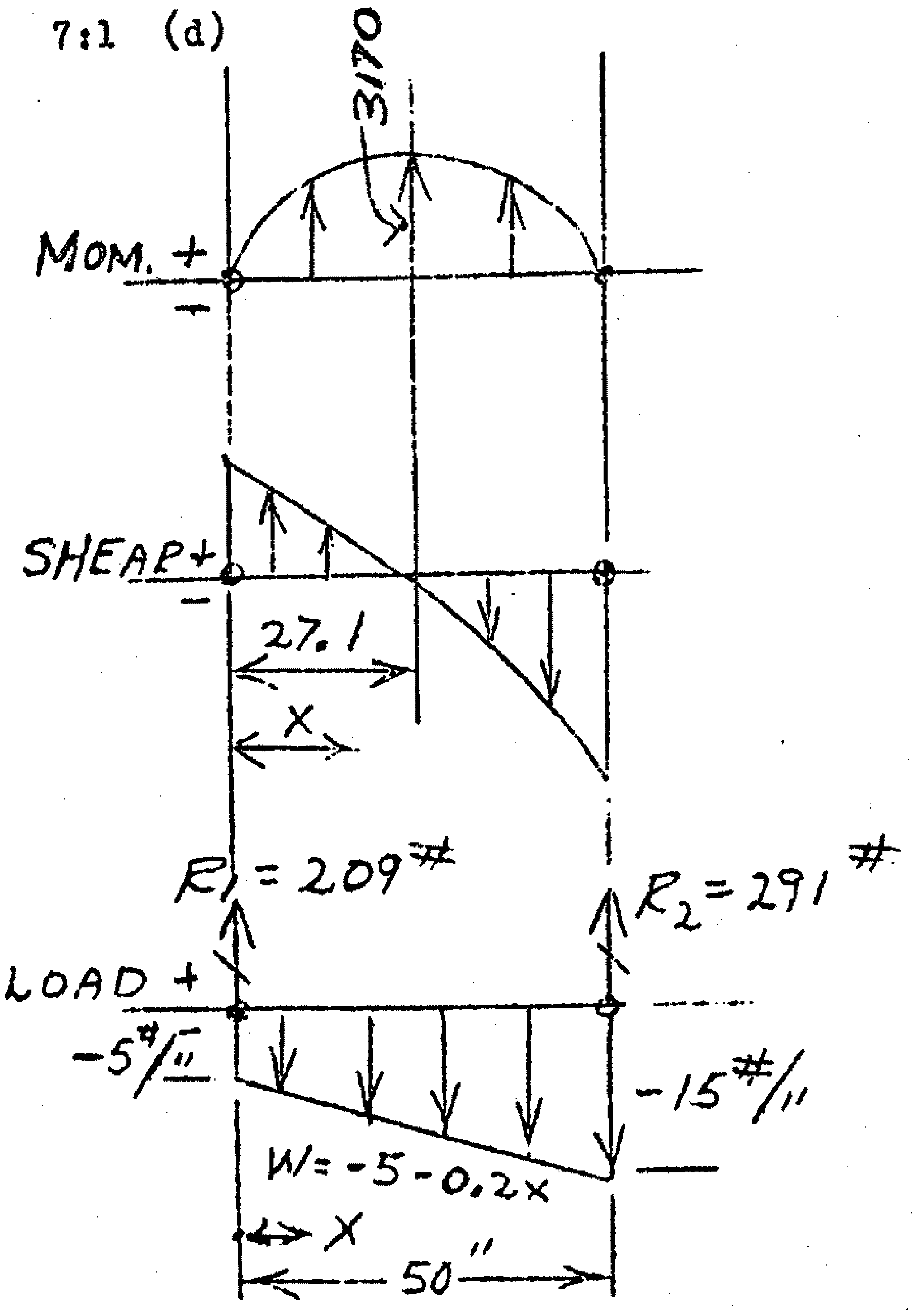
Shear a to b = +300 lbs.
Shear b to c = $300 - 25x_2$ with origin of x at point b.
Shear passes through zero 12" to right of b.

[$300 - 25x_2 = 0$, $x_2 = 12$]

Shear between c and d = −200

***

7:1 (d)



Equation of Moment.

Moment = $209x - \frac{5x^2}{2} - \frac{0.1x^3}{3}$

Where x = 27.1" Mom. = $(209)(27.1) - \frac{5}{2}(27.1)^2 - \frac{0.1}{3}(27.1)^3$ = 5670 − 1835 − 665 = +3170"#

Where x = 50, Mom. = $(209)(50) - \frac{5}{2}(50)^2 - \frac{0.1}{3}(50)^3$ = 10,450 − 6250 − 4170 = (30) = zero error in dropping decimals

***

Equation of Shear.

Shear = $209 - 5x - \frac{0.2x^2}{2} = 209 - 5x - 0.1x^2$

Shear passes through zero at x = 27.1" (see solution on next page)
Where x = 50, Shear = $209 - (5)(50) - (0.1)(50)^2$ = 209 − 250 − 250 = −291 lbs. (Check)

Equation of Load.

Where x = 0, Load = −5 lbs./in.
Where x = 50, Load = −5 − (0.2)(50) = −5 − 10 = −15 lbs./in.

In problem 7:1 (d) the solution for the distance "x" to the point where the shear passes through zero is accomplished by completing the square to solve a quadratic equation.

The general solution would be as follows:

$$\text{general form } (x + a)^2 = x^2 + 2ax + a^2$$

In our problem equating the shear to zero

$$209 - 5x - 0.1x^2 = 0, \text{ or } 0.1x^2 + 5x = 209$$

Multiplying both sides of the equation by 10

$$x^2 + 50x = 2090$$

50 in the second term of the left side of the equation represents 2a in the general equation. Then a = 25 and if we add 25^2 to each side of the equation the left hand side is a perfect square.

$$x^2 + 50x + (25)^2 = 2090 + (25)^2$$

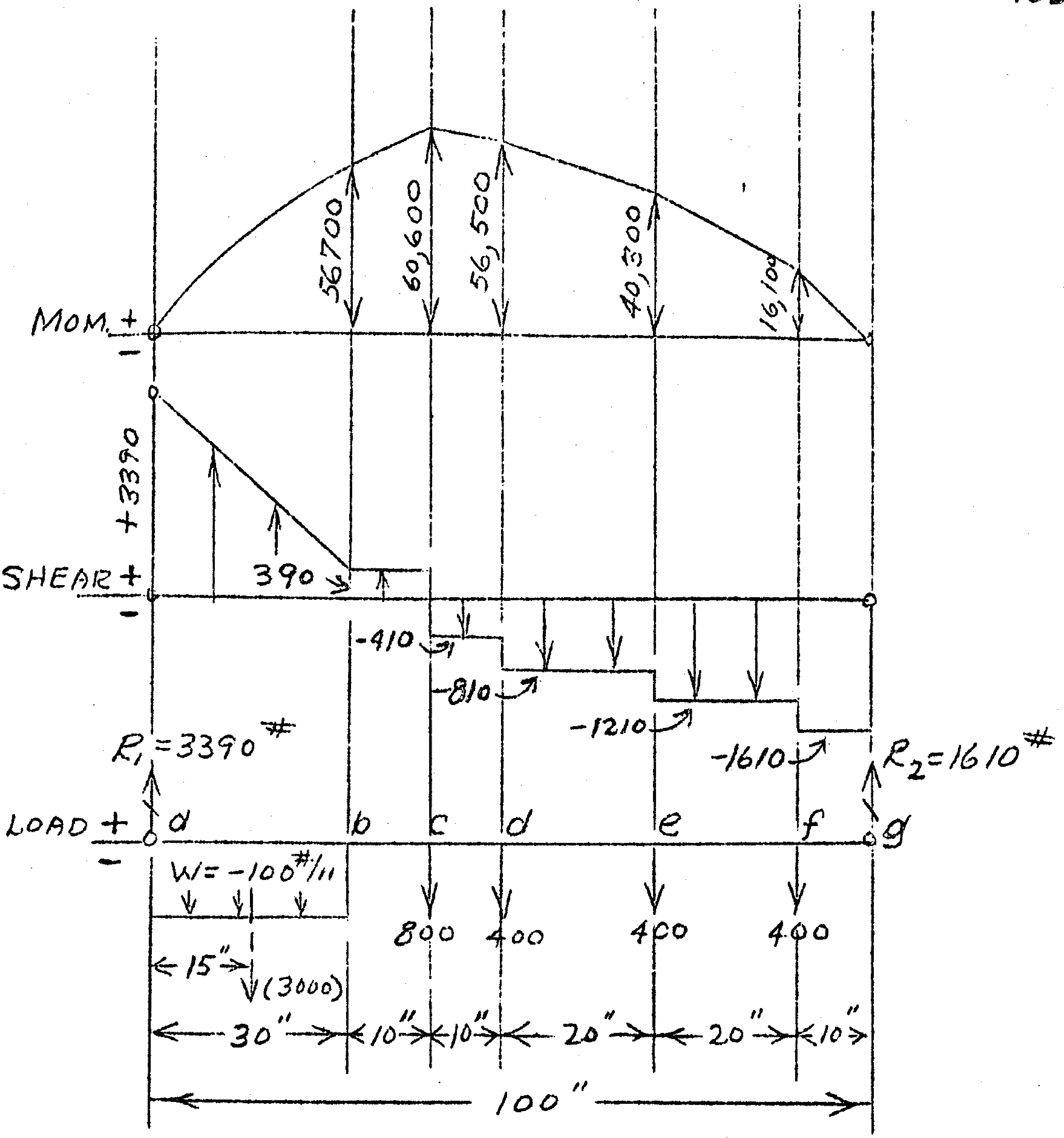
$$(x + 25)^2 = 2090 + 625 = 2715$$

Extracting the square root of each side

$$x + 25 = \sqrt{2715} = \pm 52.1$$

$$x = \pm 52.1 - 25 = +27.1 \text{ or } -77.1$$

The real solution is x = 27.1 in.



Moments (By areas of shear curve)

$$\text{at } b = \frac{(3390 + 390)}{2}(30) = 56{,}700''\#$$

at c = 56,700 + (390)(10) = 60,600"#

at d = 60,600 - (410)(10) = 56,500"#

at e = 56,500 - (810)(20) = 40,300"#

at f = 40,300 - (1210)(20) = 16,100"#

at g = 16,100 - (1610)(10) = 0 (Check)

Shears

at a = 3390 lbs., at b = 3390 - 3000 = 390 lbs.
at c = 390 - 800 = -410 lbs., at d = -410 - 400 = -810 lbs.
at e = -810 - 400 = -1210 lbs., at f = - 1210 - 400 = -1610 lbs.
This checks with reaction, R_2.

$$\sum M_{R_1} = 0 \qquad \overset{\leftarrow}{+)}$$

$$(R_2)(100) - (400)(90) - (400)(70) - (400)(50) - (800)(40) - (300)(15) = 0$$

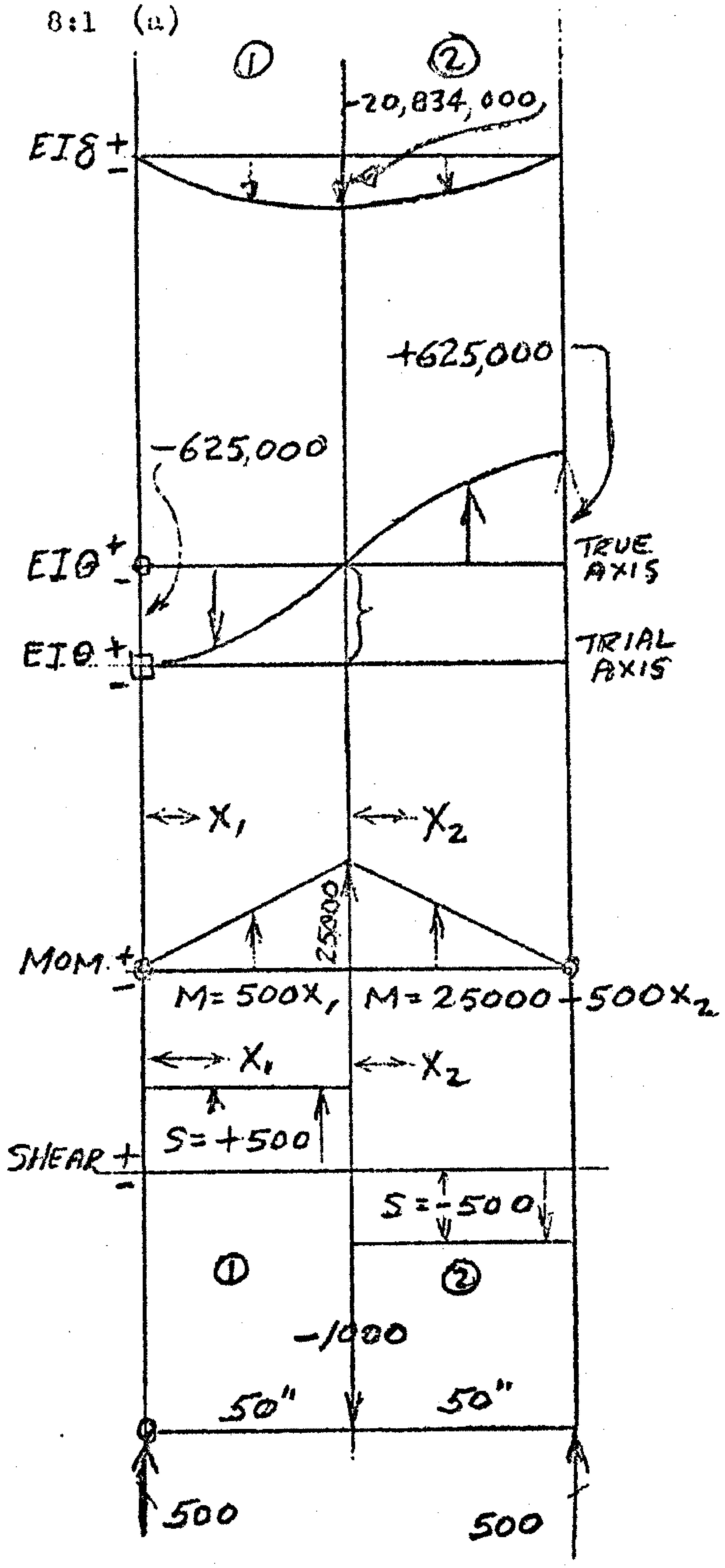
$$100R_2 - 36{,}000 - 28{,}000 - 20{,}000 - 32{,}000 - 45{,}000 = 0$$

$$100R_2 = 161{,}000; \quad R_2 = 1610 \text{ lbs.}$$

By $\sum M_{R_2}$ .... R_1 = 3390 lbs.

Solution to Problem of Assignment.

8:1 (a)



Seg. (1) $EI\delta = -625{,}000x_1 + 83.33x_1^3$

where $x_1 = 50$, $EI\delta = -20{,}834{,}000$

Seg. (2)

$EI\delta = -20{,}834{,}000 + 12{,}500x_2^2$
$-83.33x_2^2$

where $x_2 = 50$, $EI\delta = 0$ check

***

Trial $EI\theta$ Segm.(1) $= \frac{500x_1^2}{2} = 250x_1^2$

$EI\theta(\text{at } x_1 = 50) = 625{,}000$

$EI\theta$ Segm.(2) =

$625{,}000 + 25{,}000x_2 - 250x_2^2$

True

Segm.(1) $EI\theta = -625{,}000 + 250x_1^2$

Segm.(2)

$EI\theta = C_2 + 25{,}000x_2 - 250x_2^2$

$C_2 = 0$, $EI\theta = 25{,}000x_2 - 250x_2^2$

***

Segm. (1) $M = 500x_1$

Segm. (2) $M = C_1 - 500x_2$

where $x_2 = 0$, $M = C_1 = 25{,}000$

Segm.(2), $m = 25{,}000 - 500x_2$

***

Segm. (1) $S = +500$ lbs.
Segm. (2) $S = -500$ lbs.

***

Actual value of max.

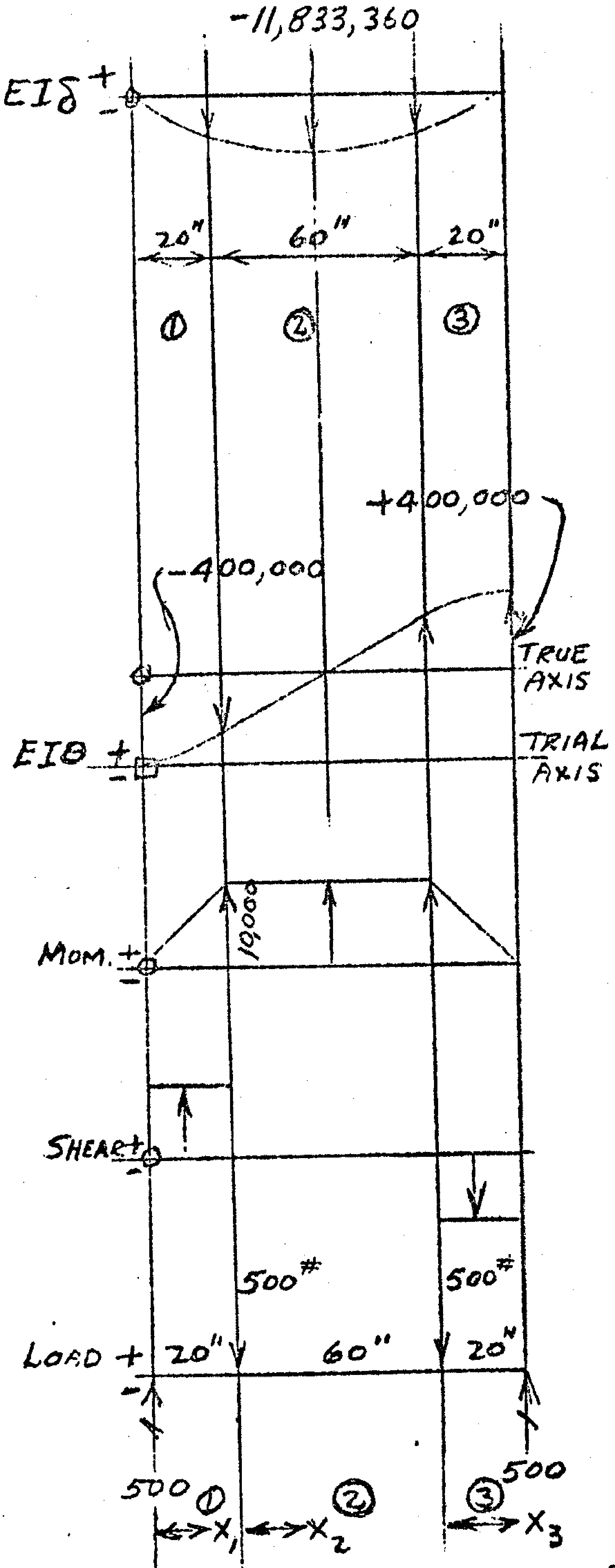
$E = 10{,}000{,}000$
$I = 3.0$

$EI\delta = -20{,}834{,}000$

$$\delta = \frac{-20{,}834{,}000}{(10{,}000{,}000)(3)} = \underline{-.694''}$$

Solution to Problem of Assignment. (Continued)

8:1 (b)



(1) $EI\delta = -400{,}000x_1 + 83.33x_1^3$

(2) $EI\delta = -7{,}333{,}360 - 300{,}000x_2 + 5000x_2^2$

where $x_2 = 30$; $EI\delta = -11{,}833{,}360$

where $x_2 = 60$; $EI\delta = -7{,}333{,}360$

(3) $EI\delta = -7{,}333{,}360 + 300{,}000x_3 + 5000x_3^2 - 83.33x_3^3$

***

Trial Segm.(1), $EI\Theta = +250x_1^2$

where $x_1 = 20$, $EI\Theta = +100{,}000$

Segm.(2), $EI\Theta = 100{,}000 + 10{,}000x_2$

where $x_2 = 30$, $EI\Theta = 400{,}000$

where $x_2 = 60$, $EI\Theta = 700{,}000$

Segm.(3), $EI\Theta = 700{,}000 + 10{,}000x_3 - 250x_3^2$

***

True

(1) $EI\Theta = -400{,}000 + 250x_1^2$

(2) $EI\Theta = -300{,}000 + 10{,}000x_2$

(3) $EI\Theta = +300{,}000 + 10{,}000x_3 - 250x_3^2$

***

Segm. (1) $M = +500x_1$
Segm. (2) $M = 10{,}000$
Segm. (3) $M = 10{,}000 - 500x_3$

***

Segm. (1) $S = +500$
Segm. (2) $S = 0$
Segm. (3) $S = -500$

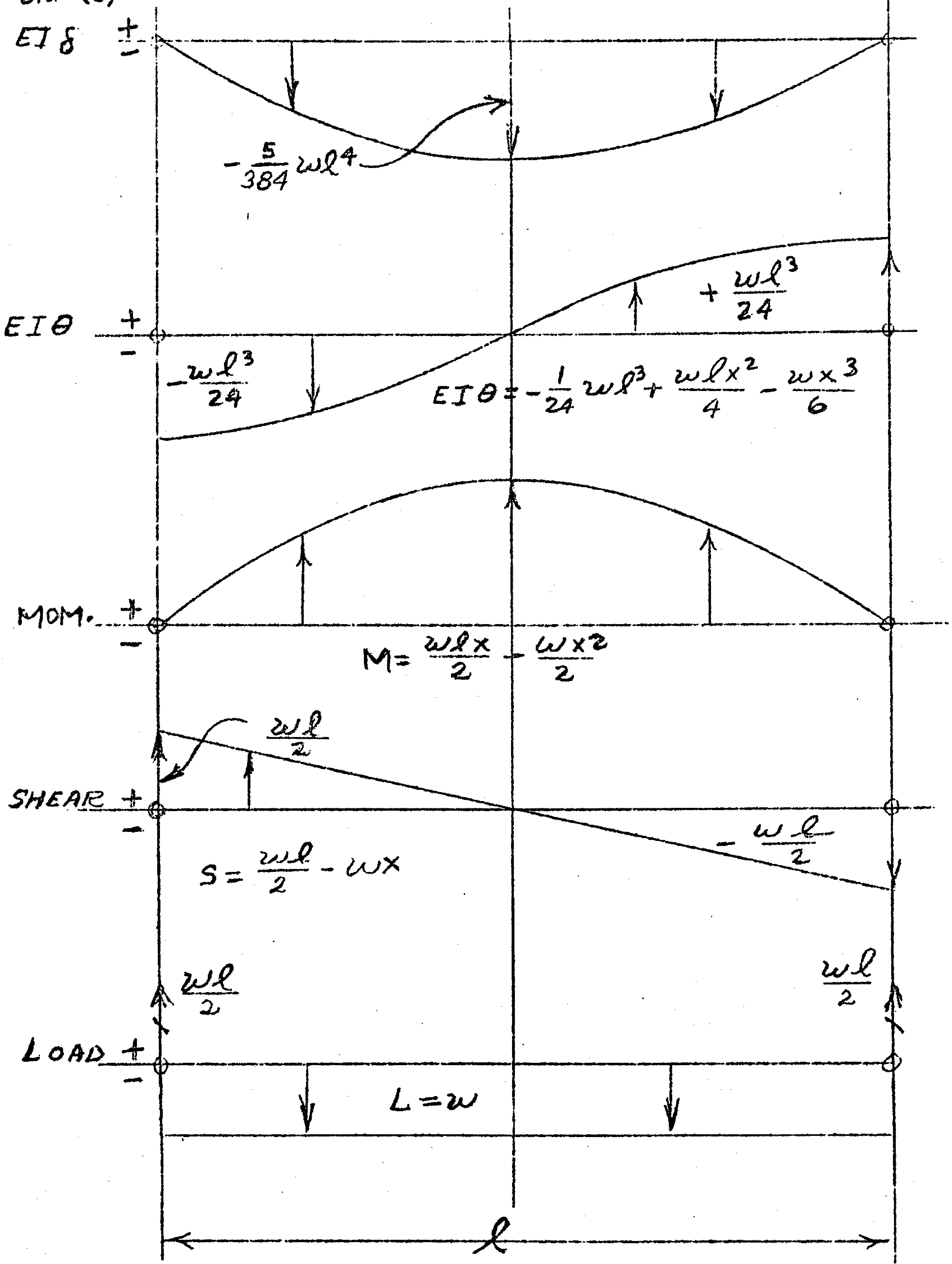
***

At Center $EI\delta = -11{,}833{,}360$

$$\delta = \frac{-11{,}833{,}360}{(10{,}000{,}000)(3)} = -.394''$$

8.1 (c)

$$EI\delta = -\frac{1}{24}w\ell^3 x + \frac{w\ell x^3}{12} - \frac{wx^4}{24}$$



8:1 (c)

$$M_{max.} = \frac{w\ell}{2}\left(\frac{\ell}{2}\right) - \frac{w}{2}\left(\frac{\ell}{2}\right)^2$$

$$= \frac{w\ell^2}{4} - \frac{w}{2}\left(\frac{\ell^2}{4}\right) = \frac{w\ell^2}{4} - \frac{w\ell^2}{8}$$

$$M_{max.} = \frac{w\ell^2}{8} \quad \longleftarrow M$$

***

Trial Eq. $EI\theta = \frac{w\ell x^2}{4} - \frac{wx^3}{6}$

$$\left[\text{at } x = \frac{\ell}{2}\right] \quad EI\theta = \frac{w\ell}{4}\left(\frac{\ell}{2}\right)^2 - \frac{w}{6}\left(\frac{\ell}{2}\right)^3$$

$$= \left(\frac{w\ell}{4}\right)\left(\frac{\ell^2}{4}\right) - \left(\frac{w}{6}\right)\left(\frac{\ell^3}{8}\right)$$

$$= \frac{w\ell^3}{16} - \frac{w\ell^3}{48} = \frac{1}{24}w\ell^3$$

True Eq. $EI\theta = -\frac{1}{24}w\ell^3 + \frac{w\ell x^2}{4} - \frac{wx^3}{6}$

***

at $x = \ell$

$$EI\theta = -\frac{1}{24}w\ell^3 + \frac{w\ell^3}{4} - \frac{w\ell^3}{6} = \frac{1}{24}w\ell^3$$

***

d max. at $x = \frac{1}{2}\ell$ =

$$-\frac{w\ell^3}{24}\left(\frac{\ell}{2}\right) + \frac{w\ell}{12}\left(\frac{\ell}{2}\right)^3 - \frac{w}{24}\left(\frac{\ell}{2}\right)^4$$

$$= -\frac{w\ell^4}{48} + \frac{w\ell}{12}\left(\frac{\ell^3}{8}\right) - \frac{w}{24}\left(\frac{\ell^4}{16}\right)$$

$$= w\ell^4\left[-\frac{1}{48} + \frac{1}{96} - \frac{1}{384}\right] = -\frac{5}{384}w\ell^4 \quad \longleftarrow EI\,d\ Max.$$

Solution to Problem of Assignment.

9:1 (1) Section properties.

No.	A	h	Ah	y	y^2	Ay^2	I_o*	Q
1	1.125	19.81	22.25	9.07	82.0	92.0		10.20
2	.445	19.44	8.67	8.70	75.8	33.7		3.86
3	1.705	9.94	16.97	- .80	+ .6	1.0	50.2	(3.33 -4.80)
4	.445	.43	.19	-10.31	106.3	47.3		-4.60
5	.750	.12	.09	-10.62	113.0	84.8		-7.99
Σ	4.470	10.74	48.17			258.8	50.2	(+17.39 -17.39)

* Note: I_o of parts No. 1, 2, 4, and 5 are so small as to be negligible.

$$I = 258.8 + 50.2 = 309^{\text{in}^4}$$

$$\frac{I}{y} \text{ (tens. side)} = \frac{309}{10.74} = 28.7$$

$$\frac{I}{y} \text{ (compr. side)} = \frac{309}{-9.26} = -33.4$$

(2) Maximum bending stresses.

Tension $f_{b_{max}} = \frac{1,200,000}{28.7} = 41,900$ lbs. per sq. in.

Compression $f_{b_{max}} = \frac{1,200,000}{-33.4} = -36,000$ lbs. per sq. in.

(3) Maximum shear stress in web.

$$f_{s_{max}} = \frac{(20,000)(17.29)}{(.091)(309)} = 12,300 \text{ lbs. per sq. in.}$$

(4) Shear along lines A, B, C, and D.

Line A $S = \frac{(20,000)(10.2)}{309} = 660$ lbs. per inch.

Line B $S = \frac{(20,000)(10.2 + 3.86)}{309} = 912$ lbs. per inch.

Line C $S = \frac{(20,000)(-4.60 - 7.99)}{309} = -815$ lbs. per inch.

Line D $S = \frac{(20,000)(-7.99)}{309} = -517$ lbs. per inch.

Allowable rivet spacing:

Line A Double row of rivets

Allowable spacing $= \frac{(2)(672)}{660} = 2.03"$ (Use 2 inches)

Solution to Problem of Assignment. (Continued)

9:1 (4)

Line B Single row of rivets

Allowable spacing $= \frac{672}{912} = .738''$ (Use 5/8 inches)

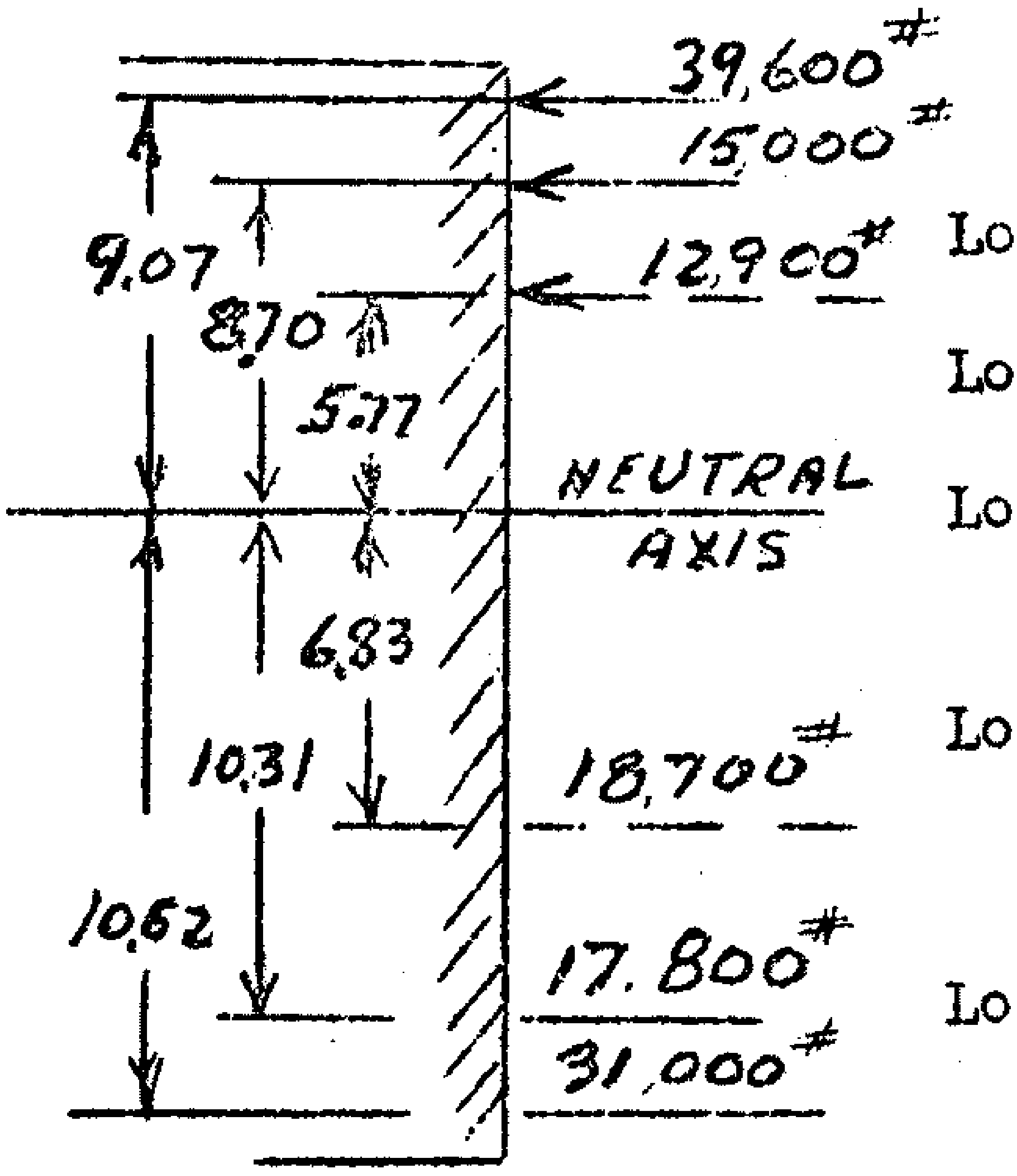
Line C Single row of rivets

Allowable spacing $= \frac{672}{815} = .825''$ (Use 3/4 inches)

Line D Double row of rivets

Allowable spacing $= \frac{(2)(672)}{517} = 2.60''$ (Use $2\frac{1}{2}$ inches)

(5)



Average stress on Segment No. 1

$f_b = \frac{(1,200,000)(9.07)}{309} = 35,200$ lbs. per sq. in.

Load on No. 1 39,600 lbs.

Load on No. 2 $\frac{(1,200,000)(3.86)}{309} = 15,000$ lbs.

Load on No. 3 above N.A.

$\frac{(1,200,000)(3.33)}{309}$ 12,900 lbs.

Load on No. 3 below N.A

$\frac{(1,200,000)(4.80)}{309}$ 18,700 lbs.

Load on No. 4 $\frac{1,200,000\ (4.60)}{309}$ 17,800 lbs.

Load on No. 5 $\frac{1,200,000\ (7.99)}{309}$ 31,000 lbs.

Check: 39,600 + 15,000 + 12,900 + 18,700 + 17,800 + 31,000

67,500 = 67,500

$\Sigma M = (39,600)(9.07) + (15,000)(8.70) + (12,900)(5.77) + (18,700)(6.83) + (17,800)(10.31) + (31,000)(10.62)$

$= 359,000 + 130,000 + 74,000 + 128,000 + 184,000 + 330,000$

$= 1,250,000''\#$ Error = 5000"# or .4% - negligible

(6) Change in Moment in 10" = (10)(20,000) = 200,000"#.
Change in loads in elements is proportional to change in moments.

Therefore change in load in No. 1 $= \frac{200,000}{1,200,000}$ (39,600) = 6,600 lbs. or 660 lbs. per in. Check.

471

9:1 (6)

Similarly - No. 2 $\Delta F = \frac{2}{12}(15{,}000) = 2{,}500$ lbs.

250 + 660 = 910 lbs. per in. Check.

- No. 3 upper - $\frac{2}{12}(12{,}900) = 2{,}150$ lbs.

250 + 660 + 215 = 1125 lbs. per in. or

$\frac{1125}{.091} = 12{,}300$ lbs. per sq. in. Check.

No. 5 $\Delta F = \frac{2}{12}(31{,}000) = 5{,}170$ lbs. or

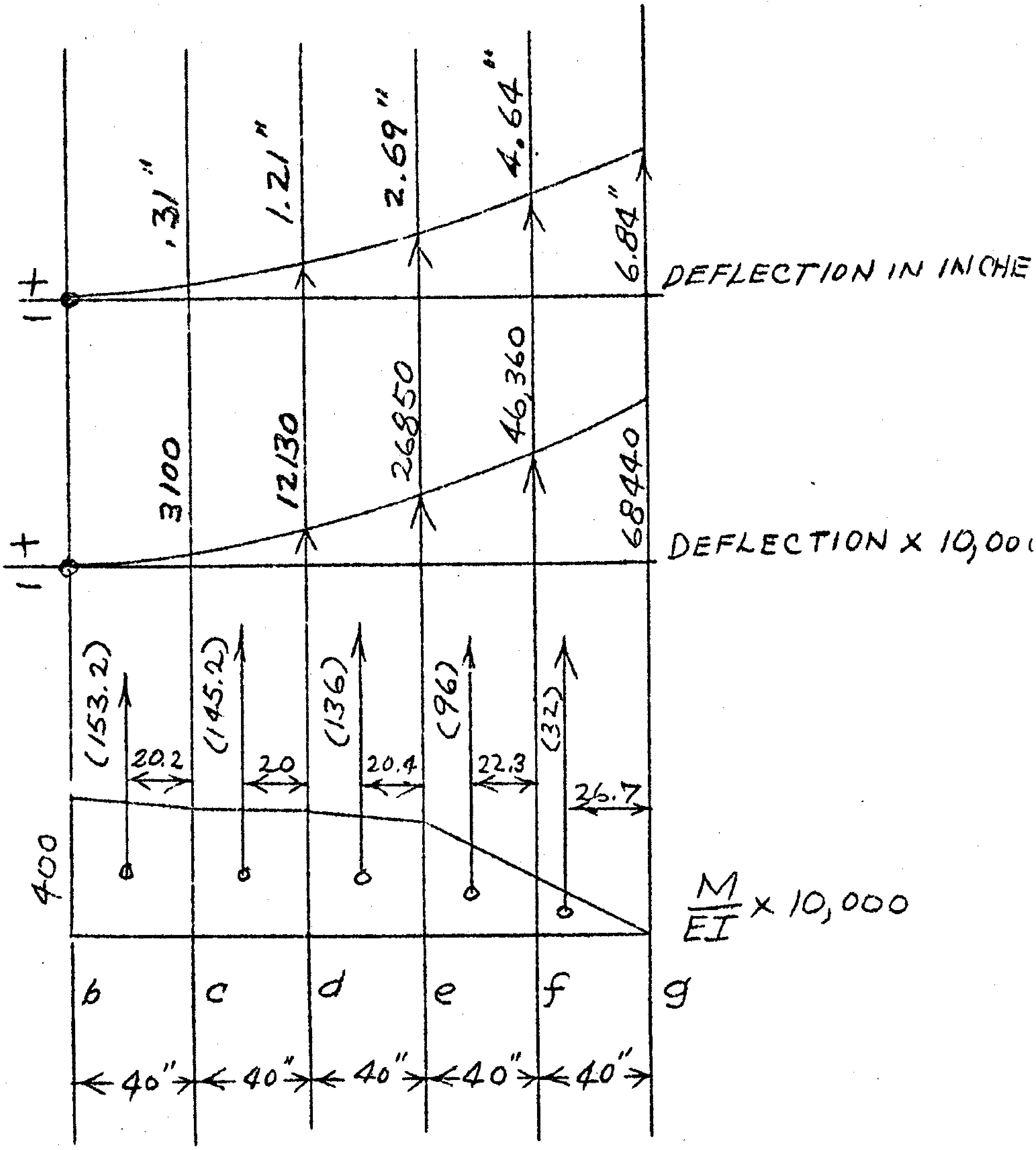
517 lbs. per in. Check.

No. 4 $\Delta F = \frac{2}{12}(17{,}800) = 2{,}970$ lbs.

297 + 517 = 814 lbs. per in. Check.

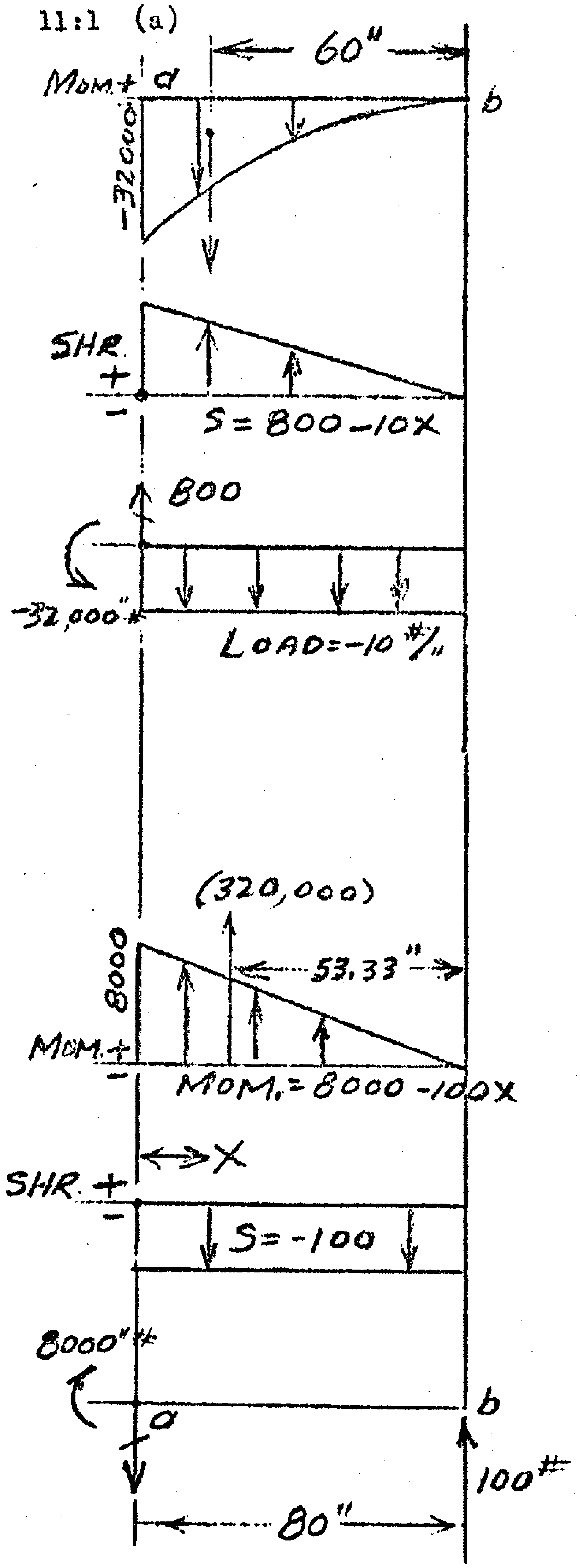
Solution to Problem of Assignment.

10:2



Solution to Problem Assignment.

11:1 (a)



For load with R_b removed:

Max. Moment = -32,000"#

Moment area = $(-32,000)(\frac{80}{3}) = -853,333$

$EI\delta_b = (-853,333)(60) = -51,200,000$

For 100 lb. unit load at R_b:

Moment area = $(8000)(\frac{80}{2}) = 320,000$

$EI\delta_b = (320,000)(53.33) = 17,066,660$

Actual Reaction at b

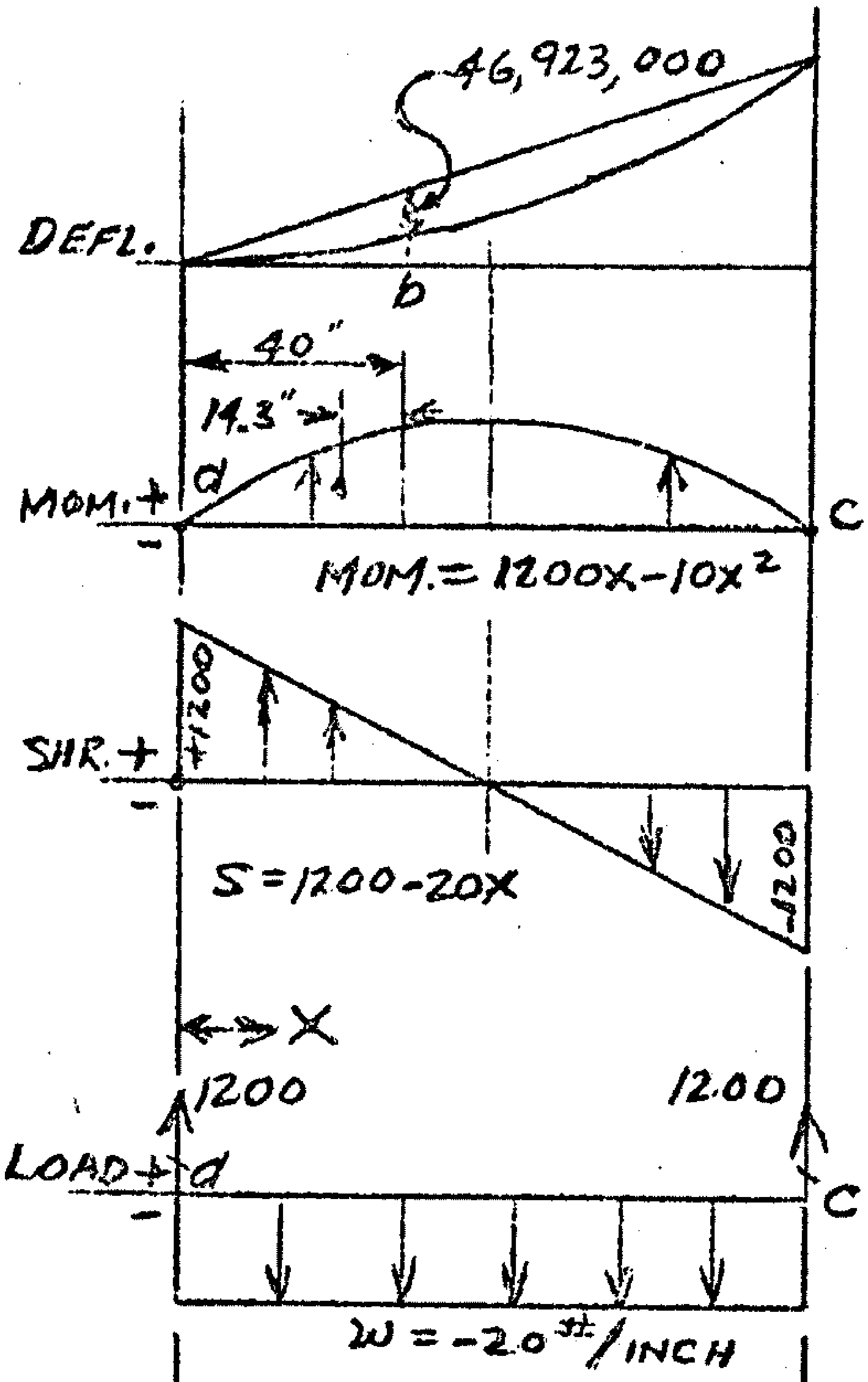
$= (100)(\frac{51,200,000}{17,066,660}) = 300$ lbs. R_b

$R_a = 800 - 300 = 500$ lbs. R_a

$M_a = (300)(80) - (800)(40) = -8000$"#

Tension on top fibers.

11:1 (b)



From Assignment 10.

Total Moment Area = $\frac{w\ell^3}{12}$

$= \frac{(20)(120)^3}{12} = 2,880,000$

EI δ_c from tangent at a

$= (2,880,000)(60) = 172,800,000$

Mom. Area between a and b = 746,670

Dist. to centroid from b = 14.3"
(see next page)

EI δ_b from tangent at "a" =

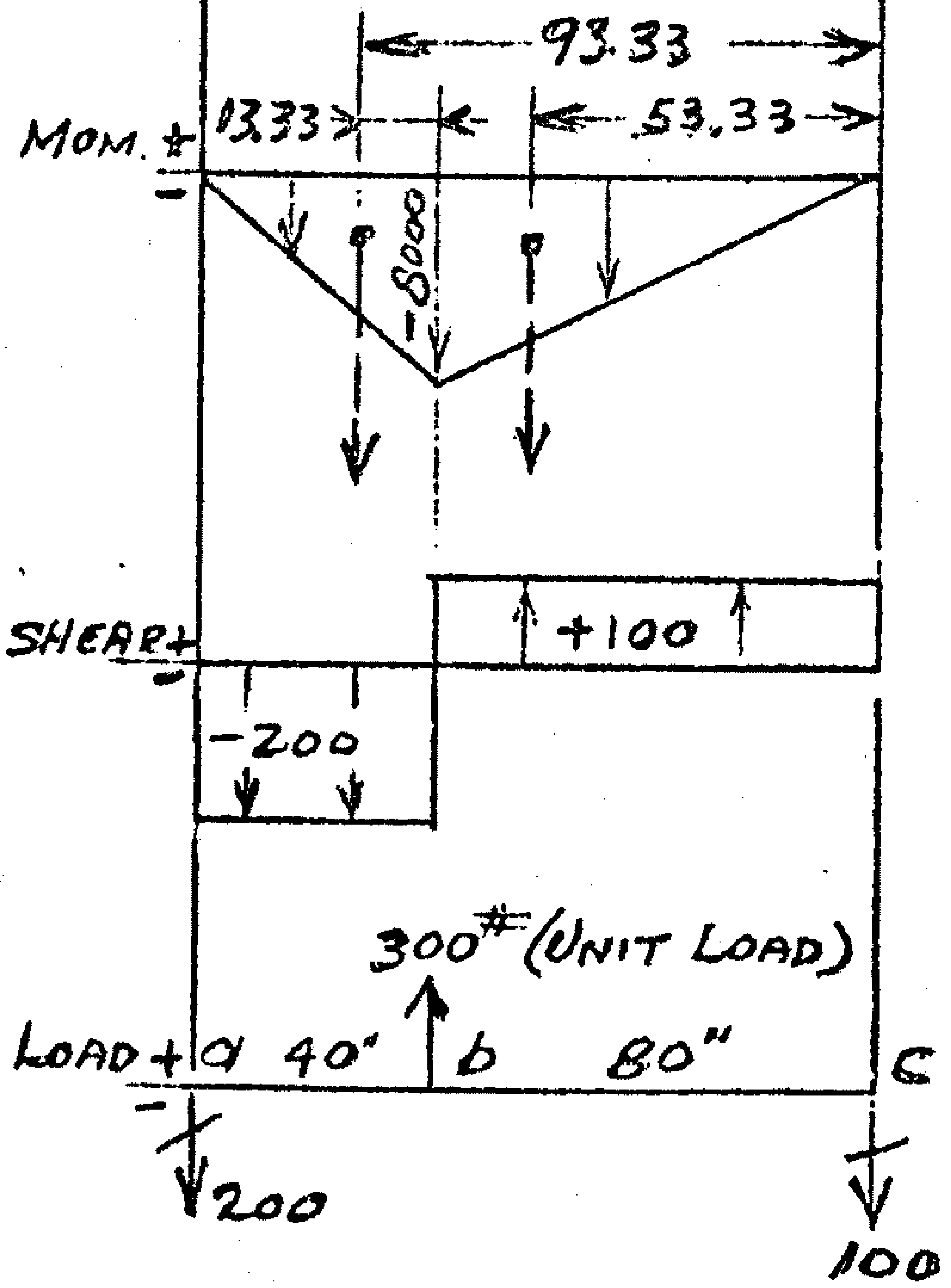
$(746,670)(14.3) = 10,677,000$

Deflection at b

$= \frac{40}{120}(172,800,000) - 10,677,000$

$= 46,923,000$ ← Due to loads.

****



Moment area (a-b) = -160,000
Moment area (b-c) = -320,000

EI δ_c from tangent at "a"

$= (-320,000)(53.33) - (160,000)$
$(93.33) = -17,065,600 - 14,932,800$
$= -31,998,400$

EI δ_b from tangent at "a"

$= (-160,000)(13.33) = -2,132,800$

Deflection at b

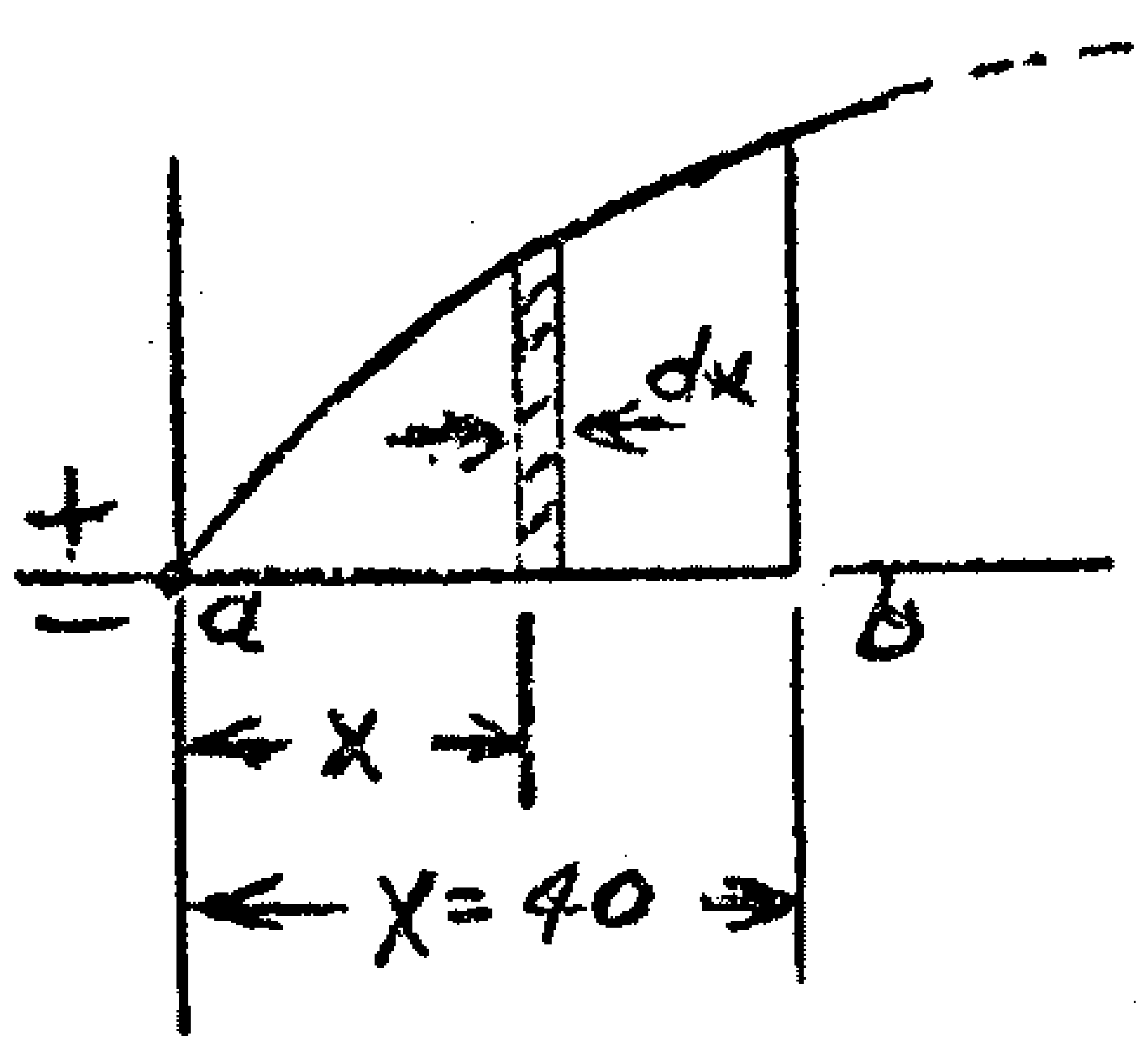
$= \frac{40}{120}(-31,998,400) - (-2,132,800)$

$= -8,533,300$

$R_b = R_2 = \frac{46,923,000}{8,533,300}(300) = 1650$ lbs.

Solution of Problem Assignment. (Continued)

11:1 (b)



Solution for areas and the center of gravity of the moment area between a and b.

Equation of moment line

$$\text{Mom.} = 1200x - 10x^2$$

$$\text{Area} = \Sigma(1200x - 10x^2)dx$$

$$= \int_0^{40}(1200x - 10x^2)dx$$

$$= \left[\frac{1200x^2}{2} - \frac{10x^3}{3}\right]_0^{40} = \left[(600)(40)^2 - \left(\frac{10}{3}\right)(40)^3\right] - \left[0\right]$$

$$= 960{,}000 - 213{,}330 = 746{,}670$$

Moment of moment area between a and b about a =

$$\int y\,dx \cdot x = \int_0^{40}(1200x - 10x^2)x\,dx$$

$$= \int_0^{40}(1200x^2 - 10x^3)dx = \left[\frac{1200x^3}{3} - \frac{10x^4}{4}\right]_0^{40}$$

$$= \left[400x^3 - 2.5x^4\right]_0^{40} = \left[(400)(40)^3 - (2.5)(40)^4\right]_0^{40}$$

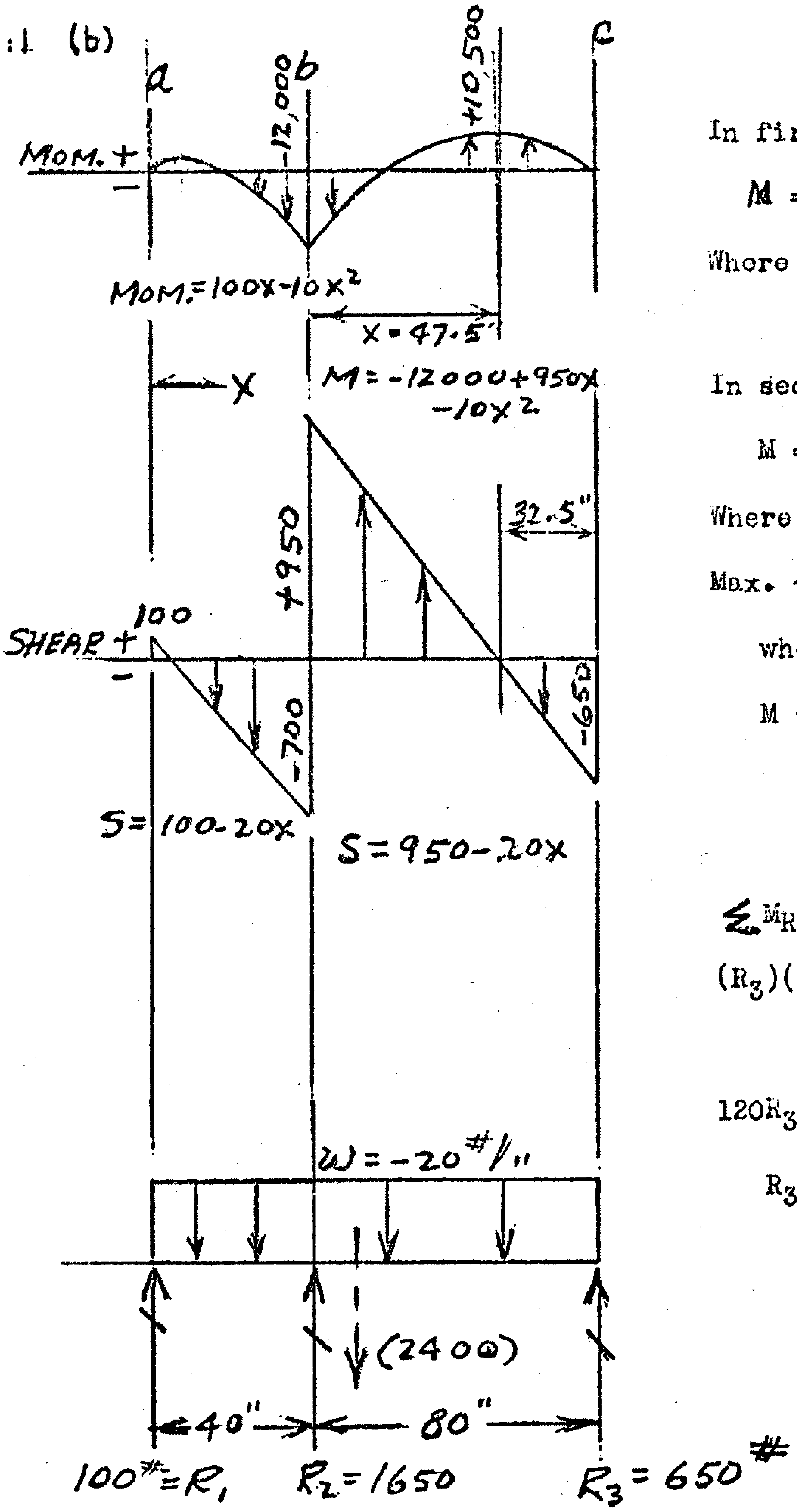
$$= \left[25{,}600{,}000 - 6{,}400{,}000\right] = 19{,}200{,}000$$

$\overline{x}$ = distance to c.g. of area from "a"

$= \frac{19{,}200{,}000}{746{,}670} = 25.7$ inches, distance from b = 14.3 inches

Solution of Problem Assignment. (Continued)

11:1 (b)



In first segment, a-b,

$M = 100x - 10x^2$

Where x = 40", M = -12,000"#

****

In second segment, b-c

$M = -12,000 + 950x - 10x^2$

Where x = 80", m = 0 check.

Max. + Mom. in span b-c occurs

where x = 47.5"

M = +10,500"#

$\Sigma M_{R_1} = 0$ ↶+

$(R_3)(120) + (1650)(40) - (2400)$

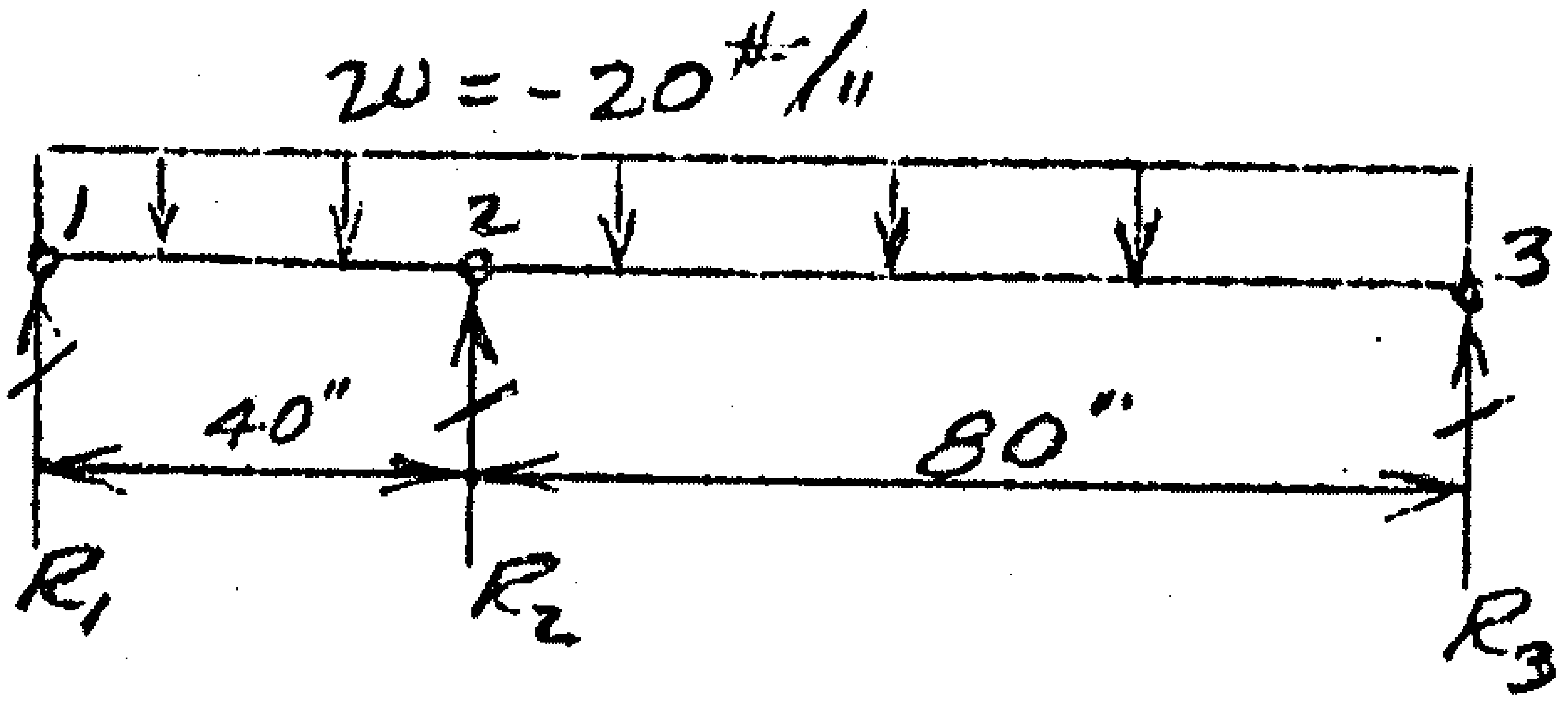
$(60) = 0$

$120R_3 = 144,000 - 66,000 = 78,000$

$R_3 = 650$ lbs., $R_1 = 100$ lbs.

Solution to Problem of Assignment.

12:2



For this problem E and I are held constant and therefore do not need to appear in the equations

$M_1 = 0, \quad M_3 = 0$

$$\frac{M_1L_1}{I_1} + 2M_2\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + \frac{M_3L_2}{I_2} = \frac{w_1L_1^3}{4I_1} + \frac{w_2L_2^3}{4I_2}$$

Simplifying:

$$0 + 2M_2(L_1 + L_2) + 0 = \frac{w_1L_1^3}{4} + \frac{w_2L_2^3}{4}$$

Noting our signs (w = -)

$$2M_2(40 + 80) = \frac{(-20)(40)^3}{4} - \frac{(20)(80)^3}{4}$$

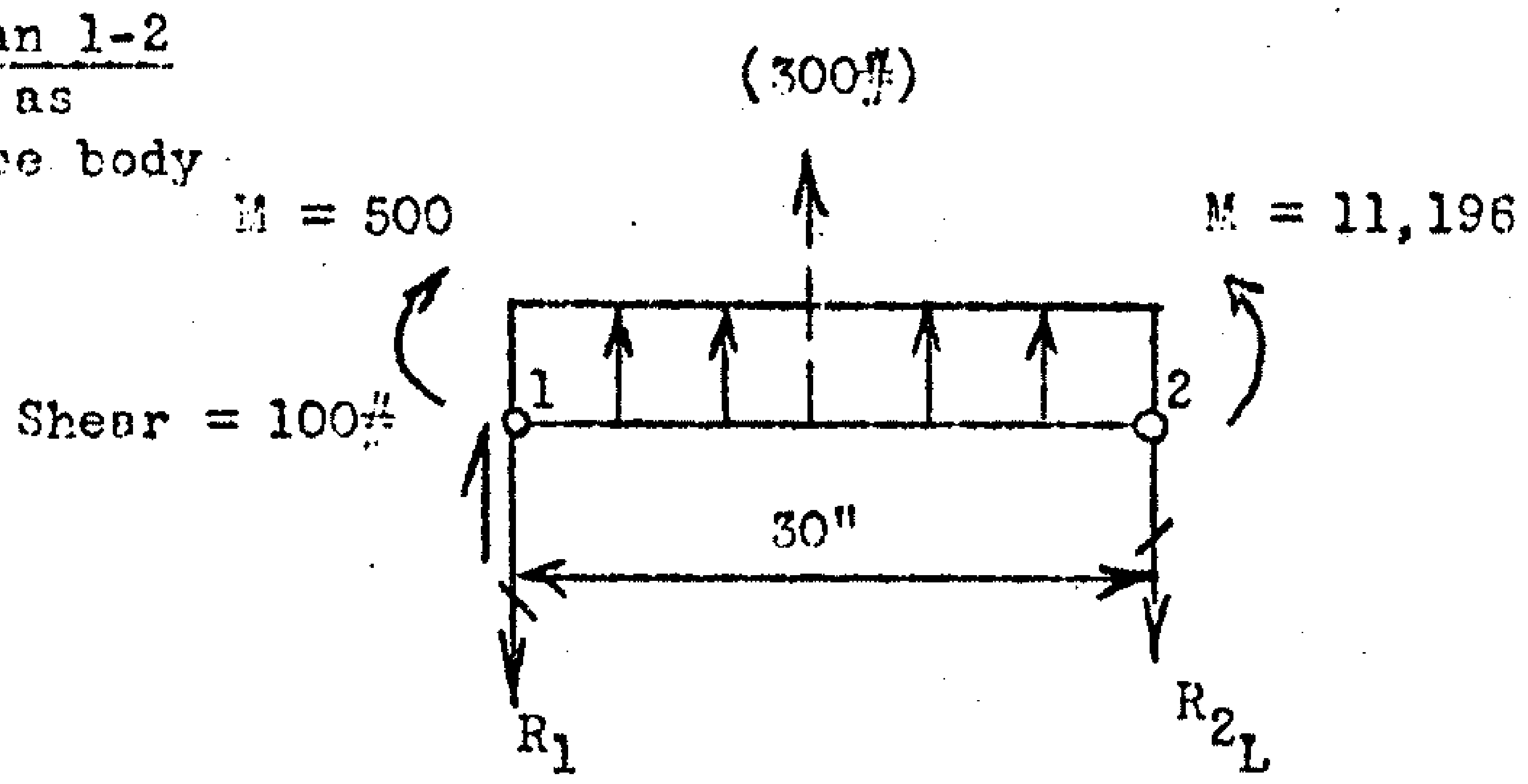
$$240M_2 = -320,000 - 2,560,000 = -2,880,000$$

$$M_2 = -\frac{2,880,000}{240} = -12,000''\#$$ (Tension on the top fibers of the beam)

The load, shear, and moment curves are shown in the solution for Problem Assignment 11.

13:1

Span 1-2
as
free body



Note that both the shear and moment at each end must be included. R_{2_L} is actually the shear to the left at 2.

$\sum M_1 = 0$ $\curvearrowright +$ taken plus

$$(R_{2_L})(30) + 500 - 11,196 - (300)(15) = 0$$

$$30R_{2_L} = 11,196 + 4500 - 500 = 15,196$$

$$R_{2_L} = 506.5 \text{ lbs.}$$

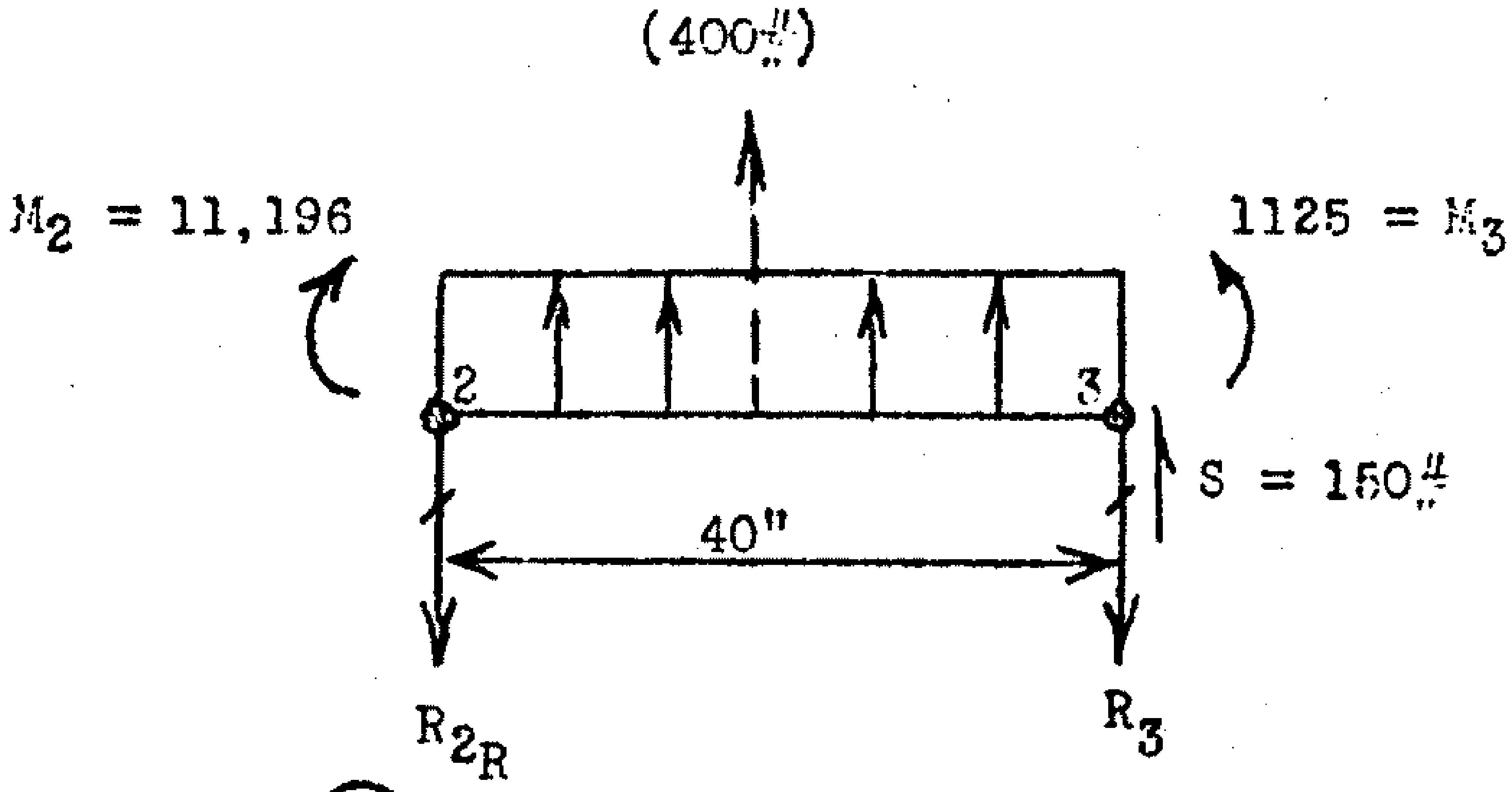
$\sum M_2 = 0$ $\curvearrowleft +$

$$(R_1)(30) - 500 - (100)(30) - (300)(15) + 11,196 = 0$$

$$30R_1 = 500 + 3000 + 4500 - 11,196 = -3196$$

$R_1 = -106.5$ lbs. Therefore R_1 acts upward instead of downward as assumed.

Span 2-3



$\sum M_{R_3} = 0$ $\curvearrowleft +$

$$(R_{2_R})(40) + 1125 - 11,196 - (400)(20) = 0$$

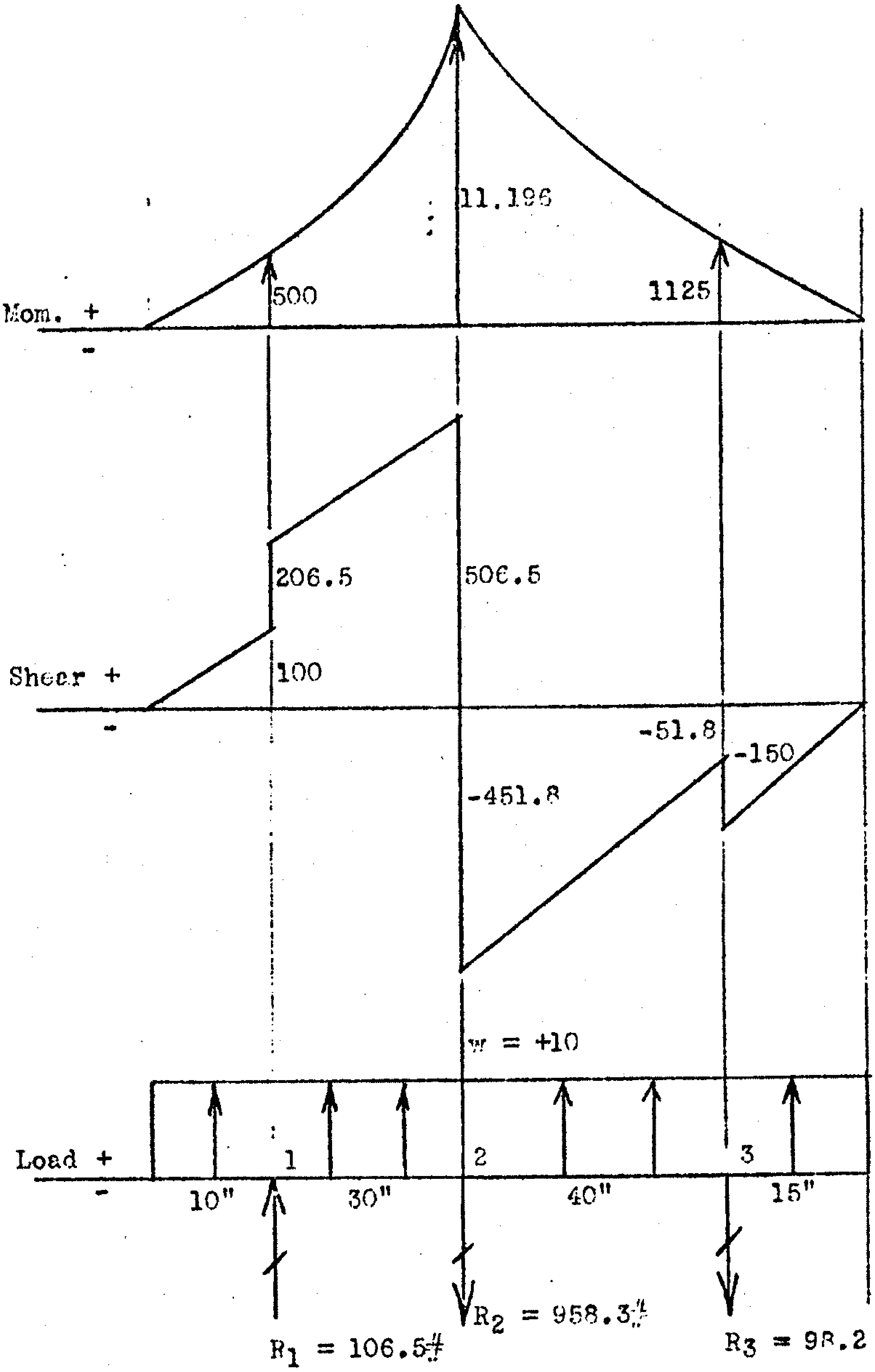
$$40R_{2_R} = 11,196 + 8000 - 1125 = 18,071$$

$$R_{2_R} = 451.8\# \qquad R_3 = 98.2\#$$

$$R_2 = R_{2_L} + R_{2_R} = 506.5 + 451.8 = 958.3\#$$

Solution to Problems of Assignment.

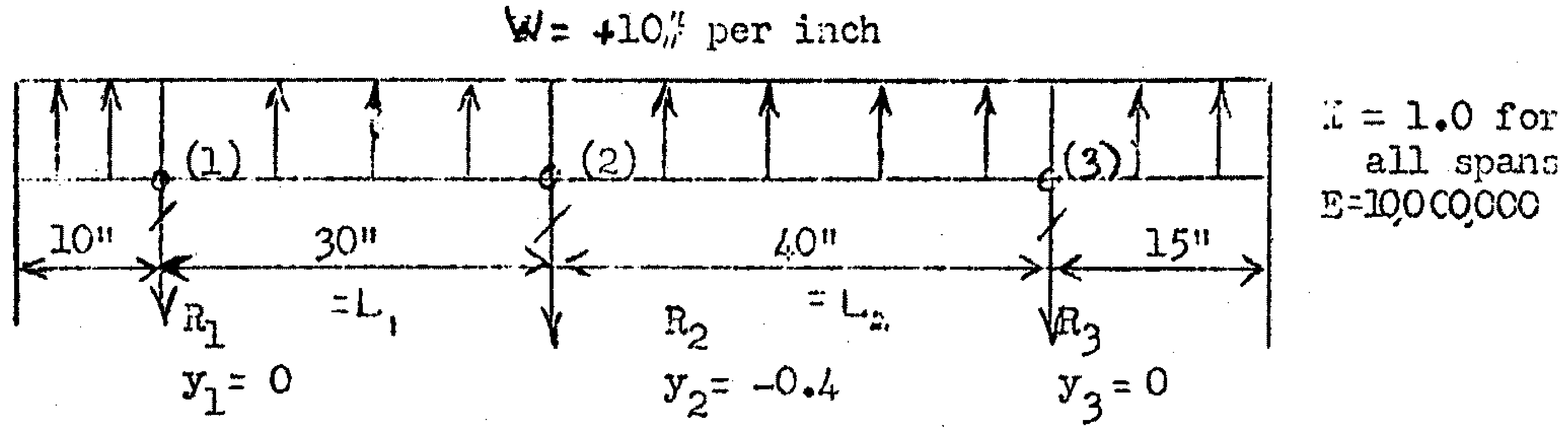
13:1



Moment checked at Supports by taking moment increments as areas in shear curve.

Solution to Problems of Assignment.

13:1



$M_1 = (10)\ (10)\ (\frac{10}{2}) = +500"\#$ $\qquad$ $M_3 = (10)\ (15)\ (\frac{15}{2}) = +1125"\#$

Important Note: When the supports deflect the moment of inertia of the beams must be used even when it is the same for all spans. The general equation is:

$$\frac{M_1L_1}{I_1} + 2M_2\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + \frac{M_3L_2}{I_2} = \frac{w_1L_1^3}{4I_1} + \frac{w_2L_2^3}{4I_2} + \frac{6E(y_1 - y_2)}{L_1} + \frac{6E(y_3 - y_2)}{L_2}$$

Substituting in the equation:

$$\frac{(500)\ (30)}{1} + 2M_2\left(\frac{30}{1} + \frac{40}{1}\right) + (1125)\ \left(\frac{40}{1}\right) = \frac{(10)\ (30)^3}{(4)\ (1)} + \frac{(10)\ (40)^3}{(4)\ (1)}$$

$$+ \frac{(6)\ (10,000,000)\ (0 + 0.4)}{30} + \frac{(6)\ (10,000,000)\ (0 + 0.4)}{40}$$

$$15,000 + 140M_2 + 45,000 = 67,500 + 160,000 + 800,000 + 600,000$$

$$140M_2 = 1,627,500 - (15,000 + 45,000) = 1,567,500$$

$$M_2 = 11,190"\#$$

See following page for solution for reactions and load, shear, and moment curves.

13:2 If there had been no deflection at 2

$$140M_2 = 67,500 + 160,000 - (15,000 + 45,000) = 167,500$$

$$M_2 = \frac{167,500}{140} = 1196"\#$$

Solution to Problems of Assignment.

14:1 (a) $M_1 = +3000 \qquad M_2 + 2000, \qquad w = 5$ lbs. per inch unif.

P = End Load = 4000 lbs., $I = 1.0$

$E = 10{,}000{,}000$ (Aluminum Alloy)

$$j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{(10{,}000{,}000)(1)}{4{,}000}} = \sqrt{2{,}500} = 50$$

$$\frac{L}{j} = \frac{80}{50} = 1.60 \qquad \sin\frac{L}{j} = 0.9996$$

$$\cos\frac{L}{j} = -0.0292$$

$$D_1 = M_1 - wj^2 = 3{,}000 - (5)(2500) = -\ 9{,}500$$

$$D_2 = M_2 - wj^2 = 2{,}000 - (5)(2500) = -10{,}500$$

At point of maximum moment $\tan\frac{x}{j} = \dfrac{D_2 - D_1 \cos\frac{L}{j}}{D_1 \sin\frac{L}{j}}$

$$\tan\frac{x}{j} = \frac{-10{,}500 - (-9{,}500)(-0.0292)}{(-9500)(0.9996)} = \frac{-10{,}780}{-\ 9{,}500} = +1.133$$

$$\frac{x}{j} = 0.849 \qquad \cos\frac{x}{j} = .660$$

$$M_{max.} = \frac{D_1}{\cos\frac{x}{j}} + wj^2 = \frac{-9500}{.660} + (5)(2500) = -14{,}400 + 12{,}500$$

$= -\ 1{,}900$ inch lbs. in span.

Location of maximum moment point in span

$\frac{x}{j} = 0.849 \qquad x = (0.849)(50) = 42.5$ inches from point (1).

14:1 (b) Same as 1 (a) except the material is steel.

$$j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{(29{,}000{,}000)(1)}{4000}} = \sqrt{7250} = 85.1$$

$$\frac{L}{j} = \frac{80}{85.1} = .94 \qquad \sin\frac{L}{j} = .808$$

$$\cos\frac{L}{j} = .590$$

$$D_1 = M_1 - wj^2 = 3000 - (5)(7250) = -33{,}250$$

$$D_2 = M_2 - wj^2 = 2000 - (5)(7250) = -34{,}250$$

14:1 (b) At point of maximum moment

$$\tan\frac{x}{j} = \frac{-34{,}250 - (-33{,}250)(.590)}{(-33{,}250)(.808)} = \frac{-14{,}630}{-26{,}850} = +.545$$

$$\frac{x}{j} = 0.50 \qquad \cos\frac{x}{j} = .878$$

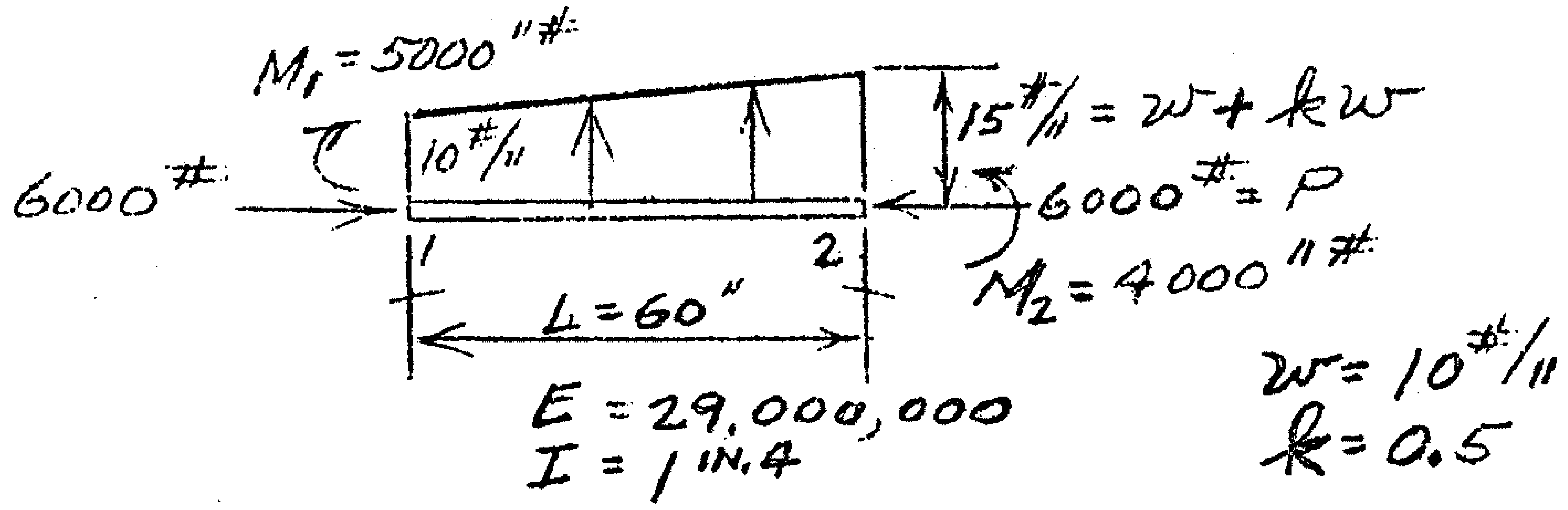
$$M_{max.} = \frac{-33{,}250}{.878} + (5)(7250) = -1570 \text{ in.lbs. in span.}$$

Location of maximum moment point in span

$$\frac{x}{j} = 0.50 \qquad x = (0.50)(85.1) = 42.5 \text{ inches}$$

The magnitude of the maximum span moment has been changed but not the location.

14:2



$$j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{(29{,}000{,}000)(1)}{6000}} = \sqrt{4830} = 69.5$$

$$\frac{L}{j} = \frac{60}{69.5} = .863; \quad \sin\frac{L}{j} = .760; \quad \cos\frac{L}{j} = .652$$

$$C_1 = \frac{M_2 - (1+k)wj^2 - (M_1 - wj^2)\cos\frac{L}{j}}{\sin\frac{L}{j}}$$

$$C_1 = \frac{4000 - (1 + .5)(10)(4830) - \left[5000 - (10)(4830)\right](.652)}{.760}$$

$$= \frac{4000 - 72{,}450 + 28{,}230}{.760} = \frac{-40{,}220}{.760} = -52{,}920 \longleftarrow C_1$$

$$C_2 = M_1 - wj^2 = 5000 - (10)(4830) = -43{,}300 \longleftarrow C_2$$

$$M_x = C_1 \sin\frac{x}{j} + C_2 \cos\frac{x}{j} + wj^2\left(1 + k\frac{x}{L}\right)$$

14:2 Taking values of x at 10 inch intervals.

$C_1 = -52,920$

$C_2 = -43,300$

$wj^2 = +48,300$

$k = 0.5$

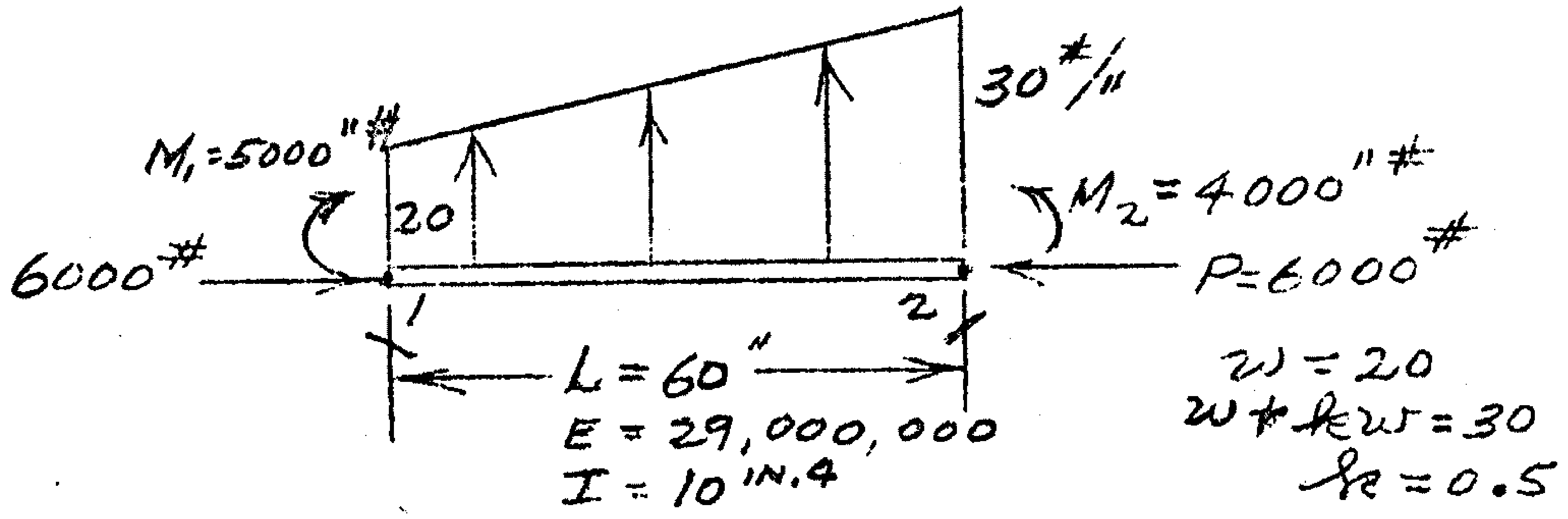
$j = 69.5$

x (1)	$\frac{x}{j}$ (2)	$\sin \frac{x}{j}$ (3)	$\cos \frac{x}{j}$ (4)	$C_1 \sin \frac{x}{j}$ (5)	$C_2 \cos \frac{x}{j}$ (6)	$1 + k \frac{x}{L}$ (7)	$wj^2(1 + k \frac{x}{L})$ (8)
10	.144	.143	.990	- 7,570	-42,870	1.083	52,320
20	.288	.284	.959	-15,030	-41,520	1.167	56,350
30	.432	.419	.907	-22,170	-39,270	1.25	60,380
40	.576	.545	.838	-28,840	-36,290	1.333	64,400
50	.720	.659	.752	-34,870	-32,560	1.417	68,430

Moment at any station = (5) + (6) + (8)

x		
10	Mom.	= - 7,570 - 42,870 + 52,320 = +1880
20	"	= -15,030 - 41,520 + 56,350 = - 200
30	"	= -22,170 - 39,270 + 60,380 = -1060
40	"	= -28,840 - 36,290 + 64,400 = - 730
50	"	= -34,870 - 32,560 + 68,430 = +1000

14:3



$$j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{(29,000,000)(10)}{6000}} = \sqrt{48,300} = 220$$

$\frac{L}{j} = \frac{60}{220} = .272$ radians; $\sin \frac{L}{j} = .269$; $\cos \frac{L}{j} = .963$

$$C_1 = \frac{M_2 - (1 + k)wj^2 - (M_1 - wj^2)(\cos \frac{L}{j})}{\sin \frac{L}{j}}$$

14:3

$$C_1 = \frac{4000 - (1 + 0.5)(20)(48,300) - \left[5000 - (20)(48,300)\right](.963)}{.269}$$

$$C_1 = \frac{4000 - 1,449,000 + 925,440}{.269} = -1,931,450 \longleftarrow C_1$$

$$C_2 = M_1 - wj^2 = 5000 - (20)(48,300) = -961,000 \longleftarrow C_2$$

$$wj^2 = +966,000$$

$$M_x = C_1 \sin\frac{x}{j} + C_2 \cos\frac{x}{j} + wj^2\left(1 + k\frac{x}{L}\right)$$

x (1)	$\frac{x}{j}$ (2)	$\sin\frac{x}{j}$ (3)	$\cos\frac{x}{j}$ (4)	$C_1 \sin\frac{x}{j}$ (5)	$C_2 \cos\frac{x}{j}$ (6)	$1 + k\frac{x}{L}$ (7)	$wj^2(1 + k\frac{x}{L})$ (8)
10	.045	.045	.999	- 86,920	-960,040	1.083	+1,046,500
20	.091	.091	.996	-175,760	-957,150	1.167	+1,127,000
30	.136	.136	.991	-262,680	-952,350	1.25	+1,207,500
40	.182	.181	.983	-349,590	-944,660	1.333	+1,288,000
50	.227	.224	.974	-432,650	-936,010	1.417	+1,368,500

Moment at any station = (5) + (6) + (8)

x

10 Mom. = - 86,920 - 960,040 + 1,046,500 = - 460
20 " = -175,760 - 957,150 + 1,127,000 = -5,910
30 " = -262,680 - 952,350 + 1,207,500 = -7,530
40 " = -349,590 - 944,660 + 1,288,000 = -6,250
50 " = -432,650 - 936,010 + 1,368,500 = - 160

The maximum moment in the span is approximately -7530 in. lbs. To obtain a closer value take values of x at 1 inch spacing on either side of x = 30 inches and plot in the additional points.

Solution to Problems of Assignment.

15:1 The lightest tube will be the one with greatest diameter and least wall thickness provided it does not buckle locally. The curves of Modulus of Rupture page 5-18 of ANC are reduced in large values of D/t to allow for local crippling. We will first try a tube $3\frac{1}{2}$" x .065 wall.

From ANC-5, Table 8-1 $A = .7014$ $I/y = .5914$ $D/t = 53.8$

Then $f_b = \pm \frac{20{,}000}{.5914} = \pm 33{,}800$ $\frac{f_b}{F_b} = \frac{33{,}800}{64{,}000} = .528$

$F_b = 52{,}500$ on Compression side

$F_b = 64{,}000$ on Tension side

ANC-5, page 5-18

$f_t = \pm \frac{16{,}000}{.7014} = +22{,}800$

$F_t = 55{,}000$ $\frac{f_t}{F_t} = \frac{22{,}800}{55{,}000} = .414$

Note that the maximum combined load will be on the tension side. Maximum unit stress in compression $= -33{,}800 + 22{,}800 = -11{,}000$ psi. only. Obviously the compression side of the tube is not critical. For the tension side:

$$\frac{f_b}{F_b} = \frac{f_t}{F_t} = .528 + .414 = .942$$

This tube is therefore satisfactory. If the allowable load had been based on the ultimate tension only, the combined unit stress in tension which is $33{,}800 + 22{,}800 = 56{,}600$ would have been shown to exceed the allowable.

15:2 The solution of this problem can be made only after a tube size has been assumed since the value of I must be known to solve the precise equation for bending. Assume a tube $3\frac{1}{2}$" O.D. by .083 wall.

$I = 1.3012$ $I/y = .7435$ $E = 10{,}000{,}000$
$A = .891$ $D/t = 42.2$ $Q = 1.2085$

****

Solution of Precise Equation.

$$j = \sqrt{\frac{EI}{P}} = \sqrt{\frac{(10)(10)^6(1.3012)}{7000}} = \sqrt{1860} = 43.1$$

$$\frac{L}{2j} = \frac{40}{(2)(43.1)} = .464 \qquad \cos\frac{L}{2j} = .893$$

$$wj^2 = (-100)(186{,}000) = -186{,}000$$

Solution to Problems of Assignment. (Continued)

15:2 M_{max} at center $= wj^2 \left[1 - \frac{1}{\cos \frac{L}{2j}}\right]$

$$= (-186,000)(1 - \frac{1}{.893}) = (-186,000)(-0.12) = 22,320$$

$$f_b = \pm \frac{22,320}{.7435} = \pm 30,000 \text{ psi} \qquad \frac{f_b}{F_b} = .55$$

$$F_b = 55,000 \text{ (ANC-5, page 5-18)}$$

$$f_c = \frac{-7000}{.891} = -7850 \text{ psi.}$$

For allowable F_c we must use the column formulas

$$\rho = 1.2085 \qquad \frac{L}{\rho} = \frac{40}{1.2085} = 33$$

Allowable $\frac{P}{A} = F_c$ from ANC-5, Table 5-1, page 5-2.

$$F_c = 42,500 - (334.5)(\frac{L}{\rho}) = 42,500 - (334.5)(33)$$

$$F_c = 42,500 - 11,050 = 31,450 \text{ psi.}$$

$$\frac{f_c}{F_c} = \frac{7,850}{31,450} = .25$$

$$\frac{f_b}{F_b} + \frac{f_c}{F_c} = .55 + .25 = .80 \text{ which is less than 1.0.}$$

The tube chosen is therefore satisfactory. The only lighter tube which has any possibility of carrying the loads is $3\frac{1}{2}$ O.D. with .065 wall. A rough check will show that this tube is too weak. Since we are checking a tube with a smaller value of I we would know that the deflection and hence the combined moment would increase slightly. The value of ρ would change very little and hence the allowable F_c would change slightly. The D/t would increase and the allowable F_b would decrease accordingly. The actual F_b = 47,000 psi. from problem 15:1. Then taking the moment from the first solution without solving a new precise equation

For the $3\frac{1}{2}$ - .065 tube $A = .7014$
$I/y = .5914$

$$\text{Then } f_b = \pm \frac{22,320}{.5914} = 37,800$$

$$F_b = 52,000 \qquad \frac{f_b}{F_b} = .73$$

$$f_c = \frac{-7000}{.7014} = -10,000$$

Use F_c = 32,000 (estimated from first solution)

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Solution to Problems of Assignment. (Continued)

15:2

$$\frac{f_b}{F_b} + \frac{f_c}{F_c} = .73 + .31 = 1.04$$

This tube is therefore too weak. The lightest standard size which is sufficient is $3\frac{1}{2}$ O.D. x .083 wall.

Solution to Problems of Assignment.

16:1 Using member 2-3 as the redundant.

Member	Size	Length	Area	Q	P_o	P_a	P_oP_aQ	P_a^2Q
1-2	1-.049	30	.146	2.05	0	-.707	0	1.025
1-3	1-.058	30	.172	1.74	-10,000	-.707	+12,300	.87
1-4	1-.049	42.4	.146	2.90	+14,140	+1.0	+41,000	2.90
2-3	1½-.083	42.4	.370	1.15	0	+1.0	0	1.15
2-4	1-.065	30	.191	1.57	0	-.707	0	.785
3-4	1-.058	30	.172	1.74	-10,000	-.707	+12,300	.87
Σ	$(Q = \frac{L}{A}(\frac{1}{100}))$						+65,600	7.600

$$X_a = - \frac{\Sigma P_oP_aQ}{\Sigma P_a^2Q} = - \frac{65,600}{7,600} = -8640 \text{ lbs.}$$

****

Design Loads for Members

Member (1)	P_o (2)	P_a (3)	P_aX_a (4)	$.25\ P_aX_a$ (5)	$P_o + P_aX_a$ (6)	$(6) \pm (5)$ (7)
1-2	0	-.707	+6110	+1530	+6110	+7640
1-3	-10,000	-.707	+6110	+1530	-3890	-5420
1-4	+14,140	+1.0	-8640	-2160	+5500	+7660
2-3	0	+1.0	-8640	-2160	-8640	-10,800
2-4	0	-.707	+6110	+1530	+6110	+7640
3-4	-10,000	-.707	+6110	+1530	-3890	-5420

Note that up to Column (6) you can check the results of the example problem. Beyond that the arbitrary correction changes the design values. For this reason an attempt should always be made to *choose as the redundant member the one carrying the lowest load.*

Solution to Problems of Assignment. (Continued)

16:1

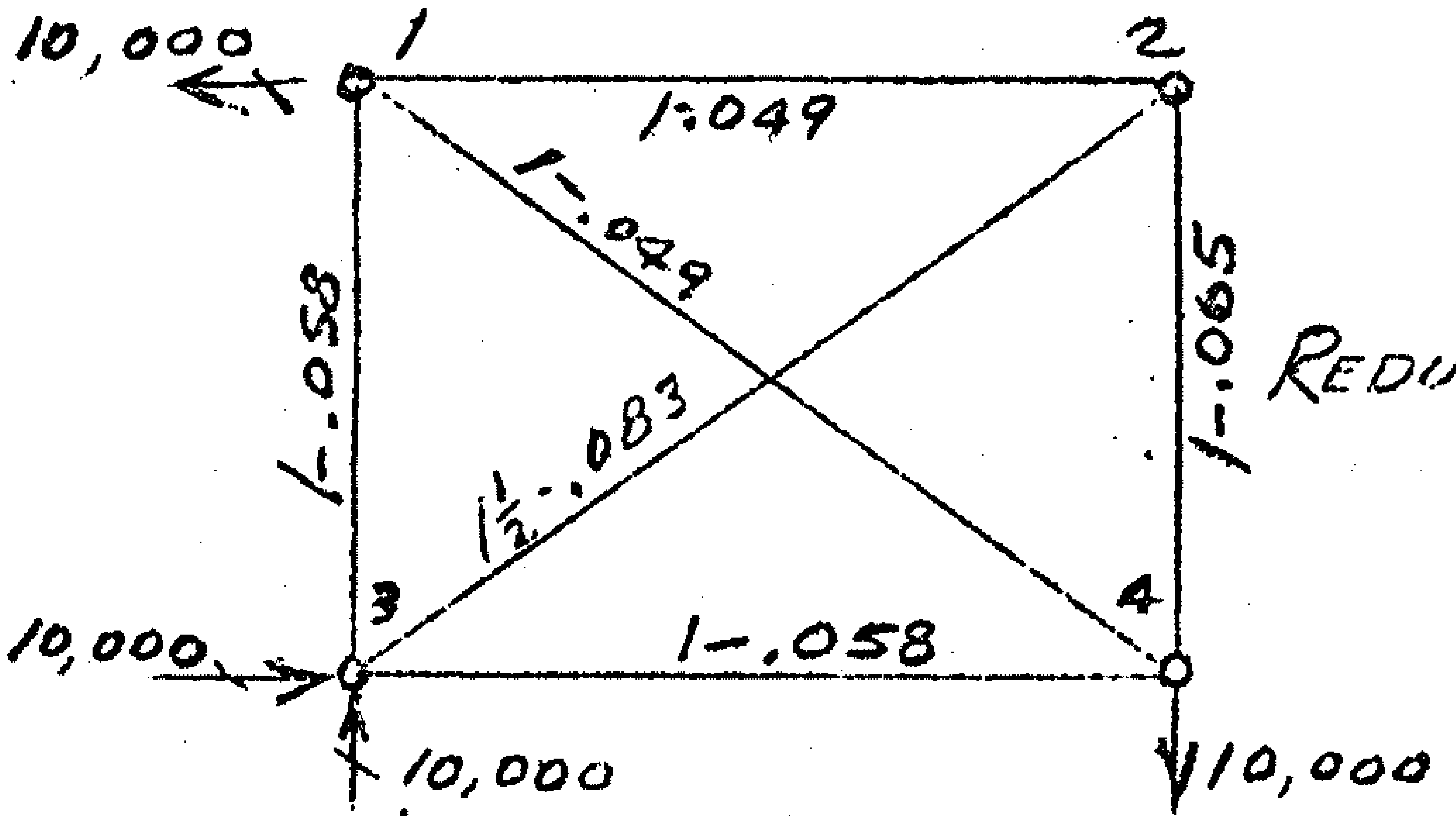


FIG. 5
REDUNDANT STRUCTURE

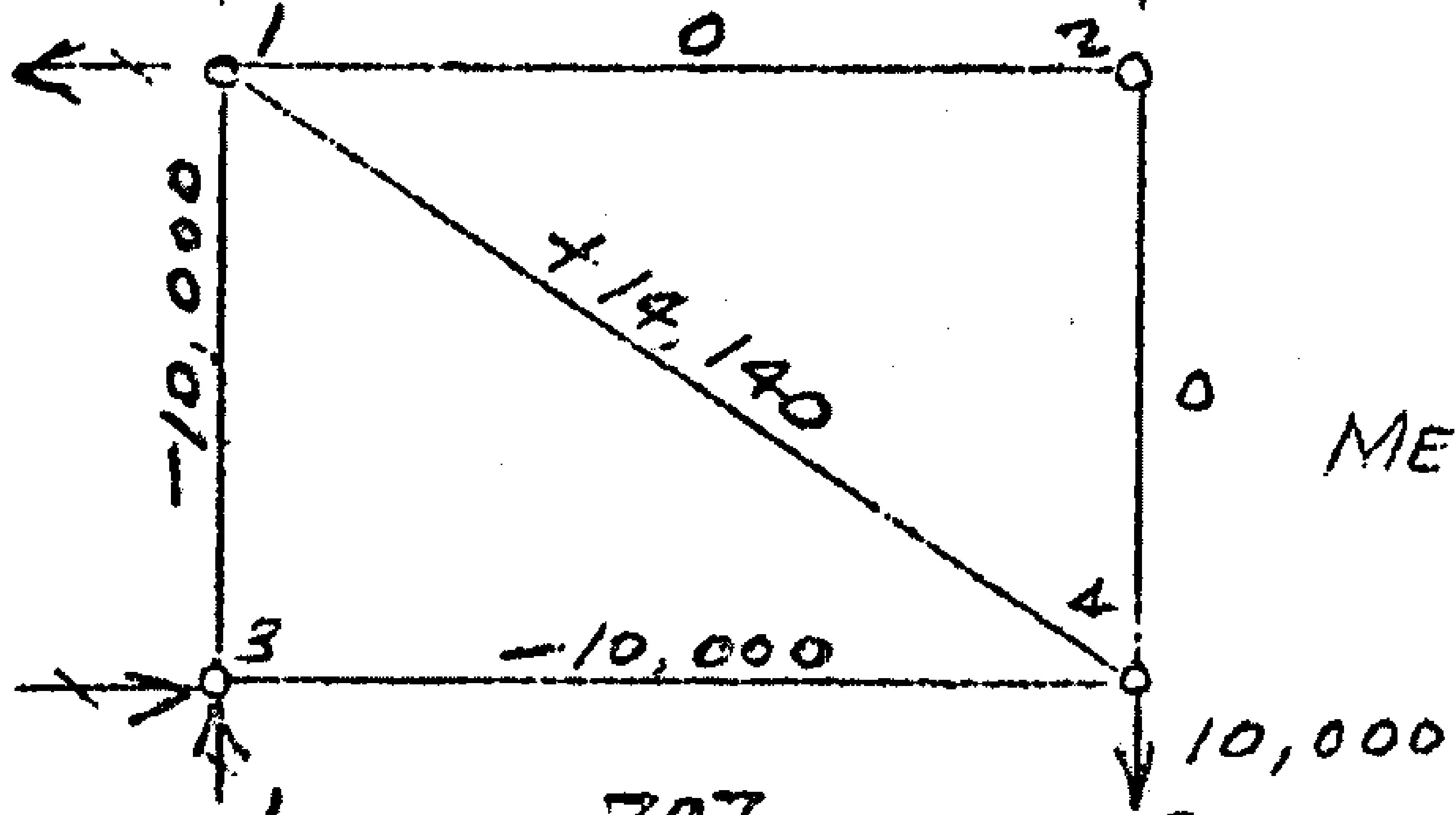


FIG. 6
MEMBER 2-3 REMOVE

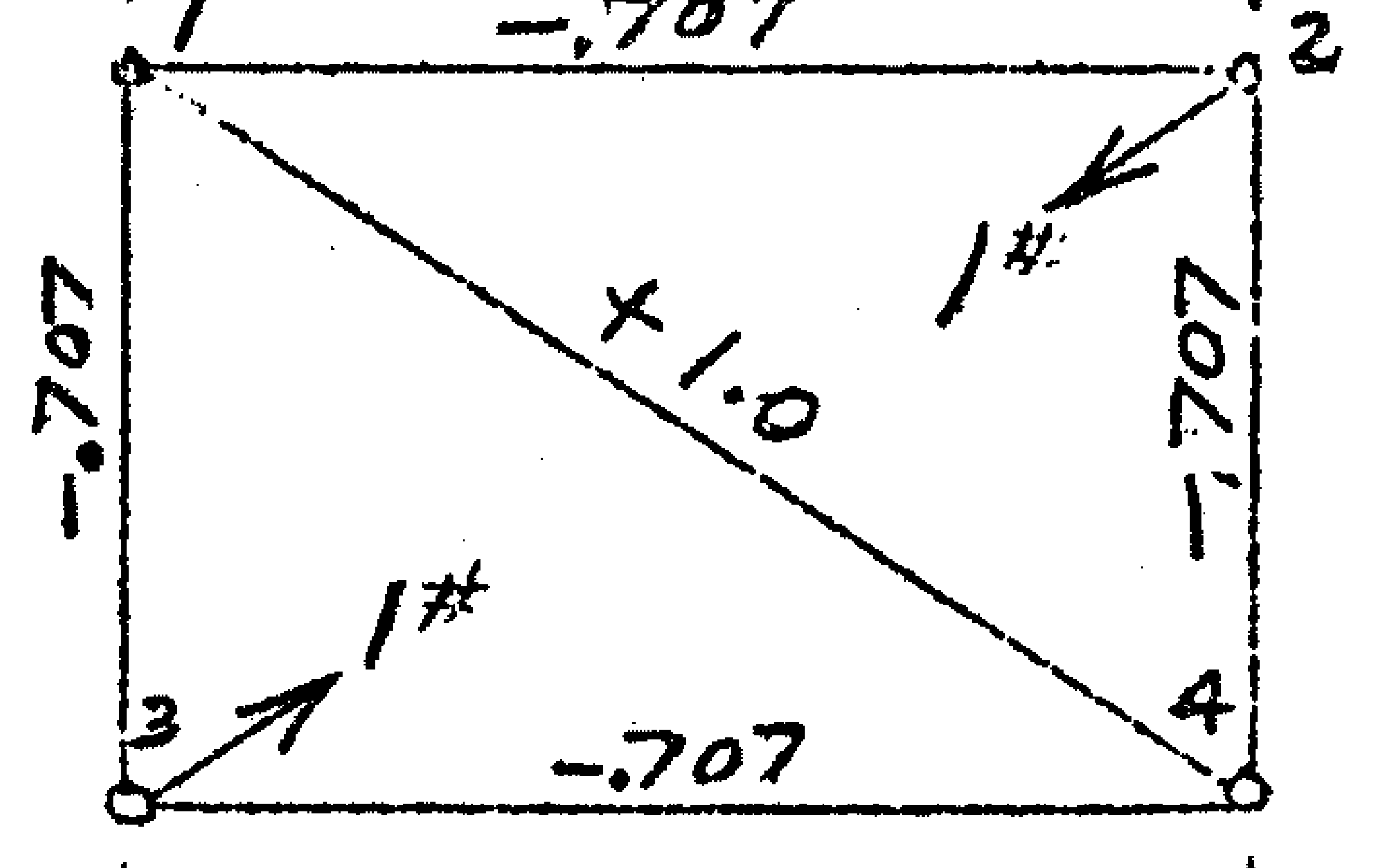


FIG. 7
UNIT LOAD IN
MEMBER 2-3

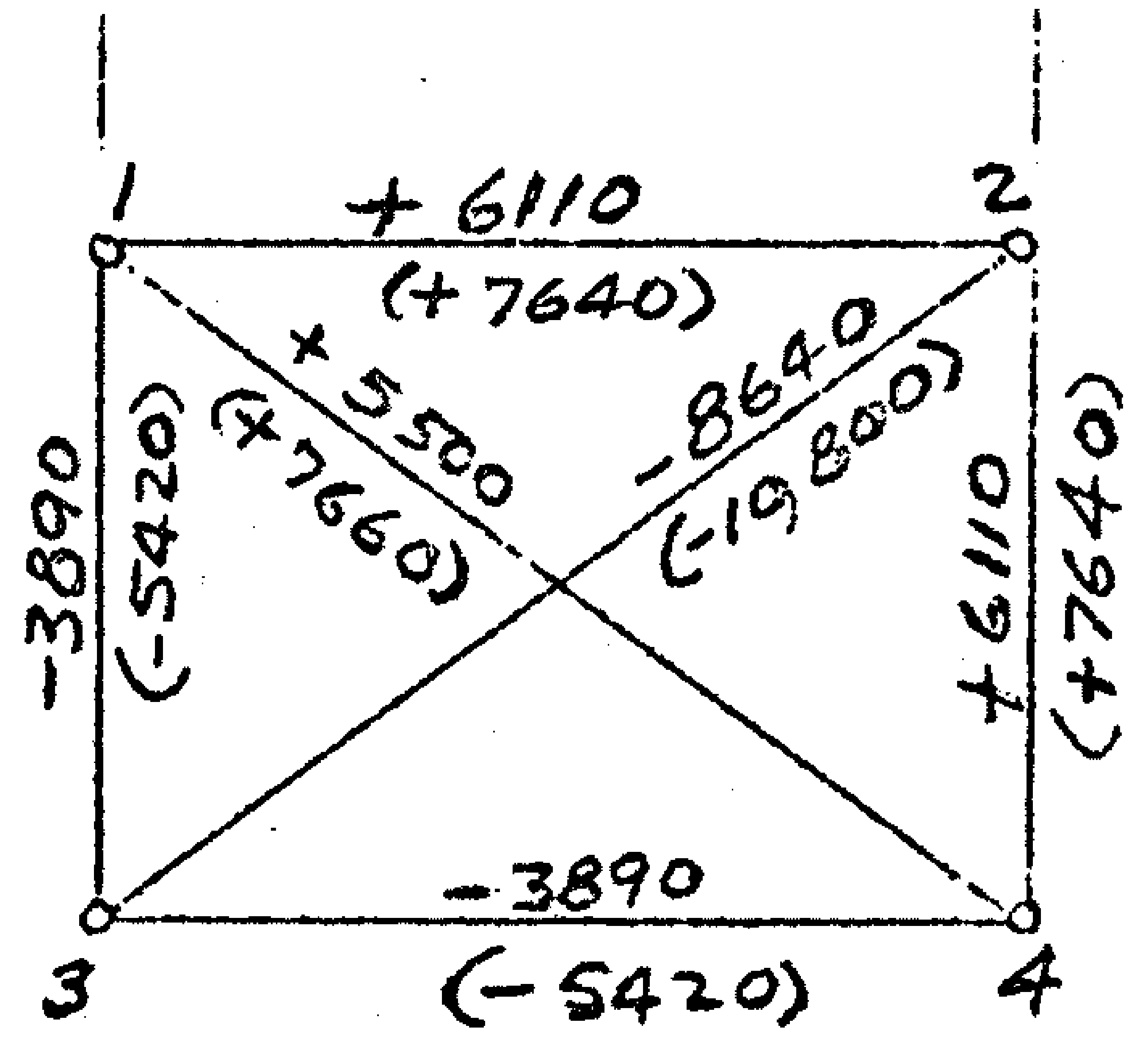


FIG. 8
CALC. AND DESIGN
LOADS
(DESIGN LOADS ARE
IN PARENTHESES)

Solution to Problems of Assignment. (Continued)

16:2 Using member 1-4 as the redundant.

Member	Size	Length	Area	Q	P_o	P_a	P_oP_aQ	P_a^2Q
1-2	1-.049	30	.146	2.05	+10,000	-.707	-14,500	1.025
1-3	1-.058	30	.172	1.74	0	-.707	0	.87
1-4	1-.065	42.4	.191	2.22	0	+1.0	0	2.22
2-3	$1\frac{1}{2}$-.065	42.4	.293	1.45	-14,140	+1.0	-20,500	1.45
2-4	1-.065	30	.191	1.57	+10,000	-.707	-11,100	.785
3-4	1-.058	30	.172	1.74	0	-.707	0	.87
Σ .	. . . . .	. . . . .	. . . .	. . . .	. . . . . .	. . . . .	-46,100	7.22

E is constant and can be omitted in solution for Q.

$Q = \frac{L}{A}\left(\frac{1}{100}\right)$ for convenience in computations.

$$X_a = -\frac{(-46,100)}{7.22} = +6380 \text{ lbs.}$$

****

Design Loads for Members

Member (1)	P_o (2)	P_a (3)	P_aX_a (4)	$.25\ P_aX_a$ (5)	$P_o + P_aX_a$ (6)	(6) ± (5) (7)
1-2	+10,000	-.707	-4510	-1130	+5490	+6620
1-3	0	-.707	-4510	-1130	-4510	-5640
1-4	0	+1.0	+6380	+1595	+6380	+7975
2-3	-14,140	+1.0	+6380	+1595	-7760	-9355
2-4	+10,000	-.707	-4510	-1130	+5490	+6620
3-4	0	-.707	-4510	-1130	-4510	-5640

Solution to Problems of Assignment. (Continued)

16:2

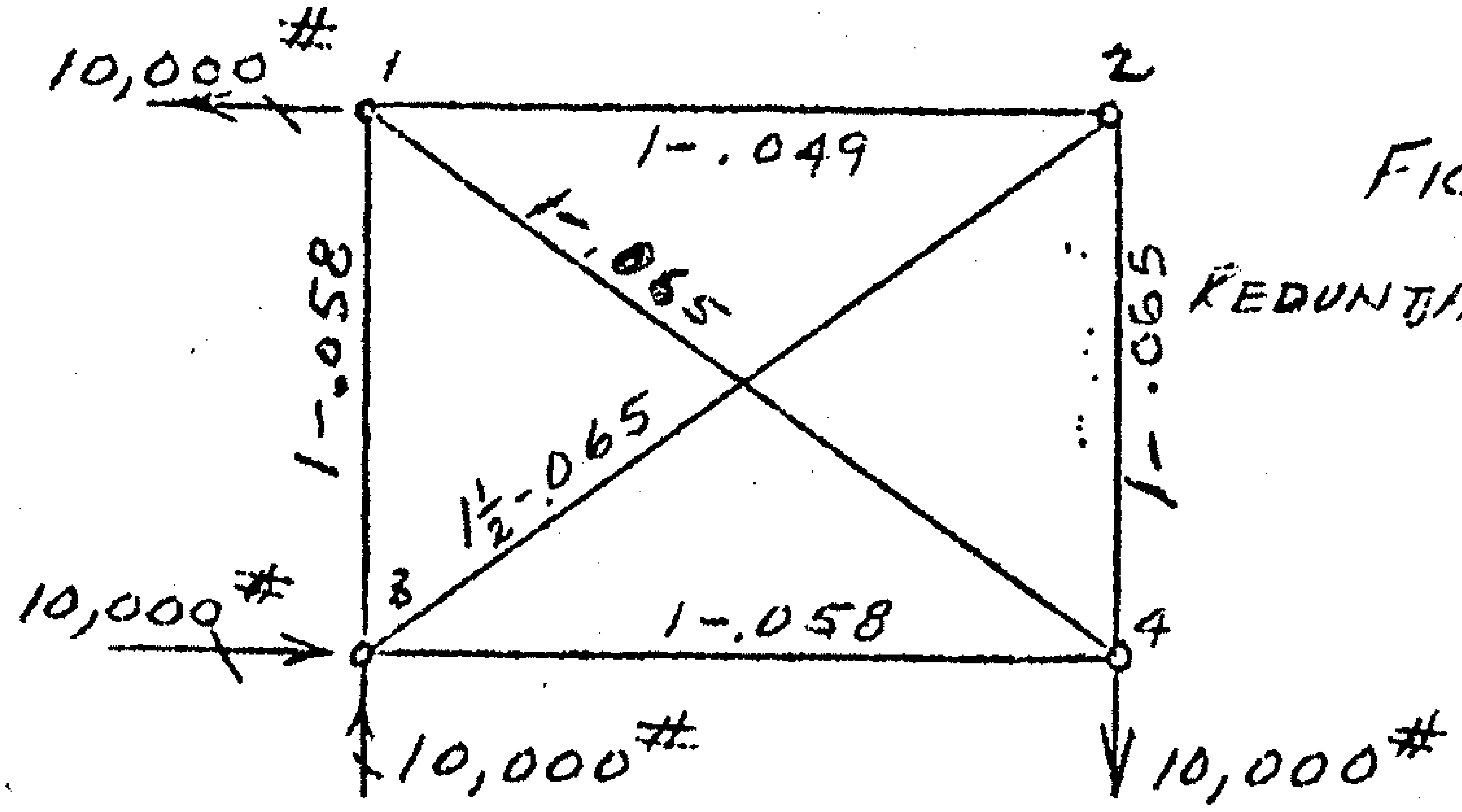


FIG. 9
REDUNDANT STRUCT.

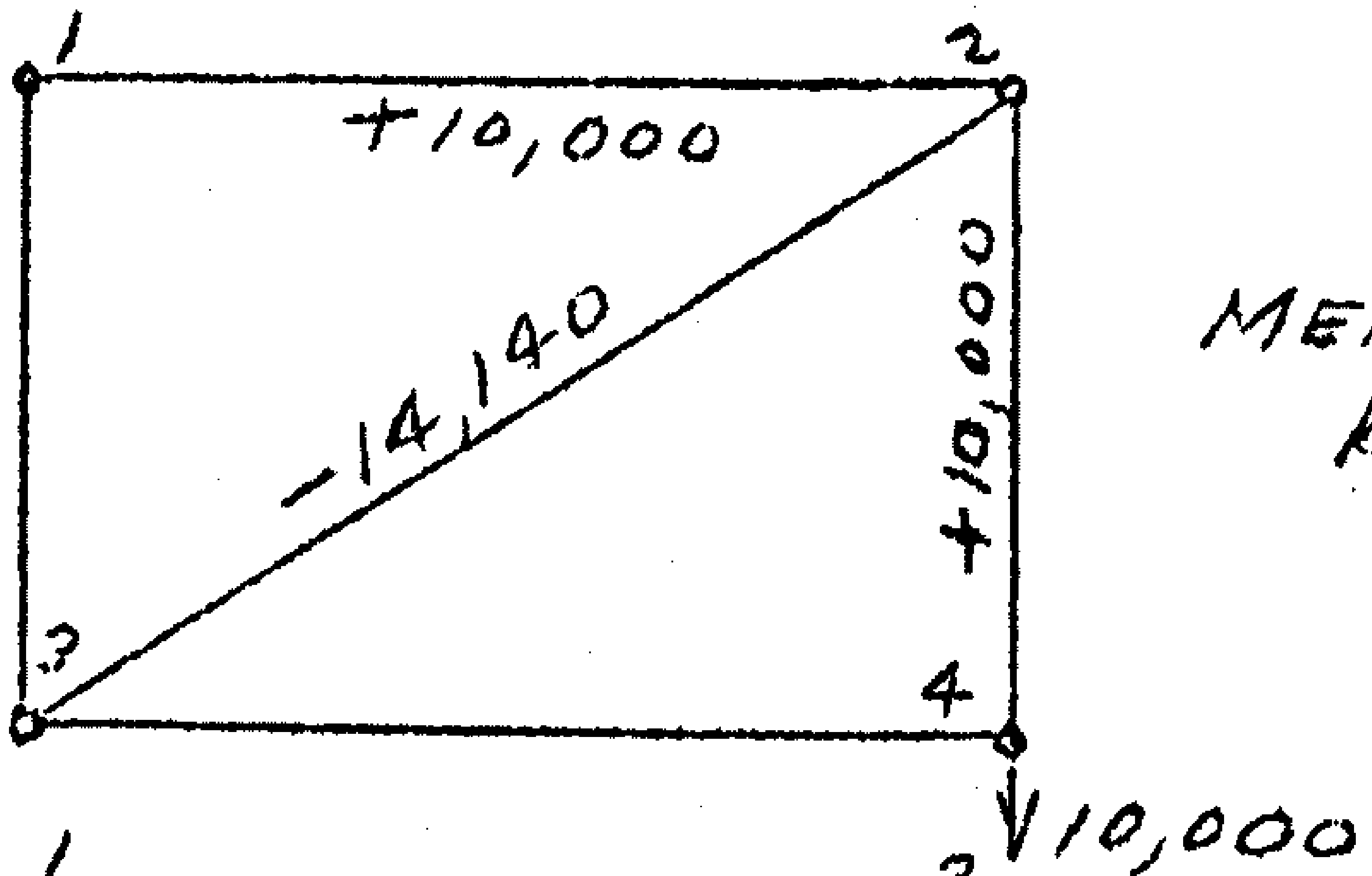


FIG. 10
MEMBER 1-4
REMOVED

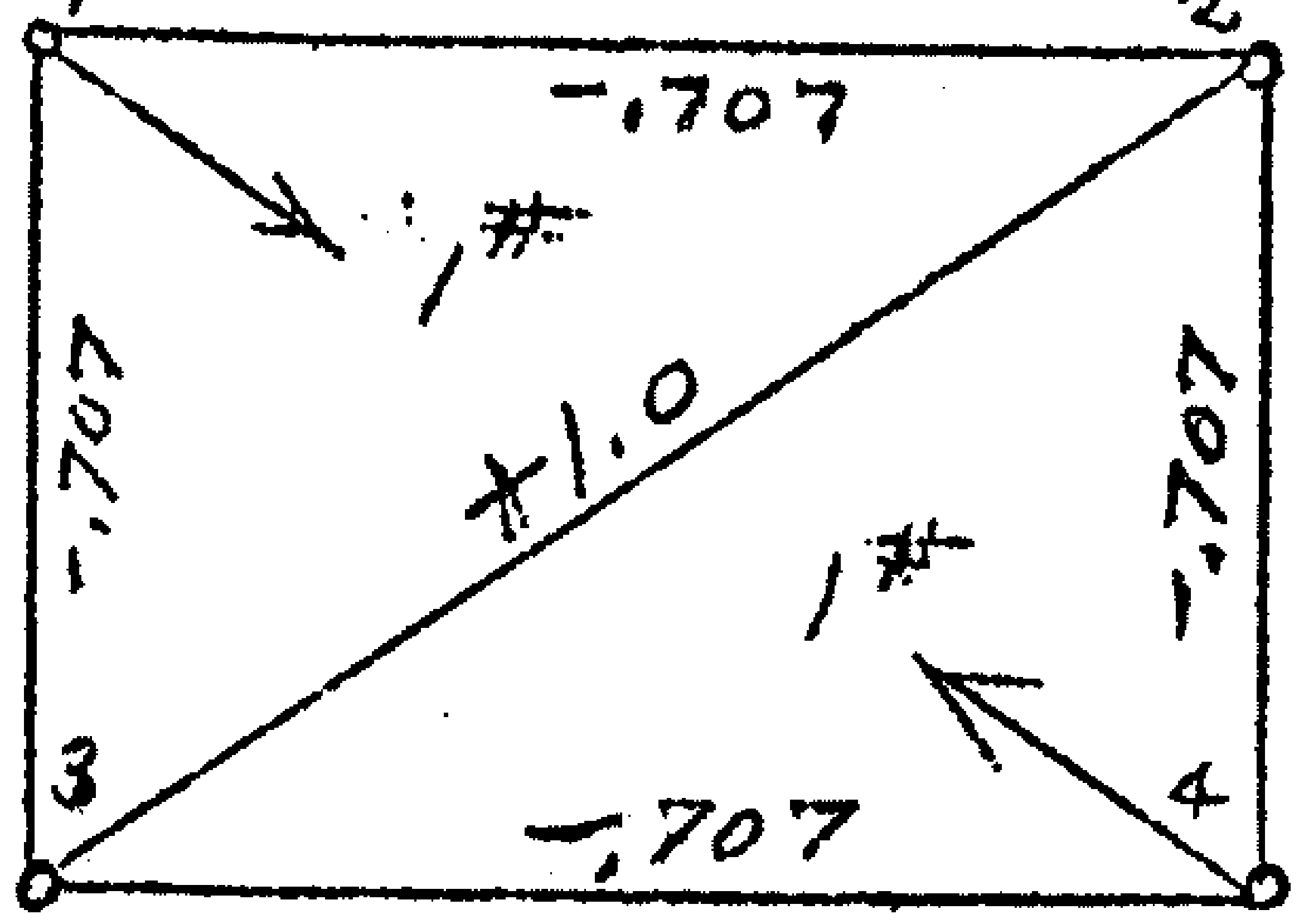


FIG. 11
UNIT LOAD IN
MEMBER 1-4

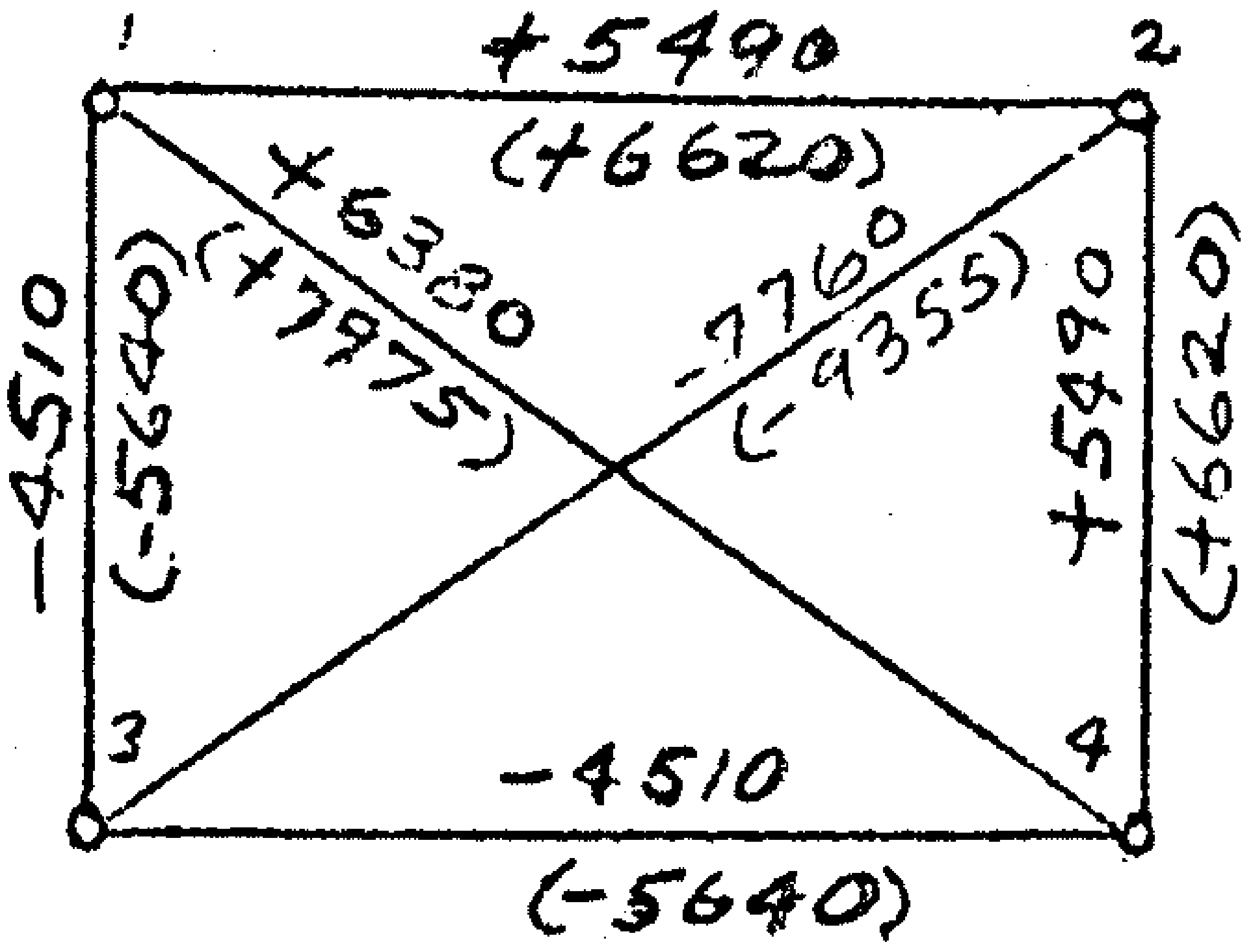


FIG. 12
CALC. AND DESIGN
LOADS
(DESIGN LOADS ARE
IN PARENTHESES)

Solution to Problems of Assignment. (Continued)

16:2

Design of Members

Member	Material	Size	ρ	L'	L/ρ	Design Load	Allow. Load	Margin of Safety
1-2	Chrome-Molyb. Steel Tube H.T. = 95,000 psi.	1-.049		30		+6620	+11710	+.77
1-3		1-.058	.334	30	90	-5640	- 6000	+.06
1-4		1-.065		42.4		+7975	+15270	+.91
2-3		$1\frac{1}{2}$-.065	.508	42.4	83.5	-9355	-11700	+.25
2-4		1-.065		30		+6620	+15270	+1.31
3-4		1-.058	.334	30	90	-5640	- 6000	+.06

Refer also to Assignment 3.

Allowable $\frac{P}{A}$ for tubes - 95,000 H.T.

Critical $\frac{L}{\rho} = 91.5$ (All tubes having $\frac{L}{\rho}$ below this value are short columns.)

From Table 4-1 ANC-5

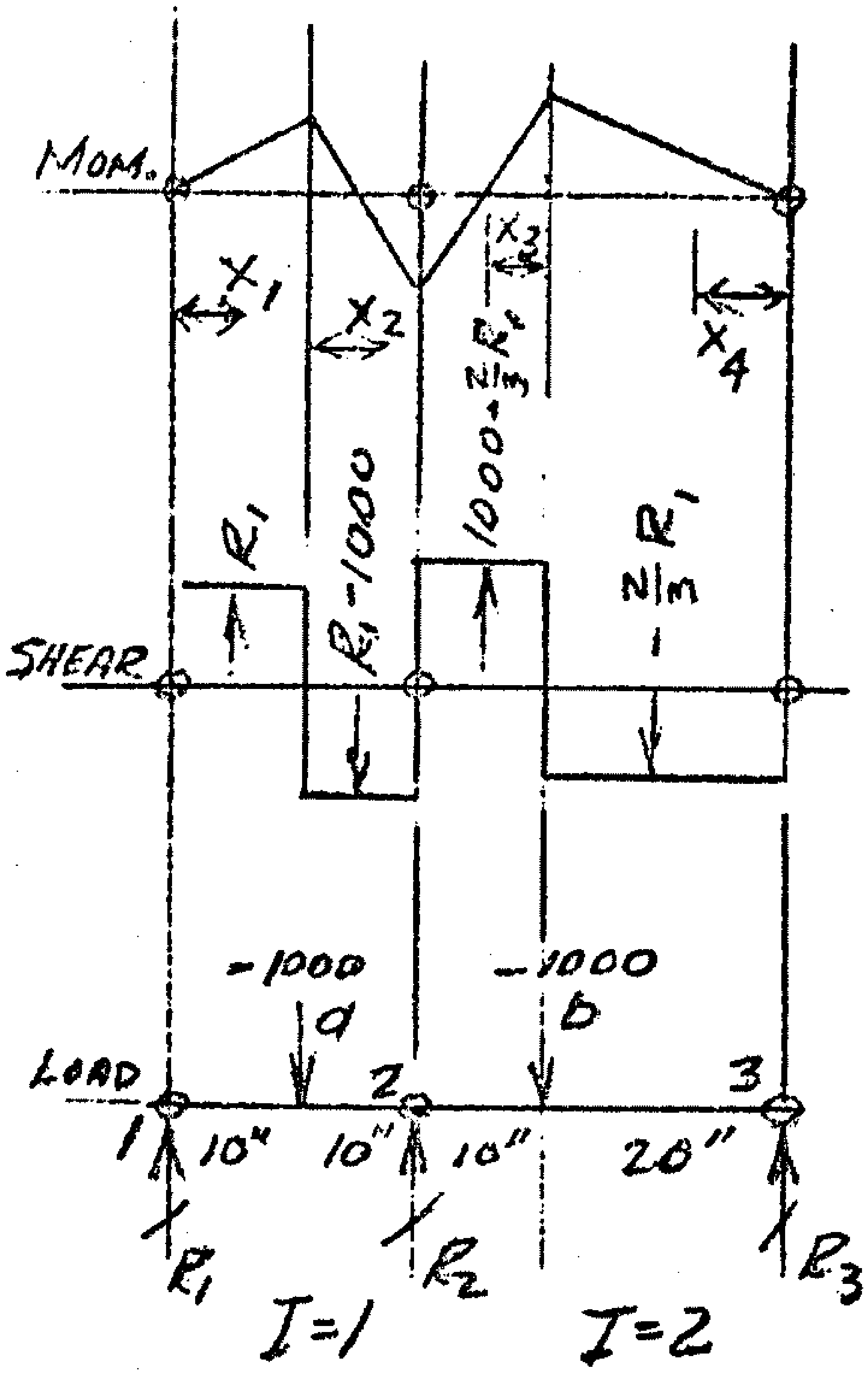
$$\text{Allowable } \frac{P}{A} = 79,500 - 51.9 \left(\frac{L}{\rho}\right)^{1.5}$$

Values are plotted on Fig. 4-2, page 4-14 of ANC-5.

Allowable tension is based on 80,000 psi. to allow for welding to end fittings.

Solution to Problems of Assignment.

17:1



Evaluating moments from both ends toward the center.

Segm. 1-a, $M = R_1x$

Segm. a-2, $M = R_1(10 + x) - 1000x$

Segm. 3-b, $M = R_3x = \frac{2}{3} R_1x$

Segm. b-2, $M = R_3(20 + x) - 1000x$

$$= \frac{2}{3} R_1(20 + x) - 1000x$$

Substituting the values of x at joint 2 to check the equations from both sides:

From the left end,

$$M_2 = R_1(10 + 10) - (1000)(10)$$
$$= 20R_1 - 10,000$$

From the right end,

$$M_2 = R_3(20 + 10) - (1000)(10)$$
$$= 30R_3 - 10,000, \text{ (but } R_3 = \frac{2}{3} R_1)$$
$$= 20R_1 - 10,000 \text{ (checks)}$$

****

Assuming R_1 unknown:

$$\Sigma M_{R_3} = 0 \quad \curvearrowleft +$$

$$(R_1)(50) + (R_2)(30) - (1000)(40) - (1000)(20) = 0$$

$$50R_1 + 30R_2 - 40,000 - 20,000 = 0$$

$$30R_2 = 60,000 - 50R_1$$

$$R_2 = 2,000 - \frac{5}{3} R_1$$

$$\Sigma M_{R_2} = 0 \quad \curvearrowright +$$

$$(R_3)(30) - (R_1)(20) - (1000)(10) - (1000)(10) = 0$$

$$30R_3 = 20R_1$$

$$R_3 = \frac{2}{3} R_1$$

$$\Sigma R = R_1 + 2000 - \frac{5}{3} R_1 + \frac{2}{3} R_1 = 2000 \text{ lbs. (checks)}$$

****

Solution to Problems of Assignment. (Continued)

17:1 Total internal work of bending is equal to

Σ W for segments 1-a, a-2, 3-b and b-2

For segment 1-a, $W = \frac{1}{2EI}\int_0^{10} (R_1 x)^2 dx = \frac{1}{2E\ (1)}\int_0^{10} R_1^2\ x^2 dx$

$$= \frac{1}{2E}\left[R_1^2\ \frac{x^3}{3} \right]_0^{10} = \frac{R_1^2}{6E}\ (10)^3 = \frac{1000R_1^2}{6E} \longleftarrow (1\text{-}a)$$

For segment a-2, $W = \frac{1}{2EI}\int_0^{10} (10R_1 + R_1 x - 1000x)^2 dx$

$$= \frac{1}{2E\ (1)}\int_0^{10} (100R_1^2 + 20R_1^2\ x - 20{,}000R_1 x + R_1^2\ x^2 - 2000R_1 x^2 + 1{,}000{,}000x^2)\ dx$$

$$= \frac{1}{2E}\left[100R_1^2\ x + \frac{20R_1^2\ x^2}{2} - \frac{20{,}000R_1\ x^2}{2} + \frac{R_1^2\ x^3}{3} - \frac{2000R_1 x^3}{3} + \frac{1{,}000{,}000x^3}{3} \right]_0^{10}$$

$$= \frac{1}{2E}\left[100R_1^2\ (10) + 10R_1^2\ (10)^2 - 10{,}000R_1 (10)^2 + \frac{R_1^2}{3}\ (10)^3 - \frac{2000R_1 (10)^3}{3} + \frac{1{,}000{,}000\ (10)^3}{3} \right]$$

$$= \frac{1}{2E}\ \left(1000R_1^2 + 1000R_1^2 - 1{,}000{,}000R_1 + \frac{1000R_1^2}{3} - \frac{2{,}000{,}000R_1}{3} + \frac{1{,}000{,}000{,}000}{3}\right)$$

$$= \frac{1}{6E}\ (3000R_1^2 + 3000R_1^2 - 3{,}000{,}000R_1 + 1000R_1^2 - 2{,}000{,}000R_1 + C)$$

$$= \frac{1}{6E}\ (7000R_1^2 - 5{,}000{,}000R_1 + C) \longleftarrow (a\text{-}2)$$

Note: Terms containing constants only may be dropped since they disappear in the partial derivative. C = a constant.

17:1 For segment 3-b, $W = \frac{1}{2EI}\int_0^{20} (\frac{2}{3} R_1 x)^2 dx$

$$= \frac{1}{2E\ (2)}\int_0^{20} \frac{4}{9} R_1^2 x^2 dx = \frac{4R_1^2}{36E}\int_0^{20} x^2 dx$$

$$= \frac{4R_1^2}{36E}\left[\frac{x^3}{3}\right]_0^{20} = \frac{4R_1^2}{36E}\left(\frac{8000}{3}\right) = \frac{8000R_1^2}{27E} \qquad \longleftarrow (3\text{-}b)$$

For segment b-2, $W = \frac{1}{2EI}\int_0^{10} \left(\frac{40R_1}{3} + \frac{2R_1 x}{3} - 1000x\right)^2 dx$

$$= \frac{1}{2E\ (2)}\int_0^{10} \left[\frac{1}{3}(40R_1 + 2R_1 x - 3000x)\right]^2 dx$$

$$= \frac{1}{4E}\int_0^{10} \frac{1}{9}(1600R_1^2 + 160R_1^2 x - 240{,}000R_1 x + 4R_1^2 x^2 - 12{,}000R_1 x^2 + 9{,}000{,}000x^2)dx$$

$$= \frac{1}{36E}\left[\left(1600R_1^2 x + \frac{160R_1^2 x^2}{2} - \frac{240{,}000R_1 x^2}{2} + \frac{4R_1^2 x^3}{3} - \frac{12{,}000R_1 x^3}{3} + \frac{9{,}000{,}000x^3}{3}\right)\right]_0^{10}$$

$$= \frac{1}{36E}\left[(1600R_1^2)(10) + 80R_1^2(10)^2 - 120{,}000R_1(10)^2 + \frac{4R_1^2}{3}(10)^3 - 4000R_1(10)^3 + C\right]$$

$$= \frac{1}{36E}(16{,}000R_1^2 + 8000R_1^2 - 12{,}000{,}000R_1 + \frac{4000R_1^2}{3} - 4{,}000{,}000R_1 + C$$

$$= \frac{1}{36E}(24{,}000R_1^2 + \frac{4000R_1^2}{3} - 16{,}000{,}000R_1 + C)\Big]$$

$$= \frac{1}{(3)(36E)}(72{,}000R_1^2 + 4000R_1^2 - 48{,}000{,}000R_1 + C)$$

$$= \frac{1}{27E}(19{,}000R_1^2 - 12{,}000{,}000R_1 + C) \qquad \longleftarrow (b\text{-}2)$$

17:1

The total work of bending is

$$W = \frac{100R_1^2}{6E} + \frac{7000R_1^2 - 5,000,000R_1 + C}{6E} + \frac{8000R_1^2}{27E} + \frac{19,000R_1^2 - 12,000,000R_1 + C}{27E}$$

$$= \frac{1}{54E}(9000R_1^2 + 63,000R_1^2 - 45,000,000R_1 + 16,000R_1^2 + 38,000R_1^2 - 24,000,000R_1 + C)$$

$$= \frac{1}{54E}(126,000R_1^2 - 69,000,000R_1 + C)$$

$$\frac{\delta W}{\delta R_1} = \frac{1}{54E}\left[(2)(126,000R_1) - 69,000,000 + 0\right] = 0$$

$$252,000R_1 - 69,000,000 = 0$$

$$R_1 = 69,000,000/252,000 = 274 \text{ lbs.}$$

$$R_2 = 1543 \text{ lbs.}$$

$$R_3 = 183 \text{ lbs.}$$

(See solution for problem 17:2 for remaining data.)

Comparison with the solution for problem 17:2 will show that the 3-moment equation is much faster and less liable to error than the method of internal work for this particular problem. The results obtained are the same.

17:2 Solution by 3-moments.

General equation:

$$\frac{M_1L_1}{I_1} + 2M_2\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + \frac{M_3L_2}{I_2} = \text{Loading terms}$$

For our case $M_1 = 0$ and $M_3 = 0$. Terms 1 and 3 drop out.

$$\text{Then } 2M_2\left(\frac{20}{1} + \frac{30}{2}\right) = \frac{(-1000)(10)(20^2-10^2)}{(1)(20)} + \frac{(-1000)(20)(30^2-20^2)}{(2)(30)}$$

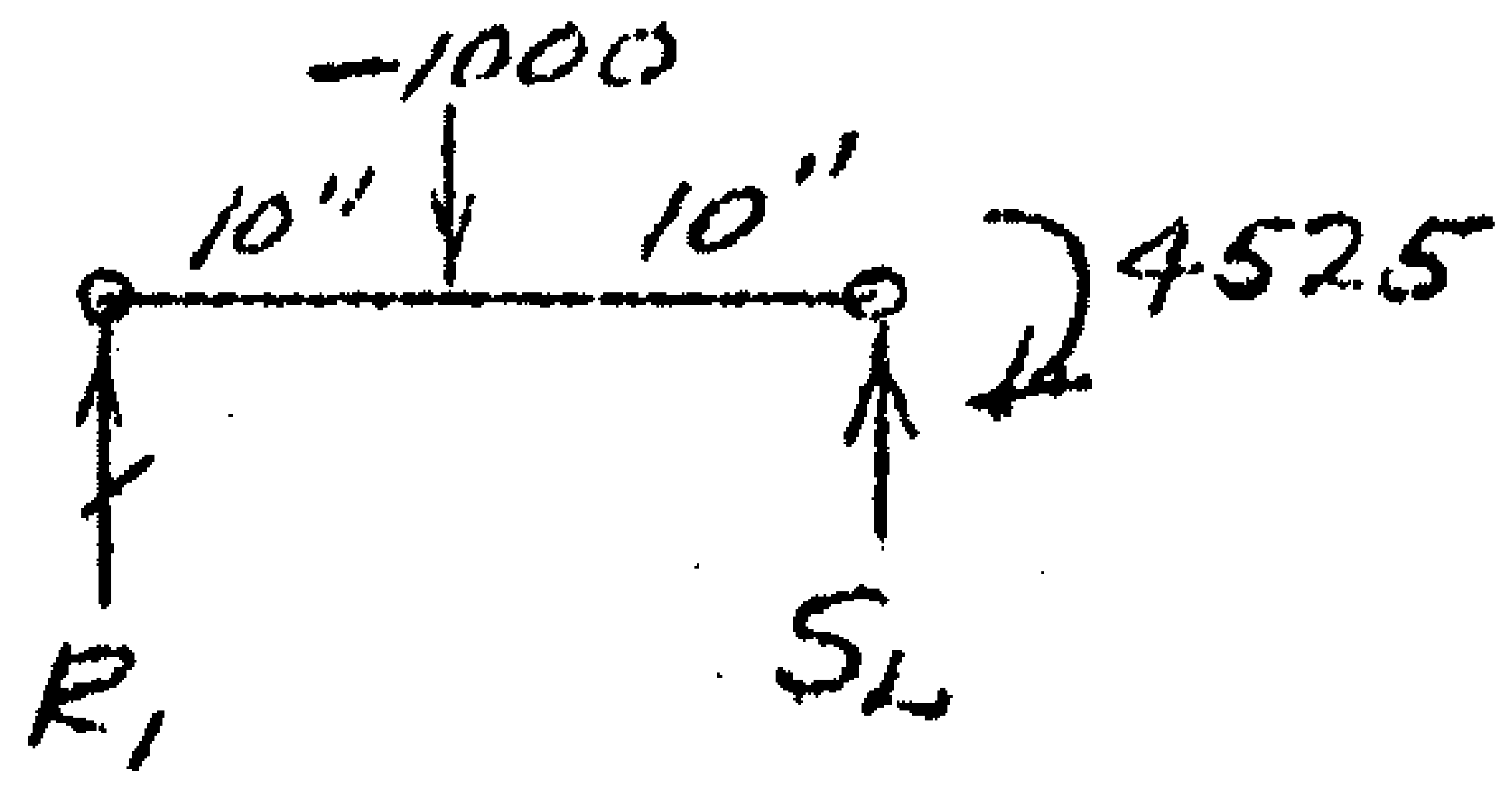
$$2M_2(20 + 15) = -150,000 - 166,700 = -316,700$$

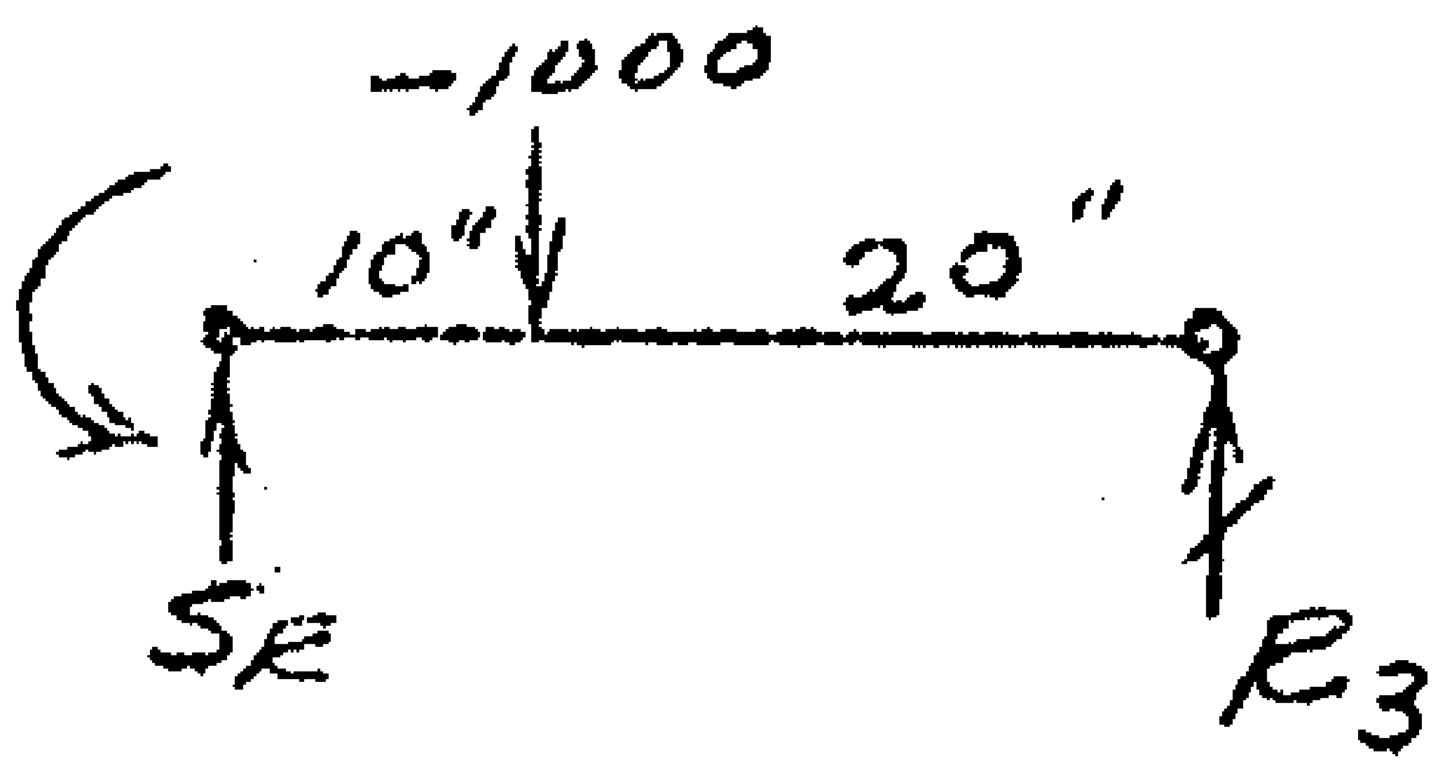
$$70M_2 = -316,700$$

$M_2 = -4,525$ in. lbs. (Tension on top fibers at joint 2.)

Solution to Problems of Assignment. (Continued)

17:2





For the left hand span $(R_1)(20) - (1000)(10) + 4525 = 0$

$$20R_1 = 10{,}000 - 4525 = 5{,}475$$

$R_1 = 274$ lbs. $S_L = 726$ lbs.

For the right hand span $(R_3)(30) - (1000)(10) + 4525 = 0$

$$30R_3 = 10{,}000 - 4525 = 5475$$

$R_3 = 183$ lbs, $S_R = 817$ lbs.

$R_2 = S_L + S_R = 726 + 817 = 1543$ lbs.

Solution to Problems of Assignment.

19:1 (a) The effective thickness of gusset which is working against each line of weld on the .065 wall tube is .125/2 = .0625. Therefore the critical thickness will be that of the gusset. Then

$$L = (4)(1.5) = 6 \text{ inches},\ t = .0625 \text{ inches}$$

$$\text{and } s = 125{,}000 \text{ psi.}$$

Then $P = .48\ Lts$

$$= (.48)(6)(.0625)(125{,}000) = 22{,}500 \text{ lbs.}$$

(b)

$L = (3)(2) = 6$ inches, $t = .093$ inches

$s = 150{,}000$ psi. (May not be usedhigher - ANC-5)

$P = (.48)(6)(.093)(150{,}000) = 40{,}000$ lbs.

(c) For tube assembly the .058 wall is critical.

The length of weld is the circumference of the 1 3/8 diameter tube = 4.32 inches

Then the weld strength is

$$P = (.48)(4.32)(.058)(90{,}000) = 10{,}800 \text{ lbs.}$$

Note: If it could be shown that the actual thickness of the weld metal were greater than .058 its actual thickness could have been used for t, except that it could not be taken greater than (1.25)(.058).

19:2 (a)

Moment = (200)(10) = 2000 in. lbs., Shear = 200 lbs.

I of rivet group = $(2)(1.5)^2 + 2(.5)^2 = 5.0$

Moment load on outer rivets = $\frac{(2000)(1.5)}{50} = 600$ lbs.

Shear load per rivet = $\frac{200}{4} = 50$ lbs.

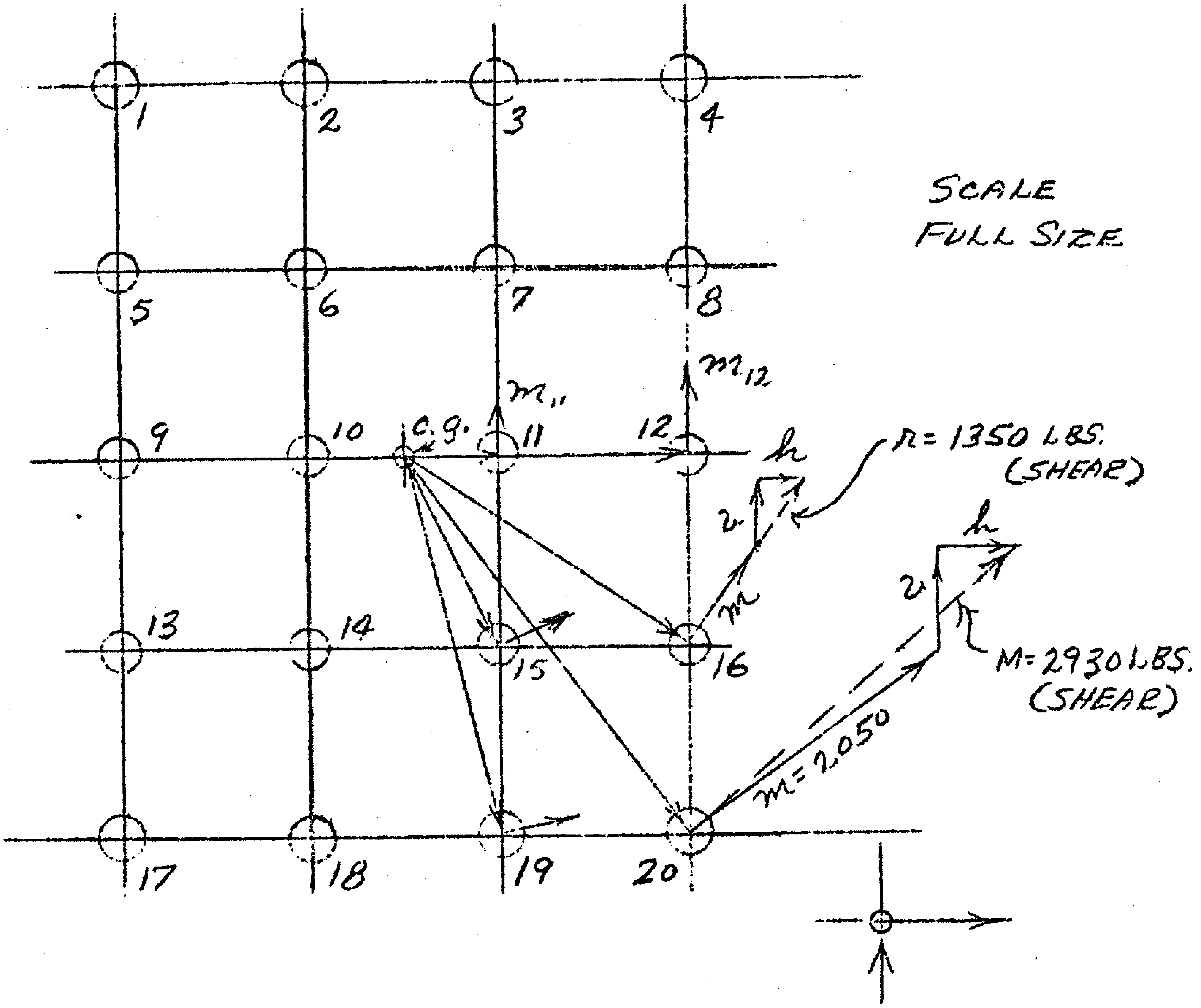
Resultant load = $(600^2 + 50^2)^{\frac{1}{2}} = 605$ lbs.

Allowable single shear = 690 lbs.

Allowable bearing in .064 24ST = 1055 lbs.

(b) The centroid of this symmetrical rivet group may be determined by inspection. Where this can not be done the position of the centroid may be established by taking moments about two reference axes at right angles such as A-A and B-B in Assignment 19:2 (b).

19:2 (b) Taking the value of a 3/16 diameter rivet as 1.0 the value k for the 1/4 diameter rivets is 1.78 when shear is considered, (the ratio of the areas); or 1.33 when bearing is considered, (the ratio of the diameters). The centroid of the symmetrical group is at the same location, if the rivets are critically loaded in shear, or if they are critical in bearing. Laying out the pattern of the joint to scale, we obtain the values for "d".



Considering Shear Values

Rivet No.	d	d^2	k	kd^2	Rivets affected	Total
1,4,17,20	2.5	6.25	1.78	11.12	Four	44.48
2,3,18,19	2.06	4.25	1.78	7.56	Four	30.22
5,8,13,16	1.80	3.25	1.00	3.25	Four	13.00
6,7,14,15	1.12	1.25	1.00	1.25	Four	5.00
9,12	1.5	2.25	1.00	2.25	Two	4.50
10,11	.5	.25	1.00	.25	Two	.50

$\Sigma I = 97.70$

$\Sigma k = 2 + 2 + 4 + 4 + 7.12 + 7.12$

$= 26.24$ = Equivalent number of 3/16 diameter rivets.

Solution to Problems of Assignment. (Continued)

19:2 (b)

Vertical Load = 10,000 lbs.
Horizontal Load = 8,000 lbs.

Moment around centroid $+\circlearrowright$

$$= (10{,}000)(2.5) + (8{,}000)(2.5) = 45{,}000 \text{ in lbs.}$$

Considering Shear values:

The vertical shear load per rivet on 3/16 rivets is $\frac{10{,}000}{26.24} = 380$ lbs.; on 1/4 rivets = (380)(1.78) = 680 lbs.

The horizontal shear load per rivet on 3/16 rivets is $\frac{8{,}000}{26.24} = 305$ lbs.; on 1/4 rivets = (305)(1.78) = 543 lbs.

The moment load on the outer 3/16 rivets (5,8,13,16) is $m = \frac{(45{,}000)(1.80)}{97.70} = 828$ lbs. each

and on the 1/4 rivets (1,4,17,20) is $m = \frac{(45{,}000)(2.5)(1.78)}{97.70} = 2050$ lbs. each.

The moment loads on the other rivets are as follows by ratios of value "d",

(2,3,18,19) = 1690 lbs. each
(6,7,14,15) = 515 lbs. each
(9,12) = 690 lbs. each
(10,11) = 230 lbs. each

At this point we should check the summations of shear and moment loads on the rivets.

Σ Vertical Shear = (8)(680) + (12)(380) = 10,000 lbs.
Σ Horizontal Shear = (8)(543) + (12)(305) = 8,000 lbs.
Σ Moment = (2050)(4)(2.5) + (1690)(4)(2.06) + (828)(4)(1.80) + (515)(4)(1.12) + (690)(2)(1.5) + (230)(2)(.5)
= 20,500 + 13,920 + 5,960 + 2,310 + 2,070 + 230
= 44,970 in lbs. (error = 30 in. lbs.)

****

The critically loaded 1/4 diameter rivet is No. 20. The resultant load on this rivet is 2930 lbs. The double shear value of the rivet is 3440 lbs. and the bearing value in 5/32 24ST plate is 3510 lbs. This rivet is therefore satisfactory for loads based on shear strength.

Solution to Problems of Assignment. (Continued)

19:2 (b) The critically loaded 3/16 diameter rivet is No. 16. The resultant load on this rivet is 1350 lbs. The allowable double shear value is 1930 lbs., and the bearing value in 5/32 24ST plate is 2630 lbs. This rivet is also satisfactory.

Minimum margin of safety on 3/16 rivet $= \frac{1930}{1350} - 1 = .43$

Minimum margin of safety on 1/4 rivet $= \frac{3440}{2930} - 1 = .17^*$

(*Note: The Bureau of Air Commerce would require .20 M.S.)

Considering Bearing Values

Rivet No.	d	d^2	k	kd^2	Rivets affected	Total
1,4,17,20	2.5	6.25	1.33	8.31	Four	33.24
2,3,18,19	2.06	4.25	1.33	5.65	Four	22.60
5,8,13,16	1.80	3.25	1.00	3.25	Four	13.00
6,7,14,15	1.12	1.25	1.00	1.25	Four	5.00
9,12	1.5	2.25	1.00	2.25	Two	4.50
10,11	.5	.25	1.00	.25	Two	.50

$\Sigma I = 78.84$

$$\Sigma k = 2 + 2 + 4 + 4 + (8)(1.33)$$
$$= 22.67$$

For a load distribution based on bearing values:

The vertical shear load per rivet on 3/16 rivets is $\frac{10{,}000}{22.67} = 440$ lbs.; on 1/4 rivets $= (1.33)(440) = 585$ lbs.

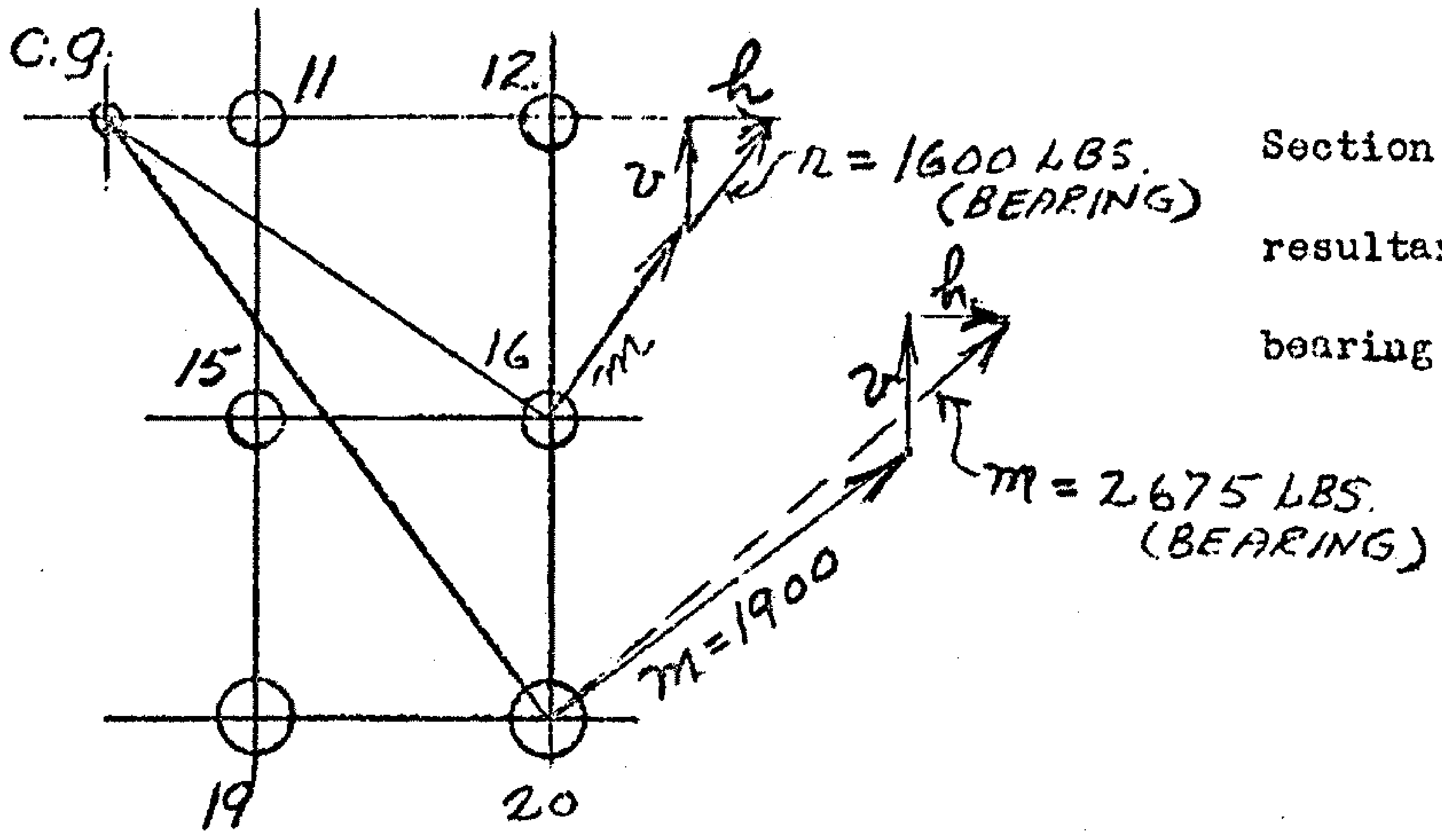
The horizontal shear load per rivet on 3/16 diameter rivets is $\frac{8000}{22.67} = 352$ lbs.; on 1/4 rivets $= (1.33)(352) = 468$ lbs.

The moment load on the outer 3/16 rivets (5,8,13,16) is $m = \frac{(45{,}000)(1.80)}{78.84} = 1028$ lbs. each

and on the 1/4 diameter rivets (1,4,17,20) is $m = \frac{(45{,}000)(2.5)(1.33)}{78.84} = 1900$ lbs. each

Solution to Problems of Assignment. (Continued)

19:2 (b)



Section of Joint showing resultant loads based on bearing distribution.

Maximum resultant load on 3/16 rivet = 1600 lbs.

Value in double shear = 1930 lbs.
in bearing = 2630 lbs.

Maximum resultant load on 1/4 rivet = 2675 lbs.

Value in double shear = 3440 lbs.
in bearing = 3510 lbs.

Minimum margin of safety

$$\text{on } 3/16 \text{ rivet} = \frac{1930}{1600} - 1 = .20$$

$$\text{on } 1/4 \text{ rivet} = \frac{3440}{2675} - 1 = .29$$

ADVANCED STRESS ANALYSIS

SOLUTION SHEETS

Solution to Problem 1:1

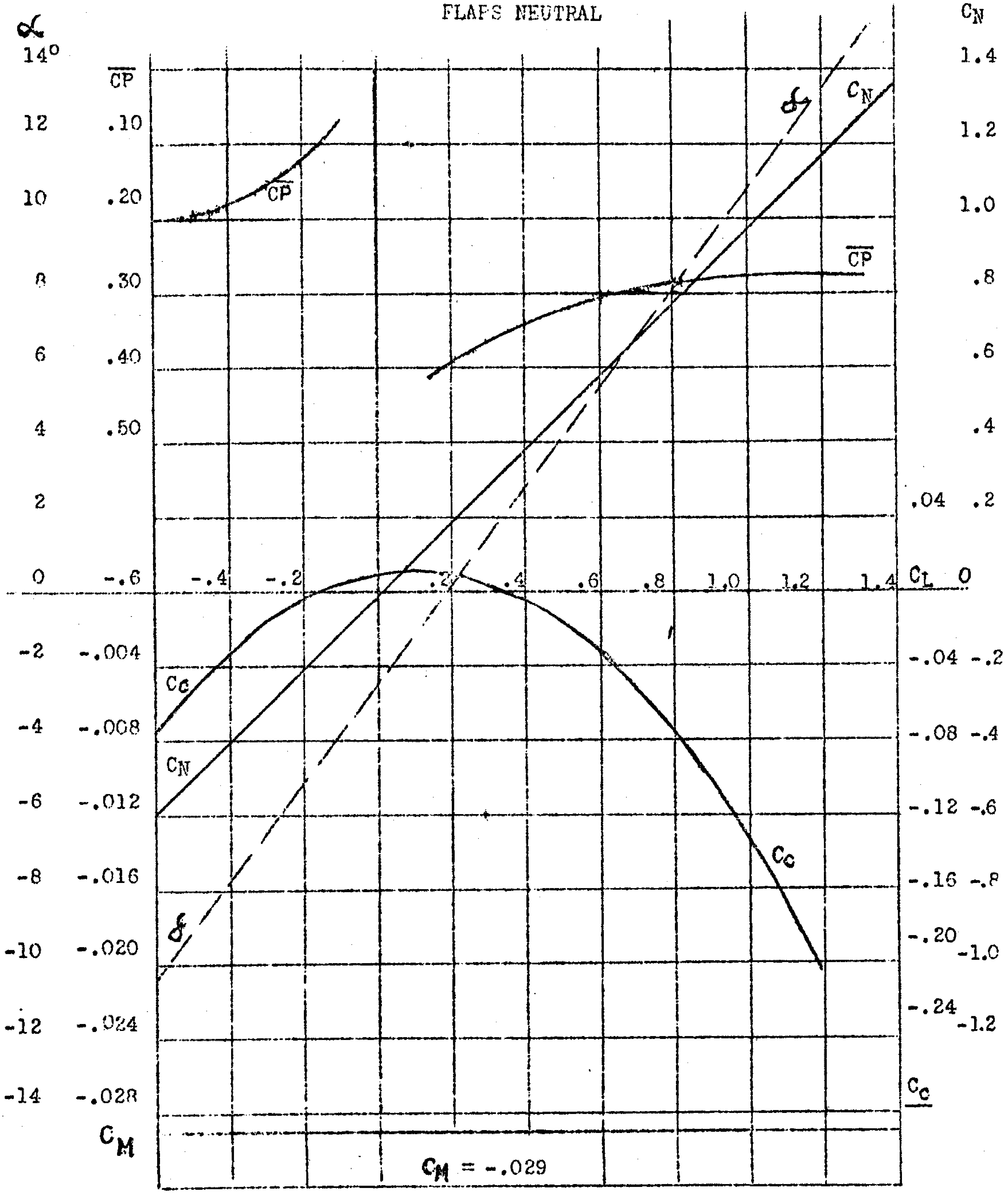
SOLUTION OF TABLE I

1	C_L	-1.0	-.8	-.6	-.4	-.2	0	.2	.4	.6	.8	1.0	1.2	1.4	1.6
2	α_6	-16.2	-13.4	-10.6	-7.9	-5.1	-2.3	.4	3.2	6.0	8.8	11.5	14.3	17.0	19.8
3	$\Delta\alpha = 18.24KC_L$	.8	.6	.4	.3	.2	0	-.2	-.3	-.5	-.6	-.8	-.9	-1.1	-1.2
4	$\alpha = (2) + (3)$	-15.4	-12.8	-10.2	-7.6	-4.9	-2.3	.2	2.9	5.5	8.2	10.7	13.4	15.9	18.6
5	C_{D_6}	.076	.050	.032	.020	.013	.010	.012	.018	.027	.046	.068	.094	.126	.158
6	$\Delta C_{D_i} = .318K_L^2$	-.013	-.009	-.005	-.002	-.001	0	-.001	-.002	-.005	-.009	-.013	-.019	-.026	-.034
7	$C_D = (5) + (6)$	.063	.041	.027	.018	.012	.010	.011	.016	.022	.037	.055	.075	.100	.124
8	$\cos\alpha$	.964	.975	.984	.991	.996	.999	1.00	.999	.995	.990	.983	.973	.962	.947
9	$\sin\alpha$	-.265	-.222	-.177	-.131	-.084	-.040	.003	.050	.096	.142	.185	.231	.273	.317
10	$C_L \cos\alpha$	-.964	-.780	-.590	-.396	-.199	0	.200	.400	.597	.792	.983	1.17	1.35	1.52
11	$C_D \sin\alpha$	-.017	-.009	-.005	-.002	-.001	0	0	.001	.002	.005	.010	.017	.03	.04
12	$C_N = (10) + (11)$	-.981	-.789	-.595	-.398	-.200	0	.200	.401	.599	.797	.993	1.19	1.38	1.56
13	$C_L \sin\alpha$	.265	.178	.106	.052	.017	0	.001	.020	.058	.114	.185	.277	.382	.508
14	$C_D \cos\alpha$	.061	.040	.027	.018	.012	.010	.011	.016	.022	.037	.054	.073	.096	.117
15	$C_C = (14) - (13)$	-.204	-.138	-.079	-.034	-.005	.010	.010	-.004	-.036	-.077	-.131	-.204	-.286	-.391
16	C_M (.25 chd)	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029	-.029
17	$\overline{CP}$	.22	.21	.20	.18	.11	---	.39	.32	.30	.29	.28	.27	.27	.27

AIRFOIL CHARACTERISTICS WITH FLAPS UP

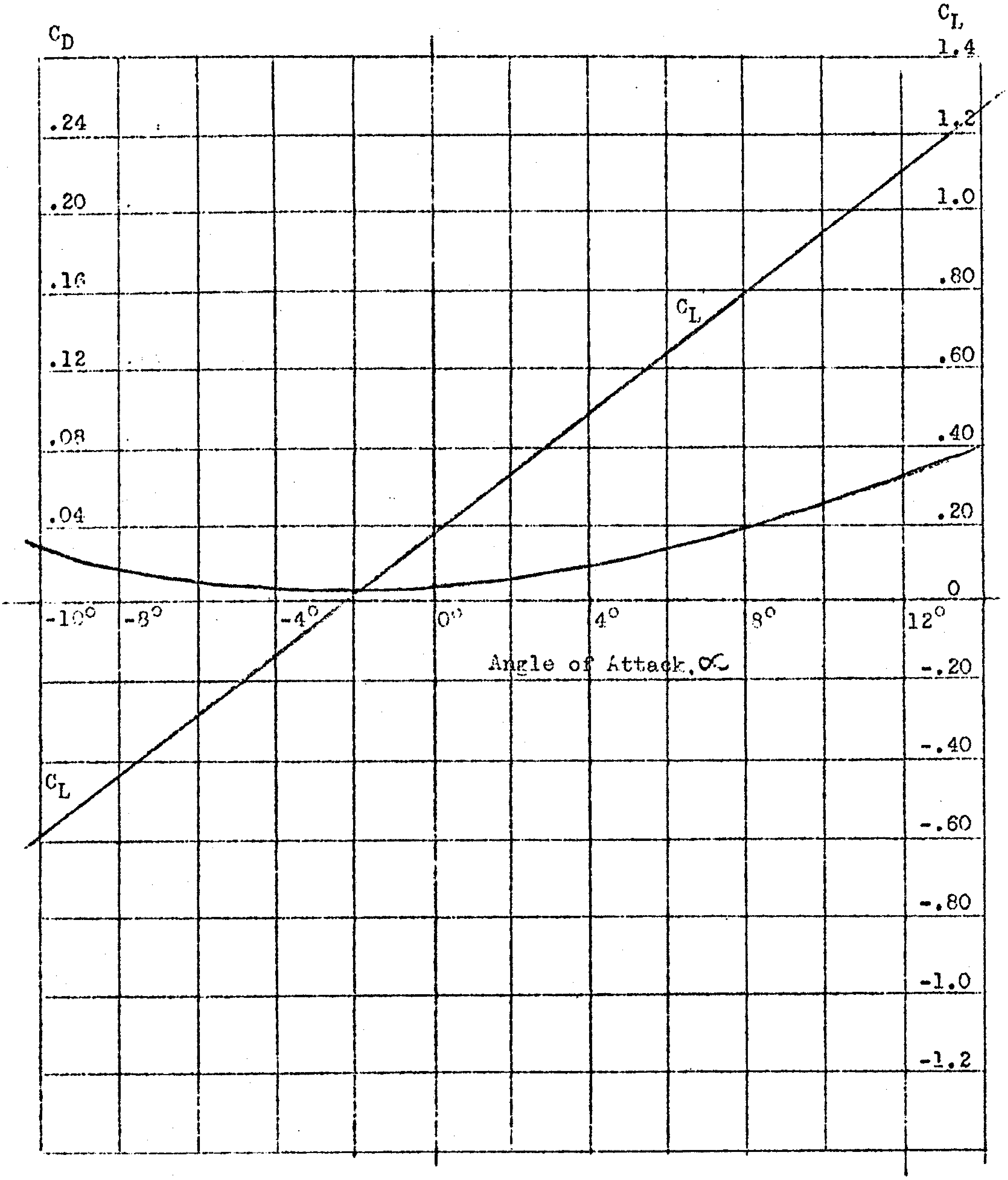
Solution to Problem 1:2

AIRFOIL CHARACTERISTICS
FLAPS NEUTRAL



Aero. Center = 25% of Chord

Solution to Problem 1:3



Airfoil Data Corrected to Aspect Ratio of 8.0

Solution to Problem 2:1

1. $V_G = 220 + .16\ (600 - 220) = 281$ mph.

   $(K_G = .16)$

2. $q_L = \dfrac{(220)^2}{391} = 123.7$

3. $q_G = \dfrac{(281)^2}{391} = 202.5$

4. $W/S = \dfrac{20{,}000}{1{,}010} = 19.9$

5. $W/P = \dfrac{20{,}000}{2{,}000} = 10.0$

q_G over portion of wing with ailerons = $(.75)(202.5) = 151.9$

Condition	I	I_1	II	III	III_1	IV
V	220	220	220	281		281
q	123.7	123.7	123.7	202.5	151.9	202.5
n (a)	2.54 (1)	2.54	-2.54	1.62		-1.62
n (b)	2.05			1.23		
n	3.54	3.54	-1.54	2.62		-.62
C_N	.570	.570	-.248	.258	.194	-.061
C_c (actual)	-.030	-.030	-.008	+.007		+.009
C_o (assumed)	-.114		0	0	0	0
CP'	.270	.301	.133	.362	.81(3)	-.225
C_M		-.029	-.029	-.029		-.029
C_M'	-.0114 (2)				-.109(3)	

Note (1) $n_I\ (a) = \dfrac{(1.0)(30)(220)(4.40)}{(575)(19.9)} = 2.54$

(2) $C_M' = .570\ (.250 - .270) = -.0114$

(3) Applies only to that portion of the span incorporating ailerons.

For Condition III, loads are to be reduced to 75 per cent of their value for Condition III. For solution see page 2-10.

Solution to Problem 2:1 (Continued)

Condition III_1

When ailerons are displaced the effect on that section of the wing in front of the aileron is to move the center of pressure forward on the side with up aileron, (assuming normal flight), and to move the center of pressure back on the side with down aileron.

For section affected by aileron

$$C_M' = -0.08 - C_M = -0.08 - 0.029 = -0.109$$

$$q_G = (.75)(202.5) = 151.9$$

$$C_N' = (.75)(.258) = .194$$

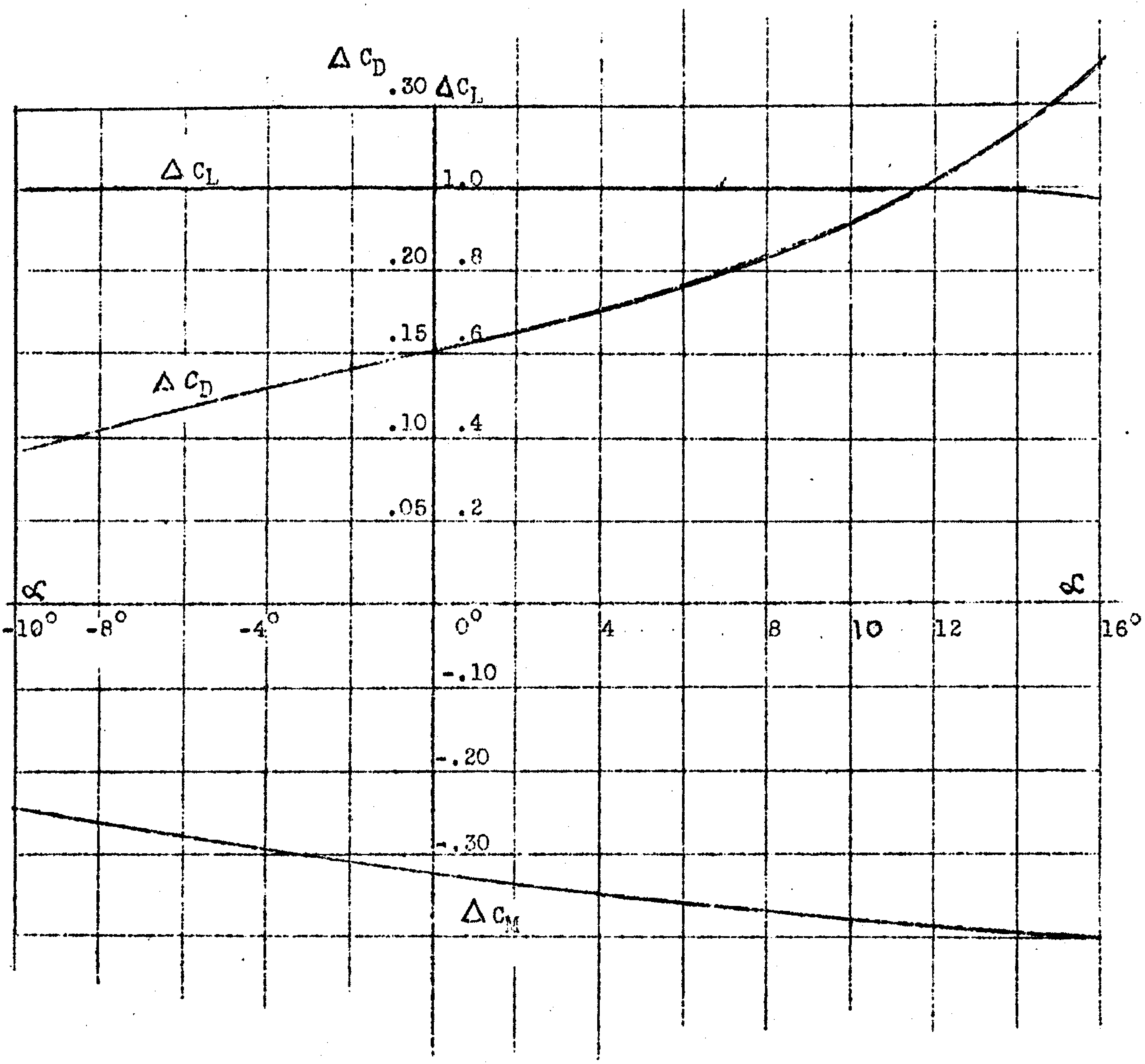
$$Cp' \text{ over aileron section} = .25 - \frac{C_M'}{C_N'}$$

$$= .25 - \left(\frac{-0.109}{.194}\right) = .25 + .56 = .81$$

It is not necessary to find the center of pressure for the entire wing. If this value is desired as a check it is necessary to know the percentage of wing area affected by ailerons. In any case this is 41.5 per cent. Then for the whole wing with aileron down the center of pressure is ---

$$\text{Average } CP = (.362)(.585) + (.81)(.415) = .548$$

Solution to Problem 3:1



INCREMENTS OVER SECTIONS AFFECTED BY FLAPS

Solution to Problem 3:2

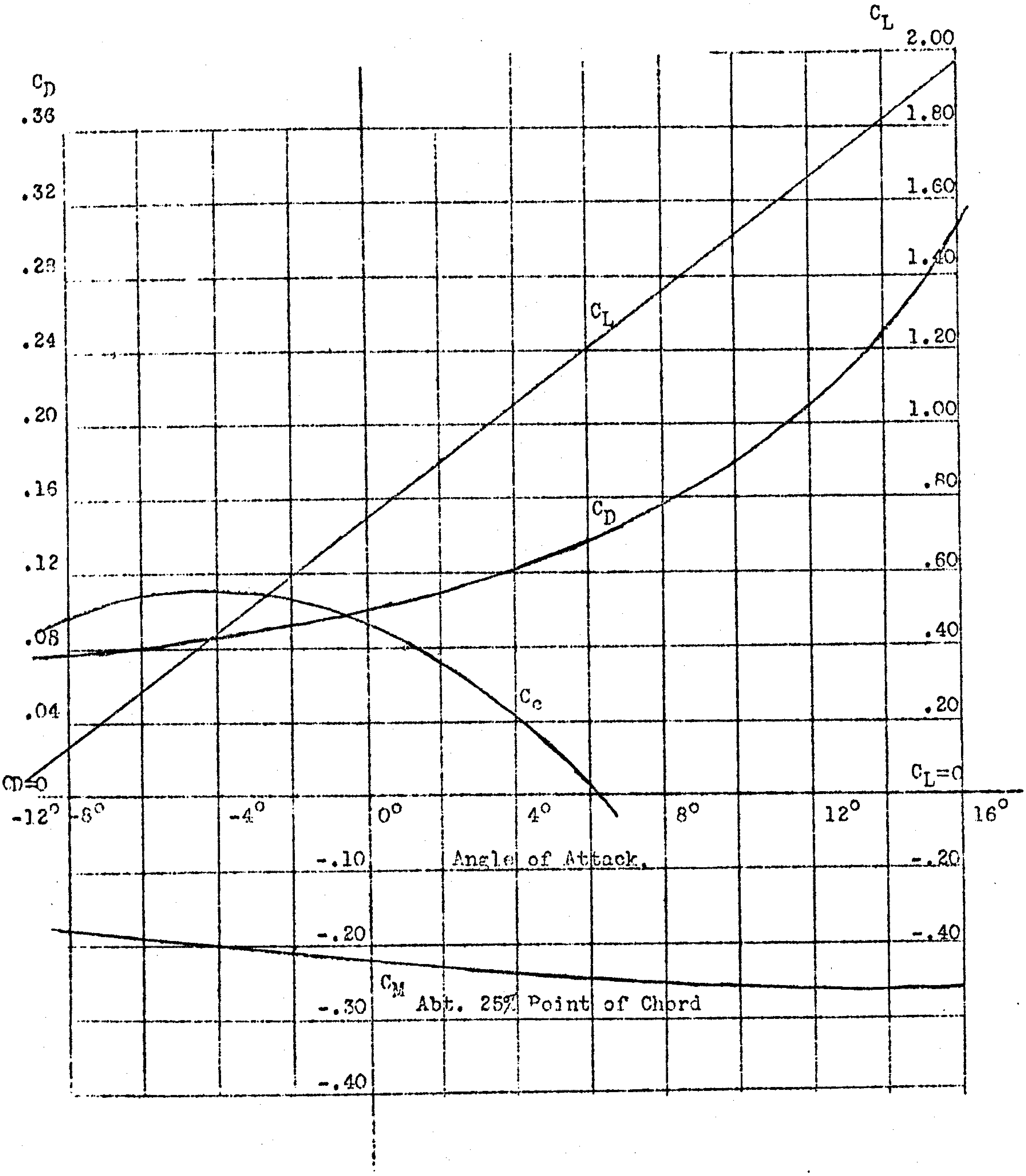
(1) α	=	-8°	-4°	0°	4°	8°	12°	16°
(2) C_L Flaps Up	=	-.44	-.13	.18	.48	.79	1.10	1.41
(3) $(.415)(C_L)$	=	-.183	-.054	.075	.199	.328	.457	.58
(4) ΔC_L	=	1.0	1.0	1.0	1.0	1.0	1.0	.98
(5) $C_L + \Delta C_L$	=	.56	.87	1.18	1.48	1.79	2.10	2.39
(6) $(.585)(C_L + \Delta C_L)$	=	.328	.508	.692	.867	1.046	1.228	1.40
(7) Avg. C_L = (3) + (6)	=	.145	.454	.767	1.066	1.374	1.685	1.98
(8) C_D Flaps Up	=	.018	.010	.010	.019	.039	.062	.100
(9) $(.415)(C_D)$	=	.007	.004	.004	.008	.016	.026	.041
(10) ΔC_D	=	.105	.130	.150	.175	.205	.250	.325
(11) $C_D + \Delta C_D$	=	.123	.140	.160	.194	.244	.312	.425
(12) $(.585)(C_D + \Delta C_D)$	=	.072	.082	.094	.114	.143	.183	.249
(13) Avg. C_D = (9) + (12)	=	.079	.086	.098	.122	.159	.209	.290
(14) C_M Flaps Up	=	-.029	-.029	-.029	-.029	-.029	-.029	-.029
(15) $(.415)(C_M)$	=	-.012	-.012	-.012	-.012	-.012	-.012	-.012
(16) ΔC_M	=	-.265	-.290	-.320	-.350	-.373	-.390	-.400
(17) $C_M + \Delta C_M$	=	-.294	-.319	-.349	-.379	-.402	-.419	-.429
(18) $(.585)(C_M + \Delta C_M)$	=	-.172	-.187	-.204	-.222	-.235	-.245	-.251
(19) Avg. C_M = (15) + (18)	=	-.184	-.199	-.216	-.234	-.247	-.257	-.263

AERODYNAMIC CHARACTERISTICS FOR ENTIRE WING

WITH FLAPS DOWN 45°

Solution to Problem 3:2 (Continued)

Plot of Characteristics for Wing with Flaps Down 45°



Solution to Problem 3:3

$V_{s_f} = 62.5$ mph. V_f = design speed for flaps down $= 2V_{s_f} = 125$ mph.

$q_{V_f} = (.00256)(125)^2 = 40$ $\qquad m = 4.4 = \frac{\Delta C_L}{\Delta \alpha}$

$\frac{W}{S} = \frac{20,000}{1,010} = 19.8$ lbs. per sq. ft.

****

For Condition VII, Positive Gust with Flaps Down.

$n_7 = 1 + \Delta n_7$

Δn_7 = gust increment $= \frac{KUV_f m}{575 \frac{W}{S}}$

$K = 1.00$, $U = 15$ fps.

$\Delta n_7 = \frac{(1)(15)(125)(4.4)}{(575)(19.8)} = 0.73$

$n_7 = 1 + \Delta n_7 = 1.73$, but 2.0 is the minimum value allowed.

Use $n_7 = 2.0$

$C_{N_7} = \frac{n_7 (\frac{W}{S})}{q_{V_f}} = \frac{(2)(19.8)}{40} = .99$

Using the permissible assumption that $C_N = C_L$, $\alpha = 3.3^\circ$, $C_D = .116$, $C_M = -.23$ for the entire wing.

$C_c = C_D \cos \alpha - C_L \sin \alpha$

$= (.116)(.998) - (.99)(.058) = .059$

$\overline{CP} = .25 - \frac{C_M}{C_N} = .25 - (\frac{-.23}{.99}) = .48$

****

Solution to Problem 3:3 (Continued)

On the portion of wing not affected by flaps:

$C_L = C_N = .41$ $\qquad$ $C_M = -.029$ $\qquad$ $C_D = .016$

$C_c = (.016)(.998) - (.41)(.058) = -0.008$

$\overline{CP} = .25 - (\frac{-.029}{.41}) = .32$

On the portion of wing affected by flaps:

$C_L = C_N = .41 + 1.0 = 1.41$

$C_D = .016 + .170 = .186$, $\quad C_c = (.186)(.998) - (1.41)(.058) = .104$

$C_M = -.029 - .345 = -.374$

$\overline{CP} = .25 - (\frac{-.374}{1.41}) = .52$

****

For Condition VIII, Negative Gust with Flaps Down.

$n_8 = 1 + \Delta n_8$

$\Delta n_8 = \text{gust increment} = \dfrac{KUV_{fm}}{575\,\frac{W}{S}}$

$K = 1.00$, $\quad U = -15$ fps.

$\Delta n_8 = \dfrac{(1)(-15)(125)(4.4)}{(575)(19.8)} = -0.73$

$n_8 = 1 - 0.73 = .27$ $\qquad$ $q_f = \dfrac{125^2}{391} = 40$ lbs. per sq. ft.

$C_{N_8} = \dfrac{(.27)(19.8)}{40} = .134$

Assume again that $C_L = C_N$, $\alpha = -8.1^\circ$, $\quad C_D = .078$, $\quad C_M = -.183$ (all for the entire wing.)

$C_c = C_D \cos\alpha - C_L \sin\alpha = (.078)(.990) - (.134)(-.140) = .096$

$\overline{CP} = .25 - \dfrac{C_M}{C_N} = .25 - (\frac{-.183}{.134}) = 1.61$, which means that the average

Solution to Problem 3:3 (Continued)

resultant is .61 x chord aft of the trailing edge of the wing. How this condition can exist will be explained more fully in later assignments.

****

On the portion of the wing not affected by flaps:

$C_L = C_N = -.43$ $\qquad C_c = -.040$ $\qquad C_M = -.029$

$$\overline{CP} = .25 - \left(\frac{-.029}{-.43}\right) = .18$$

On the portion of the wing affected by flaps:

$$C_L = C_N = -.43 + 1.00 = +.57$$

$$C_D = +.019 + .103 = .122, \quad C_c = (.122)(.990) - (.57)(-.140) = .201$$

$$C_M = -.029 - .262 = -.291$$

$$\overline{CP} = .25 - \left(\frac{-.291}{.57}\right) = .76$$

****

For Condition IX, Dive with Flaps Down.

$C_{c\ max.}$ must be determined. A curve of C_c vs. α should be plotted to determine this value.

(1)	α	-8°	-4°	-0°	$+4^\circ$
(2)	$\sin\alpha$	-.140	-.070	0	.070
(3)	$\cos\alpha$	.990	.998	1.00	.998
(4)	C_L	.145	.454	.767	1.066
(5)	C_D	.079	.086	.098	.122
(7)	$C_L \sin\alpha$	-.020	-.032	0	.075
(8)	$C_D \cos\alpha$	.078	-.079	.098	.122
(9)	$C_c = (8) - (7)$	.098	.111	.098	.047

These values are plotted on the curve on page 3-13.

Solution to Problem 3:3 (Continued)

$C_{c\ max.} = +.111$ and occurs at $= -4^\circ$.

$C_{N_9} = C_L = .454$

$n_9 = \dfrac{C_{N_9}\ qV_f}{W/S} = \dfrac{(.454)(40)}{19.8} = .918$

$C_{M_9} = -.197$

$\overline{CP} = .25 - (\frac{-.197}{.454}) = .68$

****

For the portion of the wing not affected by flaps:

$C_N = C_L = -.11.$ $C_D = .010$ $C_c = .003$

$C_M = -.029$ $C.P. = .25 - \dfrac{(-.029)}{-.11} = -.01$

= just forward of leading edge.

For the portion of the wing affected by flaps:

$C_N = -.11 + 1.00 = .89$

$C_D = .010 + .130 = .140$ $C_c = (.140)(.998) - (.89)(-.070) = .202$

$C_M = -.029 - .290 = -.319$ $\overline{CP} = .25 - (\frac{-.319}{.89}) = .61$

Solution to Problem 4:1.

The curves given in Figures B to G inclusive are illustrations of graphical integration, and as such are applicable to any variation of wing planform and of lift coefficient along the span of the wing. For our case where C_N is a constant along the wing span and the variation of the wing chord may be expressed as an equation the entire solution is possible by the use of simple calculus equations.

From the trapezoid planform of Fig. A the area of the semi-span is

$$\frac{180 + 90}{2}(540) = 72{,}900 \text{ sq. in.}$$

This area serves in our case for both A_B and A_D.

****

For A_E, Fig. E ordinates to the curve are $(R_b)(C)(b)$ but $R_b = 1.0$ so the equation of the line bounding the area A_E may be written $y = (C)(b)$. If we take the root of the wing as the origin for x, b will always be the station on the wing which is x. The equation of the chord lengths is

$$C = 180 - \frac{90}{540}x = 180 - \frac{x}{6}$$

Then y, the ordinate to curve E at any point equals $(180 - \frac{x}{6})(x) = 180x - \frac{x^2}{6}$

The area under the curve equals $\int_0^{540}(180x - \frac{x^2}{6})dx$

$$A_E = \left[\frac{180x^2}{2} - \frac{x^3}{(3)(6)}\right]_0^{540} = \left[90x^2 - \frac{x^3}{18}\right]_0^{540}$$

$$A_E = (90)(291{,}600) - \frac{(157{,}464{,}000)}{18} = 26{,}244{,}000 - 8{,}748{,}000$$

$$A_E = 17{,}496{,}000$$

$$\overline{b} = \frac{A_E}{A_D} = \frac{17{,}496{,}000}{72{,}960} = 239.8 \text{ inches}$$

****

Solution to Problem 4:1 (Continued)

In our case the calculation for $\overline{x}$ is simplified by the fact that the wing was laid out with the 25 per cent point of each chord on a line at 90 degrees with the longitudinal center line of the airplane. The 25 per cent point is also the aerodynamic center for the airfoil used. Then the aerodynamic center for the semi-span will lie on the 25 per cent point of the mean aerodynamic chord. Using line OE as the reference, the moment arm of the aerodynamic center for any chord length along the span is 45". Therefore the arm to the average aerodynamic center must also be 45 inches.

****

Length of the Mean Aerodynamic Chord.

For the trapezoidal wing the MAC length is a length of actual chord at a station 239.8 inches out from the root chord. This may be checked analytically by finding the area under curve G and dividing by A_D.

For curve G the ordinates are $y = C^2$.

Then the area under the curve is $\int C^2 dx$

$C = 180 - \frac{x}{6}$ where the origin for x is taken at the root chord.

$$C^2 = (180 - \frac{x}{6})^2 = 32,400 - 60x + \frac{x^2}{36}$$

$$\int_0^{540} (32,400 - 60x + \frac{x^2}{36}) dx = \Big|_0^{540} 32,400x - \frac{60x^2}{2} + \frac{x^3}{(3)(36)}$$

$$= (32,400)(540) - (30)(540)^2 + \frac{(540)^3}{108} = 10,206,000$$

$$\text{Length of MAC} = \frac{10,206,000}{72,900} = 140.2 \text{ inches}$$

Solution to Problem 5:1

c.g. at 15% MAC

No.	Item	V_L = 220 mph. I	II	V_G = 281 mph. III	IV
(1)	W = gross wt.	20,000	20,000	20,000	20,000
(2)	$q = \frac{V^2}{391}$ (V in mph)	123.7	123.7	202.5	202.5
(3)	$s = W$/wing area	19.8	19.8	19.8	19.8
(4)	$\frac{q}{s} = \frac{(2)}{(3)}$	6.21	6.21	10.17	10.17
(5)	n_1 = applied wing load factor	3.54	-1.54	2.62	-.62
(6)	$C_N = \frac{(5)}{(4)}$	.57	-.248	.258	-.061
(7)	corresponding to C_N	5.3°	-5.5°	1.2°	-3.0°
(8)	C_c or C'_c	-.114	0	0	0
(9)	$n_{x_1} = (8) \times (4)$	-.708	0	0	0
(10)	$n_{x_4} = \frac{F_{pr}}{W}$	-.1365	-.1365	0	0
(11)	C_M or C_M'	-.0114	-.029	-.029	-.029
(12)	$m = (11) \times (4)$	-.0708	-.180	-.294	-.294
(13)	n_3 = tail load factor	-.11	-.002	-.179	-.075
(14)	$n_2 = -(5) - (13)$ = net load factor	-3.43	+1.54	-2.44	+.70
(15)	$n_{x_2} = -(9) - (10)$ = chord load factor	.8445	.1365	0	0
(16)	$T = (13) \times W$ = tail load	-2200	-.40	-3580	-1500

Tail loads in the above table are applied loads. They must be multiplied by 1.5 for use in design.

Solution to Problem 5:1 (Continued)

c.g. at 30% MAC

No.	Item	V_L = 220 mph.		V_G = 281 mph.	
		I	II	III	IV
(1)	W = gross wt.	20,000	20,000	20,000	20,000
(2)	$q = \frac{V^2}{391}$ (V in mph)	123.7	123.7	202.5	202.5
(3)	s = W/wing area	19.8	19.8	19.8	19.8
(4)	$\frac{q}{s} = \frac{(2)}{(3)}$	6.21	6.21	10.17	10.17
(5)	n_1 = applied wing load factor	3.54	-1.54	2.62	-.62
(6)	$C_N = \frac{(5)}{(4)}$	.57	-.248	.258	-.061
(7)	corresponding to C_N	5.3°	-5.5°	1.2°	-3.0°
(8)	C_c or C_C'	-.114	0	0	0
(9)	n_{x_1} = (8) x (4)	-.708	0	0	0
(10)	$n_{x_4} = \frac{F_{pr}}{W}$	-.1365	-.1365	0	0
(11)	C_M or C_M'	-.0114	-.029	-.029	-.029
(12)	m = (11) x (4)	-.0708	-.180	-.294	-.294
(13)	n_3 = tail load factor	+.067	-.08	-.055	-.110
(14)	n_2 = - (5) - (13) = net load factor..	-3.607	+1.62	-2.56	.73
(15)	n_{x_2} = - (9) = (10) = chord load factor	.8445	.1365	0	0
(16)	T = tail load = (13) x W	+1340	-1600	-1100	-2200

Solution to Assignment:

Calculation of dead weight shears and moments for the spars.

Effect of structure:

All taken on rear spar since center of weight is at the rear spar. Values can be calculated directly from Fig. 7.

Effect of Nacelle (plus land gear) weights:

Values for front spar from Fig. 9.
Values for rear spar from Fig. 10.

Effect of Fuel and Tankage:

Front Spar takes .52 times values from Fig. 11.
Rear Spar takes .48 times values from Fig. 11.

Front Spar--Dead Weight--Load Factor = -1.0

SHEAR

STATION	145	115	Outbd. 42	Inbd.
Wing Weight.........	0	0	0	0
Nacelle.............	-3170#	-6340	-6340	0
Fuel and System*....	0	0	- 324	+186
Total...............	-3170#	-6340#	-6664	+186

*52% of total fuel and system.

MOMENT (+ = tension on bottom fibers)

Nacelle.............	0	-95,100	-558,000
Fuel and System.....	0	0	- 11,800
Total...............	0	-95,100	-569,800

Solution to Assignment (Continued)

Rear Spar - Dead Weight - Load Factor = -1.0

SHEAR

WING STRUCTURE

Station	405	270
x	135	270
x^2	18225	72890
$-.6x$	- 81	-162
$-1.5x^2/540$	- 51	-203
S wing	- 132	-365
Nac.	0	0
Fuel.	0	0
Total	- 132	-365

Station	145	115	42 Outbd.
X_1	125	155	238
X_1^2	15625	34025	52000
$-2.1x_1$	-262	-325	-480
$-2.7x_1^2/540$	- 78	-120	-260
S_{270}	-365	-365	-365
S wing	-705	-810	-1105
Nac.	+1805	+3610	+3610
Fuel *	0	0	- 299
Total	+1100	+2800	+2206

*48% of total fuel and system.

MOMENT (+ = Tension on bottom fibers)

WING STRUCTURE

x^3	2,460,000	19,680,000
$-.3x^2$	-5480	-21,900
$-.5x^3/540$	-2270	-18,200
M	-7750	-40,100
M Nac.	0	0
M Fuel	0	0
M Total	-7750	-40,100

x_1^3	1,953,000	3,724,000	11,856,000
$-365x_1$	-45,700	-56,700	-83,200
$-1\ 05x_1^2$	-16,400	-25,200	-54,600
$-9x_1^3/540$	- 3,300	- 6,200	-19,800
M_{270}	-40,100	-40,100	-40,100
M wing	-105,500	-128,200	-197,700
M Nac.	0	+ 54,150	+317,680
M Fuel*	0	0	- 10,900
M Total	-105,500	- 74,050	+109,080

Solution to Problem Assignment 6A - (2)

To compute the shears, moments and axial loads in the Rear Spar for Condition III.

Table VII

Unit Lift Shear and Moment - Condition III

(1)	Station	405	270	145	115	42
(2)	Shear - Wing	94.8	211	336	370	455
(3)	Shear F. Spar	15.2	33.8	53.8	59.2	72.8
(4)	Shear R. Spar	79.6	177.2	282.2	310.8	382.2
(5)	Mom. Wing	6,170	26,600	60,700	71,300	101,000
(6)	Mom. F. Spar	990	4,260	9,710	11,400	16,160
(7)	Mom. R. Spar	5,180	22,340	50,990	59,900	84,840

Center of Pressure for Condition III is at 36 per cent of chord. Therefore lift load is carried:

$\frac{21}{25}$ = .84 by rear spar. $\frac{4}{25}$ = .16 by front spar
(See also pg. 6A-2)

****

Dead Weight Shears and Moments for a Load Factor of -1.0 are the same as given in Table II of the text.

****

Air load factors are as follows:

For Condition III the limit load factor in the beam direction is 2.62 (Assignment 5). The design load factor is (1.5)(2.62) = 3.93. The multiplying factor for design air loads in the beam direction is therefore (3.93)(19.9) = 78.3

19.9 = wing loading in lbs. per sq. ft. for a load factor of 1.0.

Checking from C_N q, the average loading in lbs. per sq. ft. for Condition III is (.258)(202.5) = 52.2 and the design load is (1.5)(52.2) = 78.3 lbs. per sq. ft.

The chord load factor for Condition III is zero. See pages 5-7 and 5-8.

****

The vertical dead weight factor for design is (-2.44)(1.5) = -3.66. Then the multiplying factor for dead loads vertical is

$$\frac{-3.66}{-1.00} = 3.66$$

The chord load factor for dead weights is zero.

****

The design shears and moments for the rear spar for Condition III are computed in a table similar to that shown in the text for the front spar in Condition I. (See table III in text.)

Table VIII

Design Shears and Moments - Condition III - Rear Spar

(1) Station	405	270	145	115	42
(2) Unit Lift Shear	79.6	177.2	282.2	310.8	382.2
(3) Design Lift Shear	6,240	13,900	22,100	24,400	30,000
(4) Unit Dd. Wt. Shear	-132	-365	+1,100	+2,800	+2,206
(5) Des. Dd. Wt. Shear	-485	-1,350	+4,030	+10,250	+8,100
(6) Design Shear	5,755	12,550	26,130	34,650	38,100
(7) Unit Lift Moment	5,180	22,340	50,990	59,900	84,840
(8) Des. Lift Moment	406,000	1,752,000	3,998,000	4,700,000	6,650,000
(9) Unit Dd. Wt. Moment	-7,750	-40,100	-105,500	-74,050	+109,080
(10) Des. Dd. Wt. Moment	-28,400	-147,000	-386,000	-271,000	+399,600
(11) Design Moment	377,600	1,605,000	3,612,000	4,429,000	7,049,600

Column (3) = (Col. 2)(78.3)

Column (8) = (Col. 7)(78.3)

Column (5) = (Col. 4)(3.66)

Column (10) = (Col. 9)(3.66)

****

Chordwise Loads.

Since there are no chord loads from dead weight, and no propeller thrust is assumed to be acting, the only chord loads on the wing will be those due to nacelle drag.

The applied nacelle drag force $F_d = S_D q$

$S_D = 2.0 \qquad q = 202.5$

$F_D = (2)(202.5) = 405$ lbs.

Design $F_D = (1.5)(405) = 608$ lbs. acting backward.

Aft loads at stations 115 and 145 are 304 lbs. at each.

Drag shear between stations 145 and 115 = 304 lbs.

Chordwise moment at station 115 = (30)(304) = 9120 in. lbs.

Drag shear between stations 115 and 42 = 608 lbs.

Chordwise moment at station 42

$= (304)(103) + (304)(73) = 31,300 + 22,200 = 53,500$ in. lbs.

*****

Compression axial load in rear spar due to chordwise moment:

At station 115 $= -\dfrac{9120}{40.2} = -227$ lbs.

At station 42 $= -\dfrac{53,500}{43.3} = -1235$ lbs.

Solution to Problem Assignment.

Section Properties at Station 145 for Compression on the Top

No.	A	y	Ay	x	Ax	y_o	x_o	Ay_o^2	Ax_o^2	Ix_o	Iy_o	Ax_oy_o
1	2.07	-14.0	-29.	0	0	-10.16	19.2	214	764			-405
2	1.54	-15.0	-23.1	10.0	15.4	-11.16	9.2	192	130		42	-158
3	1.54	-15.2	-23.4	28.2	43.4	-11.36	-9.0	198	125		42	+157
4	2.07	-15.1	-31.3	38.6	80.0	-11.26	-19.4	263	780			+449
5	1.92	8.5	16.3	38.6	74.0	12.34	-19.4	292	722			-460
6	1.32	8.8	11.6	28.2	37.3	12.64	-9.0	212	107		36	-150
7	1.32	8.5	11.2	10.0	13.2	12.34	9.2	200	112		36	+150
8	1.92	7.8	15.0	0	0	11.64	19.2	261	708			+430
	13.70	-3.84	-52.7	19.2	263.3			1832	3448		156	+ 13

$I_{x-x} = Ay_o^2 + Ix_o = 1832$

$I_{y-y} = Ax_o^2 + Iy_o = 3448 + 156 = 3604$

Solution to Problem Assignment (Continued)

Section Properties at Station 115 for Compression on the Top

No.	A	y	Ay	x	Ax	y_o	x_o	Ay_o^2	Ax_o^2	Ix_o	Iy_o	Ax_oy_o
1	2.27	-14.6	-33.2	0	0	-10.35	20.0	243	908			-471
2	1.90	-16.0	-30.4	12.0	22.8	-11.75	8.0	262	122		63	-178
3	1.55	-16.1	-25.0	29.8	46.2	-11.85	-9.8	218	149		37	+180
4	2.27	-15.9	-36.1	40.0	90.9	-11.65	-20.0	309	908			+530
5	2.12	9.1	19.4	40.0	85.0	13.35	-20.0	377	848			-568
6	1.34	9.3	12.5	29.3	39.2	13.55	-9.3	246	116		38	-179
7	1.34	9.1	12.2	11.0	14.7	13.35	9.0	238	109		38	+161
8	2.12	8.2	17.4	0	0	12.45	20.0	330	848			+529
	14.91	-4.25	-63.2	20.0	298.8			2223	4008		176	+4

$I_{x-x} = 2223$

$I_{y-y} = 4008 + 176 = 4184$

Solution to Problem of Assignment.

(1) Referring to chart on page 10-11 we find that a line drawn horizontally at a level corresponding to a shear load of 566 lbs. per inch intersects the design curve at a point corresponding to a web thickness "t" of approximately .032. This means that a web of either .032 or .036 will be satisfactory, the latter being chosen if, as is usually the case, the designer wishes to carry through a small positive Margin of Safety in order to allow for possible future increases in gross weight. Both gauges will be checked in this example.

(2) Stiffener spacing may be either 15 inches, the rib spacing, or some divisor of 15 inches. In this case the Design Curve is based on a stiffener spacing of 10 inches, so the stiffener spacing must be either 10 inches or some smaller value which in this case may be taken as $\frac{15}{2} = 7\frac{1}{2}''$

(3) For .032 web -

$$\text{Minimum stiffener area} = .4t \times \text{stiffener spacing}$$

$$= (.4)(.032)(7.5)$$

$$= .096$$

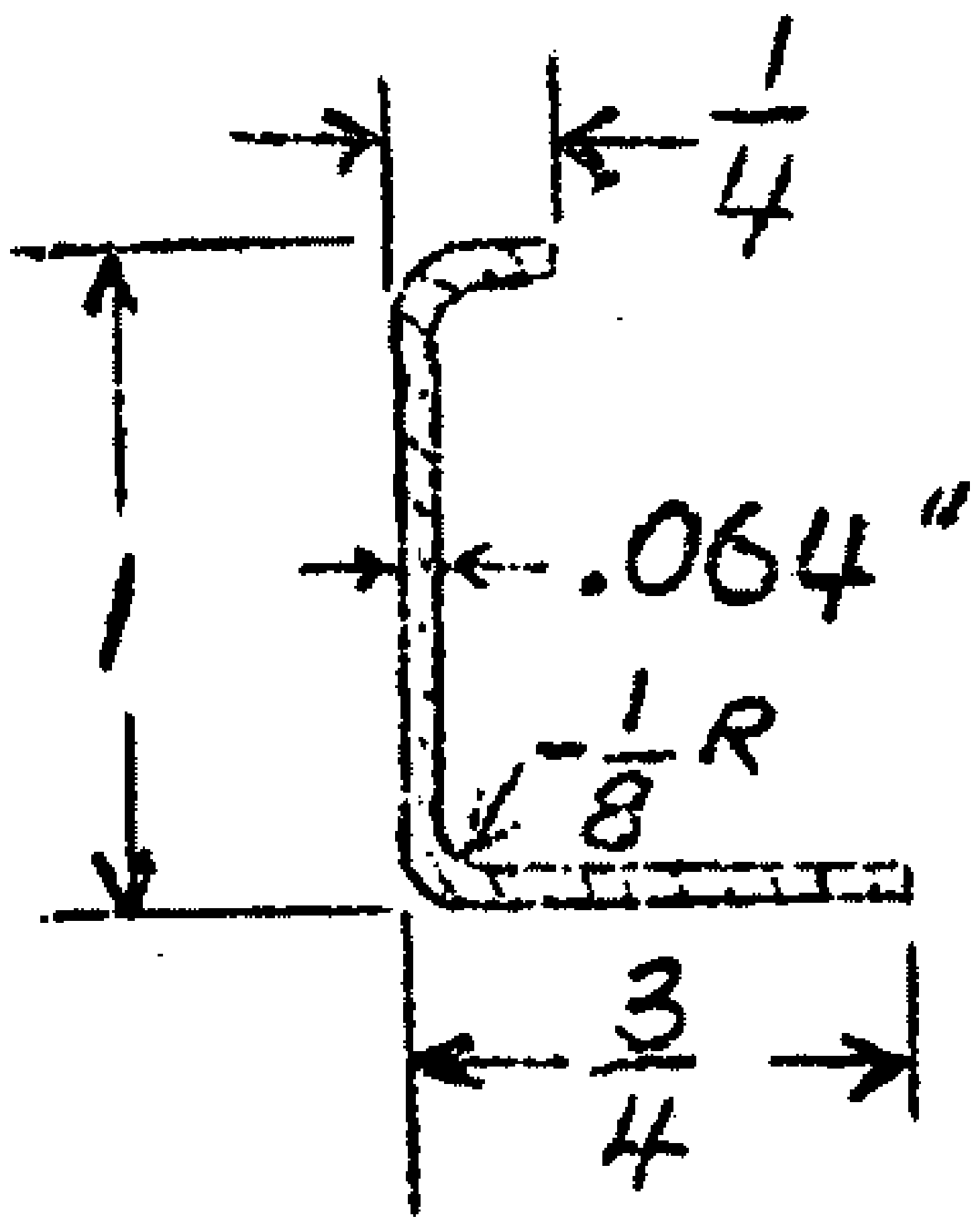
Since the web is less than .040 a stiffener on one side of the web only may be used. A formed angle of the dimensions shown may be used. A plain angle would not be satisfactory due to its low cripping strength. Developed width of angle = 1.8" (approx.)

Area = .115 sq. in. (.096 required)

For .036 web -

Approximate stiffener area = (.4)(.036)(7.5) = 108 sq. in.

The stiffener shown above will be satisfactory for this web also.



(4) Web - spar cap rivets - design load = 566 lbs. per inch-

.032 web -

Using 5/32 - A 17ST rivets,

Allowable shearing strength = 479 lbs. (See ANC-5 p. 5-19)

Allowable bearing strength - 24STAL = 409 lbs.

Required rivets per inch = $\frac{(1.2)(566)}{409}$ = 1.66

Required spacing = $\frac{1}{1.66}$ = .6 inch.

Use single row at ½ inch spacing, or preferably two rows at 1 inch spacing.

.036 web -

Using 3/16 - A 17ST rivets,

Allowable shearing strength = 690 lbs.

Allowable bearing strength = 607 lbs.

Required rivets per inch = $\frac{(1.2)(566)}{607}$ = 1.12

Required spacing = $\frac{1}{1.12}$ = .892

Use single row at 3/4 inch spacing.

Note: See page 10-6 for discussion of correction factor of 1.2 used above.

(5) End load in stiffener using Wagner beam theory -

P = stiffener spacing x shear per inch = (7.5)(566) = 4,250 lbs.

($\frac{P}{A} = \frac{4,250}{.115}$ = 37,000 lbs. per sq. inch)

Tests indicate that near the ends the stiffener stress will be much below this value--see discussion and Fig. 5 on page 10-7. Assuming a stiffener load of 20% of the above value the rivets will have to be good for (.2)(4250) = 850 lbs. This would require 2 - 5/32 diameter A-17ST rivets in single shear or 1 - 3/16 diameter 24ST rivet.

(6) No definite quantitative measure of the loads on the stiffener-web rivets is possible especially in the case of a stiffener on one side of the web only. Some conventional spacing such as 1" or $1\frac{1}{2}$" will be chosen and a rivet size will be proportioned to the web thickness. Some arbitrary rule such as balancing shear strength against bearing strength may be used. In the case of .032 or .036 webs this would point to the use of 1/8 or 5/32 diameter A-17ST rivets, the choice to be checked against the rivet sizes used in the test specimens from which the design curve was drawn up.

(7) Theoretical bending moment midway between stiffeners -

$$M = -\frac{s}{h}\frac{b^2}{24}$$
$$= -(566)\frac{(7.5)^2}{24} = -1325''\#$$

For .032 web -

$$f_s = \frac{566}{.032} = 17{,}700 \text{ lbs. per sq. in.}$$

$$\frac{f_s}{E}\left(\frac{b}{t}\right)^2 = \frac{17{,}700}{10{,}300{,}000}\left(\frac{7.5}{.032}\right)^2 = 94$$

$$\frac{M'}{M} = .4 \text{ or } M' = (.4)(-1325) = -530''\# \text{ (Fig. 7)}$$

For .036 web -

$$f_s = \frac{566}{.036} = 15{,}700 \text{ lbs. per sq. inch.} \qquad \frac{fs}{E}\left(\frac{h}{t}\right)^2 = 66$$

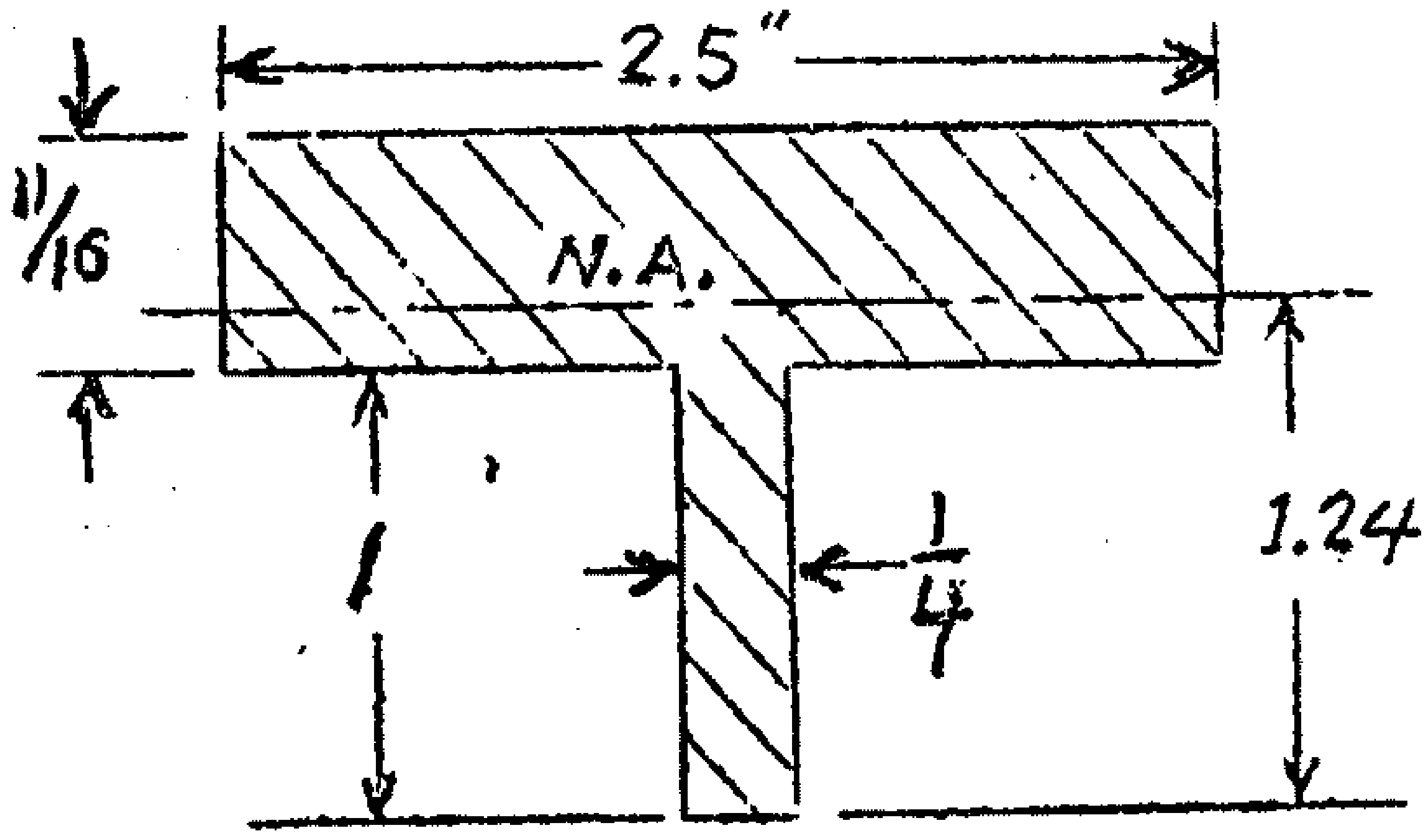
$$\frac{M'}{M} = .3 \quad \text{or} \quad M' = (.3)(-1325) = .400\ ''\#$$

If the spar cap is of the shape shown in the sketch, then I = .249;

$y_{inner} = 1.24$; $y_{outer} = .45$

$$f_{b(inner)} = \frac{530\ (1.24)}{.249} = 2{,}650 \text{ lbs. per sq. in.}$$

$$f_{b(outer)} = \frac{530\ (.45)}{.249} = 960 \text{ lbs. per sq. in.}$$



It is evident in this case that the change in unit stress at the extreme fiber is not large but should probably be considered in the determination of the allowable stress on the spar cap as a whole.

(8) Margins of safety for web-stiffener combinations -

.032 web -

$$\frac{\text{Stiffener Area}}{\text{Web Area}} = \frac{.115}{(7.5)(.038)} = \frac{.115}{.240} = .48$$

Correction factor (from curve) = 1.04
Basic allowable shear (from curve) = 560 lbs. per in.
Final allowable shear = (1.04)(560) = 582 lbs. per in.
Design shear = 566 lbs. per in.

$$\text{M.S.} = \frac{582}{566} - 1 = .03$$

.036 web -

$$\frac{\text{Stiffener Area}}{\text{Web Area}} = \frac{.115}{(7.5)(.036)} = \frac{.115}{.270} = .426$$

Correction factor (from curve) = 1.01
Basic allowable shear (from curve) = 650 lbs. per in.
Final allowable shear = (1.01)(650) = 656 lbs. per in.
Design shear = 566 lbs. per in.

$$\text{M.S.} = \frac{656}{566} - 1 = +.16$$

Web to spar cap attachment.

.032 web -

Allowable load = (2)(409) = 818 lbs. per in.
Rivet design load = (1.2)(566) = 680 lbs. per in.

$$\text{M.S.} = \frac{818}{680} - 1 = +.20$$

.036 web -

Allowable load = $(\frac{4}{3})(607)$ = 810 lbs. per in.

Rivet design load = (1.2)(566) = 680 lbs. per in.

$$\text{M.S.} = \frac{810}{680} - 1 = +.19$$

Note: It is considered good practice to keep the Margin of Safety of the rivets slightly above that of the web-stiffener combination itself.

Solution to Assignment.

SUMMARY

Condition	Point A			Point B			Point C		
	V	S	D	V	S	D	V	S	D
Level	-1,950	-13,250	-27,050 38,450	-26,500	26,500	0	-15,200	0	-21,800
Three-Point	-21,150	-10,850	34,300 -56,780	-21,700	21,700	0	26,700	0	38,300
Side	-24,720	7,250	-35,130 26,430	34,500	-34,500	0	-7,760	0	-11,100

Condition	Side Brace	Drag Brace	Axle				Axle		
	Axial Load	Axial Load	Sec. J-J	Section K-K			Section L-L		
	P_{BF}	P_{FC}	Shear	Shear	Moment	Axial Load	Shear	Moment	Torsion
Level	-37,550	-26,600	18,100	47,000	235,000	0	47,000	470,000	0
Three-Point	-30,800	46,700	15,550	40,400	202,000	0	40,400	404,000	0
Side	48,800	-13,560	21,150	22,800	275,000	-20,000	22,800	202,000	203,600

Condition	Axle	Inner Oleo Tube				Outer Oleo Tube			
	Sec. L-L	Section M-M		Fluid	Sec. N-N	Section M-M		Section F-F	
	Axial Load	Shear	Moment	Load	Shear	Shear	Torsion	Shear	Moment
Level	0	11,400	617,500	-45,600	24,300	29,100	199,500	29,100	334,500
Three-Point	0	15,650	698,000	-37,300	20,850	30,200	-274,000	30,200	347,000
Side	-20,000	20,150	312,000	-22,700	8,530	27,100	184,700	27,100	312,000

Condition	Oleo Outer Tube					
	Section F-F		Section N-N			
	Torsion	Axial Load	Axial Load	Shear	Torsion	Moment
Level	199,500	41,700	41,700	5,640	199,500	228,000
Three-Pt.	-274,000	-5,000	-500	15,350	-274,000	186,500
Side	184,700	-26,740	-26,740	9,200	184,700	83,000